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Spatial runoff updating based on the hydrologic system differential response for flood forecasting

Xiaoqin Zhang, Weimin Bao and Fei Yuan

ABSTRACT

Runoff controls water volume in the rainfall–runoff process and plays a dominant role in flood forecasting. This study proposes a spatial runoff updating approach based on the hydrologic system differential response (HSDR). The first-order partial derivative is employed to express the hydrologic response to runoff change. A stepwise approximation for the HSDR is suggested to reduce the effect of linearization of the nonlinear hydrologic system. The regularized least square algorithm is used to calculate the estimated errors of runoff. The HSDRs for spatial distributed runoff (SDR) updating and areal mean runoff (AMR) updating are examined to correct the predictions of the Xinanjiang model in two basins in China. The case results show that the HSDR for runoff updating can improve flood predictions; the HSDR with stepwise approximation outperforms that without it; the HSDR_SDR performs better than the HSDR_AMR; and with increasing lead time, the HSDR method exhibits more stable performance than the autoregressive (AR) technique on streamflow correction. The proposed HSDR_SDR method can decompose the information of residuals between observations and calculations to update spatial runoff through the response matrix for each sub-basin. With simple structure and stable performance, the HSDR_SDR is convenient and effective for real-time flood forecasting.

Key words | error updating, flood forecasting, spatial runoff, system differential response, Xinanjiang model

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HIGHLIGHTS

- Propose an error updating method based on the hydrologic system differential response to correct spatial distributed runoff.
- Use a stepwise approximation for the proposed method to reduce the effect of the linearization of the nonlinear hydrologic system.
- Results show that the proposed method exhibits more stable performance than the autoregressive technique with increasing lead time in flood forecasting.

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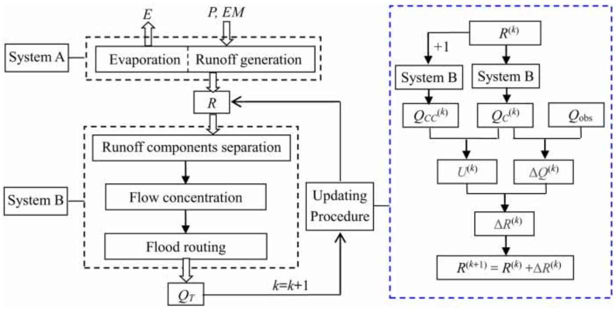
GRAPHICAL ABSTRACT



INTRODUCTION

Hydrological processes are highly nonlinear, time-dependent and spatially varying due to spatial–temporal variability of basin characteristics, precipitation and evaporation (Beven 2002; McDonnell 2003; McDonnell *et al.* 2007; Chang *et al.* 2019). The processes of rainfall–runoff and runoff–riverflow are complex. Reliable hydrological modeling remains an ongoing challenge; there exist inevitably various errors in flood forecasting (e.g. Wagener & Gupta 2005; Pebesma *et al.* 2007; Kavetski & Clark 2011; Montanari & Baldassarre 2013; Pianosi & Wagener 2016; Seibert *et al.* 2018; Brunnera & Sikorska-Senoner 2019). An error updating procedure is essential for a reliable and accurate real-time flood forecasting system (WMO 1992).

Currently, many techniques are available to correct model predictions, such as input variables updating (e.g. Si *et al.* 2015), state variable updating (e.g. Spaaks & Bouten 2013), parameter updating (e.g. Moradkhani *et al.* 2005) and output updating (e.g. Valipour *et al.* 2013). Moreover, from the perspective of information utilization, some effective updating strategies are developed (e.g. Hsu *et al.* 2003; Bao *et al.* 2011; Tiedeman & Green 2013; Zhang & Bao 2013; Li *et al.* 2016). The hydrologic system differential response (HSDR) between errors of model outputs (dependent variables) and related factors (independent variables) is useful for correcting independent variables (Kirchner 2009). The dynamic system response curve method based on the HSDR has been proposed to update areal mean variables in the whole basin, including areal mean runoff (AMR; Bao *et al.* 2014; Zhang *et al.* 2014), areal mean rainfall (Si *et al.* 2015), areal mean state variables (Sun *et al.* 2018a) and areal mean initial conditions (Sun *et al.* 2018b). In previous works, little research has been conducted on the spatial variability of updated variables. This prompts us to examine the performance of the HSDR method on updating spatial distributed variables in flood forecasting.

The calculation of runoff generation controls water volume in rainfall–runoff models; it plays an especially important role in flood forecasting. The spatial–temporal variabilities in basin underlying, precipitation, evaporation and runoff production significantly affect runoff calculated by hydrological models (Singh 1997; Smith *et al.* 2004; Huza *et al.* 2014; Fraga *et al.* 2019; Guse *et al.* 2019; Visser-Quinn *et al.* 2019). The effects of all error sources are compressed into model output; it is difficult to distinguish and identify error sources from the residuals between observations and calculations (Renard *et al.* 2011; Li *et al.* 2017; Zhang *et al.* 2019). Extracting the spatial–temporal properties of runoff errors from discharge residuals is important for flow updating. How to consider the spatial–temporal variability in runoff updating by using the HSDR method is an interesting problem that needs to be investigated.

The main objective of the present study is to extend the HSDR method on spatial error updating in flood forecasting. The HSDR with stepwise approximation is proposed to gradually update spatial distributed runoff (SDR) and AMR, referred to as HSDR_SDR and HSDR_AMR, respectively. Furthermore, the performance of the HSDR and the autoregressive (AR) technique with increasing lead time is compared. This study can provide an investigation of the HSDR on spatial runoff updating; it could help improve real-time flood forecasting and extend our understanding of error feedback in the hydrological process.

METHODS

Xinanjiang model

The Xinanjiang (XAJ) model developed by Zhao (1992) is a simple conceptual rainfall–runoff model based on the concept of runoff formation upon the repletion of storage. It has been extensively applied for flood forecasting in humid and semi-humid regions in China (e.g. Jayawardena & Zhou 2000; Li *et al.* 2009; Liu *et al.* 2009; Lü *et al.* 2013; Yao *et al.* 2014). The model structure consists of evaporation, runoff production, runoff component separation, flow concentration and flood routing. The evaporation is represented by a three-layer model. The runoff production occurs on the repletion of storage. The total runoff is separated into surface runoff, subsurface runoff and groundwater components. The flow concentration for each runoff component to the outflow of each sub-basin is routed by using the linear reservoir technique. The outflow is finally routed to the basin outlet by using the Muskingum method.

