

# A bankruptcy method for pollution load reallocation in river systems

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## ABSTRACT

This study forms the basis and sets practical guidelines for developing river water quality management strategies for resolving conflicts related to the allocation of pollution discharge permits using bankruptcy methods. This approach was implemented by changing the concepts and considering the river self-purification potential (capacity) as an asset which is to be shared among various beneficiaries. The beneficiaries are the point sources which release their wastewater to the river with minimum treatment costs. Four commonly used bankruptcy methods in the water resources allocation literature are used here to develop new river bankruptcy solution methods for allocating pollution share to the riparian parties of river systems. For this purpose, the Qual2 K river water quality simulation model is integrated with a particle swarm optimization (PSO) model while various pollution loadings discharge policies have been determined based on the bankruptcy method. This method was employed in one of the most polluted rivers of northern Iran, which is the source of eutrophication for Anzali International Wetland. The results show that the application of this method could facilitate the conflict resolution among different beneficiaries in order to improve the conditions of river water quality.

**Key words** | conflict resolution, point source, river water quality management, Qual2k

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## INTRODUCTION

The discharge of different urban, industrial, and agricultural pollution, more than the river self-purification capacity, leads to the deterioration of the river ecosystem and increase of water treatment price in lower bounds. In allocating pollution load to different pollutant sources, it is necessary to take environmental standards as well as treatment costs into consideration.

In common models of river water quality management, the permitted wastewater discharge rate for each pollutant source is determined based on minimization of treatment costs by considering downstream water quality limitation as a constraint, or minimizing the water quality violation from the standard by considering a treatment budget limitation as the constraint (Niksokhan *et al.* 2009; Ghosh & Mujumdar 2010; Barati 2011; Nikoo *et al.* 2012, 2013; Barati

2013; Huashan *et al.* 2013; Barati *et al.* 2014; Liu *et al.* 2014; Joonwoo *et al.* 2015; Tavakoli *et al.* 2015; Hosseini *et al.* 2016; Jie *et al.* 2016; Shakibaeina *et al.* 2016; Wang *et al.* 2016; Alizadeh *et al.* 2017; Chounlamany *et al.* 2017; Saberi & Niksokhan 2017; Zeferino *et al.* 2017).

In recent years, with the development of game theory application in water resource management, the use of conflict resolution approaches in river water quality management have been taken into consideration. Some recent works which successfully implemented conflict resolution methods in river water quality management are discussed as follows: Kampas & White (2003) obtained certain rules for permitted agriculture pollution discharge using a bargaining solution. For this purpose, bankruptcy resolution methods were applied for providing various types of

allocation licenses for a small river basin in southwestern England. The results showed that allocation rates depend on the beneficiaries' power of bargaining. On the other hand, it was found that different allocation rules, according to various interpretations of equity, have distinct effects on benefit sharing among different beneficiaries. Mahjouri & Ardestani (2010a, 2010b) and Mahjouri & Bizhani-Manzar (2013) established certain river water quality management policies by using conflict resolution methods and optimization models. The comparison of the two approaches represents the importance of cooperation performance to achieve maximum benefit from using surface water resources when considering the qualitative and quantitative aspects of river water. Based on the results, the cooperative participation of water consumers has a higher final net benefit in the game.

Abed Elmdoust & Kerachian (2012) established two cooperative fuzzy games for equal and effective modeling of river water allocation among the consumers inside and outside the basin by considering the water quality conditions in their research. In this regard, they used Hukuhara-Shapley for allocating the created benefit by crisp through fuzzy attributes functions. In their study, Malakpour *et al.* (2016) expanded the environmental penalty functions for river quality management using an  $n$ -person evolutionary game. The results of the model show that the penalty functions can force pollutants to treat their released wastewater to meet river water quality standards. Finally, they recommended a sustainable treatment strategy in which the pollutants are in mutual agreement.

In recent years, a few researchers have studied the application of bankruptcy theory in various natural resources and allocation problems, such as underground water resources management, multi-objective resources allocation, and fisheries (Ansink & Ruijs 2008; Sheikmohammady & Madani 2008; Ansink & Weikard 2012; Madani & Zarezadeh 2012; Madani & Dinar 2013; Madani *et al.* 2014a, 2014b, 2014c; Mianabadi *et al.* 2014).

Ansink & Weikard (2012) determined different water allocation scenarios for border rivers by using four classic rules of the bankruptcy method. Proportional Rules (PR), Constrained Equal Award (CEA), Constrained Equal Loss (CEL), and Talmud Rule were used as different methods of bankruptcy resolution.

Madani *et al.* (2014d) suggested bankruptcy optimization models for water allocation based on four bankruptcy rules, considering the critical importance of water delivery time during the planning horizon. They studied the bankruptcy method in resolving water disputes using a broad range of rules in order to solve water conflicts over transboundary rivers. Moreover, Mianabadi *et al.* (2014) analyzed the relationship between the bankruptcy theory and shared river problems. They implemented PR, CEA, CEL, and AP methods and developed a program for fair water allocation in transboundary rivers.

As mentioned before, in recent years, game theories have developed significantly in developing policies of pollution loading discharge into the river. Accordingly, the concept of the bankruptcy method among various beneficiaries in water quantity allocation has been developed but it is not used in water quality management. The novelty of this paper is an application of the bankruptcy method in river water quality management. So, by integrating the Qual2K quality simulation model with particle swarm optimization (PSO) model, different policies of pollution discharge to the river are determined based on the bankruptcy method. Therefore, the application of bankruptcy rules in the context of water quality management and the array of technical tools (e.g. water quality simulation, PSO, etc.) employed is the contribution of this work.

