

Fuzzy reliability-based optimization of a hydropower reservoir

Sukanya J. Nair and K. Sasikumar

ABSTRACT

Reservoir operation modeling and optimization are inevitable components of water resources planning and management. Determination of reservoir operating policy is a multi-stage decision-making problem characterized by uncertainty. Uncertainty in inflows and power demands lead to varying degrees of the working of a reservoir from one period to another. This transition, being ambiguous in nature, can be addressed in a fuzzy framework. The different working states of the reservoir are described as fuzzy states. Based on the degree of success in meeting the power demand and randomness associated with inflows, hydropower production is considered as a random fuzzy event. This paper examines the scope of profust reliability theory, a theory used in the reliability analysis of manufactured systems, in the performance optimization of a hydropower reservoir system. The operating policy derived from a profust reliability-based optimization model is compared with a simulation model. The model is then used to derive the optimal operation policy for a hypothetical reservoir fed by normally distributed inflow, for a period of five years. The results show that the model is useful in deriving optimal operating policies with improved reliabilities in hydropower production.

Key words | fuzziness, optimization, profust reliability, reservoir operation, stochastic process

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NOTATION

A_1 to A_4	Coefficients in the equation for membership value of transitions	P_{ij}^t	Probability that the transition from X_i to X_j occurs during period t
a	Slope of head storage curve	R_t	Release from the reservoir during time period t
b	Intercept of head storage curve	S_t	Storage during time period t
D_t	Demand during time period t	S_{min}	Minimum storage
F	Fuzzy event <i>the system has failed</i>	S_{max}	Maximum storage
g	Acceleration due to gravity	T_{ij}	Transition from i to j
H_t	Average head in the reservoir during period t	T_{SF}	Fuzzy set representing failure transitions
I_t	Inflow during period t	T	Time period
N	Total number of fuzzy states	W	Fuzzy event <i>the system is working</i>
O_t	Spill from the reservoir during period t	X	Fuzzy set representing possible states of the system
P_t	Power produced during period t	x	Parameter value corresponding to discrete state at period t
P_i^t	Probability that the system is in state X_i during period t		

y	Parameter value corresponding to discrete state at period $t + 1$
μ_i	Representative value of degree of failure for state X_i
$\mu_{T_{ij}}$	Membership value of the transition from state X_i to state X_j
μ_F^t	Membership value of degree of failure during period t
ε	Efficiency of the power plant
ρ	Density of water

INTRODUCTION

Reservoirs are vital components of water resources management. The successful operation and management of reservoirs are important for the sustainability of water resource systems. Reservoir operation optimization aims to minimize the failures or the risk associated with those failures. According to Koutsoyiannis (2005), the failure of a system may be attributed to structural failure or inability to perform the intended function. The failure of a system can be defined using various criteria or factors. Structure, performance, cost, and even subjective intention may be used for defining failure. Operational failures, in other words, performance degradation, are more significant among these failures. As far as reservoir operation is concerned, operational failure may be the inability to meet the specified target in terms of drinking water supply, agricultural water, hydropower, or flood control. Whatever a failure is, if the effect of it tends to be critical, research on it becomes essential (Cai 1996).

The failure of a system such as a reservoir can be ascertained in terms of performance measures. As such, reliability is an important performance measure to study system failure. Reliability has been defined in different ways. Time-based reliability, volume-based reliability, and occurrence-based reliability are different forms of reliability (Kundzewicz & Kindler 1995). Time-based reliability is defined as the probability of no failure performance in a particular interval. Occurrence-based reliability is the ratio of the number of periods the system has entered into the satisfactory state to the total number of periods of operation.

Volume-based reliability is the ratio of the volume of water supplied to the volume of water demanded, and is relevant in the case of water supply reservoirs.

Reservoir reliability studies can be broadly classified into two types – analytical studies and simulation-based studies. Some of the studies based on analytical methods include reliability analyses of water supply reservoirs by Vogel & Stedinger (1987), Vogel & Bolognese (1995), and Kuria & Vogel (2015). Determination of reliability of a reservoir by analytical methods is difficult, especially when it is fed by seasonal flows. Another approach to reliability analysis, called chance constrained programming, was proposed by ReVelle (1999) in which the probability of occurrence of a constraint is fixed *a priori*. A modification of this method is the reliability programming approach, in which the reliability levels are considered as decision variables (Moy *et al.* 1986). Although reliability studies of water supply reservoirs are common, only a few studies have investigated the reliability of hydropower reservoirs in detail. Raje & Mujumdar (2010) studied the reliability of hydropower as part of a study on the effect of climate change on the reliability of a multipurpose reservoir. Guolei *et al.* (2010) used the reliability of hydropower production as a performance measure, along with three other measures, to compare operating policies derived using inflow forecasts with different lead times.