The flowchart of the XAJ model is given in Figure 1. All symbols inside the box are variables including inputs, outputs and state variables. The model inputs are rainfall P and pan evaporation EM; and the outputs are actual evaporation E and discharge Q_T at the basin outlet. The variables outside the box are parameters, including evapotranspiration parameters (K, WUM, WLM, C), runoff production parameters (WM, B, IM), runoff separation parameters (SM, EX, KI, KG), runoff concentration parameters (CS, CI, CG) and Muskingum parameters (KE, XE). More details of the symbols can be found in the literature (Zhao 1992).

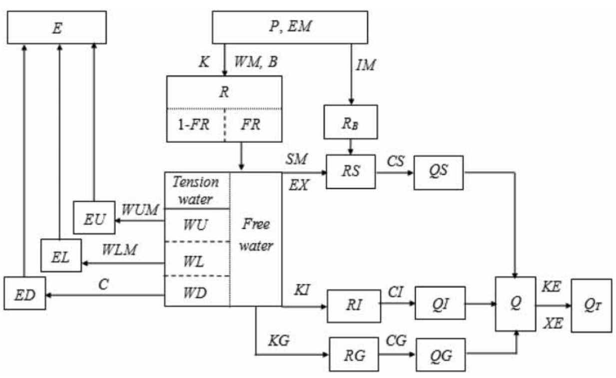


Figure 1 | Flowchart of the Xinanjiang (XAJ) model (Zhao 1992).

In the application of the XAJ model, the study basin is commonly divided into a set of sub-basins to represent spatial characteristics. The basic inputs are the areal mean rainfall and pan evaporation in each sub-basin. The areal mean rainfall of each sub-basin is represented by the point measurements obtained from the rain gauges in the corresponding sub-basin. The rainfall–runoff modeling is conducted in each sub-basin. The outflow of each sub-basin is routed down to the outlet of the whole basin.

HSDR method

A hydrological model can be considered as a composition of a set of equations with various variables and parameters. It can be simplified as a mathematical function:

$$Q = f(X, t) \tag{1}$$

where Q is the time-order-dependent variable (model output discharge); X is the vector of independent variables (inputs, state variables and parameters) and t is the time.

The first-order partial derivative of $Q = f(X, t)$ can be expressed as:

$$dQ = \left.\frac{\partial f}{\partial X}\right|_{X=Xc} dX \tag{2}$$

where X_C is the vector of current best estimated variables and $\frac{\partial f}{\partial X}$ is the differential response of variable X.

When the change of independent variable is small enough, the matrix form of Equation (2) can be written as:

$$\Delta Q = U \Delta X + \xi \tag{3}$$

where U is the Jacobian matrix ($m \times n$) representing the system differential response; m is the number of output observations used for updating and n is the length of the variable that needs to be updated ($m \geq n$); ΔQ is the vector of the residuals between observations and calculations; ΔX is the vector of estimated variable errors and ξ is the vector of residuals, obeying white noise and zero mean value.

Using the regularized least square algorithm, we can obtain:

$$\Delta X = (U^T U + \beta I)^{-1} U^T \Delta Q \tag{4}$$

where β is the regularization coefficient.

The updated variable X' can be expressed as:

$$X' = X_C + \Delta X \tag{5}$$

For better inversion, a successive approximation approach for the HSDR is employed. The estimate of X with the stepwise approximation can be rewritten as:

$$X^{(k+1)} = X^{(k)} + \Delta X^{(k)} \tag{6}$$

where $X^{(k)}$ is the estimated variable in the k iteration; $\Delta X^{(k)}$ is the error estimate of X in the k iteration and the superscript k represents the number of iterations.

The schematic of the HSDR for runoff updating in the XAJ model is displayed in Figure 2. Every process in the XAJ model can be generalized as a system, and several related processes also can be regarded as a bigger system. The procedure of evaporation and runoff production is referred as the System A, and that including runoff component separation, flow concentration and flood routing is referred as the System B. In flood forecasting, we can use real-time residuals between calculations and observations to update variables through the basic mathematical foundation of the hydrologic system. The streamflow prediction can be improved by re-running the hydrologic model with updated variables. We adopt a stepwise approximation for the HSDR to reduce the effect caused by the linearization of the nonlinear system, which is different from the previous studies without iteration (Bao *et al.* 2014; Zhang *et al.* 2014; Si *et al.* 2015; Sun *et al.* 2018a, 2018b). We employ the root mean square error *RMSE* as the objective function for iteration termination. The error updating is operated when the newly corrected performance is better than the original; it can assure that there is no deterioration in the model performance when using the HSDR method.

Spatial runoff updating using the HSDR

According to the hydrological response of outflow discharge variations to runoff, Equation (1) can be written as:

$$Q_T = f(R_i, t) \quad i = 1 \text{ to } b \tag{7}$$

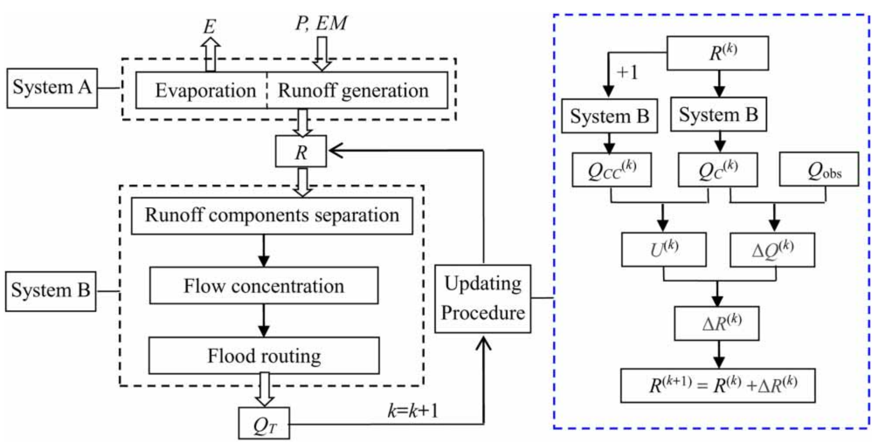


Figure 2 | Schematic of the HSDR for runoff updating in the XAJ model. (Note: P is the precipitation; EM is the measured pan evaporation; E is the actual evaporation; R is the runoff; Q_T is the discharge at the basin outlet; k is the iteration number; Q_{obs} is the observed discharge; $Q_C^{(k)}$ is the discharge calculated using $R^{(k)}$; $Q_{CC}^{(k)}$ is the discharge calculated using $R^{(k)}$ plus one unit; $U^{(k)}$ represents the system response matrix in the k iteration; $\Delta Q^{(k)}$ represents the residuals between observations and calculations; $\Delta R^{(k)}$ represents the estimate of runoff error in the k iteration). Please refer to the online version of this paper to see this figure in colour: http://dx.doi.org/10.2166/hydro.2020.045.