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## METHODOLOGY

In this study, an approach based on the bankruptcy problem resolution rule has been offered for waste load allocation in river systems. Considering the non-uniform spatial and temporal variability of water flows and wastewater discharged to the river, non-linear optimization models were proposed for solving river bankruptcy problems. In order to simulate the superposition of wastewater discharge from different point sources, and also considering the re-aeration and disposal of the pollution along the river, a river water quality simulation model is linked with the optimization model. In this study, a river water quality simulation model (Qual2k) used is coupled with the optimization model in order to simulate the dissolved oxygen (DO) variation along the river due to different pollution discharge scenarios.

### River water quality simulation model

In this study, the Qual2kw model is utilized for the simulation of river quality variables based on different waste load allocation scenarios developed by the optimization model. Qual2kw is a one-dimensional model for the simulation of river and stream water quality variations. This model simulated the transport and fate of several constituents using the advection-diffusion equation. The model uses the finite-difference numerical method for the solution of the adjective-dispersive mass transport and reaction equations. Equation (1) shows the variation of constituent concentration variation based on a general mass balance equation for a water column (Pelletier *et al.* 2006):

$$\frac{dc_i}{dt} = \frac{Q_{i-1}}{V_i} c_{i-1} - \frac{Q_i}{V_i} c_i - \frac{Q_{ab,i}}{V_i} c_i + \frac{E_{i-1}}{A_i} (c_{i-1} - c_i) + \frac{E_i}{A_i} (c_{i+1} - c_i) + \frac{W_i}{V_i} + S_i \quad (1)$$

where  $Q_i$  is the flow at reach  $i$  (L/day),  $Q_{ab,i}$  is the abstraction flow at reach  $i$  (L/day),  $V_i$  is the volume of reach  $i$  (L),  $A_i$  is the area of reach  $i$  (m<sup>2</sup>),  $W_i$  is the external loading of the constituent to reach  $i$  (mg/day),  $S_i$  is the sources or sinks of the constituent due to reactions and mass transfer mechanisms (mg/L/day),  $E_i$  and  $E_{i-1}$  are bulk dispersion coefficients between reaches  $i-1$  and  $i$  and  $i$  and  $i+1$ , respectively (m<sup>2</sup>/day),  $c_i$  is the concentration of water quality constituted in reach  $i$  (mg/L),  $t$  is time (day) and  $S$  includes the external sources (positive) or sink and pit (negative) as the constituents.

$\frac{\partial c}{\partial t}$  is the concentration variation rate in time including the temporal variation of concentration based on diffusion, advection, reaction and rate of changes due to pollution load discharged into the river. This model includes an automatic calibration system which minimizes the objective function  $f(x)$  via the genetic algorithm. Based on this objective function, differences between observed data and simulated data are decreased for each water quality variable simulated in the model.

$$f(x) = \left( \sum w_i \right) * \left\{ \sum \frac{1}{w_i * \left( \frac{\sum (p_{ij} - o_{ij})^2 / m}{\sum o_{ij} / m} \right)^{0.5}} \right\} \quad (2)$$

where  $O_{ij}$  represents the observed values,  $P_{ij}$  is regarded as the simulated values,  $m$  indicates the number of observed and predicted values,  $W_i$  is the weight coefficients and  $n$  is considered as the number of different existing variables in RMSE (root mean square error).

### Pollution load allocation using bankruptcy method

The main question frequently asked is how to allocate available resources among the members during system bankruptcy. There are various solutions to this problem. The theory of bankruptcy is regarded as one of the analytic methods which could be used in source allocation disputes. The aim of this method is to distribute an asset to a group of creditors when this amount is not adequate to meet their credit's claim. In recent years, several bankruptcy rules have been developed. Some of these rules are based on the cooperative bankruptcy game. Among the most frequently used bankruptcy rules, we can refer to Consistency, Constrained Equal Losses (CEL), and Constrained Equal Awards (CEA) rules, which are in equal proportions of claims, losses, and awards.

In the next section, the relationships and principles of certain current methods of bankruptcy games are discussed. The above-mentioned methods will provide satisfactory or acceptable results while having convenience in computations. The difference between these methods is derived from their different definitions of equity in source distribution among the players. The main players in this study are river riparian point sources (consisting of wastewater of urban areas and different industries) which discharge their wastewater to the river with minimum treatment cost. The current bankruptcy methods in specifying treatment level and permitted discharge rate for pollutant units to include the rules discussed below.

### Constrained Equal Award rule (CEA rule)

Resources allocation based on the CEA rule will do based on the following steps:

1. The initial allocation to all beneficiaries is equal to the lowest, considering that the total allocations do not exceed the demand.

2. The fully satisfied creditor is then excluded.
3. After updating the remaining resources, the process continues with the remaining creditors considering their unsatisfied claims.

In each stage, when the sum of allocated amounts to all remaining beneficiaries (which are equal to the lowest claim) is more than the remaining resources, the remaining resource is distributed equally among all remaining beneficiaries.