Reliability study of any system is based on probability. The probability density functions of failure characteristics are to be known for determining the reliability of any system. Due to the absence of past records of failure data, and the uncertainty of human perception, the use of fuzzy techniques to deal with the ambiguities is justifiable. The work of El-Baroudy & Simonovic (2004) deals with the ambiguity in the boundary of the failure region of a water resource system, which is flexible with the choice of the decision-maker. They proposed three fuzzy reliability indices for assessing the performance of such water resource systems. Suresh & Mujumdar (2004) have used a fuzzy approach for developing a new performance indicator called fuzzy risk of low yield of a crop, to study the implications of operation policy for an irrigation reservoir system, considering the uncertainty in the response of crop yield to different factors. The uncertainties in inflows and variable discretization, as well as non-commensurate

nature of objectives, were addressed in a work by Akbari *et al.* (2011).

The profust reliability theory, developed by Cai *et al.* (1993), is prominent in the reliability analysis of electrical systems. Bowles & Pelaez (1995) discussed the application of fuzzy techniques in reliability engineering and used it to find the reliability of an electrical system. Since then many reliability studies have been reported on repairable and degradable electrical systems using this approach. The work of Pandey & Tyagi (2007), which obtained the reliability estimates of a power loom plant, is notable among these. Studies show that this theory is applicable to natural systems as well. One of the best examples of the application of profust reliability theory to a natural system is the work by Mirakbari & Ganji (2010), in which they assessed the reliability of a rangeland system by considering vaguely defined states of drought severity.

One of the important characteristics of profust reliability theory is that, for any system, it allows intermediate states between the fully working and fully failed states. A reservoir system can also be analyzed using this theory as follows.

A reservoir can have three types of states – a fully working state, a fully failed state, and several partially working states in between these two. If the hydropower demand is met completely the reservoir can be considered to be in a fully working state. If the specified demand is not at all satisfied the reservoir is in a fully failed state. If the demand is satisfied partially it is in a partially working state. If 50% of the demand is satisfied, the reservoir state is such that it is 50% working and 50% failed. Similarly, if 25% of the demand is satisfied the reservoir is 25% working and 75% failed. However, the proportion of hydropower demand that can be satisfied in a particular period, with the available inflow, is uncertain. The partially working states can be considered as fuzzy states as the reservoir in this state is working to a certain degree and at the same time is failed to a certain degree. Thus, the reliability of a reservoir which has partially satisfied its hydropower demand needs to be analyzed in the context of fuzziness.

With the passage of time the transition of the system occurs from one fuzzy state to another if the proportions

of demand satisfied in the two successive time periods are different. Otherwise, the system continues in its earlier state. During the operational time period, three types of transitions are possible:

1. The system can move from a less working state to a more working state.
2. The system can go from a more working state to a less working state.
3. The system can remain in the previous state.

The system is considered failed if its transition occurs from a less failed state (for instance, a state in which 75% of the demand is satisfied) to a more failed state (a state in which 50% of the demand is satisfied). Out of the three transitions mentioned above, the continuation of the system in the least working state is the most undesirable transition. The transition from the most working state to the least working state is also undesirable. These transitions are graded relatively based on their severity, and for improving the reliability of reservoir operation the probability of the undesirable transitions (failure transitions) are to be minimized.

The concept of fuzzy reliability analysis has not been applied to a reservoir so far, except the work of Ganji & Jowkarshorijeh (2012). The definition of operational failure used in the present work is similar to that used by Vogel (1987), in which failure of a water supply system is defined as the condition in which a pre-specified yield cannot be delivered. This work tests the applicability of profust reliability theory in the development of optimal operating policy for a hydropower reservoir.