where Q_T is the discharge at the basin outlet; the subscript 'i'' represents the sub-basin code; b is the number of sub-basins; $R_i = [r_{i,1}, \cdots, r_{i,j}, \cdots, r_{i,n}]^T$ is the AMR in the 'i' sub-basin and n is the length of runoff needed to be updated.

Equation (7) determines how basin outlet discharge responds to spatial runoff. It also can be used to understand how error in runoff accumulates as it propagates through the System B. Figure 3 displays two strategies for runoff updating by using the HSDR, namely the AMR updating in the whole basin (referred as HSDR_AMR) shown in Figure 3(a) and the SDR updating that gradually updates the AMR in each sub-basin (referred as HSDR_SDR) shown in Figure 3(b). Figure 3 is the key part of the HSDR method shown in the right blue dash line box in Figure 2. The major differences between the HSDR_SDR and HSDR_AMR are the consideration of differential response matrix U for each sub-basin or the whole basin (blue dash line boxes in Figure 3) and the assignment of the estimated incremental runoff for each sub-basin (black dash line boxes in Figure 3).

As the HSDR_AMR shown in Figure 3(a), the differential response matrix U for the whole basin is calculated from the change of outlet discharge to the perturbations of AMR in the whole basin. The procedure of the HSDR_AMR mainly includes:

Step 1: Calculate runoff R_i ($i = 1$ to b) at the 'j' time in each sub-basin via the System A;

Step 2: Generate a perturbed runoff series $R_i^{'}$ by increasing the R_i ($i = 1$ to b) at the 'j' time by one unit in each sub-basin, while the runoff at other time remains unchanged;

Step 3: Calculate the difference between the flow series via the System B using the R_i and $R_i^{'}$;

Step 4: Repeat Steps 1–3 at different times ($j = 1$ to n) to obtain all the columns of matrix U. Thus, using Equation (4), we can compute the ΔR for the whole basin. According to the area proportion of each sub-basin to the whole basin, the estimate of runoff error ΔR_i in the 'i' sub-basin can be calculated by using Equation (8).

$$\Delta R_i = \Delta R \cdot \eta_i \tag{8}$$

where $\eta = [\eta_1, \cdots, \eta_i, \cdots, \eta_b]$ represents the vector of sub-basin area weight that is the area proportion of each sub-basin to the whole basin.

As the HSDR_SDR shown in Figure 3(b), the differential response matrix U for each sub-basin is calculated according to the response of outlet discharge to runoff change in the corresponding sub-basin. The main procedure of the HSDR_SDR is shown as follows:

Step 1: Calculate runoff R_i ($i = 1$ to b) at the 'j' time in each sub-basin via the System A;

Step 2: Generate a perturbed runoff series $R_i^{'}$ by increasing the runoff in the 'i' sub-basin at the 'j' time by one unit while keeping the runoff at other times in the 'i' sub-basin and all runoff in other sub-basins unchanged;

Step 3: Compute the difference between the flow series via the System B using the R_i and $R_i^{'}$;

Step 4: Repeat Steps 1–3 at different times ($j = 1$ to n), all the columns of matrix U for the 'i' sub-basin are obtained.

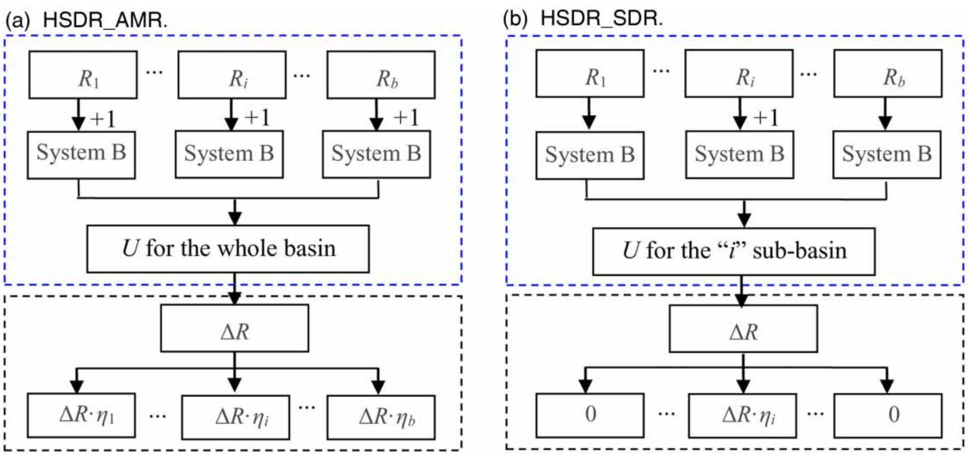


Figure 3 | Strategies for runoff updating with the HSDR. (a) HSDR_AMR. (b) HSDR_SDR. Please refer to the online version of this paper to see this figure in colour: http://dx.doi.org/10.2166/hydro.2020.045.

By using the U for the 'i' sub-basin, we can estimate ΔR in the whole basin. The estimate of ΔR_i for the 'i' sub-basin is assigned according to the area proportion of the 'i' sub-basin to the whole basin.

Step 5: Repeat Step 4 for each sub-basin ($i = 1$ to b), we can obtain the estimated error of AMR in each sub-basin.

In previous works, the HSDR_AMR without stepwise approximation was used (Bao *et al.* 2014; Si *et al.* 2015). In the HSDR_SDR, the information of residuals between observations and calculations can be decomposed by using the differential response due to SDR in each sub-basin. Compared with the HSDR_AMR, the HSDR_SDR can account for spatial variability of runoff, especially in basins with various spatial characteristics.

