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Optimal water allocation integrated with water supply, replenishment, and spill in the in-series reservoir based on an improved decomposition and dynamic programming aggregation method

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ABSTRACT

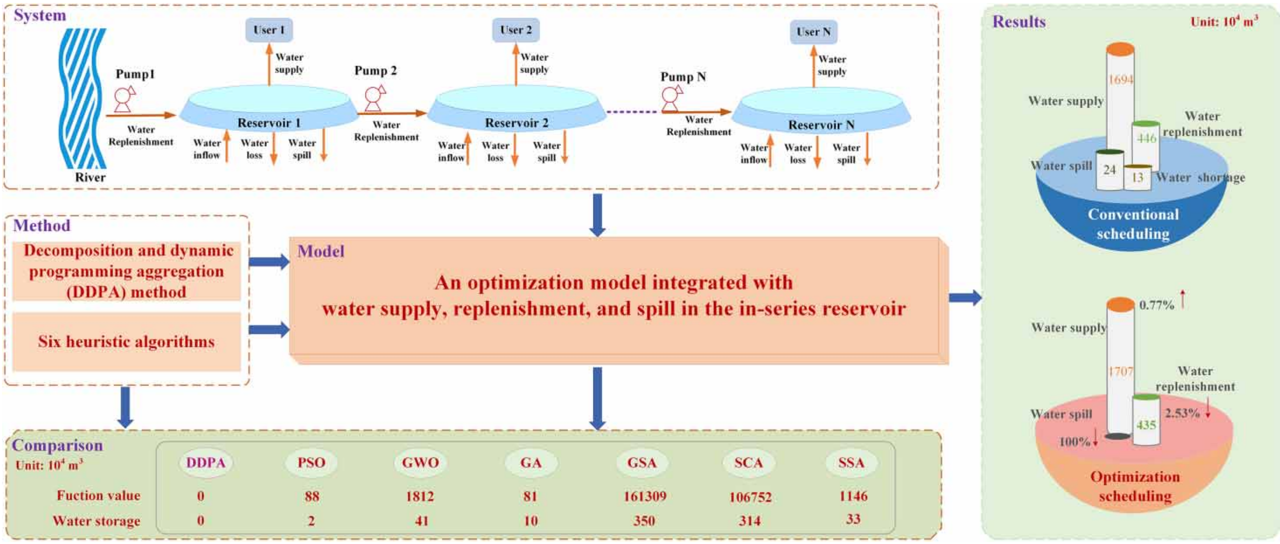
In humid regions with the monsoon climate, seasonal water shortages and water spills occur alternately because of uneven temporal and spatial distributions of water resources. An optimization model for the in-series reservoir (ISR) with replenishment pumping stations was developed to obtain the minimum annual sum of water shortage and systematically considered the reservoir operation rule of water spill and replenishment. This model features multiple dimensions; dynamic programming (DP) may cause a 'curse of dimensions', while the decomposition-coordination method has difficulty in judging logic conditions in the reservoir operation rules. So, an improved decomposition and DP aggregation (DDPA) method was proposed. The proposed model and the method were applied to a real case in the humid region of southern China. Compared to a conventional scheduling method, the water supply was increased by 0.8% and replenishment was reduced by 2.5%. Moreover, a comparison between DDPA and six heuristic algorithms was discussed. All heuristic algorithms' objective function values only obtained local optimal solutions, and the water shortage of the system was 0.12–20.5%. The obtained results demonstrated that DDPA was the better choice for highly complex multi-reservoir systems. The proposed optimization algorithm enriched the optimization theory of multi-dimensional and multi-variable complex systems.

Key words: dynamic programming, heuristic algorithms, in-series reservoir, multi-dimensional, optimal scheduling, water resources allocation

HIGHLIGHTS

- An optimization model integrated with water supply, replenishment, and spill in the in-series reservoir (ISR) was proposed.
- An improved decomposition and dynamic programming aggregation (DDPA) method was proposed.
- The performance of DDPA was superior to the six heuristic algorithms.
- The water utilization efficiencies of the ISR were improved.

GRAPHICAL ABSTRACT



1. INTRODUCTION

The reservoir is an essential infrastructure for allocating water resources, alleviating regional water scarcity, and adjusting the uneven allocation of time and space for water resources. The optimal allocation method of water resources for reservoirs has attracted wide attention, such as in Southern China. The mean annual precipitation is about 1,000 mm in these humid regions, but 60–70% of that occurs during the flood due to the monsoon climate (Quinn *et al.* 2018). Although the total annual inflows into the reservoirs are usually more than what is demanded, many water inflows are spilled during the flood season due to the uneven distribution of time and space, resulting in seasonal water shortages. So, it is required to replenish water by pumping stations from rivers or other reservoirs to achieve multi-reservoir joint operation during the dry season (Ming *et al.* 2017).