The paper is organized in the following manner. In the methodology section, the fundamentals of profust reliability theory are explained. The proposed method is used to form an optimization model and it is compared with a simulation model in the second section. The third section is the application of the optimization model to a hypothetical reservoir operated to maximize the reliability of monthly power production. The results obtained are discussed in the next section. Finally, the conclusions derived from the study are presented.

METHODOLOGY

Profust reliability

Conventional reliability is based on binary state assumption. It examines whether the system is working or failed. There are only two possible states. By contrast, profust reliability is based on multiple states which are fuzzy in nature. According to this theory, at any given instant of time, the system may be in one of the possible states of working in the range of fully working to fully failed states as explained earlier. The term ‘profust’ is a combination of probability and fuzziness. In the analysis of system failure, fuzziness describes the ambiguous nature of failure whereas probability characterizes the randomness associated with the failure event. The system is considered to be in different discrete states, the transitions between which are gradual and uncertain.

Let $X = \{X_1, X_2, X_3, X_4, \dots, X_n\}$ be the set of possible states of a system. Let W be the event that the system is working and F be the event that the system has failed. Each one of the states $X_1, X_2, X_3, \dots, X_n$ is a fuzzy state with a certain degree of membership μ_i in the set F and consequently a membership value of $1-\mu_i$ in set W .

$$F = \{(X_1, \mu_1), (X_2, \mu_2), (X_3, \mu_3) \dots, (X_n, \mu_n)\} \tag{1}$$

Initially the system is in a particular state X_i . Depending on the input to the system and the target output for the subsequent time period the system can enter into another state X_j , which may be either a more working state or a less working state compared to X_i . The transition is considered as failure if the system enters into a less working state from a more working state. Since W and F are fuzzy sets, the transitions between them are fuzzy in nature.

If we denote the failure transitions as T_{SF} , it may be defined as a fuzzy event.

$$T_{SF} = \{(T_{ij}, \mu_{T_{ij}}), i, j = 1, 2, 3, \dots, n\} \tag{2}$$

where $\mu_{T_{ij}}$ is the membership value of failure transition.

In the present study, four partially working states are considered (i.e., $n = 4$) and these states are designated as X_1, X_2, X_3 and X_4 . X_4 is the most working state and X_1 is the most failed state. The states are discretized based on

the degree of failure. The continuation of the system in the fully failed state X_1 is the worst failure, with a membership value of 1 in the set T_{SF} . Continuation of the system in the fully working state X_4 is not considered as failure, and hence, the membership value is assumed as zero for this transition. Similarly, transition from X_1 to X_4 is given a membership value of zero and transition from X_4 to X_1 is assigned a membership value of 0.9, a value close to 1.

In order to derive the membership values of other state transitions, each transition is assigned two parameters – (x, y) . x represents the parameter value for the discrete state at period t and y represents the parameter value for the discrete state at period $t + 1$. The parameter values for each transition are given in Table 1. The parameter value pair $(0, 0)$ corresponds to the system remaining in the fully failed state, whereas $(1, 1)$ corresponds to the continuation of the system in the fully working state. The two intermediate states 2 and 3 are assigned parameter values $x = 1/3$ and $x = 2/3$. The same values are assigned for the parameter y also.

The membership value for the transition from current state i with parameter value x to the next state j with parameter value y is assumed to be of the form

$$\mu_{T_{ij}} = A_1 + A_2x + A_3y + A_4xy \tag{3}$$

On applying the boundary conditions,

$$\mu_{T_{11}} (x = 0, y = 0) = 1$$

$$\mu_{T_{14}} (x = 0, y = 1) = 0$$

$$\mu_{T_{41}} (x = 1, y = 0) = 0.9$$

$$\mu_{T_{44}} (x = 1, y = 1) = 0$$

the values of A_1 to A_4 are obtained as $A_1 = 1, A_2 = -0.1, A_3 = -1$ and $A_4 = 0.1$

Table 1 | Parameter values (x, y) of transitions T_{ij}

j	1	2	3	4
i				
1	(0, 0)	(0, 1/3)	(0, 2/3)	(0, 1)
2	(1/3, 0)	(1/3, 1/3)	(1/3, 2/3)	(1/3, 1)
3	(2/3, 0)	(2/3, 1/3)	(2/3, 2/3)	(2/3, 1)
4	(1, 0)	(1, 1/3)	(1, 2/3)	(1, 1)

Table 2 | Membership values of transitions T_{ij}

j	1	2	3	4
i				
1	1	0.667	0.333	0
2	0.967	0.644	0.322	0
3	0.933	0.622	0.311	0
4	0.9	0.60	0.30	0

Therefore, the general equation for membership value of transition T_{ij} is

$$\mu_{T_{ij}} = 1 - 0.1x - y + 0.1xy \tag{4}$$

The values of $\mu_{T_{ij}}$ obtained by substitution of parameter values mentioned in Table 1 in Equation (4) are given in Table 2.