CASE STUDY

Study area and data

The proposed method is evaluated in the Baishuikeng (BSK) reservoir basin and the Shaowu (SW) basin in China (Figure 4). The characteristics of the two basins are summarized in Table 1. The BSK is a reservoir basin. The 'observed'

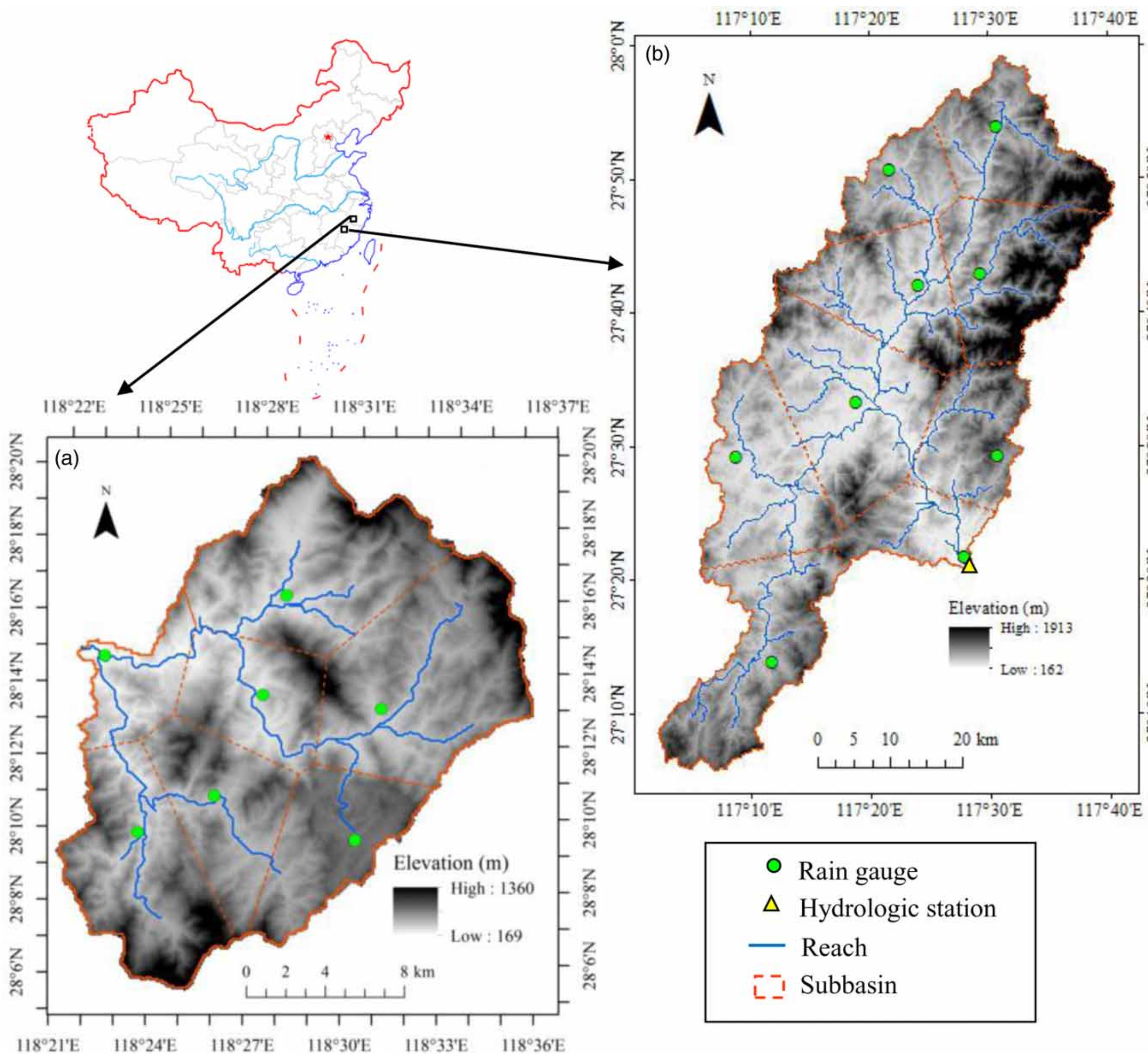


Figure 4 | Location of the study areas. (a) Baishuikeng (BSK) basin; (b) Shaowu (SW) basin.

Table 1 | Basin information

Basin code	Basin area (km^2)	Raingauge number	Raingauge area (km^2/gauge)	Data period (year)	Flood events number
Baishuikeng (BSK)	330	7	47.14	2004–2010	27
Shaowu (SW)	2745	9	305	1988–2000	34

reservoir inflows in the BSK basin are calculated according to the reservoir water level and outflow discharge; the 'observed' hydrographs are serrated displaying fluctuation. Most hydrographs at the outlet of the SW basin have multi-peaks. According to the collected history of floods, the average duration time from the beginning of the flood to the occurrence of flood peak is 34.6 h in the BSK basin and 61.1 h in the SW basin, which can affect the length of observations used for updating with different lead times.

The BSK and SW basins are divided into seven sub-basins and nine sub-basins, respectively. The rainfall, pan evaporation and discharge data are provided by the local hydrological bureau. The XAJ model parameters are calibrated by using historical daily data and hourly data during flood events. Twenty of 27 flood events in the BSK basin and 25 of 34 flood events in the SW basin are used for parameter calibration and the remainder of the flood events for validation. The data for all flood events are at an hourly time step. The calibrated model parameters are listed in Table 2.