The basic principle of joint operation for a multi-reservoir system is to redistribute water resources among the reservoirs through hydraulic connections (Ahmad *et al.* 2014), which can take full advantage of the storage capacity of each reservoir. Thus, the operation purpose of in-series reservoirs (ISRs) is to reduce water spill and improve runoff utilization; the decision variable is the amount of water supply in each period, which is subject to the reservoir's capacities for both water storage and water demand (Chang *et al.* 2019; Rani *et al.* 2020). It has made significant progress in the optimization of reservoir system management and operations (Labadie 2004). However, most of the reservoirs in these studies are in arid or semi-arid regions, so the role of the operation rule of the reservoir is ignored to simplify the models (Celeste & Billib 2009). In humid regions, the rule, which is water replenishment in the shortage season and water spill in the flood season, has a critical role in improving the utilization efficiency of water resources and energy and the safe operation of reservoirs.

Water allocation models of reservoirs are a typical multi-dimensional optimization problem, and the solution methods mainly have mathematics programming and heuristic algorithms. Dynamic programming (DP) based on Bellman's principle (Bellman & Dreyfus 1964) applies to this multi-stage decision-making process. However, the curse of dimensionality may be induced when dealing with multiple reservoirs (Cheng *et al.* 2017). Decomposition (Turgeon 1981) is the mainstream idea for dealing with multi-dimensional optimization problems using DP. The decomposition-coordination method (Mahey *et al.* 2017; Tan *et al.* 2019) is the most commonly used and achieves the whole system's optimal state through successive iterative calculations between the large-scale system and subsystems according to the coordinating variables. The Lagrange multiplier method (Duan *et al.* 2022) is used to deal with the constraints when adopting the decomposition-coordination approach. However, this method is restricted by the differentiability and convexity of the objective function, and it fails in judging logic conditions in the operation rule constraint.

Recently, with the maturity of heuristic search theory and the development of computing power, modern heuristic algorithms with global search ability have gained their advantages, such as the genetic algorithm (GA; Allawi *et al.* 2018), the particle swarm optimization (PSO) algorithm (Azadeh *et al.* 2012), and the Grey Wolf Optimizer (GWO; Moeini & Afshar 2013) have become considerably popular optimization methods. These algorithms feature random sampling and can solve multi-dimensional problems without decomposition. When heuristic algorithms are adopted, a constrained optimization problem usually transforms into an unconstrained one by penalty functions (Pina *et al.* 2017; Wan *et al.* 2018). Similarly, the penalty function method may fail to handle the complicated logic conditions in the reservoir operation rule (Gong *et al.* 2020). What is worse, the algorithm parameters, such as the crossover and mutation rates in GA and the inertia weight and acceleration coefficients in PSO, cannot be determined even though these values are essential factors that can affect the performances of algorithms (Zhang & Dong 2019). There are only empirical value ranges for most algorithm parameters. Although several approaches have been applied for setting parameters, such as the deterministic strategy (Kavoosi *et al.* 2019) and the adaptive strategy (Cui *et al.* 2016; Nunes *et al.* 2018), they are not proven to be of universal significance (Karafotias *et al.* 2015).

To summarize, the model and algorithm for allocating water resources in reservoirs have not been sufficiently addressed. In this regard, this paper investigated the optimal scheduling strategy integrated with water supply, replenishment, and spill in the ISR. An optimization model of the ISR was proposed to minimize each reservoir's annual sum of water shortages. Then, an improved decomposition and DP aggregation (DDPA) method was proposed. The method is to transform the $N+1$ dimensional DP into the $N+1$ iterative calculations of one-dimensional DP. Both the subsystem models and the aggregation model are one-dimensional DP models. This method effectively solved the multi-variable and multi-dimensional reservoir optimization problem. The optimization scheduling scheme increased the utilization efficiency of inflow water and reduced energy consumption. Finally, a comparison between DDPA and six heuristic algorithms, including GA, PSO, GWO, Sine Cosine Algorithm (SCA), Salp Swarm Algorithm (SSA), and Gravitational Search Algorithm (GSA), was discussed from optimality, adaptability, and efficiency. The results demonstrated that the DDPA algorithm was better to choose for this kind of reservoir optimization model. The proposed model and the method enriched the optimization theory of multi-dimensional and multi-variable complex systems.

2. MODEL AND METHOD

2.1. Description of the system

The system of the ISR with pumping stations is a common water supply system in humid regions with a monsoon climate. These usually operate jointly but have their water users. From Figure 1, the system is composed of *N* reservoirs. Each reservoir undertakes the task of water supply for its users, water spill of exceeding water during the flood season, and water replenishment from other reservoirs or a transit river by pumping during the water shortage.