This approach was implemented by changing the concepts and considering the river self-purification potential (capacity) as an asset which is to be shared among various beneficiaries. The beneficiaries are the point sources which like to release their wastewater to the river with minimum treatment cost. In order to obtain the CEA rules, the objective function is to maximize the permitted pollution concentration of each pollutant source (Equation (3)). Equation (4) represents the permitted pollution concentration which equals a minimum of CEA decision variables and pollution concentration before treatment. The dissolved oxygen concentration at the control point due to discharged pollution loads is obtained via the river water quality simulation model which is shown as a function in Equation (5):

$$\text{Max } \alpha \quad (3)$$

$$C_i^{CEA} = \min(\alpha, c_i^P) \quad (4)$$

$$F^{Do}(C_i^{CEA}) \geq Do_s \quad (5)$$

where  $\alpha$  is the maximum equal pollution discharge which is equal for all point sources,  $C_i^{CEA}$  is the permitted pollution concentration discharge,  $c_i^P$  is the pollution potential of point source  $i$  and  $Do_s$  is the standard amount for dissolved oxygen concentration at a control point.  $F^{Do}$  is the DO concentration at a control point obtained using the river water quality simulation model.

### Constrained equal loss rule (CEL rule)

The CEL rule can be viewed as the opposite of the CEA rule because it gives priority to satisfying the highest claim (pollution discharge permit) first. Once the highest claim is satisfied, the process is repeated with the remaining resources and creditors. The process stops at any stage

(including the first stage) if the available resource is not sufficient to satisfy the highest claim of the remaining creditors. At this stage, the remaining resource is split equally among the remaining creditors.

In this state, the system reduces the pollutant concentration levels of each of the resources equally while the dissolved oxygen concentration at the control point reaches the standard limit. The objective function of this section is to minimize the amount of pollution concentration reduction in which each pollutant should reduce it from its initial pollution concentration. Equation (7) represents the dischargeable concentration which equals the pollutant concentration rate before treatment subtracted by the amount of concentration reduction rate for each of the pollutant sources. The dissolved oxygen concentration at the control point due to pollution discharged to the river is obtained via Equations (6)–(8):

$$\text{Min } \beta \quad (6)$$

$$C_i^{CEL} = \max(C_i^P - \beta, 0) \quad (7)$$

$$F^{Do}(C_i^{CEL}) \geq Do_s \quad (8)$$

where  $\beta$  is the amount of pollution that should be reduced by each pollutant (it is equal for all point sources),  $C_i^{CEL}$  is the pollution concentration after treatment. The other parameters and variables are as described previously.

### Proportional bankruptcy rule (P-rule)

In this method, the pollution concentration rate related to each of the pollutant sources is decreased equally with a constant ratio, as presented in Equations (9)–(11):

$$\text{Min } \gamma \quad (9)$$

$$C_i^{PR} = \gamma * C_i^P \quad (10)$$

$$F^{Do}(C_i^{PR}) \geq Do_s \quad (11)$$

where  $\gamma$  is the ratio in which each pollutant should reduce its pollution concentration in order to reach water quality standards in the river (it is equal for all point sources),  $C_i^P$  is the pollution concentration of pollutant resource  $i$

before treatment and  $C_i^P$  is the pollution concentration of pollutant resource  $i$  after  $\gamma$  percent of pollution reduction.

### Talmud rule (Tal rule)

This method is, in fact, a combination of two equations of CEL and CEA. Each player's share is estimated from Equations (12) or (13):

$$C_i^{TR} = CEA(0.5 * C_i^P) \quad \text{if} \quad F^{Do}(0.5 * C_i^P) < Do_s \quad (12)$$

$$C_i^{TR} = 0.5 * C_i^P + CEL(0.5 * C_i^P) \quad \text{if} \quad F^{Do}(0.5 * C_i^P) > Do_{\text{standard}} \quad (13)$$

where  $CEA(0.5 * C_i^P)$  means to use CEA Rule in which  $C_i^P$  is replaced by  $0.5 * C_i^P$ ,  $CEL(0.5 * C_i^P)$  means to use CEL in which  $C_i^P$  is replaced by  $0.5 * C_i^P$ .

Finally, confirming each of the bankruptcy methods is not feasible for all beneficiaries and they may achieve the highest allocation rate by employing different methods. In this way, the superior method seems to be distinguished from each of the beneficiaries' points of view. There are different solutions for evaluating sustainability and accepting a method in the game for conflict resolution.

In order to calculate Equations (5), (8) and (11), it is necessary to run a river water quality model (Qual2k) coupled with the optimization process model to simulate water quality variation along the river. Therefore, the PSO algorithm is used as the optimization method. Various stages of research operation are presented based on the flowchart in Figure 1.

### Implementing a pollution load allocation policy

The proposed bankruptcy optimization models provide different pollution reduction scenarios, based on various notions of fairness. Therefore, acceptability of different solutions are always questionable because there is always at least one point pollution source which finds one of the given alternatives unfair (because they can gain more under another rule). As one of the most commonly used social choice (voting) methods, the plurality index can be considered as an indicator of potential acceptability of a decision rule in multi-participant decision-making problems. Based on this index, the number of stakeholders who prefer