Study design

The performance of the HSDR on runoff updating is examined to correct the flood predictions by the XAJ model in two basins with different rain gauge density and different flood characteristics. The comparisons of the HSDR updating with and without stepwise approximation are presented. The performances of the HSDR_AMR and HSDR_SDR are compared. Moreover, the performance of the HSDR and the second-order AR technique with different lead times is demonstrated. The AR technique based on

Table 2 | Calibrated parameter values of the XAJ model

Parameter	Description	Unit	BSK	SW
K	Ratio of potential evapotranspiration to pan evaporation	/	0.78	0.98
WUM	Tension water capacity of upper layer	mm	20	20
WLM	Tension water capacity of lower layer	mm	80	80
C	Coefficient of deep evapotranspiration	/	0.14	0.16
WM	Area mean tension water capacity	mm	150	150
B	Exponent of the tension water capacity curve	/	0.3	0.25
IM	Ratio of impervious area	/	0.01	0.01
SM	Area mean free water capacity	mm	11	17
EX	Exponent of the free water capacity curve	/	1.5	1.5
KI	Outflow coefficient of interflow	/	0.45	0.32
KG	Outflow coefficient of groundwater	/	0.25	0.38
CS	Recession constant of surface storage	/	0.6	0.75
CI	Recession constant of interflow storage	/	0.85	0.889
CG	Recession constant of groundwater storage	/	0.995	0.995
KE	Storage time constant of the Muskingum method	h	1	1
XE	Dimensionless weighting factor of the Muskingum method	/	0.3	0.3

residuals between observations and calculations is a traditional and widely used correction method. The lead time in this study represents the time ahead of flood peak timing. We set different lead times by using different numbers of observations for updating. Shorter lead time means more observations used for updating. Note that the parameter values remain unchanged during both the prediction and updating processes.

The absolute relative peak error (*ARPE*), *RMSE* and Nash–Sutcliffe efficiency (*NSE*) (Nash & Sutcliffe 1970) are employed to evaluate the model performance. They are calculated as follows:

$$\text{Absolute relative peak error}\,(ARPE) = \left|\frac{QC_p - Q_p}{Q_p}\right| \times 100\% \tag{9}$$

$$\text{Root mean square error}\,(RMSE) = \sqrt{\frac{\sum_{t=1}^{N}[Q(t) - QC(t)]^2}{N}} \tag{10}$$

$$\text{Nash–Sutcliffe efficiency}\,(NSE) = 1 - \frac{\sum_{t=1}^{N}[QC(t) - Q(t)]^2}{\sum_{t=1}^{N}[Q(t) - \overline{Q}]^2} \tag{11}$$

where Q_p and QC_p represent the observed and calculated peak discharges, respectively; $Q(t)$ and $QC(t)$ represent the observed and calculated discharges at time t, respectively; N is the total number of the observations for one flood event and $\overline{Q}$ is the mean value of the observations for one flood event.

RESULTS AND DISCUSSION

Results

The results by using the XAJ model without correction are shown in Table 3. The mean results for the HSDR with and without stepwise approximation are shown in Figure 5 for the BSK basin and Figure 8 for the SW basin. The mean evaluation measures by the HSDR with stepwise approximation are labeled as 'AMR' and 'SDR', respectively, and those without it are represented as 'AMR′' and 'SDR′', respectively. The mean results for the HSDR_SDR,

Table 3 | Mean evaluation measures by using the XAJ model

	BSK basin			SW basin		
Period	*ARPE* (%)	*RMSE* (m^3/s)	*NSE*	*ARPE* (%)	*RMSE* (m^3/s)	*NSE*
Calibration	7.82	30.5	0.846	9.46	154.3	0.837
Validation	8.06	34.8	0.862	4.82	116.1	0.876

HSDR_AMR and AR with different lead times are compared in Figure 6 for the BSK basin and Figure 9 for the SW basin. The 'XAJ' represents the original results calculated by the XAJ model without correction. The '*IN*' represents the number of average iterations. Some hydrographs with different lead times are displayed in Figures 7 and 10. The 'Q_{obs}' represents the observed discharge at the basin outlet.

As expected, the results for the HSDR_SDR and HSDR_AMR with stepwise approximation are improved compared with those without it. Both in the BSK and SW basins shown in Figures 5 and Figure 8, the *RMSE* and *ARPE* values are reduced and the *NSE* values are increased. It indicates that the HSDR with iteration can reduce the effect of the linearization of nonlinear system in error updating.

From Figures 6 and 9, in comparison with the HSDR_AMR, the values of *RMSE* for the HSDR_SDR are all smaller and the values of *NSE* for the HSDR_SDR are all greater. It shows that the HSDR_SDR obviously performs better than the HSDR_AMR with an increasing lead time. The iterations for both the HSDR_AMR and HSDR_SDR are all fewer than 10. Moreover, when increasing lead time, the performance of the AR technique is considerably decreased as compared with the HSDR method. In the BSK basin, the corrected results by the AR are even poorer than the original XAJ results when the lead time is larger than 6 h. When the lead time is greater than 1 h in the BSK basin and 3 h in the SW basin, the HSDR_SDR performs better than the AR.

As Figures 7 and 10 exhibited, the hydrographs for the HSDR_SDR and AR agree well with the observations when lead time is 1 h. When increasing lead time, the simulations by the HSDR_SDR are generally better matched to the observations and especially better capture flood peaks. It is found that there exist large errors around flood peak

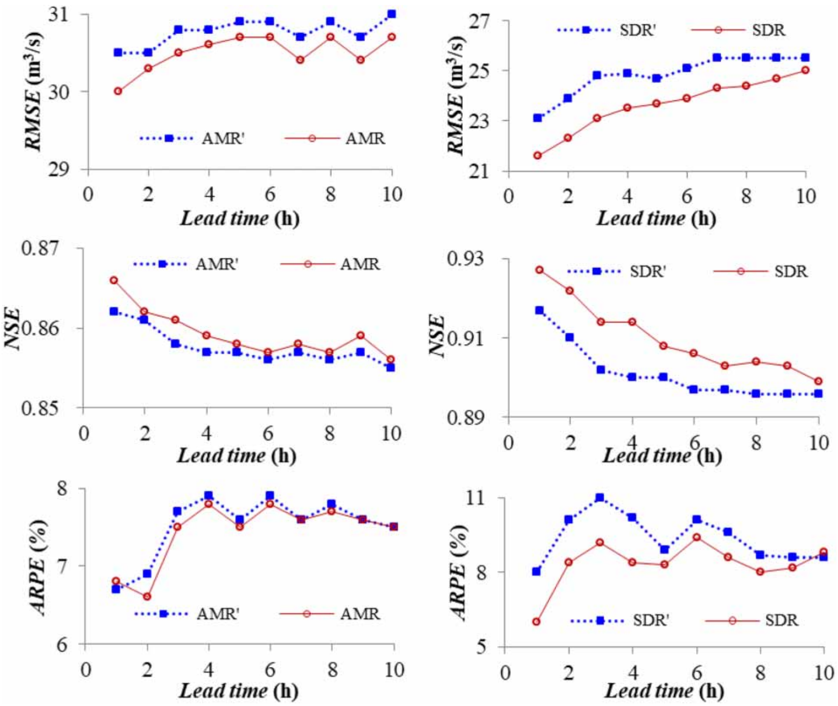


Figure 5 | Mean evaluation measures with and without stepwise approximation in the BSK basin.