In Figure 1, $X_{i,t}$ (L^3), $LS_{i,t}$ (L^3), $PS_{i,t}$ (L^3), and $EF_{i,t}$ (L^3) are water supply, local inflow, water spill, and water loss of the reservoir *i* during the period *t*, respectively; $YS_{i,t}$ (L^3) is the water demand of user *i* during the period *t*; $Y_{i,t}$ (L^3) is water replenishment for reservoir *i* by the pumping station *i* during the period *t*; *N* is the total number of reservoirs in the system; *i* is the sequence number of the reservoir, $i = 1, 2, \dots, N$; *t* is the sequence number of each period, $t = 1, 2, \dots, T$.

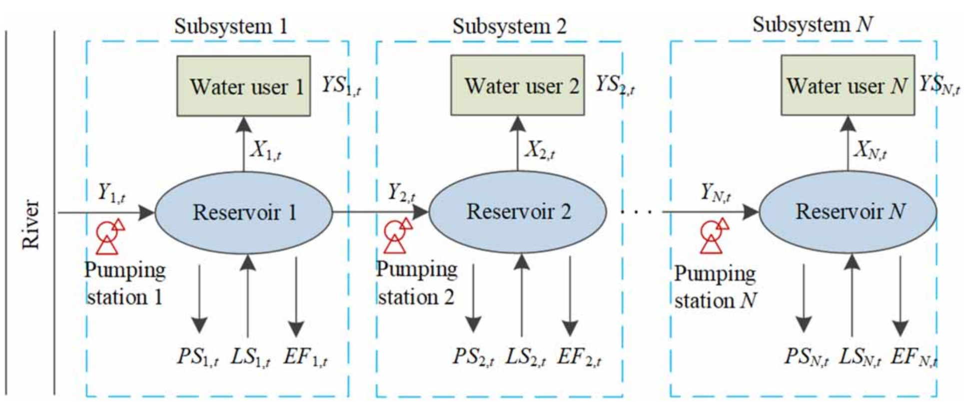


Figure 1 | Schematic of the ISR with the replenishment pumping station.

2.2. Optimization model

2.2.1. Objective function

The objective function of the system is to minimize the annual sum of squared water shortages of each reservoir, which is expressed as Equation (1):

$$\min F = \sum_{i=1}^{N} \sum_{t=1}^{T} (X_{i,t} - YS_{i,t})^{2} \tag{1}$$

where F is the annual sum of squared water shortages of each reservoir.

2.2.2. Constraints

① Annual available water for the system

The annual available water includes the total water provided by reservoirs and the maximum pumping volume from the transit river, which is expressed as Equation (2):

$$\sum_{i=1}^{N} \sum_{t=1}^{T} X_{i,t} \le SK + BZ \tag{2}$$

where SK (L^3) is the total annual available water of reservoirs and BZ (L^3) is the maximum pumping volume from the river.

② Annual available water of the reservoir i

The annual available water of each reservoir is the sum of local inflows and water replenishment by pumping, which is expressed as Equation (3):

$$\sum_{t=1}^{T} X_{i,t} = W_i \quad (i = 1, 2, \ldots, N) \tag{3}$$

where W_i (L^3) is the annual available water of the reservoir i. Substituting Equation (3) into Equations (2) and (4) can be obtained:

$$\sum_{i=1}^{N} W_i \le SK + BZ \tag{4}$$

③ The operation rule of the reservoir

The lower and upper bounds of each reservoir's water storage during each period can be expressed as Equation (5):

$$V_{i,t}^{\min} \le V_{i,t} \le V_{i,t}^{\max} \tag{5}$$

where $V_{i,t}$ (L^3) is the water storage of the reservoir i during the period t; $V_{i,t}^{\min}$ (L^3) and $V_{i,t}^{\max}$ (L^3) are the lower and upper bounds of storage during the period t, respectively.

According to the water balance equation, the water storage of the reservoir i during the period t can be calculated by Equation (6):

$$V_{i,t} = \begin{cases} V_{i,t-1} + LS_{i,t} - X_{i,t} + Y_{i,t} - PS_{i,t} - EF_{i,t} & i = N \\ V_{i,t-1} + LS_{i,t} - X_{i,t} + Y_{i,t} - Y_{i+1,t} - PS_{i,t} - EF_{i,t} & i \ne N \end{cases} \tag{6}$$

where $LS_{i,t}$ (L^3), $PS_{i,t}$ (L^3), and $EF_{i,t}$ (L^3) are local inflow, water spill, and water loss of the reservoir i during the period t, respectively; $Y_{i,t}$ (L^3) is water replenishment for the reservoir i by the pumping station i during the period t.

