Estimation of maximum scour depth downstream of an apron under submerged wall jets

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ABSTRACT

An analysis of laboratory experimental data pertaining to local scour downstream of a rigid apron developed under wall jets is presented. The existing equations for the prediction of the maximum scour depth under wall jets are applied to the available data to evaluate their performance and bring forth their limitations. A comparison of measured scour depth with that computed by the existing equations shows that most of the existing empirical equations perform poorly. Artificial neural network (ANN)- and adaptive neuro-fuzzy interference system (ANFIS)-based models are developed using the available data, which provide simple and accurate tools for the estimation of the maximum scour depth. The key parameters that affect the maximum scour depth are densimetric Froude number, apron length, tailwater level, and median sediment size. Results obtained from ANN and ANFIS models are compared with those of empirical and regression equations by means of statistical parameters. The performance of ANN (RMSE = 0.052) and ANFIS (RMSE = 0.066) models is more satisfactory than that of empirical and regression equations.

Key words | ANN, hydraulic structures, rigid apron, scour, wall jets

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NOTATION

The following symbols are used in this paper:

A	Sluice opening
a_0–a_n	Equation parameters
b_0–b_n	Equation parameters
D_{50}	Median sediment size
D_{90}	90% finer sediment size
D_{95}	95% finer sediment size
d_s	Depth of maximum scour at equilibrium
d_t	Tailwater depth
F	Jet Froude number
F_d	Particle densimetric Froude number
$F_{d(95)}$	Particle densimetric Froude number based on D_{95}
g	Gravitational acceleration
K_L'	Factor
L	Length of apron downstream of sluice gate
N	Number of data
o_n	Network output
s	Specific gravity of sediment particles
t_n	Target output
V	Issuing jet velocity
W	Weight between any two nodes
Δw_n	Change in weight at nth iteration
Δw_{n-1}	Change in weight at $(n-1)$th iteration
X_i	Observed value of the variable
X_{max}	Maximum value of the variable
X_{min}	Minimum value of the variable
X_{norm}	Normalized value of the variable
X_1–X_n	Independent variables
x_s	Distance of the maximum scour depth from the apron
Y	Observed value
Y'	Predicted value
Y_1	Response variable
α	Momentum factor

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Δ	$(s-1)$
η	Learning rate
σ_g	Geometric standard deviation of the bed material size
ϕ	Angle of repose

INTRODUCTION

The phenomenon of scour downstream of hydraulic structures has been a subject of interest for many researchers owing to its utmost importance while determining the safety of hydraulic structures. Persistent scouring exposes the foundations of these structures, thereby threatening their stability. Local scour downstream of a rigid apron under wall jets commences when the erosive capacity of the jet exceeds the threshold bed shear stress for the instigation of the sediment motion. Jet issuing under a sluice gate develops into a wall jet as it moves over the rigid apron. As soon as it encounters the erodible bed, the process of scouring is initiated. The erosive capacity of the jet is reduced as it moves further downstream of the erodible bed. Hence, a dune formation occurs at the end of the scour profile. Figure 1 demonstrates a definition sketch of the scour hole developed under a wall jet. In this figure, d_s = maximum equilibrium scour depth, x_s = distance from the end of apron to maximum scour depth, a = sluice opening, V = issuing jet velocity, d_t = tailwater depth, L = length of the rigid apron. The maximum scour depth depends on various parameters, namely sluice opening, sediment size, jet Froude number, tailwater depth, and length of the rigid apron.

Brief review of literature

Rouse (1939) performed pioneering investigation on scour due to a jet. Scour due to two-dimensional horizontal wall jets was investigated by Laursen (1952), Tarapore (1956), and Hogg *et al.* (1997). Investigations on scour due to impinging jets have also been undertaken by Akashi & Saito (1984), Aderibigbe & Rajaratnam (1996), Beltaos (1972, 1974, 1976a, 1976b), Beltaos & Rajaratnam (1973, 1974, 1977), Kobus *et al.* (1979), Mazurek *et al.* (2001), and Dugad & Pani (2002). Iwagaki *et al.* (1958) studied scour promoted by a three-dimensional jet and proposed an analytical model. The experimental data of Laursen (1952) were critically analyzed by Carstens (1966), and an empirical equation was further developed to account for the sediment transport rate. Altinbilek & Okyay (1973) and Francis & Ghosh (1974) studied scour that took place due to impinging plane jets. Aamir & Ahmad (2016) have put forward a comprehensive review on scour under wall jets. Equations for the prediction of the maximum scour depth under wall jets have been proposed by Valentin (1967), Altinbilek & Basmaci (1973), Chatterjee *et al.* (1994), Aderibigbe & Rajaratnam (1998), Lim & Yu (2002), Sarkar & Dey (2005), Dey & Sarkar (2006), and Aamir & Ahmad (2017).

Recently, researchers have expressed keen interest in favor of using soft-computing techniques to predict the scour depth near various hydraulic structures. Studies have been carried out on scour depth prediction under bridge piers and pile groups using neural networks and the genetic programming approach by Kambekar & Deo (2003), Azamathulla *et al.* (2006), Bateni *et al.* (2007a, 2007b),

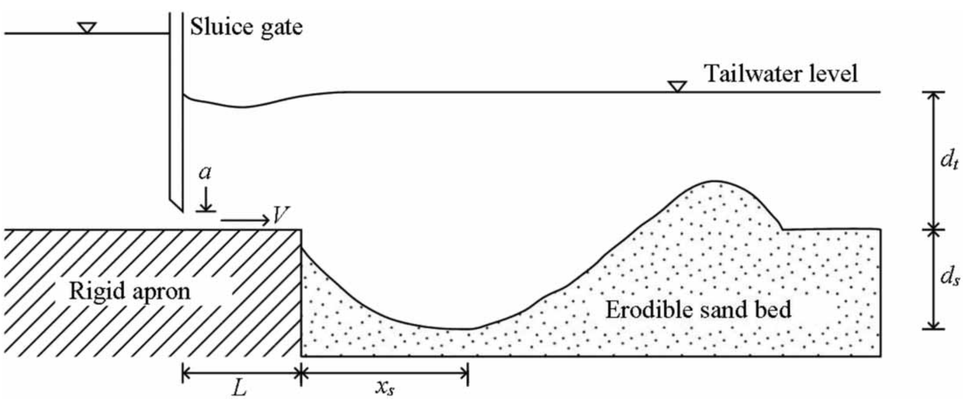

Figure 1 | Definition sketch of the scour hole under a wall jet.

Lee *et al.* (2007), Guven & Gunal (2008), Firat & Gungor (2009), Azamathulla *et al.* (2010), Kaya (2010), Najafzadeh & Barani (2011), Hashemi *et al.* (2011), Guven & Azamathulla (2012), Ismail *et al.* (2013), Najafzadeh & Azamathulla (2014), and Najafzadeh *et al.* (2016a, 2016b). To estimate scour below spillways, alternative neural networks were used by Azamathulla *et al.* (2008). Characteristics of scour downstream of stilling basins were predicted by Farhoudi *et al.* (2010) by applying the neuro-fuzzy model. Azamathulla (2012) developed a gene expression programming (GEP)-based model to predict scour depth downstream of sills, producing satisfactory results. Azamathulla & Ahmad (2012) also developed a GEP-based model for transverse mixing coefficient with competent results. Najafzadeh *et al.* (2014) and Najafzadeh (2015) used the group method of data handling (GMDH) to predict scour downstream of the ski-jump bucket spillway and grade control structures. Najafzadeh & Lim (2014) and Najafzadeh *et al.* (2017a, 2017b) used data-driven models to predict scour downstream of sluice gates and developed an improved neuro-fuzzy-based GMDH using the particle swarm optimization (NF-GMDH-PSO) as an adaptive learning network. The performance of the NF-GMDH network for the training stage indicated that the proposed NF-GMDH-PSO network provides accurate predictions. In the testing stage, the NF-GMDH-PSO network yielded better scour predictions with relatively lower error than that calculated using the empirical equations. Karbasi & Azamathulla (2017) used five different soft-computing techniques, namely artificial neural network (ANN), support vector regression, GEP, GMDH neural network, and adaptive neuro-fuzzy interference system (ANFIS), to predict the maximum scour depth under wall jets. The results obtained from the soft-computing techniques were found to be superior to those of empirical and regression equations. A comparison of various soft computing techniques showed that the accuracy of the ANN model is higher than other models. A new GEP-based equation was also proposed, given by the following equations:

$$\frac{d_s}{a} = a_1 + a_2 + a_3 \tag{1}$$

$$a_1 = \tanh\left[\left(2.75F\left(\frac{D_{50}}{a}\right)^{1/5}\right) - \left(\frac{L}{a}\frac{D_{50}}{a} + \sigma_g^{L/a}\right)\right] \tag{2}$$

$$a_2 = \left|\sinh\left(\frac{D_{50}}{a}\right) + \left(\frac{F}{\tanh(F - 8.689)}\right)\right| \tag{3}$$

$$a_3 = \ln(F)\left[\frac{1}{\left(\left(1.51\frac{D_{50}}{a}\right)\left(\frac{L}{a} + \sigma_g\right)\right)^{1/3}}\right] \tag{4}$$

Ebtehaj *et al.* (2017) used a self-adaptive extreme learning machine to predict scour depth around bridge piers. Pourzangbar *et al.* (2017) predicted scour depth at seawalls using GP and ANNs. Results revealed that the developed models are more accurate as compared to empirical relations. Lee *et al.* (2019) developed a new three-phase model for sediment transport problems with a water–air interface and verified a three-phase model for local scour caused by submerged wall jets.