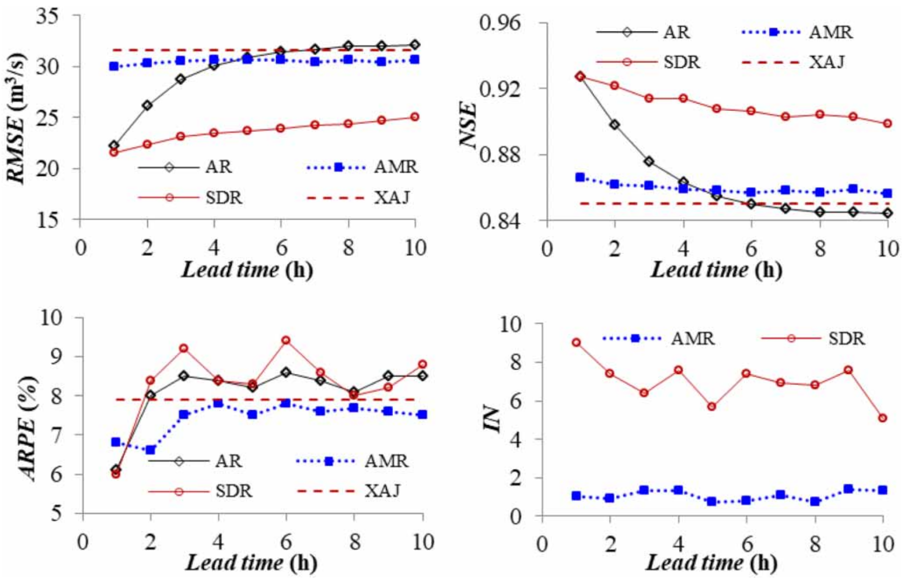


Figure 6 | Mean evaluation measures with different lead times in the BSK basin.

for some floods by using the AR, such as Figure 10 with the lead time of 6 h, which may be due to poor correlation between the discharge residuals around flood peak with increasing lead time.

For all the flood events in the study basins, the model performances are improved using the proposed method as compared with the original predictions using the XAJ. Overall, both in the large basin SW with sparse-density rain

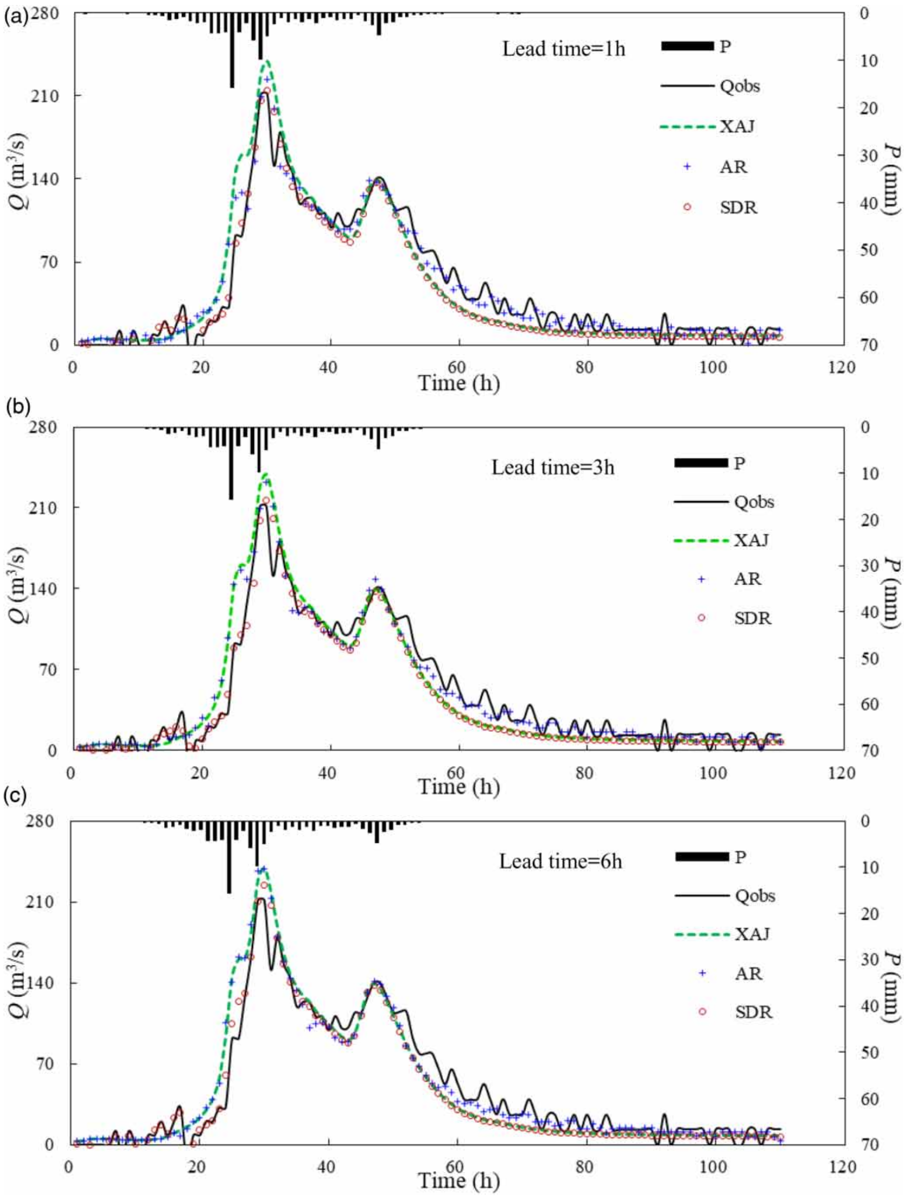


Figure 7 | Hydrographs for the flood event 20071006 with different lead times in the BSK basin.

gauge and small reservoir basin BSK with high-density rain gauge, the HSDR_SDR and HSDR_AMR with iteration outperform those without it; the performances of the HSDR_SDR are generally superior to the HSDR_AMR; and the HSDR for runoff updating exhibits more stable performance with increasing lead time as compared with the AR technique.