The water replenishment $Y_{i,t}$ and water spill $PS_{i,t}$ during each period are determined by the lower and upper bounds of water storage as follows:

a. If $V_{i,t} < V_{i,t}^{\min}$, the reservoir should be replenished with water. $Y_{i,t}$ and $PS_{i,t}$ can be calculated by Equations (7) and (8), respectively.

$$Y_{i,t} = V_{i,t}^{\min} - V_{i,t} \tag{7}$$

$$PS_{i,t} = 0 \tag{8}$$

b. If $V_{i,t} > V_{i,t}^{\max}$, the excessive water in the reservoir should be released. $Y_{i,t}$ and $PS_{i,t}$ can be calculated by Equations (9) and (10), respectively.

$$Y_{i,t} = 0 \tag{9}$$

$$PS_{i,t} = V_{i,t} - V_{i,t}^{\max} \tag{10}$$

c. If $V_{i,t}^{\min} \leq V_{i,t} \leq V_{i,t}^{\max}$, then both $Y_{i,t}$ and $PS_{i,t}$ should be zero (Equation (11)).

$$Y_{i,t} = PS_{i,t} = 0 \tag{11}$$

④ The maximum water supply can be expressed as Equation (12)

$$X_{i,t} \leq YS_{i,t} \tag{12}$$

⑤ The maximum water replenishment of the reservoir during each period is restricted by the pumping capacity, which can be expressed as Equation (13):

$$Y_{i,t} \leq Y_{i,t}^{\max} \tag{13}$$

where $Y_{i,t}^{\max}$ (L^3) is the maximum pumping volume of the station i during the period t.

⑥ Initial and boundary conditions

The restriction was imposed to make the final storage of the reservoir equal to the initial.

$$V_{i.0} = V_{i,T}(i = 1, 2, \cdots, N) \tag{14}$$

⑦ Nonnegativity restriction

2.3. Solving methods

According to the characteristics of the above mathematical model, a multi-dimensional and multi-stage nonlinear model, including water supply $[X_{1,t}, X_{2,t}, \cdots, X_{L,t}]$, replenishment water $[Y_{1,t}, Y_{2,t}, \cdots, Y_{L,t}]$, and spill water $[PS_{1,t}, PS_{2,t}, \cdots, PS_{L,t}]$, contains a total of $3L$ variables and each variable has N dimensions. It will cause a 'curse of dimensionality'. So, this study proposed an improved decomposition and dynamic programming aggregation (DDPA) method. The method aims to transform the original system into several subsystems and then solve both the subsystem models and the aggregation model with DP.

2.3.1. Decomposition and dynamic programming aggregation

① Decomposition of the original system

First, the original large-scale system will be decomposed into several subsystems, and each subsystem only consists of a reservoir and a pumping station as shown in Figure 1.

For each subsystem, the pumping station lifts water to replenish the reservoir from another, except that pumping station 1 lifts water to reservoir 1 from a river.

② Optimization of subsystems

The objective function in the optimization model of each subsystem is expressed as:

$$\min f_i = \sum_{t=1}^{T} (X_{i,t} - YS_{i,t})^2 \tag{15}$$

where f_i is the annual sum of squared water shortage in each period of the subsystem i. In this manner, one-dimensional DP can be adopted to solve each subsystem model as shown in Figure 2(a).

The constraints of the subsystem include Equations (3) and (5)–(14). Equation (3) is the coupling constraint of decision variables $X_{i,t}$, which can be transformed into the state transition equation. Equations (5)–(11) compose the operation rule of the reservoir. The operation rule is integrated into the recursive procedure of DP to correct water storage $V_{i,t}$ and obtain water spill $PS_{i,t}$ or water replenishment $Y_{i,t}$ of each period simultaneously as shown in Figure 2(a). Equations (12) and (13) restrict the solution spaces of decision variables.

For each subsystem, the annual available water W_i can be discretized between [0, $SK + BZ$], and the objective function values $\tilde{f}_i(W_i)$ and ($\tilde{X}_{i,t}$, $\tilde{Y}_{i,t}$, $P\tilde{S}_{i,t}$) can be obtained with DP according to each discrete value of W_i, which can be expressed as Equation (16):

$$W_i \sim \tilde{f}_i(W_i) \sim (\tilde{X}_{i,t}, \tilde{Y}_{i,t}, P\tilde{S}_{i,t}) \quad i = 1, 2, \cdots, N \tag{16}$$

③ Aggregation of the system

The aggregation model can be formed directly using the optimization results of subsystems $\tilde{f}_i(W_i)$. The objective function of the aggregation model is expressed as follows:

$$\min F = \sum_{i=1}^{N} \tilde{f}_i(W_i) \tag{17}$$

The coupling constraint of the aggregation model is expressed as Equation (4).