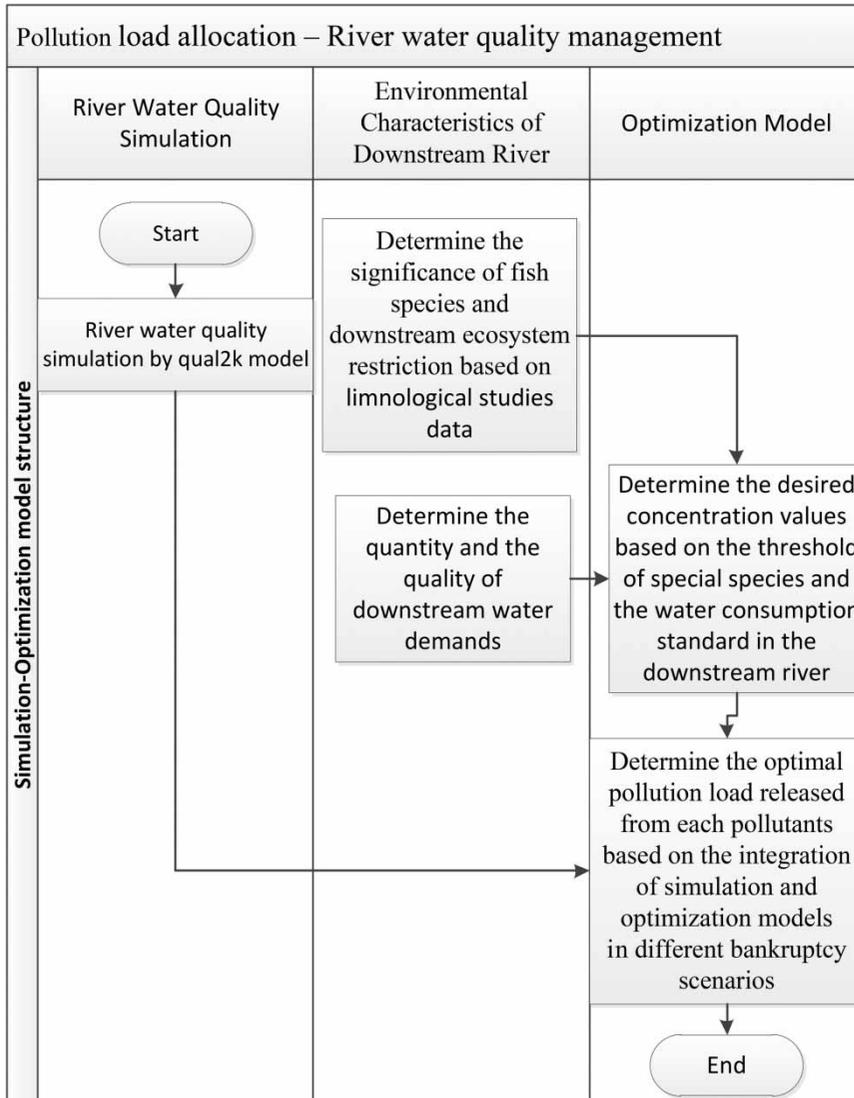
one method to the others is simply an indicator of the degree of acceptance of that method (Madani & Dinar 2013).

Acknowledging the possibility of the rejection of suggested allocations and the difference in the notion of fairness by the beneficiaries (pollution source) who find certain allocation rules of pollution permits unfair, there is a need for evaluating the acceptability of different bankruptcy solutions. The popularity of each solution is a simple indicator of its potential acceptability (in the case of asymmetric powers), and the majority cannot necessarily determine the feasible solution (when powerful parties do not support the most popular solution).

In some cases, a pollutant source located at a large distance from the other sources may have a negligible impact on the violation of the standard, and should therefore be exempted from the application of a discharge reduction factor. It is therefore necessary to first evaluate the contribution of each pollutant source to the violation of the standard, and to apply the bankruptcy method only to those sources that significantly contribute to it.

### CASE STUDY

In order to examine the efficiency of the recommended method, one of the most polluted rivers of Iran recognized by the Environment Protection Organization was selected as the case study. Zarjub River is one of the important rivers discharged into the Anzali international wetland in the north of Iran (Figure 2). This wetland is also severely affected by the decline of river water quality. If not polluted, this river could be a suitable environment for fish propagation. It could also have an economic significance from fishing, commercial and athletic aspects. The entrance of pollutions due to urban and industrial activities into the river has not only destroyed the river ecosystem, but it has also led to the extreme eutrophication of Anzali Wetland. According to library and field studies conducted at Zarjub River basin, the main pollutants of this basin are urban wastewaters which enter the river through local wastewater treatment systems. Due to the growth in population and urbanization development in the Zarjub watershed, urban wastewater constitutes over 90% of river pollution loading. Extensive pollutions into Zarjub River justify the necessity of studying the quality of this river and analyzing



**Figure 1** | Various stages of research operation.

the dissolved oxygen rate of the river when flowing into Anzali Wetland. The main players of this study are 11 point sources which discharge their pollution with a low level of treatment to the river. The point sources of pollution are listed in Table 1 and are shown in Figure 2.