DISCUSSION

Flow at a basin outlet is contributed by all related spatial–temporal information. Qualifying the relative contribution of error sources to model outputs is a challenging task. Runoff distribution is spatially and temporally uneven in a basin. The HSDR_SDR can account for the influence of

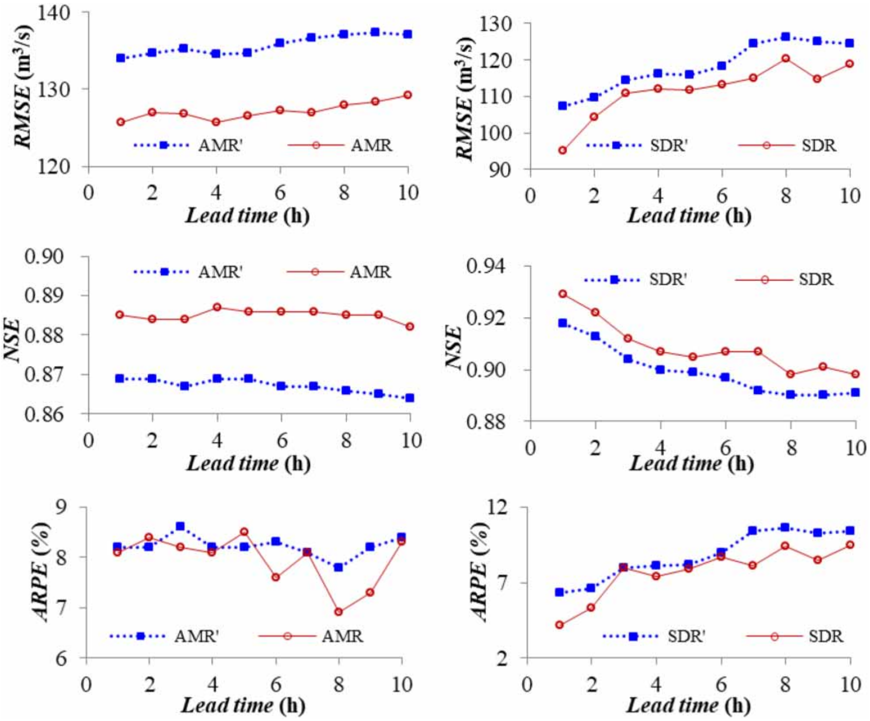


Figure 8 | Mean evaluation measures with and without stepwise approximation in the SW basin.

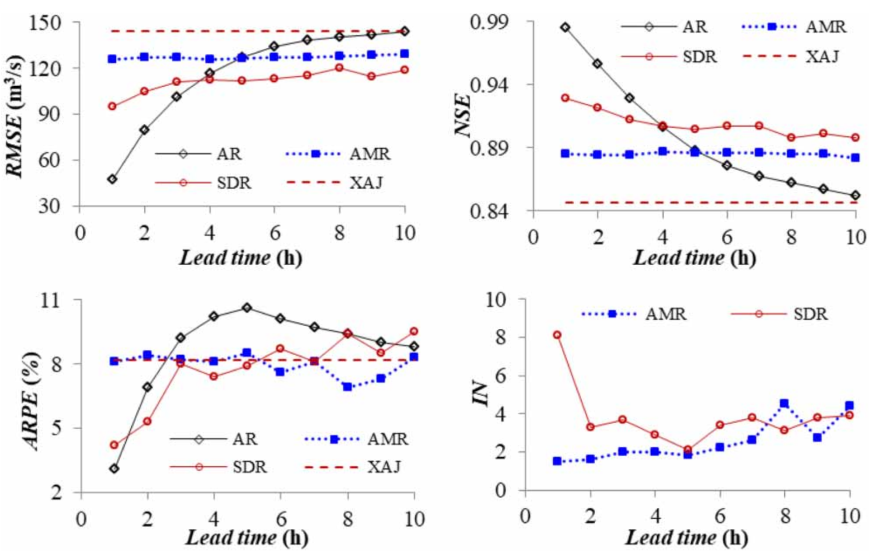


Figure 9 | Mean evaluation measures with different lead times in the SW basin.

spatial variability in runoff updating on the hydrologic response through the differential response matrix for each sub-basin. To some extent, the HSDR_SDR can apportion the total response error to contributing factors for runoff updating in each sub-basin. It is simple to operate with no more parameters nor change in the hydrological model. Theoretically and practically, compared with the HSDR_AMR that uses the differential response matrix for