Conventional reliability of the system during the interval $(t_1, t_2) = P$ (failure transitions do not occur in the interval

$$(t_1, t_2) = 1 - P(\text{failure transitions occur in the interval } (t_1, t_2)) \tag{5}$$

In profust reliability, failure transition is the fuzzy event T_{SF} . Therefore, the profust reliability for the time interval $(t_1, t_2) = 1 - P(T_{SF} \text{ occurs in the interval } (t_1, t_2))$

$$1 - \sum_{i=1}^n \sum_{j=1}^n (\mu_{T_{ij}}) P(T_{ij} \text{ occurs during } (t_1, t_2)) \tag{6}$$

where $\mu_{T_{ij}}$ is the membership value of transition from state X_i to state X_j without passing via any intermediate state (Cai 1996).

Fuzzy reliability-based optimization model for hydropower reservoir system

Consider a hypothetical reservoir which is operated for hydropower generation. The optimum monthly releases from the reservoir for satisfying a constant monthly power demand at maximum reliability are to be determined. This can be formulated as an optimization problem with the objective of minimizing the probability of failure transitions for the operational time horizon. In the present study, the

Table 3 | State discretization

State	Interval of degree of failure	Representative value, μ_i
X_1	$0.1875 < \mu_F^t < 0.25$	0.21875
X_2	$0.125 < \mu_F^t < 0.1875$	0.15625
X_3	$0.0625 < \mu_F^t < 0.125$	0.09375
X_4	$0 < \mu_F^t < 0.0625$	0.03125

reservoir operation is restricted such that the degree of failure is limited to the range 0 to 0.25. This also corresponds to a range of degree of working from 0.75 to 1. Based on this degree of failure, reservoir operation is divided into four states, $X_1, X_2, X_3,$ and X_4 . The interval representing the degree of failure for each state and the corresponding representative values (i.e., the mid values of the range) for the interval are given in Table 3. The partially working states X_1 to X_4 are made very close to the fully working state by limiting the membership values of degree of failure from 0 to 0.25.

Figure 1 represents the state transition diagram. The transitions from more working states to less working states and continuation in less working states are designated as failure and are marked with solid lines. The transitions occur in each time period of the whole operational period. Therefore, to improve the reliability of hydropower production for the operational time horizon, it is necessary to minimize the sum of failure transitions for all the time periods.

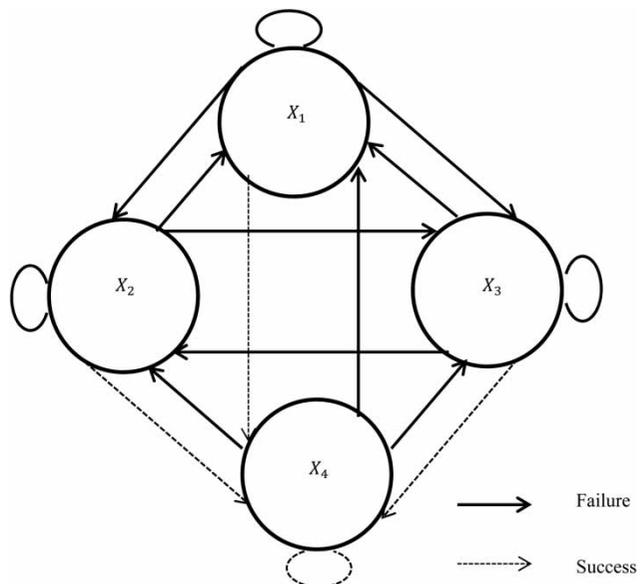


Figure 1 | State transition diagram.