Objectives

The principal aim of this article is to analyze the laboratory data for local scour depth developed under wall jets. Published experimental data are taken from Dey & Sarkar (2006), Verma & Goel (2005), Lim & Yu (2002), Aderibigbe & Rajaratnam (1998), Lee (1995), Chatterjee *et al.* (1994), Rajaratnam & Macdougall (1983), Rajaratnam (1981), and Iwagaki *et al.* (1965). Prediction equations proposed by previous researchers are analyzed for their performance against the available data. Although the application of ANN and ANFIS techniques in scour prediction has been of interest to many researchers because of their simplicity and accuracy in the prediction of scour depth under various hydraulic structures such as piers, abutments, and spur dikes, but there has not been substantial work undertaken to develop soft-computing techniques for the prediction of scour depth under wall jets. Reported studies have used only a narrow range of available experimental data to develop artificial intelligence models. Considering the seriousness of the issue of scour under wall jets, which presents a manifest threat to the foundations of hydraulic structures, a sincere effort has been made in this study to develop ANN and ANFIS models with a much wider range of available data as input parameters, to facilitate better and more accurate prediction of scour depth under wall jets, having a broader application focused on practical problems. A flowchart demonstrating a brief and logical flow of the work carried out in this paper is shown in Figure 2.

METHODOLOGY

Description of collected data

A large volume of experimental data has been collected from the literature in respect to scour under wall jets. A range of different parameters of the collected data is summarized in Table 1. In this table, F = issuing jet Froude number ($=V/(ga)^{0.5}$) and g = acceleration due to gravity. The length of the rigid apron/sluice opening ratio, i.e. $L/a = 0$, indicates the absence of a rigid apron when the jet directly encounters the erodible bed as soon as it emerges from the sluice opening.

Dey & Sarkar (2006) performed experiments in a glass-walled flume of 10 m long, 0.6 m wide, and 0.71 m depth, having a sediment recess of 2 m length and 0.3 m depth downstream of a rigid apron made of perspex sheet. Verma & Goel (2005) conducted experiments in a flume of 3 m length and 0.23 m width, with a 1 m long test section. Aderibigbe & Rajaratnam (1998) used an experimental flume of 5 m long, 0.32 m wide, and 0.65 m depth. The test section was 1.3 m long having a sand bed, and there was no rigid apron used, i.e. the emerging jet directly encountered the sand bed. Chatterjee *et al.* (1994) performed experiments in a glass-walled flume of 9 m long, 0.6 m wide, and 0.69 m depth, with a 3 m long and 0.25 m deep sediment recess. Rajaratnam & Macdougall (1983) and Rajaratnam (1981) conducted experiments in a 5.5 m long flume having a width of 0.31 m and a depth of 0.66 m. The sediment recess in both cases was 3.5 m long and 0.23 m depth. Mostly, the authors have used sluice gate opening within the range of 5–30 mm. Rajaratnam (1981) and Rajaratnam & Macdougall (1983) have worked in the range of 0.36–3.81 mm sluice opening, whereas Chatterjee *et al.* (1994) have gone up to 50 mm sluice opening.

Factors affecting maximum scour depth

As reported in the literature, the maximum equilibrium scour depth is dependent on jet Froude number (F), tailwater depth (d_t), sediment size (D_{50}), and length of the rigid apron (L). The maximum scour depth is found to increase with the increase in the issuing jet Froude number (Ali & Neyshaboury 1991; Dey & Sarkar 2006). Increasing tailwater depth causes a reduction in the maximum scour depth up to a critical tailwater depth, after which there is an increase in the maximum scour depth (Ali & Lim 1986; Dey & Sarkar 2006). The maximum scour depth decreases with an increase in sediment size, which can be represented by D_{50} (Ali & Neyshaboury 1991), while it decreases with an increase in the length of the rigid apron (Dey & Sarkar 2006).

To find out the significance of various independent variables on predicting the maximum scour depth, a correlation matrix was developed among the dependent and independent variables, as given in Table 2, which indicates the type of correlation (positive or negative) between d_s/a and the other independent variables. d_s/a shows positive correlation with F, d_t/a and F_d (densimetric Froude number = $V/(g\Delta D_{50})^{0.5}$, where $\Delta = s - 1$; s = relative density of sediments), indicating that the value of d_s/a increases with an increase in F, d_t/a or F_d; and negative correlation with L/a, D_{50}/a and σ_g (=geometric standard deviation of the bed material size), indicating that the value of d_s/a decreases with an increase in L/a, D_{50}/a or σ_g.

An F-test was carried out to find out the significance of each parameter on predicting the maximum scour depth d_s/a, as shown in Figure 3. It can be observed from Figure 3 that F_d is the most significant parameter which affects the predicted maximum scour depth, followed by L/a, d_t/a, D_{50}/a, and F. σ_g is relatively insignificant.

Scour depth prediction equations

A number of prediction equations have been proposed by various investigators for local scour under wall jets based on experimental analyses. Table 3 presents such equations for the prediction of the maximum scour depth. These equations were analyzed in the present study for their performance with the available laboratory data.

Regression analysis

Regression analysis is generally used to apply quantitative connections between a dependent variable and one or