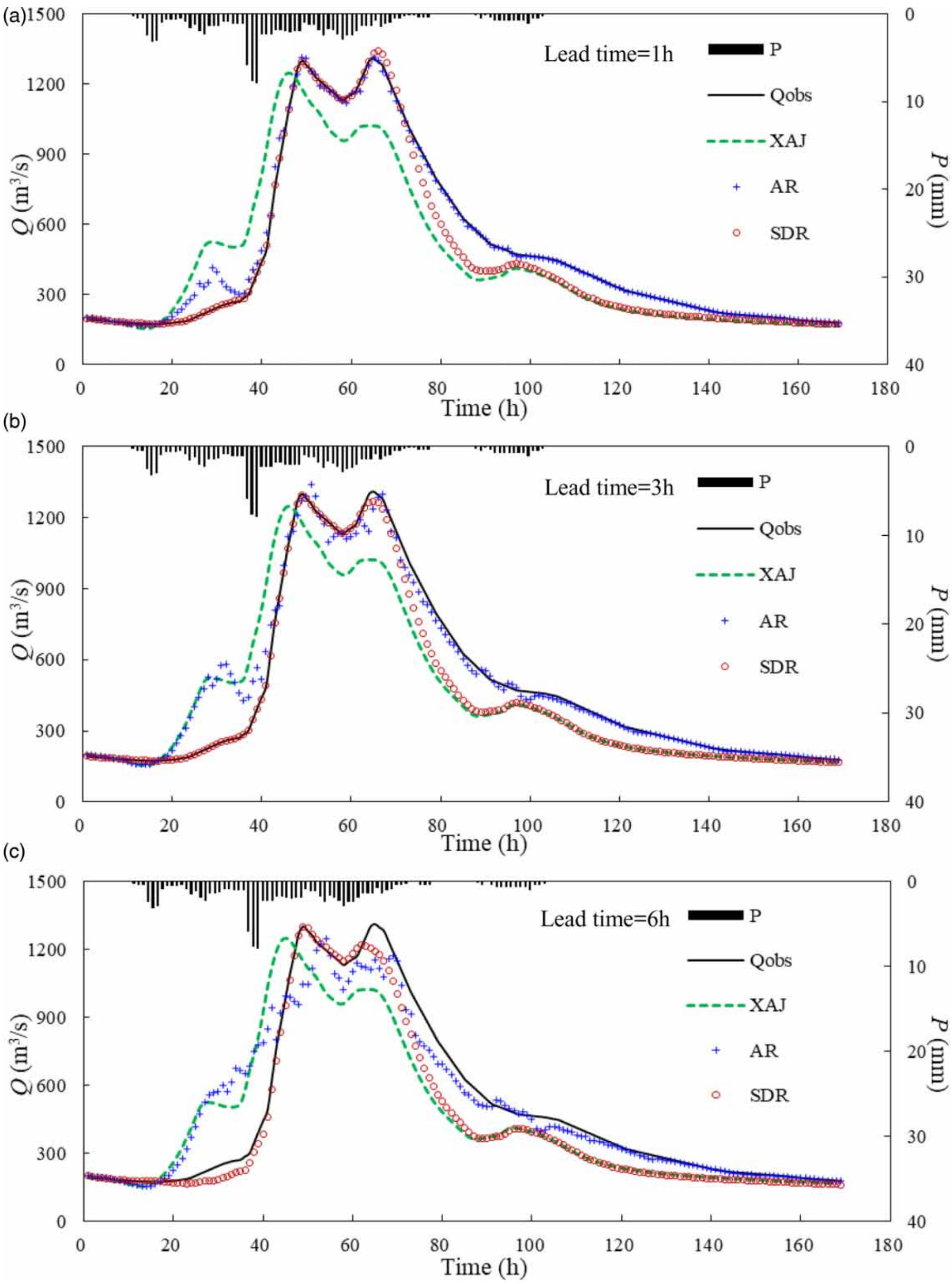


Figure 10 | Hydrographs for the flood event 19890526 with different lead times in the SW basin.

the whole basin, the HSDR_SDR can better improve flood predictions due to the consideration of spatial variability in runoff updating on the hydrological response. Operational flood forecasting requires accurate forecasts with a suitable lead time for the issuance of a flood warning. We suggest the use of the HSDR_SDR with stepwise approximation for long lead time predictions.

The first-order partial derivative is adopted to reflect the hydrological response to the change of variables in a nonlinear hydrologic system, and a stepwise approximation approach is adopted to reduce the error caused by the linearization of the nonlinear system in model solution. The case studies demonstrate that the proposed method based on the HSDR can produce satisfactory discharge correction by updating runoff. Even in a reservoir basin such as the BSK, the HSDR_SDR can obtain satisfactory correction for reservoir inflow. The HSDR performance decreases slowly when increasing lead time; it indicates that the HSDR for runoff updating is effective with strong convergence and stable performance. The more observed data used for updating the better the HSDR performs. Moreover, the HSDR approach is more robust than the AR technique on streamflow correction with increasing lead time.

The residuals between observed and calculated discharges at the basin outlet are comprehensively caused by various errors such as model structure, parameters and states. We use the HSDR to correct flood predictions by adjusting runoff based on the information of the outlet discharge residuals. Unfortunately, there exist various errors in hydrological modeling that we cannot quantify. It may be improper that all errors of discharge at the basin outlet is primarily attributed to runoff, which is the possible reason for why there is little improvement in the model performance for several of the flood events in the study basins. The performance of the proposed approach depends on whether the corrected variables are the main error sources of the discrepancy between observed and calculated streamflow. From the *RMSE* and *NSE* in Figure 9, the AR performs better for the 1-, 2-, and 3-h predictions in the SW basin. It is possible due to the fact that the AR technique takes all errors into consideration from a black-box perspective. Runoff controls the water volume in the rainfall–runoff process; it is also the dominant error source in operational forecasting. Future research should identify and examine the principle sources of error.

In the study case, the *RMSE* is set as the objective function for iteration termination. We find that the *RMSE* and *NSE* for the corrected results are all improved by using the HSDR, but the *ARPE* for several of the flood events are not reduced. The multi-objective functions that accout for both flood peak and *RMSE* will be investigated in the application of the proposed method (e.g. Engeland *et al.* 2006; Fowler *et al.* 2018; Her & Seong 2018).

Generally, the proposed method for spatial runoff updating has the capacity to improve real-time flood forecasting without increasing model complexity and model parameters, which make it easy to operate in real-time forecasting. It is worth pointing out that the HSDR method is general rather than a specific one and can be extended for other variables updating (such as rainfall, soil moisture, parameter and state) in other models to produce reasonable flood forecasting results.

CONCLUSIONS

The HSDR can represent the relation between model outputs and influencing variables. We present an enhanced HSDR approach with stepwise approximation for SDR updating in flood forecasting. The study area is divided into a number of sub-basins in the application of the XAJ model. The HSDR method is used to gradually update area mean runoff in each sub-basin (SDR) and AMR in the whole basin, respectively. The results are as expected. The HSDR_SDR outperforms the HSDR_AMR considering the spatial variability of runoff. Furthermore, compared with the AR technique, the HSDR method exhibits less deterioration on the improvement of model performance when increasing lead time.

It is highlighted that the HSDR for error updating is based on the hydrologic response to the updated variable through the adopted hydrological model; the HSDR for runoff updating is specifically based on the runoff–riverflow process expressed in the hydrological model. Obviously, the HSDR method has good theoretical foundations and strong potential for improving real-time flood forecasting. It can provide an insight of how basin outflow responds to variable changes through the physical mechanism described by hydrological models. The proposed approach based on the HSDR in this study can identify the total response error to the contributing factors; it provides an effective approach for error decomposition and correction. The performance of the HSDR on other spatial variables updating will be explored; it would help evaluate the proposed method and ultimately extend our understanding of error feedback in the hydrologic process. Future work will continue to further examine the proposed method through physically based distributed hydrologic models and quantify the uncertainty of updated variables through uncertainty estimation techniques.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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