Obviously, Equation (17) is another expression of Equation (1). The aggregation model can also be solved by one-dimensional DP and the decision variable is W_i.

The inverse recursion of DP should be applied as shown in Figure 2(b), so that the water replenishment $Y_{i,t}$ from the subsystem $i - 1$ to i can be known when the model of subsystem $i - 1$ is optimized. The recursion equation and state transition equation can be expressed as Equations (18) and (19):

$$h_i(\xi_i) = \min [\tilde{f}_i(W_i) + h_{i+1}(\xi_{i+1})] \tag{18}$$

$$\xi_{i+1} = \xi_i - W_i \quad (i = 1, 2, \ldots, N - 1) \tag{19}$$

where ξ_i (L^3) is the sum of the annual total replenishment volume from the subsystem i to N; $h_i(\xi_i)$ (L^6) is the sum of squared water deviations from the subsystem i to N according to each ξ_i.

After obtaining the optimal results of the aggregation model F* and [W_1*, W_2*, … , W_N*], the optimization results [$X_{i,t}$, $PS_{i,t}$, $Y_{i,t}$]* of each subsystem can be acquired by searching the results of subsystem models according to W_i*, which can finally compose an optimal operation scheme of the whole system.

Overall, the key of DDPA is to transform the $N + 1$ dimensional DP into the $N + 1$ iterative calculations of one-dimensional DP. Both the subsystem models and the aggregation model are one-dimensional DP models. Different from traditional DDPA (Gong & Cheng 2018), the improved method adopts nested mode and embeds the recursive process of the subsystem model in the reverse recursive process of the aggregation model. For each subsystem, the calculation of