The optimization model is formulated to derive the operating policy for a small hydropower reservoir, with the objective of maximizing the fuzzy reliability of power production. It is assumed that the head-storage relationship is linear and evaporation losses are negligible. Since the probability of a fuzzy event is the product of the membership function of the fuzzy event and the probability of the event, the product of $\mu_{T_{ij}}$ and P_{ij} is to be minimized for all time periods. The values of $\mu_{T_{ij}}$ are taken from Table 2.

The objective function is

$$\text{Min } Z = \sum_{t=1}^{12} \sum_{i=1}^4 \sum_{j=1}^4 \mu_{T_{ij}} * P_{ij}^t \tag{7}$$

The different constraints associated with the problem are discussed next.

1. The degree of failure associated with each state:

$$\mu_F^t = 1 - \frac{P_t}{D_t} \tag{8}$$

The degree of failure during any time period t is defined such that it is unity when the power produced P_t is equal to demand, D_t and zero when the power produced is zero (Figure 2). However, since the working states are defined as fuzzy, the cases of complete working and complete failure are not discussed, or in other words, extreme values of the degree of failure are not significant.

2. Expected value of degree of failure:

$$\mu_F^t = \sum_{i=1}^4 \mu_i * P_i^t \tag{9}$$

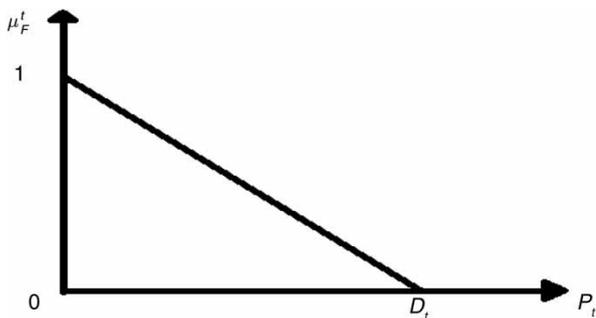


Figure 2 | Membership function of degree of failure.

The exact value of the degree of failure for a particular period is unknown. Thus, the expected value is considered. The expected value of μ_F^t is given by the above constraint, where P_i^t is the probability that the system is in state X_i during period t and μ_i is the representative value of degree of failure corresponding to state X_i , as given in Table 3.

3. Sum of state probabilities:

$$\sum_{i=1}^4 P_i^t = 1 \tag{10}$$

This constraint indicates the fact that the system is in any one of the possible states in a particular time period or, in other words, the sum of probabilities is equal to 1.

4. State transition equation:

$$P_j^{t+1} = \sum_{i=1}^4 P_i^t * P_{ij}^t, \quad j = 1, 2, 3, 4 \tag{11}$$

The transition of the reservoir from one state to another can be represented as a non-homogenous Markov chain. P_j^{t+1} is the probability that the system is in state j during period $t + 1$, P_i^t is the initial probability and P_{ij}^t is the probability that the transition from i to j occurs during period t .

5. Storage continuity equation:

The fifth constraint is the storage continuity equation. S_{t+1} , S_t are reservoir storages at time periods t and $t + 1$, respectively, in Mm^3 , R_t is the release during period t in Mm^3 , I_t is the inflow during period t in Mm^3 and O_t is the spill from the reservoir during period t in Mm^3 .

$$S_{t+1} - S_t + R_t + O_t = I_t \tag{12}$$

6. Hydropower equation:

$$P_t = \frac{\rho g e H_t R_t}{30 * 24 * 60 * 60} \tag{13}$$

This constraint gives the monthly power production in megawatts, ρ is the unit weight of water, g is the acceleration

due to gravity, H_t is the average head during $(t, t + 1)$ in m and ε is the efficiency of the turbine.

7. Equation for spill from reservoir:

$$O_t = \frac{1}{2} \{ |S_{t-1} + I_t - R_t - S_{max}| + (S_{t-1} + I_t - R_t - S_{max}) \} \quad (14)$$

This constraint limits the value of spill to zero when net storage is less than capacity, to the amount exceeding capacity when net storage is more than capacity.

8. Storage limits:

$$S_{min} \leq S_t \leq S_{max} \quad (15)$$

The storage during any period must lie within the limits of S_{min} and S_{max} , where S_{min} is the dead storage and S_{max} is the reservoir capacity.

9. Limits on release:

$$0 \leq R_t \leq S_{max} \quad (16)$$

The lower limit of release is fixed as zero and the upper limit is fixed as the reservoir capacity as no other downstream flow requirements are considered.