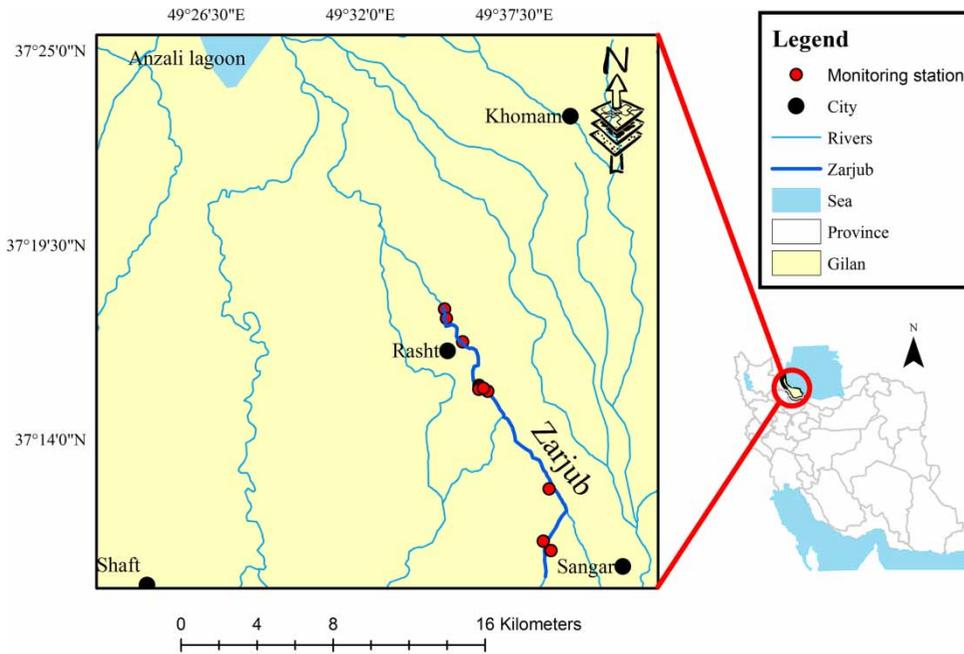
## RESULTS

The present research aimed to study the quality analysis of Zarjub River basin in Gillan Province of Iran. The Qual2k model was used for the simulation of river water quality changes in this area. In this regard, the length of Zarjub

River was divided into seven reaches during which 11 point pollutants discharge pollution into the river at different intervals. The river water quality model is calibrated based on observed data in the study area. Then, different pollution discharge scenarios were evaluated by coupling PSO and bankruptcy models.

### River water quality simulation model calibration

According to the quality examination of Zarjub River, there are 11 points of pollutant sources across the river. In current conditions, the dissolved oxygen rate is 6 mg/L upstream of



**Figure 2** | Study area map.

the river and approximately 1 mg/L downstream of the river. The river water quality simulation model was calibrated and verified for current quality conditions. The results of the calibration and verification of dissolved oxygen simulation along the river are illustrated in Figures 3 and 4. Figures 5 and 6 display the calibration and verification of BOD<sub>5</sub> simulation along the river. The results represent an acceptable calibration and verification for the river quality simulation model.

### Results of the optimization model with bankruptcy approach

The dischargeable concentration rate by each of the point pollutant sources was determined based on different approaches of the bankruptcy method. These methods include: CEA, CEL, Proportional Bankruptcy, and Talmud. Table 1 illustrates BOD<sub>5</sub> permitted discharge rates for each of the pollutant sources. Based on different

**Table 1** | Biological oxygen demand (BOD) permitted discharge rates for each of the pollutant resources based on different approaches of bankruptcy method

Pollutant number	Initial BOD (mg/L)	Constrained equal award	Constrained equal loss	Proportional bankruptcy	Talmud
1	100	89.78	31.5	70	65
2	8	8	0	5.6	4
3	130	89.78	61.5	91	80
4	120	89.78	51.5	84	75
5	180	89.78	111.5	126	105
6	90	89.78	21.5	63	60
7	110	89.78	41.5	77	70
8	90	89.78	21.5	63	60
9	100	89.78	31.5	70	65
10	180	89.78	111.5	126	105
11	180	89.78	111.5	126	105

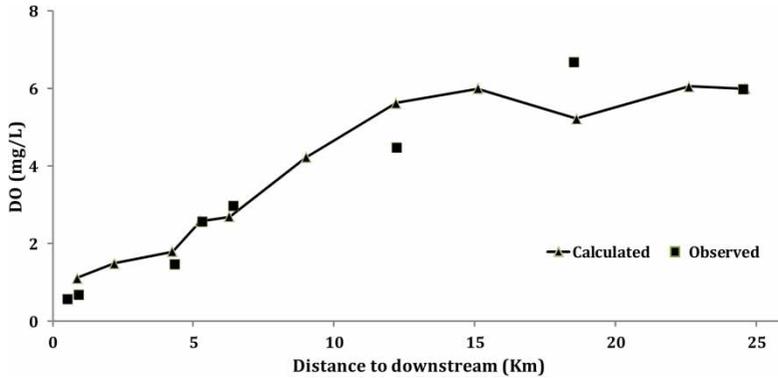


Figure 3 | Variation of dissolved oxygen along the river (simulated versus observed data).

approaches of bankruptcy including CEA, CEL, Proportional Bankruptcy, and Talmud, the dissolved oxygen rate at the control (check) point, located downstream of the river, is 4.02, 4.25, 4.6, and 4.1, respectively.

Based on the results in Table 1, according to CEA rules, if the BOD<sub>5</sub> concentration is greater than 89.78 mg/L, the pollutants must discharge 89.78 mg/L of biological oxygen demand; otherwise, its primary rate is entered into the river. Based on the results, the pollution source No. 2 (which discharges the lowest BOD<sub>5</sub> concentration into the river) does not require treating its own wastewater based on the Equal Award method, while it should completely treat its wastewater in the Equal Loss method. In the Equal Award method, all pollutant sources except the second pollution source should reduce BOD<sub>5</sub> concentration up to 89.78 mg/L, which is considered as a disadvantage for large pollutant units.