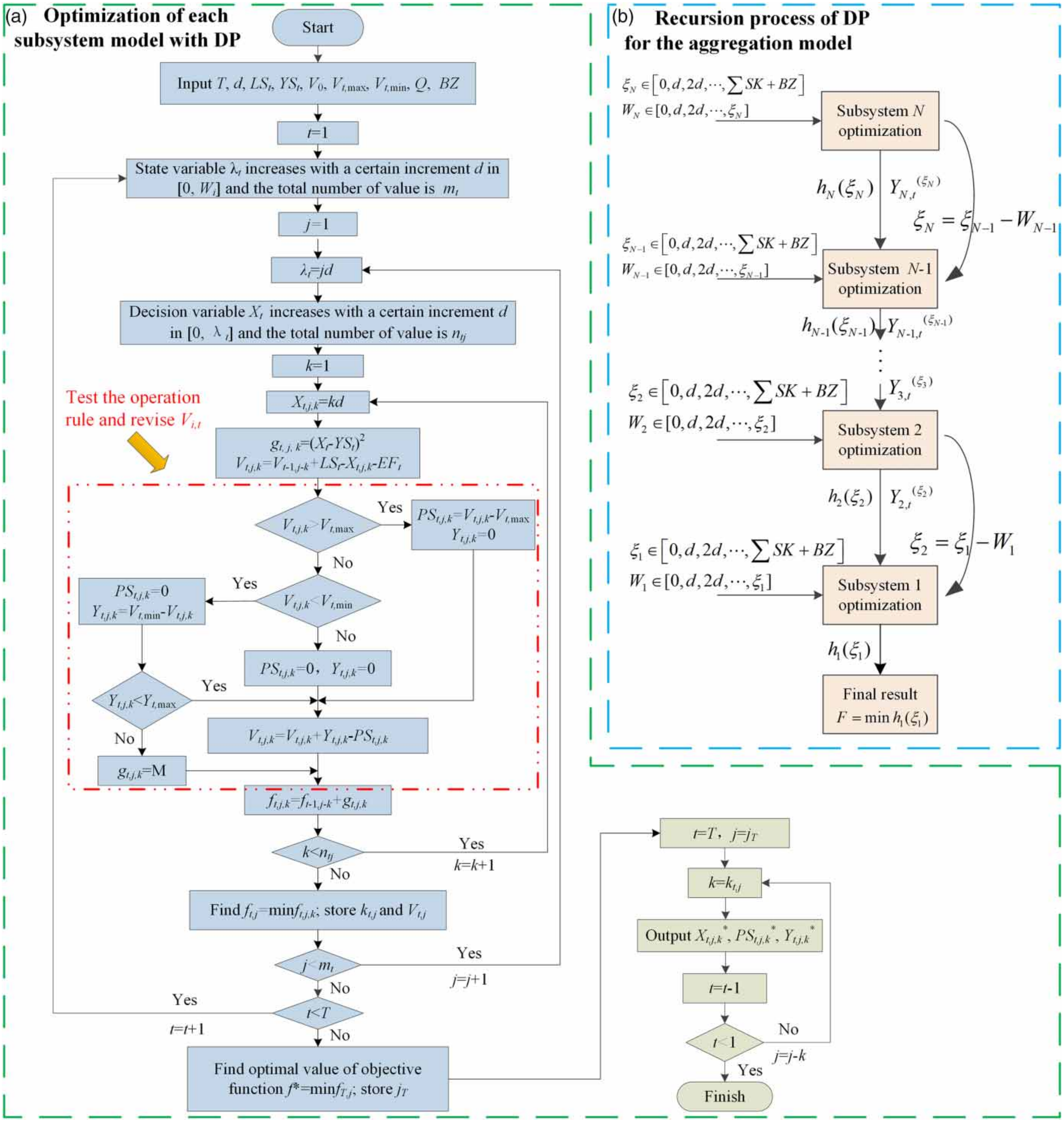


Figure 2 | Flow chart of DDPA.

each state decision variable is coupled with the reservoir water storage inspection mechanism, so that the water spill and replenishment can be calculated according to the rule, and the constraint conditions with the judgment logic can be handled.

Therefore, this method has the potential to reduce the curse of dimensionality and effectively obtain the results of global optimal.

3. CASE STUDY

3.1. Study area

The model was applied to a dual-reservoir-and-dual-pumping-station system in Nanjing, Jiangsu province, China, which consists of Shanhu (SH) reservoir, Hewangba (HWB) reservoir, and two pumping stations, Huzhang (HZ) and Xiaozhuang (XZ), as shown in Figure 3. The system is located at 32° 27′ N, 118° 46′ E in the subtropical monsoon climate zone. The mean annual precipitation is 1,002.7 mm, and 73.6% of that occurs between June and September.

3.2. Input data

The system mainly provides irrigation water. The characteristics of reservoirs and pumping stations are shown in Table 1. The average topography of the HWB reservoir is much higher than that of the SH reservoir. During water shortages, the HWB reservoir can lift water from the SH reservoir by the HZ pumping station, and the SH reservoir can lift water from the Zao river by the XZ pumping station. The annual water rights of the XZ pumping station, which means the maximum volume that can be pumped from the Zao river, are allocated by the local water authority as shown in Table 1. The HZ pumping station is an internal station in the system and is not limited by any water rights.

The operation cycle of the system is divided into 20 stages in a water conservancy year from October of the current year to September of the following year. Among them, to improve scheduling accuracy, June to September is the local flood season and water consumption peak (rice irrigation), with the stage every 10 days corresponding to stages 9–20. Other stages 1–8 are divided into stages by month. The inflow, water demand, and water loss during each period at a 75% probability of drought years are shown in Table 1.

4. RESULTS AND DISCUSSION

In this section, the optimization results of the DDPA and the heuristic algorithms will be revealed.

4.1. Optimization results of the DDPA

According to the optimization results, the water shortage is reduced from 13×10^4 m^3 to 0, while the total amount of external water extraction is from 446×10^4 to 435×10^4 m^3. The water supply is increased by 0.8%, and water spill and energy consumption are reduced by 100 and 2.5%, respectively. Figure 4 shows the results of the conventional and optimization operation of the ISR.