10. Head-storage relation:

$$H_t = a \left(\frac{S_{t-1} + S_t}{2} \right) + b \quad (17)$$

H_t is the average head during the interval $(t, t + 1)$ and a and b are the slope and intercept of the head-storage curve.

11. Limits on degree of failure:

$$0 \leq \mu_F^t \leq 0.25 \quad (18)$$

The degree of failure, being defined as a fuzzy quantity, should lie between 0 and 1. But, in order to ensure that 75% of the demand is satisfied in all periods, the degree of failure is limited to a maximum value of 0.25 as already mentioned.

12. Limits on probability:

$$0 \leq P_{ij}^t \leq 1 \quad (19)$$

$$0 \leq P_i^t \leq 1 \quad (20)$$

The probability of the system in state i during period t and the probability that the transition from i to j occurs during period t must lie between 0 and 1.

The results of the proposed optimization model are compared with that of a simulation model developed by the National Institute of Hydrology, India (Jain *et al.* 1996–97). In the simulation model, the release policy is formulated such that power produced is equal to the constant demand of 2 MW, if possible. If available water is less than S_{min} , no release is made. During any period, if $S_{t-1} + I_t > S_{max}$, then extra water after meeting power demand is spilled. If there is not enough water to generate the required power, power is generated to the extent possible.

Hypothetical case study

The optimization model is now applied to the same hypothetical reservoir with monthly inflows that are generated using the Thomas-Fiering model. The flows in all the months are assumed to follow normal distribution with statistical parameters listed in Table 4. The reservoir details are given in Table 5. The optimal operating policy for the five-year period under consideration is obtained by solving the resulting non-linear optimization model using LINGO 17.0 successively for each year. The maximum constant demand D_t that can be satisfied with the available inflow and the allowable degree of failure are determined by increasing the value of D_t uniformly until the optimization problem becomes infeasible.

RESULTS AND DISCUSSION

The results of the validation are presented in Figures 3 and 4. The power produced as per the simulation and optimization models are compared with the specified demand in Figure 3. It can be observed that the power produced as per the optimization model is greater than or equal to 1.68 MW in all the periods. Although the power produced

Table 4 | Streamflow statistics

Month	Mean flow (in Mm ³)	Standard deviation (in Mm ³)	Correlation coefficient with the previous month
June	31.58	28.87	0.1563
July	104.07	93.60	-0.0378
Aug	196.95	200.00	0.1778
Sept	200.00	142.59	0.1802
Oct	116.58	73.82	-0.0388
Nov	25.24	30.80	0.1005
Dec	10.68	6.03	0.7869
Jan	7.46	2.70	0.8485
Feb	5.02	2.46	0.3395
Mar	4.91	3.94	0.8201
Apr	3.84	3.20	0.0591
May	1.76	0.88	0.5164

Table 5 | Reservoir details

Parameter	Value
Storage capacity	473 Mm ³
Dead storage	50 Mm ³
Operation period	5 years
Turbine efficiency	85%
<i>a</i>	0.2438
<i>b</i>	25.615
Initial storage	60 Mm ³

by the simulation model matches the demand for most of the time periods, there is greater shortage of power during the third to sixth periods compared to the power produced as per the optimization model. This pattern is reflected in the degree of failure also, as shown in Figure 4(b). The maximum value of degree of failure is 0.33 for the simulation model whereas this value is 0.16 for the optimization model. Figure 4(a) shows the storage variation. The storage obtained by optimization and simulation is more or less the same, with slightly greater values for the optimization model. Figure 4(c) shows the releases obtained as per the optimization and simulation models for the same inflow. The release obtained by optimization model is more or less uniform, whereas more fluctuations are observed in the release from simulation model. The release rule obtained from optimization distributes available storage in such a

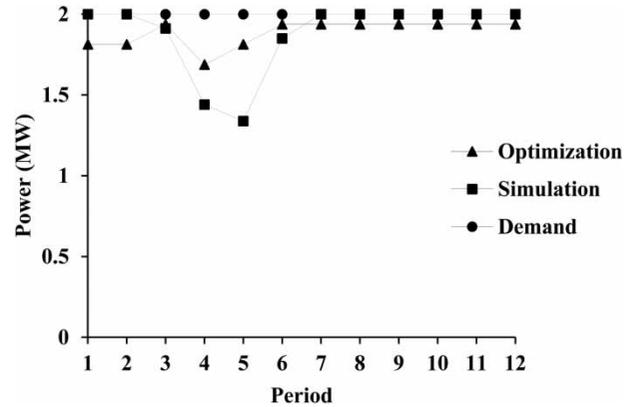


Figure 3 | Results of validation.