According to CEL rules, the pollution sources fail to send any pollutants into the river if the BOD<sub>5</sub> concentration is lower than 68.46 mg/L. In case of greater rates, the difference

rate of the primary concentration, along with the 68.46 mg/L amount, can be discharged into the river. Further, in the Equal Loss Rule, pollution source No. 2 fails to send any pollution into the river and the remaining resources discharge their pollution with different proportions.

According to the proportional bankruptcy rule, all dischargers must multiply the primary BOD<sub>5</sub> concentration with 0.70 and can discharge wastewater with this amount of BOD<sub>5</sub> into the river.

In the Talmud rule, which follows a combination of both methods of Equal Award and Equal Loss, the requested amount is compared with the required rate. For this purpose, the river simulation model is implemented with the new BOD<sub>5</sub>, which equals to half of the initial BOD<sub>5</sub>, and the resulting DO at the control (check) point is compared with the standard DO. Here, as the concentration of DO at the control point, obtained from half of the BOD concentration, is slightly more than the standard DO, the CEA method is used for half the amount of waste load allocation.

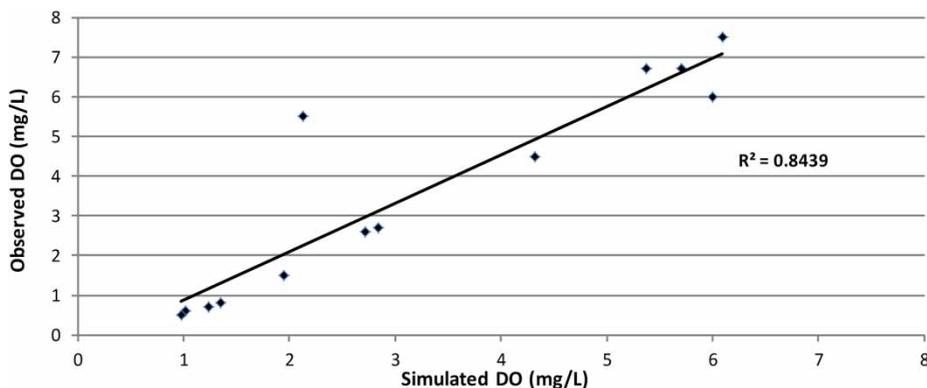
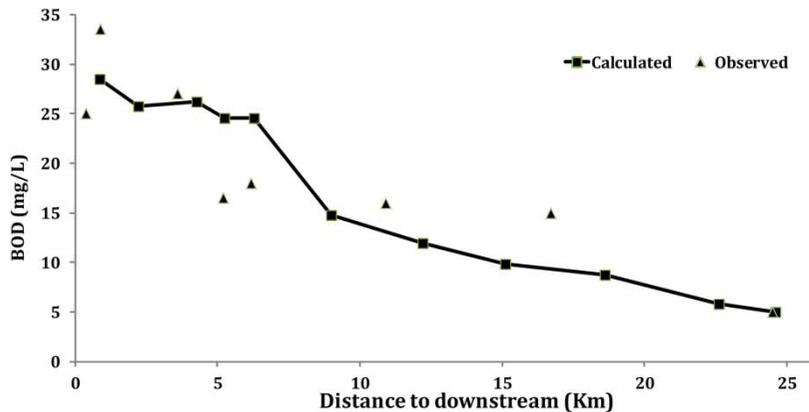


Figure 4 | Results of DO verification.



**Figure 5** | Variation of biological oxygen demand (BOD) along the river (simulated versus observed data).

Finally, the best-permitted discharge rate for each pollutant resource that can be discharged into the river is estimated.

According to the obtained results, the CEA method is better for those resources having fewer pollutant rates and consequently gains more awards (which means they should pay less money for wastewater treatment). In the CEL method those resources which discharge higher pollutants experience better conditions. In the Proportional Bankruptcy method, as an equal percentage is enforced for the discharge of all resources, they experience similar or equal conditions. In the Talmud method, the results are between the CEA and CEL approaches. The evaluation of the stability of different bankruptcy allocation solutions for different pollution discharge scenarios in this study suggested that the Talmud rules are more acceptable by beneficiaries.

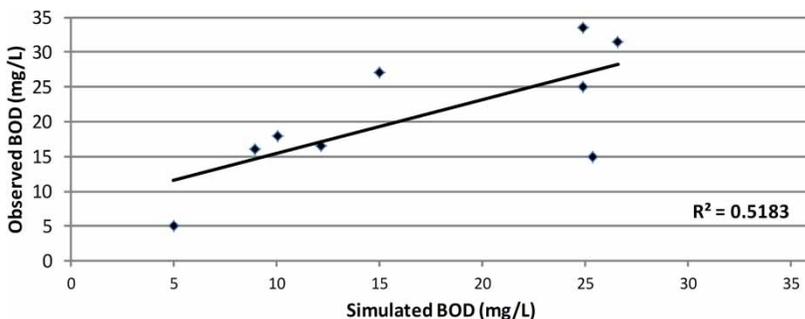
## CONCLUSIONS

Various urban, industrial, and agricultural pollution discharges, more than river self-purification potential,

damages river ecosystem and increase water treatment costs. As different decision-makers and stakeholders are involved in the water quality management in river systems, a new bankruptcy form of the game theory is used to resolve the existing conflict of interests related to waste load allocation downstream. The river restoration potential can allocate to the conflicting parties with respect to their claims, by using the bankruptcy solution methods.