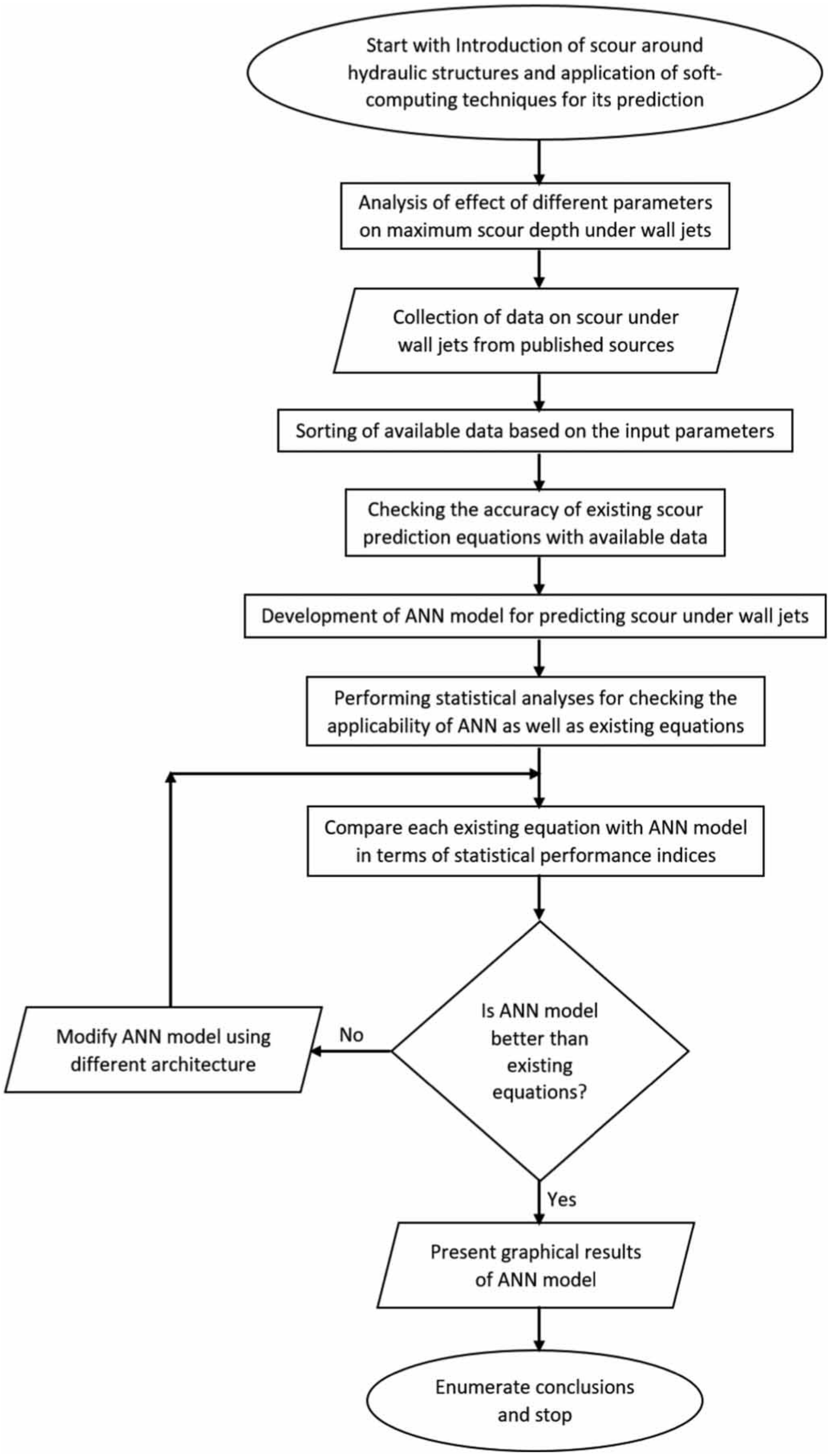

Figure 2 | Flowchart demonstrating a brief and logical flow of the work carried out.

Table 1 | Range of parameters for data used in the present analysis

Investigator	Number of data	Range of parameters							Flume dimensions		
		d_s/a	F	d_t/a	D_{50}/a	L/a	σ_g	Hours	Length (m)	Width (cm)	Depth (cm)
Dey & Sarkar (2006)	205	2.27–8.16	2.37–4.87	6.57–13.85	0.02–0.44	26.67–55	1.1–3.9	12–24	10	60	71
Verma & Goel (2005)	29	0.5–4.2	1.24–4.62	3.67–29.7	0.1–0.3	9.33–60	–	–	3	23	–
Lim & Yu (2002)	63	0.6–20.33	1.09–7.99	10–25.4	0.08–1.64	0	1.1–1.3	>50	–	–	–
Aderibigbe & Rajaratnam (1998)	30	1.32–24.4	1.21–21.54	12–60	0.05–1.35	0	1.3–3.1	5–52	5	32	65
Lee (1995)	20	1.42–5.52	0.83–2.59	5.5–9	0.07	0	1.2–1.5	44–215	–	–	–
Chatterjee *et al.* (1994)	28	0.9–4.1	1.02–5.46	5.82–15.5	0.02–0.22	13.2–33	1.2–1.4	0.2–4.7	9	60	69
Rajaratnam & Macdougall (1983)	12	2.1–15.83	1.56–9.3	≈1	0.06–0.19	0	1.3	6– > 41	5.5	31	66
Rajaratnam (1981)	14	3.34–31.29	0.65–3.79	15.27–106.86	0.1–0.67	0	1.3	6–65	5.5	31	66
Iwagaki *et al.* (1965)	19	2.65–27	1.58–9.26	25–62	0.19–0.37	–	1.5	–	–	–	–
Total	420	0.5–31.29	0.65–21.54	≈1–106.86	0.02–1.64	0–60	1.1–3.9	–	–	–	–

Table 2 | Correlation matrix among different variables

	d_s/a	L/a	F_d	d_t/a	D_{50}/a	F	σ_g
d_s/a	1						
L/a	−0.45	1					
F_d	−0.65	0.81	1				
d_t/a	0.19	0.32	0.32	1			
D_{50}/a	−0.15	0.39	0.39	0.19	1		
F	0.20	−0.19	−0.08	−0.16	−0.44	1	
σ_g	−0.03	−0.27	−0.26	−0.11	−0.17	0.06	1

more independent variables (Karbasi & Azamathulla 2017). In multiple linear regression (MLR), the function is a linear mathematical statement, i.e. straight-line, of the following form:

$$Y_1 = a_0 + a_1X_1 + a_2X_2 + \cdots + a_nX_n \quad (5)$$

where Y_1 is the response variable, a_0–a_n are the equation parameters for the linear equation, and X_1–X_n are the independent variables.

Multiple nonlinear regression (MNLR) is an illustration of regression analysis in which observational information is modeled by a function, which is a nonlinear combination of the model parameters and depends on one or more independent variables. Dissimilar to MLR, which is limited to estimating linear models, MNLR can estimate models with

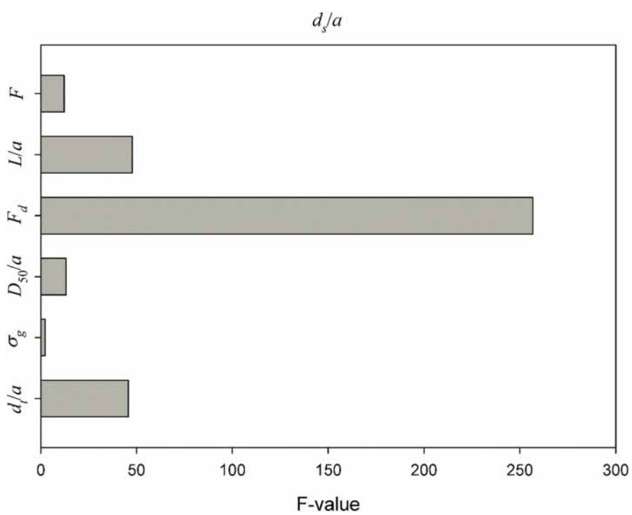

Figure 3 | Results of the *F*-test.

Table 3 | Scour depth prediction equations

Investigator	Equation
Valentin (1967)	$\log\left(\frac{d_s}{a}\right) = \frac{F-2}{4.7} - 0.55\log\left(\frac{D_{90}}{a}\right)$, where D_{90} = particle size for which 90% are finer by the weight
Altinbilek & Basmaci (1973)	$\frac{d_s}{a} = \left(\frac{a}{D_{50}}\tan\phi\right)^{0.5} F_d^{1.5}$, where $F_d = V/(\Delta g D_{50})^{0.5}$, where V = issuing jet velocity, g = acceleration due to the gravity, $\Delta = s-1$, s = relative density of the sediment; ϕ = angle of repose
Chatterjee *et al.* (1994)	$\frac{d_s}{a} = 0.775F$
Aderibigbe & Rajaratnam (1998)	$\frac{d_s}{a} = 3.35F_{d(95)} - 6.11$, where $F_{d(95)}$ = densimetric Froude number based on D_{95}
Lim & Yu (2002)	$\frac{d_s}{a} = 1.04F_d^{1.47}\sigma_g^{-0.69}\left(\frac{D_{50}}{a}\right)^{0.33}K'_L$, where K'_L = factor to account for the effects of an apron downstream from the outlet, which is given by $K'_L = e^{-0.004F_d^{-0.35}\sigma_g^{-0.5}\left(\frac{D_{50}}{a}\right)^{-0.5}\left(\frac{L}{a}\right)^{1.4}}$; σ_g = geometric standard deviation
Sarkar & Dey (2005)	$\frac{d_s}{a} = 0.42F_d^{0.49}\left(\frac{L}{a}\right)^{-0.36}\left(\frac{d_t}{a}\right)^{1.08}$
Dey & Sarkar (2006)	$\frac{d_s}{a} = 2.59F_d^{0.94}\left(\frac{L}{a}\right)^{-0.37}\left(\frac{d_t}{a}\right)^{0.16}\left(\frac{D_{50}}{a}\right)^{0.25}$

nonlinear relationships between input and response variables (Karbasi & Azamathulla 2017). The general presentation of the nonlinear relation is assumed to be the following:

$$Y_1 = b_0 X_1^{b_1} \cdot b_2 X_2^{b_2} \cdots b_n X_n^{b_n} \tag{6}$$

where b_0–b_n are the equation parameters.

Artificial neural network

ANN is a type of data-driven model used for data mapping between a set of input and output variables by simulating the biological cognition process of a human brain (Azamathulla *et al.* 2005). A typical neural network consists of three layers of neurons. The first layer is the input layer, the second is the hidden layer, and the last one is the output layer. The books of Kosko (1992) and Wassermann (1993) can be referred to get an understanding of the detailed functioning of the neural networks.