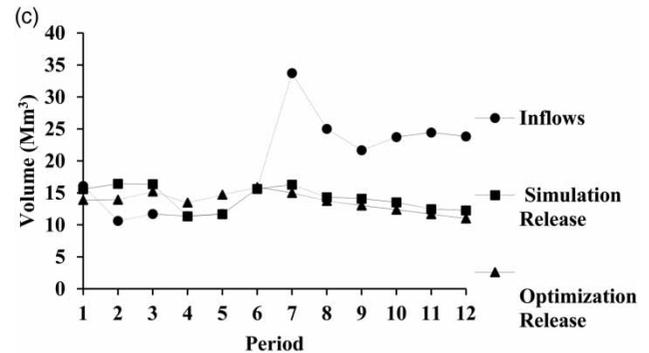
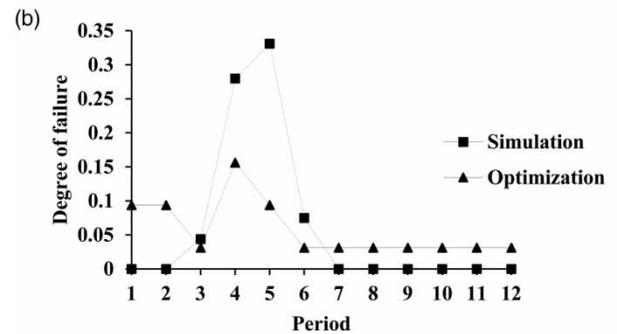
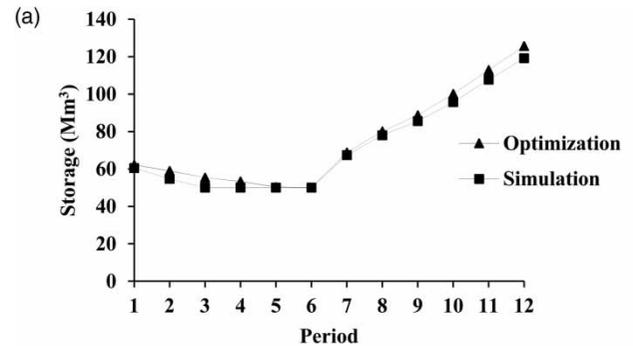


Figure 4 | Results of validation. (a) Storage variation. (b) Degree of failure. (c) Inflow vs release.

manner that it is available for release during periods of low flows. However, simulation and optimization models perform alike during periods receiving high inflow.

Figure 5 shows the year-wise results of the application of the optimization model to the hypothetical reservoir. The monthly storage variation, degree of failure, and release for each year of the five-year period under consideration are shown in Figure 5(a)–5(c), respectively. The release, storage variation, and degree of failure show a similar pattern in the second and fourth years. The results in the first, third, and fifth years also are identical. The maximum and minimum power produced, maximum power demanded, and corresponding initial storage for each year are given in Table 6.