In the present study, a new approach was introduced for dispute resolution of pollution loading allocation to various pollutant sources based on the bankruptcy method. As was also described in the review of the literature, different methods of dispute resolution have been provided to determine the pollution share of each point source. However, the application of the bankruptcy method was evaluated for the first time in the area of river water quality management. This method was already used among various beneficiaries in the allocation of pollution share of each pollutant.

This approach was implemented by changing the concepts and considering the river self-purification potential (capacity) as an asset which is to be shared among various



**Figure 6** | Results of BOD verification.

beneficiaries. The beneficiaries are the point sources who like to release their wastewater to the river with minimum treatment costs. It should be noted that the suggested method does not necessarily minimize the total cost of wastewater treatment in the basin and may result in sub-optimal allocations from an economic optimization method. However, it should be emphasized that this method can be used to develop practical solutions when utility information is not available or reliable, side payments are not feasible, and parties are not highly cooperative.

Considering the non-uniform spatial and temporal variability of water flows and wastewater discharged to the river, non-linear optimization models were proposed for solving river bankruptcy problems. Four river bankruptcy network flow optimization models were developed based on four conventional bankruptcy rules, i.e. proportional (P), Talmud, CEA, and CEL, for river water quality management.

In order to examine the efficiency of the above method, Zarjub River, as one of the most polluted rivers of Iran, was considered as the case study of the present research. The results of implementation indicated that the Bankruptcy Rule was applicable in river quality management with the dispute resolution approach, which provides a diverse combination of scenarios including pollution loading reduction for decision-making and negotiation among beneficiaries. The logic of the obtained results in different approaches is nearly consistent with the application of this method in water quantity allocation problems.

The models can be applied to any river system (or bankruptcy network) problem, irrespective of its characteristics and pollution sources variability conditions. Acknowledging the possibility of the rejection of suggested allocations and the difference in the notion of fairness by the beneficiaries (pollution source), who find certain allocation rules of pollution permits unfair, there is a need for evaluating the acceptability of different bankruptcy solutions. Besides, the popularity of each solution is a simple indicator of its potential acceptability (in the case of asymmetric powers), and the majority cannot necessarily determine the feasible solution (when powerful parties do not support the most popular solution). The evaluation of the stability of different bankruptcy allocation solutions for different pollution discharge scenarios in this study suggested that the Talmud rules are more acceptable to beneficiaries.

In the presence of multiple point sources of pollution along a river, several control points should be considered, and for each of them the bankruptcy should be applied considering only those sources that significantly contribute to the violation of the limit at the same points.

## ACKNOWLEDGEMENTS

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## CONFLICT OF INTEREST

There is no conflict of interest.

## REFERENCES

- AbedElmdoust, A. & Kerachian, R. 2012 *Water resources allocation using a cooperative game with fuzzy payoffs and fuzzy coalitions*. *Water Resour. Manage.* **26**, 3961–3976.
- Alizadeh, M. J., Shahheydari, H., Kavianpour, M. R., Shamloo, H. & Barati, R. 2017 *Prediction of longitudinal dispersion coefficient in natural rivers using a cluster-based Bayesian network*. *Environ. Earth Sci.* **76** (2), 86–97.
- Ansink, E. & Ruijs, A. 2008 *Climate change and the stability of water allocation agreements*. *Environ. Resour. Econ.* **41**, 249–266.
- Ansink, E. & Weikard, H. P. 2012 *Sequential sharing rules for river sharing problems*. *Soc. Choice Welfare* **38**, 187–210.
- Barati, R. 2011 *Parameter estimation of nonlinear Muskingum models using the Nelder–Mead simplex algorithm*. *J. Hydrol. Eng.* **16** (11), 946–954.
- Barati, R. 2013 *Application of excel solver for parameter estimation of the nonlinear Muskingum models*. *KSCE J. Civil Eng.* **17** (5), 1139–1148.
- Barati, R., Neyshabouri, S. A. A. S. & Ahmadi, G. 2014 *Development of empirical models with high accuracy for estimation of drag coefficient of flow around a smooth sphere: an evolutionary approach*. *Powder Technol.* **257**, 11–19.
- Chounlamany, V., Tanchuling, M. A. & Inoue, T. 2017 *Spatial and temporal variation of water quality of a segment of Marikina River using multivariate statistical methods*. *Water Sci. Technol.* **76** (6), 1510–1522.
- Ghosh, S. & Mujumdar, P. 2010 *Fuzzy waste load allocation model: a multi-objective approach*. *J. Hydroinform.* **12**, 83–96.