In the present analysis, a feed-forward back-propagation neural network with one hidden layer was used, in which the input data are fed into the input layer and the target patterns are associated with the output unit; the error is propagated back to the network for readjustment of weights. Each of the variables in the dataset was normalized using Equation (7) to make the range of input data fall within the interval (0,1).

$$X_{norm} = \frac{X_i - X_{min}}{X_{max} - X_{min}} \tag{7}$$

where X_{norm} is the normalized value of the observed variable X_i, X_{min} is the minimum value, and X_{max} is the maximum value of the variable.

Based on the published literature on scour under wall jets, and also based on the results of correlation analysis and the F-test performed in this study, the input parameters to the ANN model were taken as F_d, L/a, d_t/a, and D_{50}/a. The enumeration technique was employed to optimize the network in terms of the number of hidden layers (either 1 or 2) and corresponding neurons in each hidden layer. Different learning rate values were tested and based on an *RMSE* criterion, an optimized network topology of 4-9-1 with a learning rate fixed at 0.06 was found to be more suitable than the other tested network topologies, with $RMSE$ = 0.052. Since the length of the rigid apron L was considered as an input parameter, only the dataset with known values of L was used to train the model. Training and cross-validation of the ANN model were done using 75% of the available data (15% data from the training dataset were used for cross-validation), while the remaining 25% data (which was not used for the training purpose) were used for the testing of the trained network.

Adaptive neuro-fuzzy interference system

ANFIS is a fuzzy Sugeno model put in the framework of adaptive systems to facilitate learning and adaptation (Jang 1993). The neuro-fuzzy model combines ANN and a fuzzy inference system (FIS) to facilitate the process of learning and adaption. In neuro-fuzzy models, a multilayer feed-forward neural network is used to identify the parameters of an adaptive network FIS. Importantly, fuzzy logic allows the communication between the input space and output space with a list of If-Then sentences, called law.

In this research, the hybrid learning algorithm, which combines the least-squares method and the back-propagation, was utilized to train and adapt the FIS. Detailed information about ANFIS can be found in Jang (1993).

Statistical error analysis

Statistical performance indices were used to check the accuracy of existing equations, which are a measure of the extent of agreement between the observed and predicted data (Najafzadeh *et al.* 2017a, 2017b). If N is the number of data, Y [$=(d_s/a)_{\text{measured}}$] is the observed value and Y' [$=(d_s/a)_{\text{predicted}}$] is the corresponding predicted value, the different performance indices may be defined as follows.

Coefficient of correlation,

$$R = \frac{N\Sigma YY' - \Sigma Y \Sigma Y'}{\sqrt{N\Sigma Y^2 - (\Sigma Y)^2}\sqrt{N\Sigma Y'^2 - (\Sigma Y')^2}} \tag{8}$$

Root-mean-square error,

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(Y_i - Y_i')^2}{N}} \tag{9}$$

Mean absolute percentage error,

$$\text{MAPE} = \frac{100}{N}\sum_{i=1}^{N}\frac{|Y_i - Y_i'|}{|Y_i|} \tag{10}$$

$$\text{BIAS} = \frac{1}{N}\sum_{i=1}^{N}(Y_i - Y_i') \tag{11}$$

Scatter index,

$$\text{SI} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[(Y_i - \overline{Y_i}) - (Y_i' - \overline{Y_i'})\right]^2}}{\frac{1}{N}\sum_{i=1}^{N} Y_i} \tag{12}$$

RESULTS AND DISCUSSION

Accuracy of existing scour depth prediction equations

The dataset used in this paper was applied to the existing equations, and a comparison of measured scour depths with scour depths predicted by the existing equations was plotted in Figure 4(a)–4(g). The equations which consider the effect of the length of apron L were analyzed with only the set of data for which the values of L were known. For this purpose, the dataset was primarily divided into two sets; one with values of L and the other for which $L = 0$. The solid line in each figure represents the line of perfect agreement between the observed and the predicted values of the non-dimensional maximum scour depth d_s/a. It is observed from Figure 4 that the predicted scour depths from the equations proposed by Valentin (1967), Altinbilek & Basmaci (1973), and Aderibigbe & Rajaratnam (1998) deviate positively from the line of perfect agreement, whereas a negative deviation can be observed in case of the equation given by Chatterjee *et al.* (1994). Scour depths predicted from equations proposed by Lim & Yu (2002), Sarkar & Dey (2005), and Dey & Sarkar (2006) are found to give lesser deviations from the line of perfect agreement than the others – the least by Dey & Sarkar's (2006) equation. It is to be noted that only the set of data which lies within the range of applicability of a particular equation was used to test that equation.

The equation proposed by Chatterjee *et al.* (1994) considers the maximum scour depth to be a function of the

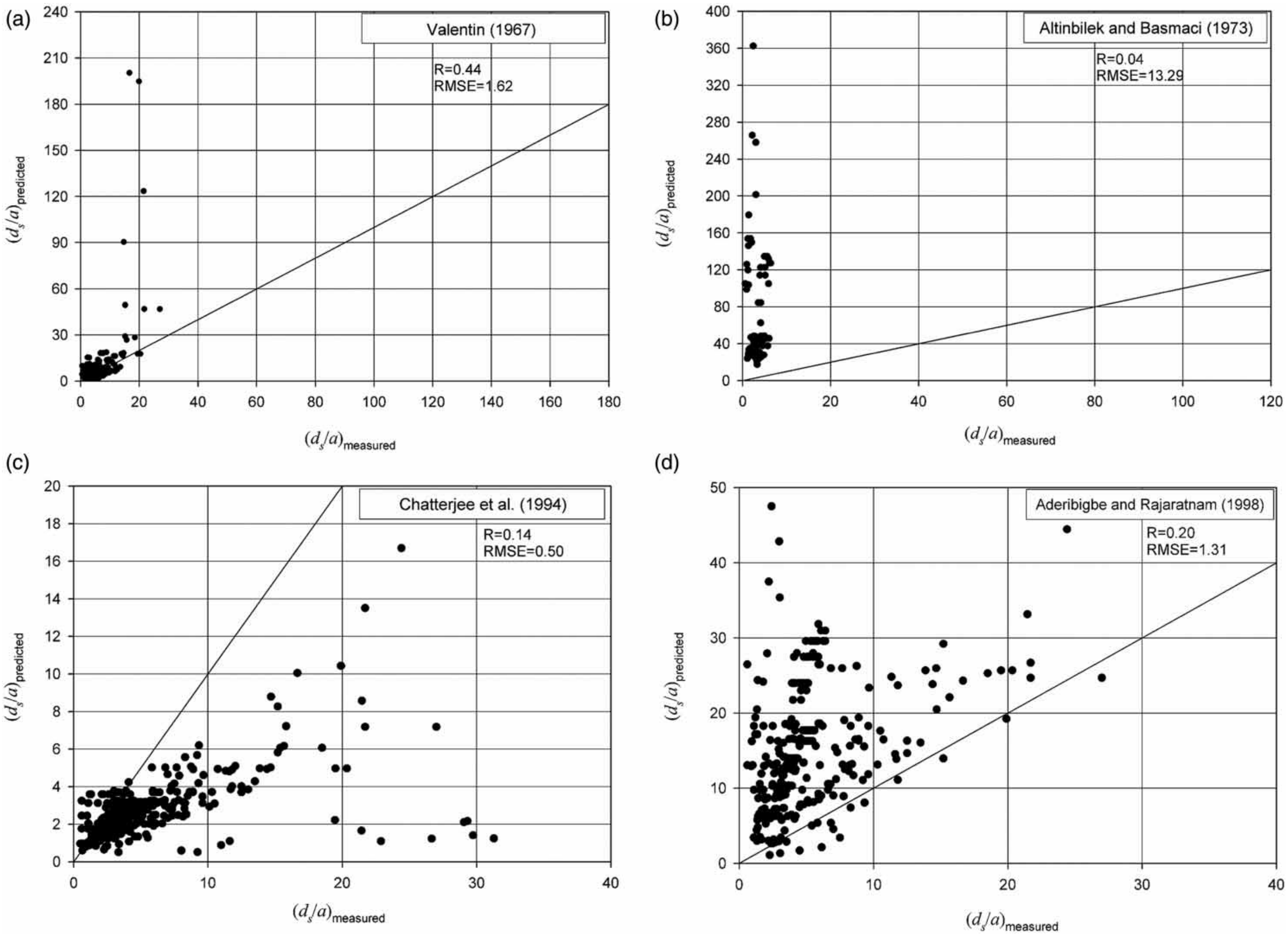

Figure 4 | Comparison of measured scour depths with scour depths predicted by various equations. *(Continued.)*

issuing jet Froude number only, neglecting the effect of other parameters. On the other hand, the equation proposed by Dey & Sarkar (2006) takes into account the effect of a number of parameters on the maximum scour depth, namely densimetric Froude number F_d, length of the rigid apron L, tailwater depth d_t, and sediment size represented by D_{50}, thus increasing the accuracy of the predicted scour depth since more number of parameters are considered. However, the applicability of Dey & Sarkar's (2006) equation also becomes limited to a range of $6.57 \leq (d_t/a) \leq 13.85$ and $(L/a) > 26$. The less number of parameters involved in Chatterjee *et al.*'s (1994) equation makes its applicability more versatile.