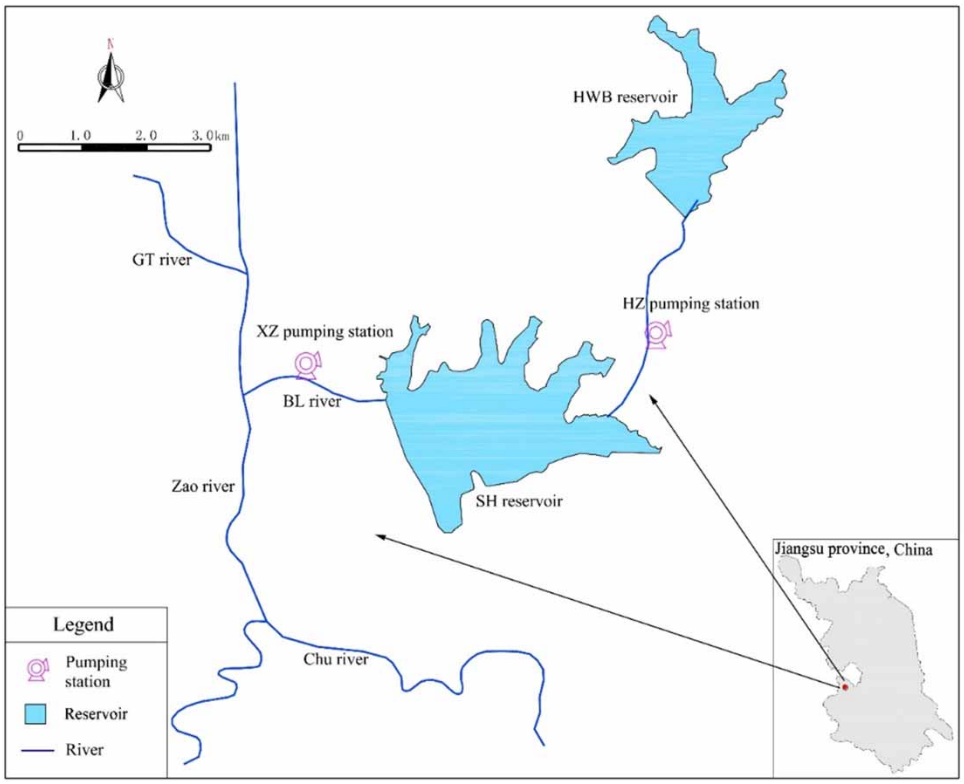


Figure 3 | The system of the dual-reservoir-and-dual-pumping-station in Jiangsu province, China.

Table 1 | Characteristics of reservoirs and pumping stations (unit: 10^4 m^3)

Reservoir	Categories	Period 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
SH	Inflow	115	45	11	32	12	28	32	187	157	106	82	87	73	52	91	75	91	56	43	24	1399
	Water loss	22	18	16	9	10	14	19	29	11	12	10	12	12	12	13	13	13	9	10	10	274
	Water demand	37	33	27	17	11	25	19	33	13	315	76	87	22	68	128	109	97	45	27	21	1210
	Water shortage																					85
HWB	Inflow	8	5	1	0	0	2	13	16	10	27	3	27	21	23	19	14	4	4	4	2	203
	Water loss	5	3	2	1	1	3	3	5	2	2	3	3	4	3	3	4	3	2	2	2	56
	Water demand	13	9	8	3	1	10	10	13	37	130	50	18	15	19	17	18	51	52	13	10	497
	Water shortage																					350
Pumping station		Design discharge (m^3/s)							Design pumping head (m)				Maximum daily operation duration (h)						Water rights (10^4 m^3)			
XZ		2.1							19.4				20						446			
HZ		0.7							22.3				20						/			

Note: These data are provided by Liuhe Water Authority, Nanjing, China.

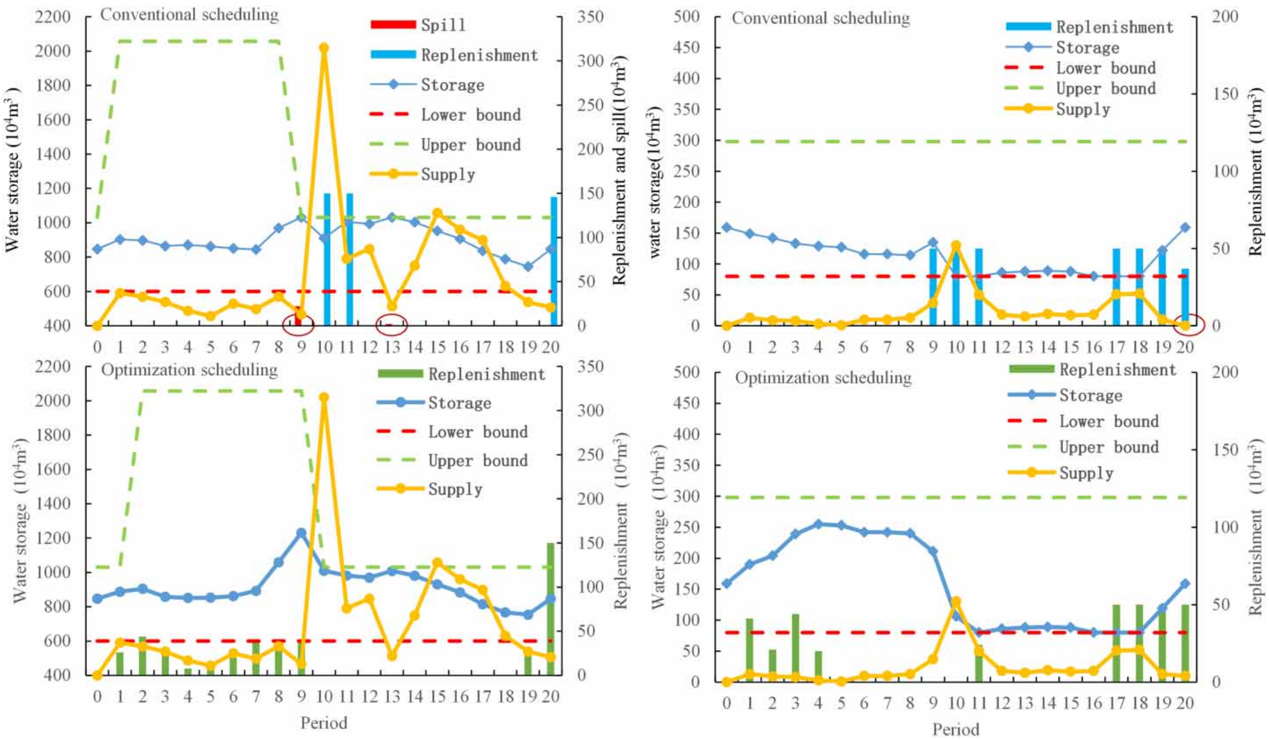


Figure 4 | Operation curves of reservoirs for conventional and optimization.