- Hosseini, K., Nodoushan, E. J., Barati, R. & Shahheydari, H. 2016 Optimal design of labyrinth spillways using meta-heuristic algorithms. *KSCE J. Civil Eng.* **20** (1), 468–477.
- Huashan, M. A., Sujal, I. M. & Nasly, M. A. 2013 Application of QUAL2Kw for water quality modeling in Tunggak River, Kuantan, Pahang, Malaysia. *Res. J. Recent Sci.* **3** (6), 6–14.
- Jie, G., Cheng Fei, H., Cui-ping, K. & Olaf, K. 2016 A water quality model applied for the rivers into the Qinhuangdao coastal water in the Bohai Sea, China. *J. Hydrodynam.* **28** (5), 905–913.
- Joonwoo, N., Hyungu, C. & Sangjin, L. 2015 Water quality projection in the Geum River basin in Korea to support integrated basin-wide water resources management. *Environ. Earth Sci.* **73**, 1745–1756.
- Kampas, A. & White, B. 2003 Probabilistic programming for nitrate pollution control: comparing different probabilistic constraint approximations. *Eur. J. Oper. Res.* **147**, 217–228.
- Liu, D., Guo, S., Shao, Q., Jiang, Y. & Chen, X. 2014 Optimal allocation of water quality and waste load in the northwest Pearl river delta, China. *Stoch. Environ. Res. Risk Assess.* **28** (6), 1525–1542.
- Madani, K. & Dinar, A. 2013 Exogenous regulatory institutions for sustainable common pool resource management: application to groundwater. *Water Resour. Econ.* **2–3**, 57–76.
- Madani, K. & Zarezadeh, M. 2012 Bankruptcy methods for resolving water resources conflicts. In: *2012 World Environmental and Water Resources Congress* (E. D. Loucks, ed.). ASCE, Albuquerque, New Mexico, pp. 2247–2252.
- Madani, K., Sheikhmohammady, M., Mokhtari, S., Moradi, M. & Xanthopoulos, P. 2014a Social planner's solution for the Caspian Sea conflict. *Group Decision Negotiat.* **23**, 579–596.
- Madani, K., Rouhani, O. M., Mirchi, A. & Gholizadeh, S. 2014b A negotiation support system for resolving an international transboundary natural resource conflict. *Environ. Model. Softw.* **51**, 240–249.
- Madani, K., Read, L. & Shalikian, L. 2014c Voting under uncertainty: a stochastic framework for analyzing group decision making problems. *Water Resour. Manage.* **28**, 1839–1856.
- Madani, K., Zarezadeh, M. & Morid, S. 2014d A new framework for resolving conflicts over transboundary rivers using bankruptcy methods. *Hydrol. Earth Syst. Sci.* **18**, 3055–3068.
- Mahjouri, N. & Ardestani, M. 2010a Application of cooperative and non-cooperative games in large-scale water quantity and quality management: a case study. *Environ Monit. Assess.* **172**, 157–169.
- Mahjouri, N. & Ardestani, M. 2010b A game theoretic approach for interbasin water resources allocation considering the water quality issues. *Environ. Monit. Assess.* **167** (1–4), 527–544.
- Mahjouri, N. & Bizhani-Manzar, M. 2013 Waste load allocation in rivers using fallback bargaining. *Water Resour. Manage.* **27**, 2125–2136.
- Malakpour, S., Kerachian, R. & Nikoo, R. 2016 Developing water quality management policies for Chitgar urban lake: application of fuzzy social choice and evidential reasoning methods. *Environ. Earth Sci.* **75**, 404–416.
- Mianabadi, H., Mostert, E., Zarghami, M. & Giesen, N. 2014 A new bankruptcy method for conflict resolution in water resources allocation. *J. Environ. Manage.* **144**, 152e159.
- Nikoo, M. R., Kerachian, R. & Niksokhan, M. H. 2012 Equitable waste load allocation in rivers using fuzzy Bi-matrix games. *Water Resour. Manage.* **26** (15), 4539–4552.
- Nikoo, R., Kerachian, R., Karimi, A., Azadnia, A. & Jafarzadegan, K. 2013 Optimal water and waste load allocation in reservoir-river systems: a case study. *Environ. Earth Sci.* **71**, 4127–4142.
- Niksokhan, M. H., Kerachian, R. & Karamouz, M. 2009 A game theoretic approach for trading discharge permits in rivers. *Water Sci. Technol.* **60** (3), 793–804.
- Pelletier, G. J., Chapra, S. C. & Tao, H. 2006 QUAL2Kw, a framework for modeling water quality in streams and rivers using a genetic algorithm for calibration. *Environ. Model. Softw.* **21**, 419–425.
- Saberi, L. & Niksokhan, M. H. 2017 Optimal waste load allocation using graph model for conflict resolution. *Water Sci. Technol.* **75** (6), 1512–1522.
- Shakibaenia, A., Kashyap, S. H., Dibike, Y. B. & Prowse, T. D. 2016 An integrated numerical framework for water quality modeling in cold-region rivers: a case of the lower Athabasca River. *Sci. Total Environ.* **569–570**, 634–646.
- Sheikhmohammady, M. & Madani, K. 2008 Sharing a multi-national resource through bankruptcy procedures. In: *Proceeding of the 2008 World Environmental and Water Resources Congress, Honolulu, Hawaii* (R. W. Babcock & R. Walton, eds). American Society of Civil Engineers, Reston, VA, USA, pp. 1–9.
- Tavakoli, A., Nikoo, M. R., Kerachian, R. & Soltani, M. 2015 River water quality management considering agricultural return flows application of a non-linear two-stage stochastic fuzzy programming. *Environ. Monit. Assess.* **187** (4), 158–171.
- Wang, G., Wang, S., Kang, Q., Duan, H. & Wang, X. 2016 An integrated model for simulating and diagnosing the water quality based on the system dynamics and Bayesian network. *Water Sci. Technol.* **74** (11), 2639–2655.
- Zeferino, J. A., Cunha, M. C. & Antunes, A. P. 2017 Adapted optimization model for planning regional wastewater systems: a case study. *Water Sci. Technol.* **76** (5), 1196–1205.

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