From Figure 4(c), it is evident that Chatterjee *et al.*'s (1994) equation underpredicts the scour depth in most of the cases when tested with the available data. Dey & Sarkar's (2006) equation, on the other hand, overpredicts the scour depth when tested with the available data. Thus, it can be well stated that Dey & Sarkar's (2006) equation performs much better than the others. However, this result might have been affected by the fact that almost 50% of the data was taken from Dey & Sarkar (2006).

Table 4 shows the values of performance indices for the existing prediction equations when tested with the available data.

The value of R is maximum in case of Dey & Sarkar (2006) and Valentin (1967). However, Dey & Sarkar's (2006) equation has the least values for RMSE and MAPE. Therefore, it is evident from the above error analysis that the equation proposed by Dey & Sarkar (2006) performs better than others when tested with the available data. Altinbilek & Basmaci's (1973) equation has the least value

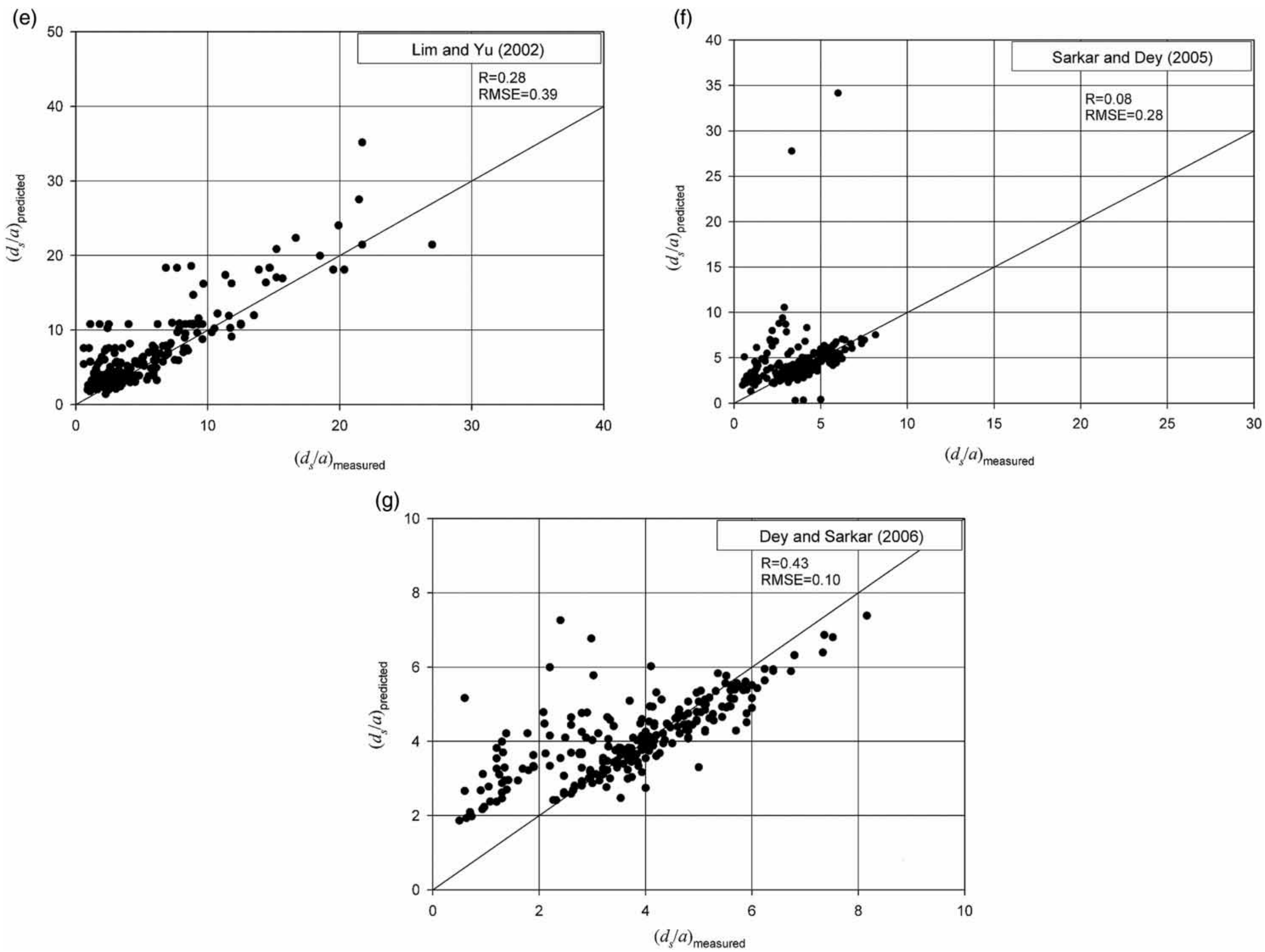

Figure 4 | Continued.

Table 4 | Values of performance indices for the existing equations

Investigator	*R*	RMSE	MAPE	BIAS	SI
Dey & Sarkar (2006)	0.425	0.102	0.332	−0.003	0.268
Sarkar & Dey (2005)	0.080	0.280	0.489	−0.007	0.736
Lim & Yu (2002)	0.282	0.388	0.828	0.027	0.903
Aderibigbe & Rajaratnam (1998)	0.197	1.313	2.986	−0.035	3.119
Chatterjee *et al.* (1994)	0.144	0.496	0.427	−1.012	6.740
Altinbilek & Basmaci (1973)	0.045	13.292	58.017	−0.107	2.480
Valentin (1967)	0.442	1.617	0.770	−0.020	0.618

for *R*, whereas it gives whimsically high values for RMSE and MAPE. Therefore, this equation cannot be recommended to predict scour in practical cases.

Overpredictions and underpredictions

The percentage of data with underpredictions and overpredictions is presented in Table 5. Each equation was tested against the data which fall within the range of applicability of that equation. However, Chatterjee *et al.*'s (1994) equation was tested against the complete dataset, since it takes into consideration only the effect of the jet Froude number for the calculation of the maximum scour depth. Hence, a large number of underpredictions was observed in this case. The equations given by Valentin (1967), Lim & Yu (2002) and Sarkar & Dey (2005) also feature a significant number of underpredictions. Aderibigbe & Rajaratnam's (1998) equation has a large number of overpredictions more than 200%, whereas Altinbilek & Basmaci's (1973)

Table 5 | Comparison of measured scour depths with scour depths predicted using existing equations

	Percentage of overpredictions			Percentage of underpredictions		
Investigator	**Greater than 200%**	**Greater than 100%**	**Greater than 50%**	**Total underpredictions**	**Greater than 25%**	**Greater than 50%**
Dey & Sarkar (2006)	6.7	13.7	19.8	40.6	1	0
Sarkar & Dey (2005)	11.5	15.7	21.1	39.6	5.4	1
Lim & Yu (2002)	9	19.1	33.7	26.1	5	0
Aderibigbe & Rajaratnam (1998)	63.2	77.4	86.6	4.6	1.6	1.3
Chatterjee *et al.* (1994)	0.4	1.5	3	89	70.5	24
Altinbilek & Basmaci (1973)	100	100	100	0	0	0
Valentin (1967)	6.5	18.1	42.2	21.9	12.2	1.4

equation predicts unrealistically high scour depth, giving overpredictions greater than 200% for the complete dataset applied to it.

Multiple linear regression

MLR analysis of the complete dataset (Table 1) yields the following equation of the maximum scour depth under wall jets:

$$\frac{d_s}{a} = 1.38 + 0.24F_d - 0.004\left(\frac{L}{a}\right) + 0.025\left(\frac{d_t}{a}\right) + 2\left(\frac{D_{50}}{a}\right) \quad (13)$$

The value of R for the MLR equation was obtained as 0.54, which shows that this equation poorly predicts the maximum scour depth.