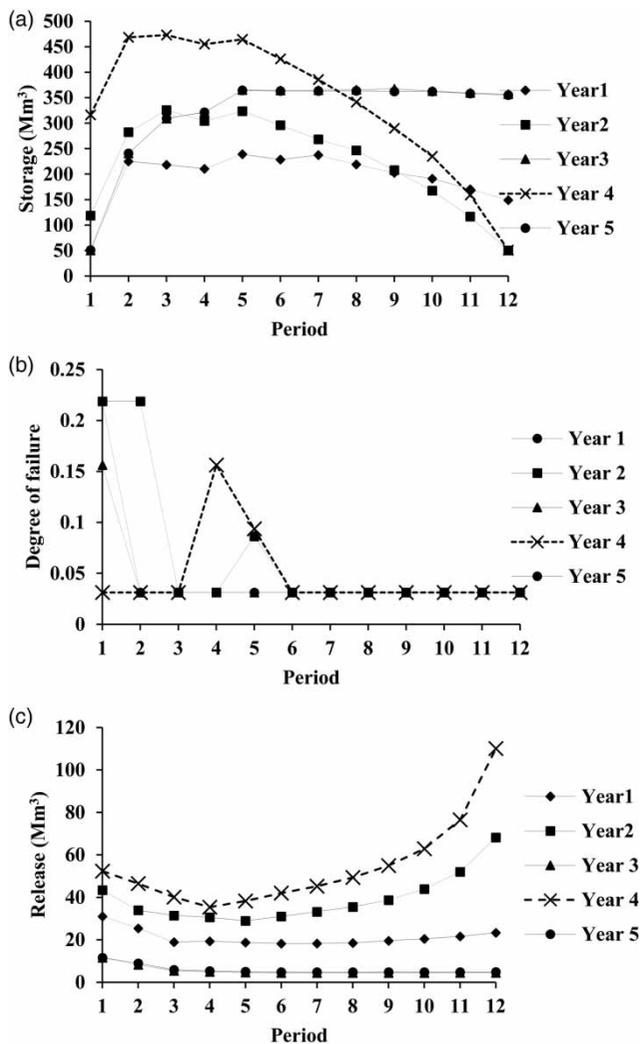


Figure 5 | Results of case study. (a) Storage variation. (b) Degree of failure. (c) Releases.

Table 6 | Maximum power, demand, and initial storages for different years

Year	Minimum power produced (MW)	Maximum power produced (MW)	Maximum power demanded (MW)	Initial storage (Mm ³)
1	3.91	4.84	5	60
2	8.44	9.69	10.4	149
3	1.41	1.61	1.7	50
4	15.83	18.16	18.75	357
5	1.41	1.75	1.8	50

The optimization model is run with an assumed initial storage of 60 Mm³. The maximum power produced during the first year is 4.84 MW. This maximum power is produced in all the periods of the first year except the first month. The power produced in the first month is 3.91 MW. The optimization model is then run with the corresponding ending storage of 149 Mm³ in the first year which is also the starting storage for the second year. The same procedure is adopted for the remaining years. The maximum and minimum powers that are produced in the second year are 9.69 MW and 8.44 MW, respectively. Maximum power is produced in all the months of the second year except the first two. In the third year, the maximum power of 1.61 MW is produced in all periods except the first period. The maximum power of 18.16 MW is produced in all periods of the fourth year except periods 4 and 5. In the fifth year the power produced is minimum in the first month and maximum in the remaining periods. The maximum power produced and the power demanded show similar alternately increasing and decreasing trends, as observed from Figure 6.

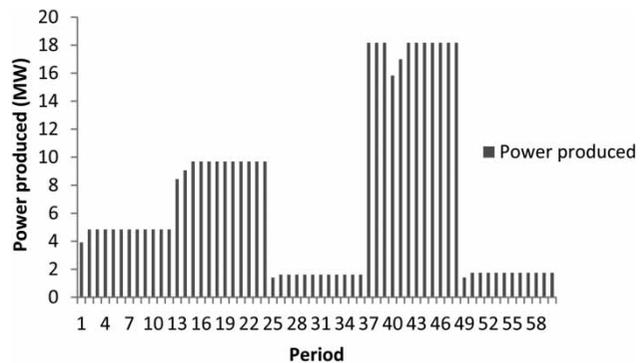


Figure 6 | Power produced during the operational period.

SUMMARY AND CONCLUSIONS

A profust reliability-based optimization model is developed to determine the optimal operating policy for a hydropower reservoir with the objective of minimizing the transition from more working state to less working state. The results from the model are compared with those from a simulation model for a hypothetical reservoir. The optimal operating policy is obtained for a period of five years, on a monthly time scale basis, with the assumed normally distributed inflows and specified power demand. The following conclusions may be drawn from the study:

- The optimization model performs better than the simulation model.
- The model developed is capable of incorporating gradual state transitions. Further, these transitions are assigned different degrees of membership depending on whether the transition is from a less failed state to a more failed state or vice versa.
- The working of the reservoir is discretized into states from the functional point of view unlike in a traditional approach where the discretization is based on storage in the reservoir.
- The results of the case study demonstrate the applicability of profust reliability in reservoir operation.

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First received 11 July 2018; accepted in revised form 7 January 2019. Available online 1 February 2019