Multiple nonlinear regression

MNLR analysis of the complete dataset (Table 1) yields the following equation of the maximum scour depth under wall jets:

$$\frac{d_s}{a} = 2.19(F_d)^{1.18}\left(\frac{L}{a}\right)^{-0.31}\left(\frac{d_t}{a}\right)^{0.04}\left(\frac{D_{50}}{a}\right)^{0.43} \quad (14)$$

The value of R for the MNLR equation was obtained as 0.62, which shows that this equation can also not be relied upon to predict the maximum scour depth.

It can be concluded from the above error analyses that the equation proposed by Dey & Sarkar (2006) performs better than the other existing equations. However, this equation, as well as other equations, is based on regression analysis, which does not explicitly account for nonlinear correlation among various parameters. Also, when tested against the available data, these equations were not found to be very accurate in predicting the maximum scour depth. It is, therefore, emphasized that a further improvement in the prediction of the maximum scour depth needs to be explored using soft-computing techniques that take into account the nonlinear behavior of the parameters. Therefore, ANN- and ANFIS-based models were developed for the better prediction of the maximum scour depth.

Artificial neural network

Figure 5 shows the comparison of measured and predicted values of normalized scour depth using the ANN model for (a) training and (b) testing dataset. The results presented in Figure 5 show actual values of observed and predicted scour depth (in contrast to the normalized values used for training and testing the ANN model), which are denormalized to increase practical understandability. The value of R comes out to be 0.95 for training and 0.96 for testing, which shows that the model can be used efficiently to predict equilibrium scour depth under wall jets. For the purpose of comparison with the existing equations, the normalized values (X_{norm}) of results obtained from the ANN model are reconverted into the non-dimensional form of d_s/a (X_i) using Equation (7).

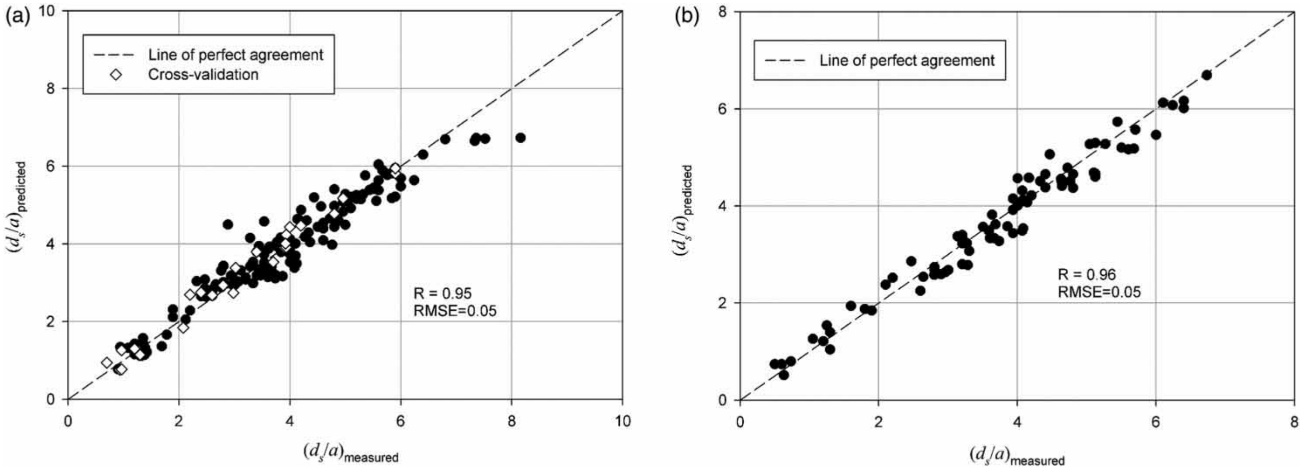

Figure 5 | Comparison of measured and predicted scour depths using the ANN model for (a) training and (b) testing.

Table 6 | Results of sensitivity analysis for the ANN model

Functions	*R*	RMSE	MAPE
$d_s/a = f(L/a, d_t/a, D_{50}/a)$	0.54	1.17	0.83
$d_s/a = f(L/a, d_t/a, F_d)$	0.89	0.37	0.48
$d_s/a = f(L/a, D50/a, F_d)$	0.83	0.49	0.67
$d_s/a = f(d_t/a, D_{50}/a, F_d)$	0.79	0.57	0.74

Sensitivity analysis

A sensitivity analysis was carried out for the ANN model in order to assign the most effective parameters for the model. The analysis was conducted such that one parameter from the ANN model was eliminated each time to evaluate the effect of that input on the output. The results indicate that the parameter F_d ($R = 0.54$, RMSE = 1.17, and MAPE = 0.83) is the most effective parameter on the scour depth and D_{50}/a ($R = 0.89$, RMSE = 0.37, and MAPE = 0.48) has the least influence. The other effective parameters are L/a and d_t/a. The statistical error parameters obtained from the sensitivity analysis are given in Table 6.

Adaptive neuro-fuzzy interference system

Figure 6 shows the comparison of measured and predicted values of scour depth using the ANFIS model for (a) training and (b) testing dataset. The value of *R* comes out to be 0.94 for training and 0.92 for testing, which shows good

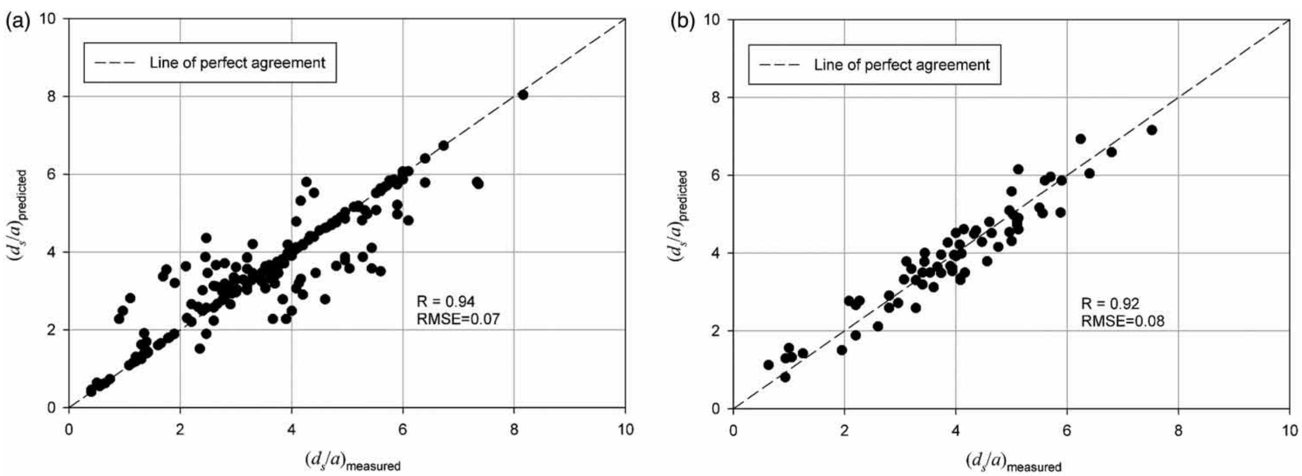

Figure 6 | Comparison of measured and predicted scour depths using the ANFIS model for (a) training and (b) testing.

performance of the model to predict equilibrium scour depth under wall jets.

Table 7 gives the values of performance indices, while Table 8 gives overpredictions and underpredictions for the proposed ANN and ANFIS models. These models perform better than any of the existing equations. On comparison with the values of performance indices of Dey & Sarkar's (2006) equation (given in Table 4), the ANN and ANFIS models are found to have a higher value of R and lower values of RMSE and MAPE than Dey & Sarkar's (2006) equation. This makes the proposed models slightly advantageous as compared to Dey & Sarkar's (2006) equation for the prediction of the maximum scour depth. In addition to better prediction of the maximum scour depth, the ANN and ANFIS models also have a wider range of applicability since they were developed using a wide range of experimental data. The number of overpredictions is considerably less in case of these models. Although total underpredictions are high, underpredictions greater than 50% is negligible. Overall, the comparison of Dey & Sarkar's (2006) equation with the ANN and ANFIS models clearly indicates that the two artificial intelligence models are advantageous. Hence, the proposed models can be used as a prediction tool for the maximum scour depth under wall jets.

Table 7 | Values of performance indices for ANN and ANFIS models

Model	Stage	*R*	RMSE	MAPE	Bias	SI
ANN	Training	0.954	0.052	0.181	−0.004	0.167
	Testing	0.962	0.050	0.168	0.002	0.146
ANFIS	Training	0.940	0.066	0.184	0.002	0.226
	Testing	0.920	0.085	0.197	0.003	0.120

Parametric study

As concluded from the sensitivity analysis, F_d is found to be the most effective parameter in the prediction of the maximum scour depth. The effect on the results of the proposed model for varying F_d was investigated. The discrepancy ratio (DR), defined as the ratio of predicted and observed values, was used to quantify the sensitivity of the proposed model to the F_d parameter (Najafzadeh *et al.* 2016a, 2016b). Unity valued DR shows a perfect agreement, while values greater (or smaller) than unity indicate over- (or under-) prediction of the maximum scour depth. Variations of DR values were plotted against the logarithm of F_d.

The results of the (a) ANN and (b) ANFIS models are illustrated in Figure 7. The minimum, mean, and maximum values of DR for the ANN model were obtained as 0.85, 0.98, and 1.16, respectively. For $3.31 < F_d < 5.08$, DR values were found to be around 1.0, showing good agreement between the observed maximum scour depth and that predicted using the ANN model. The ANN model gives slight overpredictions for $9.84 < F_d < 12.61$, while it gives relatively underpredicted values for $6.13 < F_d < 8.69$.

The results of other prediction equations are shown in Figure 8(a)–(g). It can be observed that the equations of Dey & Sarkar (2006) (Figure 8(a)) and Sarkar & Dey (2005) (Figure 8(b)) give DR values relatively closer to 1.0 when compared to other equations, but not as good as the ANN or ANFIS model. Lim & Yu's (2002) equation (Figure 8(c)) gives relatively high overpredictions for $7.08 < F_d < 12.61$. Aderibigbe & Rajaratnam's (1998) equation generally overpredicts the maximum scour depth, as is observed from Figure 8(d). Chatterjee *et al.*'s (1994) equation (Figure 8(e)) generally underpredicts the maximum scour depth, but gives DR values almost near to 1.0 for

Table 8 | Overpredictions and underpredictions for ANN and ANFIS models

		Percentage of overpredictions			Percentage of underpredictions		
Model	**Stage**	**Greater than 200%**	**Greater than 100%**	**Greater than 50%**	**Total underpredictions**	**Greater than 25%**	**Greater than 50%**
ANN	Training	0	0	0.6	28.0	0	0
	Testing	0	1.4	1.4	30.4	13.0	0
ANFIS	Training	0	0	2.0	32.0	7.4	0
	Testing	0	0	2.8	42.3	1.4	0

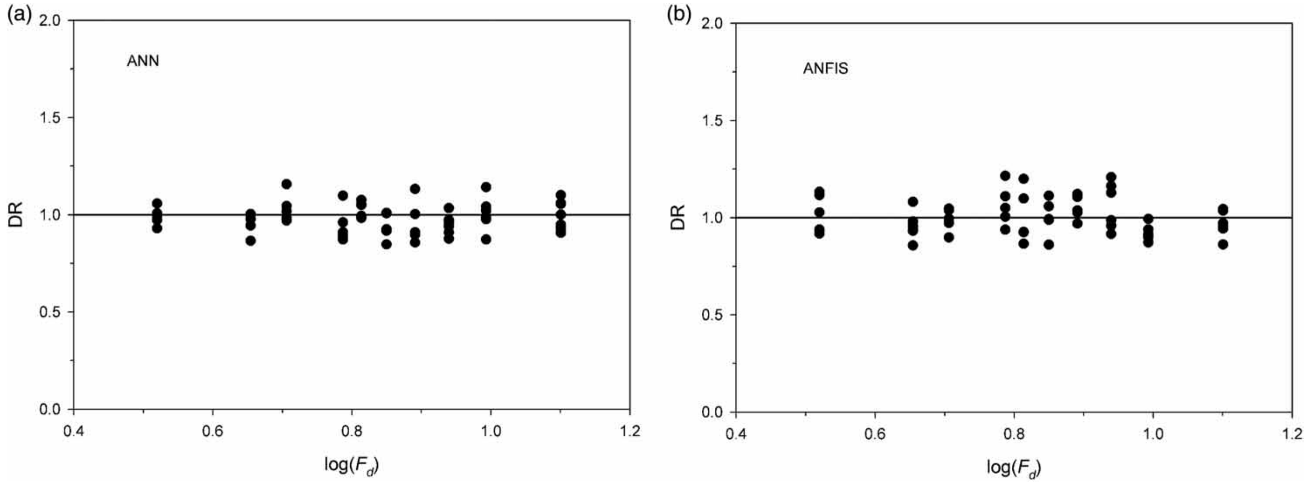

Figure 7 | Results of DR analysis for (a) ANN and (b) the ANFIS model.

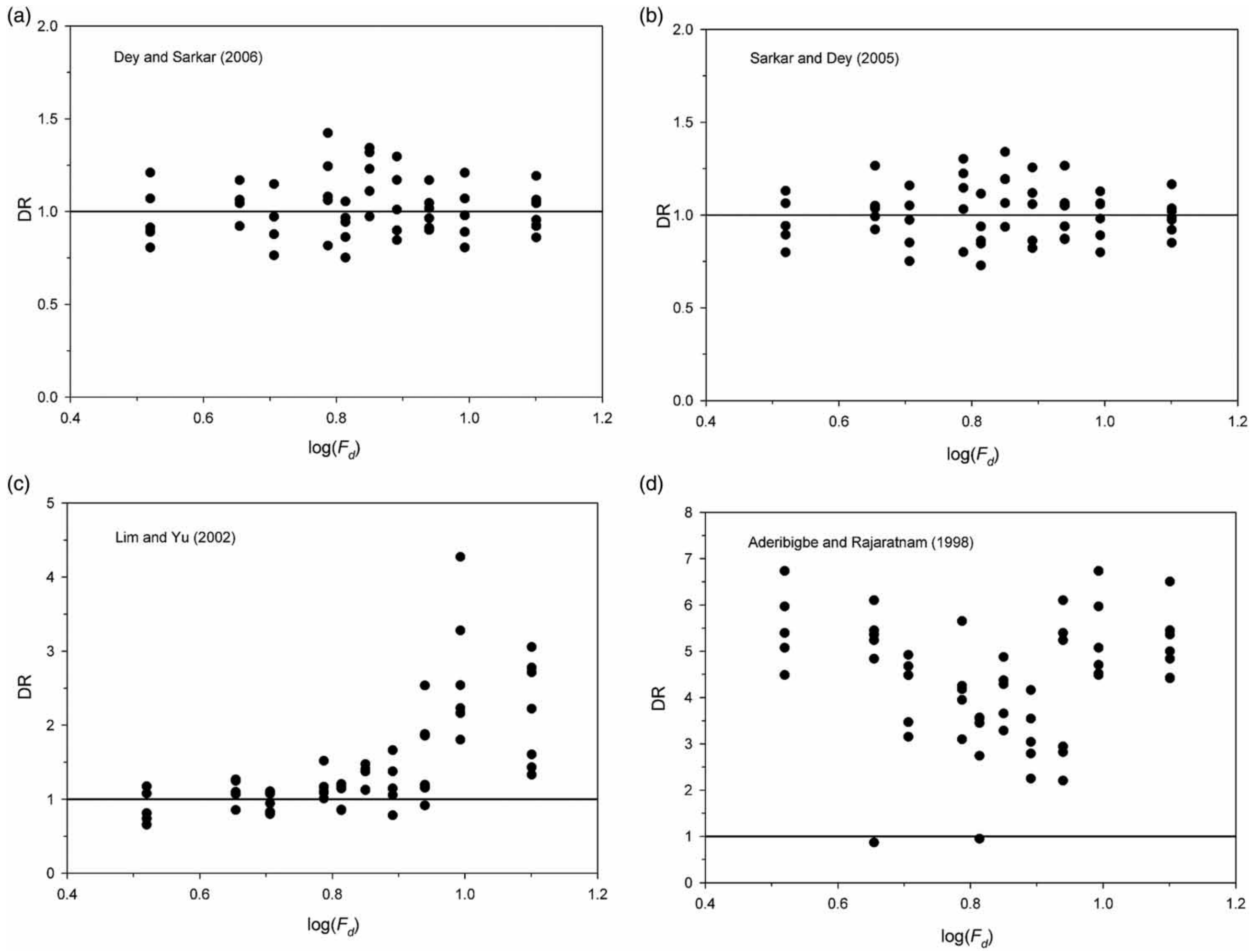

Figure 8 | Results of DR analysis for various equations. *(Continued.)*

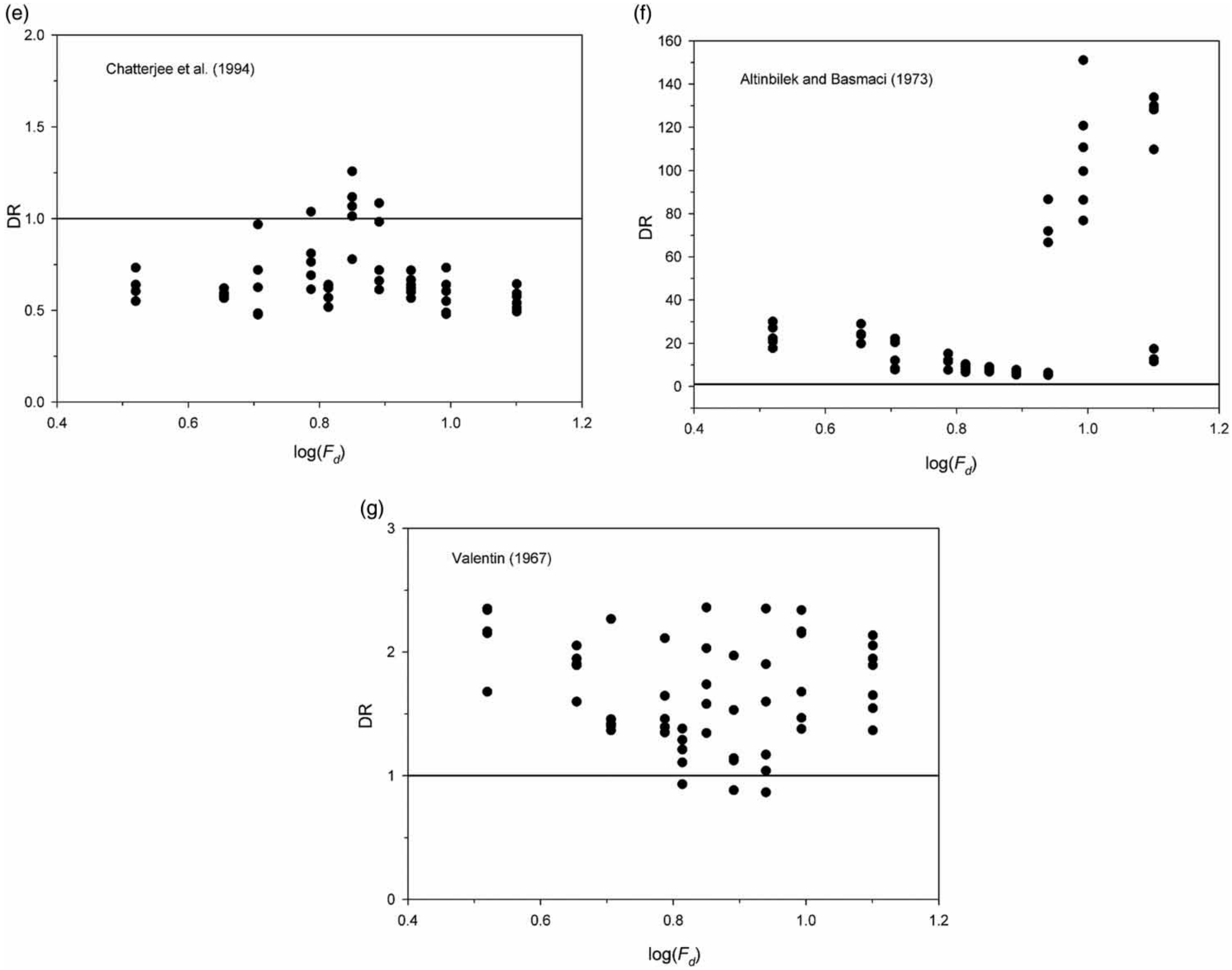

Figure 8 | Continued.

$6.13 < F_d < 7.78$. Altinbilek & Basmaci's (1973) equation (Figure 8(f)) always overpredicts the maximum scour depth for all values of F_d. Valentin's (1967) equation (Figure 8(g)) also gives generally overpredicted values, except for a few values between $6.51 < F_d < 8.69$. The mean, minimum, and maximum values of DR for all the existing equations and the ANN and ANFIS models are given in Table 9.

Figure 9 presents a percentage error graph showing a comparison of the percentage errors against the percentage of data analyzed, for the ANN and ANFIS models, and Dey & Sarkar (2006) and Valentin (1967) equations, since these equations are better than others. It is found from Figure 9

Table 9 | Values of DR for existing equations, and ANN and ANFIS models

	DR values		
Investigator	**Mean**	**Minimum**	**Maximum**
Present study (ANN model)	0.98	0.85	1.16
Present study (ANFIS model)	1.00	0.86	1.22
Dey & Sarkar (2006)	1.02	0.75	1.42
Sarkar & Dey (2005)	1.01	0.73	1.34
Lim & Yu (2002)	1.47	0.66	4.27
Aderibigbe & Rajaratnam (1998)	4.37	0.87	6.73
Chatterjee *et al.* (1994)	0.68	0.48	1.26
Altinbilek & Basmaci (1973)	35.50	5.25	151.04
Valentin (1967)	1.67	0.86	2.36

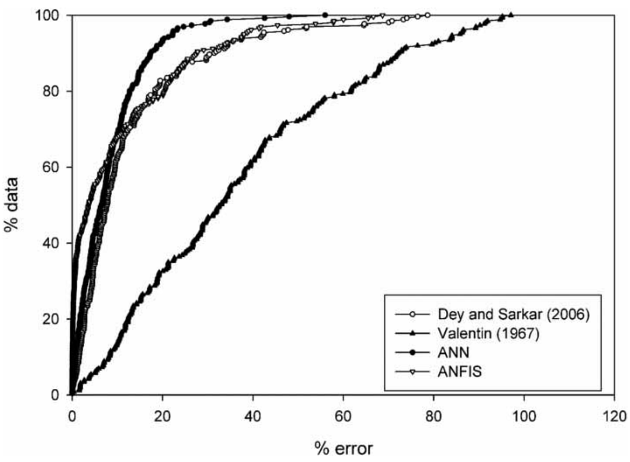

Figure 9 | Percentage error graph.

that the percentage error of the ANN and ANFIS models is slightly less than the Dey & Sarkar's (2006) equation, while the equation proposed by Valentin (1967) gives higher values of percentage error.

CONCLUSIONS

An attempt was made to evaluate the existing equations to predict scour depth under wall jets by applying these equations to a large set of published experimental data. Most of these empirical or regression equations do not perform well when tested against the available data. ANN- and ANFIS-based models were proposed as simple and accurate tools for the prediction of the maximum scour depth. The following conclusions are drawn from this study:

1. The maximum scour depth was found to be positively correlated to F, d_t/a and F_d, whereas it was negatively correlated to L/a, D_{50}/a, and σ_g. Results of the F-test indicated that F_d was the most significant parameter which affects the predicted maximum scour depth, followed by L/a, d_t/a, D_{50}/a, and F. σ_g was relatively insignificant.
2. Accuracy of the existing predictive equations was checked with the available data. Most of the existing equations did not perform well when tested with the available data. Statistical performance indices (R, RMSE, MAPE, BIAS, and SI) were found to be poor for the existing equations. The equation proposed by Dey & Sarkar (2006) performed better than others when judged against the available laboratory data.
3. The results of sensitivity analysis indicated that the parameter F_d ($R = 0.54$, RMSE $= 1.17$, and MAPE $= 0.83$) was the most effective parameter on the scour depth and D_{50}/a ($R = 0.89$, RMSE $= 0.37$, and MAPE $= 0.48$) was the least effective. The other effective parameters were L/a and d_t/a, ranked from high to low values, respectively.
4. The proposed ANN and ANFIS models were advantageous compared to the existing predictive equations. The value of the coefficient of correlation (R) for the ANN model was 0.95 for the training dataset and 0.96 for the testing dataset. Other performance indices for the ANN model were obtained as RMSE $= 0.05$ and MAPE $= 0.168$. For the ANFIS model, the value of R was 0.94 for the training dataset and 0.92 for the testing dataset, with RMSE $= 0.085$ and MAPE $= 0.197$.

This study successfully evaluates the performance of ANN and ANFIS models, whereas newer methods of soft-computing such as GMDH and GEP may be applied and compared as future work. Experimental data for attached wall jets only were used, whereas published data for impinging and circular jets may also be considered in future studies. The proposed ANN and ANFIS models are recommended for practice in the design of apron and other hydraulic structures, since they are developed using a wide range of data, providing a larger range of applicability.

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CONFLICT OF INTEREST

There is no conflict of interest at present for this manuscript.

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