The optimization results reflect the regulation capacity of the system, forming an effective joint operation mechanism of the 'dual-reservoir-and-dual-pumping-station' system, which achieved the goal of optimization scheduling between reservoirs.

4.2. Practical comparison of DDPA and heuristic algorithm

For the sake of demonstrating the performance of DDPA, six heuristic algorithms were used to solve the reservoir operation problem under the same environment, including GA, PSO, GWO, SCA, SSA, and GSA. The main controlling parameters of all heuristic algorithms, the number of search agents and maximum iteration, are equal to 100 and 500, respectively. And, the values were obtained through many experiments for other controlling parameters of each algorithm to ensure the best performance. Moreover, the constraints were transformed into unconstrained one by penalty functions. Table 2 shows all the results, and heuristic algorithms selected the best of 30 independent runs.

From Table 2, it can be observed that no algorithms found water spills due to the consideration of the reservoir operation rules in the constraints. The objective function value of the DDPA algorithm is 0, water shortage is 0, and the total pumping volume from the Zao river by the XZ station is 435×10^4 m^3. The DDPA algorithm obtained the global optimal solution.

However, all of the heuristic algorithms only obtained the local optimal solution. The objective function values are 81–16,1309 $\times 10^4$ m^3. As can be found in Table 2, water shortage is 2–350 $\times 10^4$ m^3. And only the water replenishment of PSO, GWO, and SCA is low than DDPA, but GA, GSA, and SSA increased by 4.5,15.3, and 8.9%, respectively.

Of course, analyzing the supply–demand relationship in each stage of all heuristic algorithms is also necessary. These results are presented in Figure 5, and the water resource including reservoirs and replenishment is not reasonably allocated to each demand stage by heuristic algorithms. On the other hand, it is well known that using heuristic algorithms it is notoriously difficult to deal with strict equality constraints. Therefore, none of the heuristic algorithms in Table 2 satisfy the constraint that the initial storage is equal to the final storage required by constraint (6), except for the SH reservoir with the GA algorithm.

Therefore, the heuristic algorithms have shortcomings in solving the optimization problem of series reservoirs. This comparison is significant and would help future researchers select the suitable optimization method for their case study.

Table 2 | The results of all the algorithms (unit: 10^4 m^3)

Algorithm	Reservoir	Water demand	Water spill	Water replenishment	Water shortage	Final storage	Objective function
DDPA	SH	1,210	0	435	0	847	0
	HWB	497	0	350	0	159	
	Total	1,707	0	/	0	/	
PSO	SH	1,210	0	377	0	774	88
	HWB	497	0	337	2	148	
	Total	1,707	0	/	2	/	
GWO	SH	1,210	0	263	10	795	1812
	HWB	497	0	240	31	80	
	Total	1,707	0	/	41	/	
GA	SH	1,210	0	463	5	847	81
	HWB	497	0	357	5	171	
	Total	1,707	0	/	10	/	
GSA	SH	1,210	0	632	247	1021	161309
	HWB	497	0	273	103	185	
	Total	1,707	0	/	350	/	
SCA	SH	1,210	0	524	170	900	106752
	HWB	497	0	203	144	156	
	Total	1,707	0	/	314	/	
SSA	SH	1,210	0	438	15	798	1146
	HWB	497	0	417	18	244	
	Total	1,707	0	/	33	/	

4.2.1. Comparison of optimality

In this section, the comprehensive comparison of the performances of DDPA and heuristic algorithms from optimality, adaptability, and efficiency will be analyzed and discussed.

It can compare intuitively by the water supply reliability and vulnerability indexes. Reliability represents the satisfaction degree of water demand of the system, which is expressed by the average value of the ratio of actual water supply to water demand at each stage, and vulnerability represents the severity of water shortage. The water supply reliability index value of the system should be as large as possible, and the water supply vulnerability should be as small as possible.

From Table 3, the reliability of DDPA is 100% and the vulnerability is 0. However, the reliability and vulnerability of heuristic algorithms are 54.37–100% and 0–100%. It can be observed that the DDPA method is the most optimal. The PSO, GA, and SSA algorithms also show better optimization performance than other heuristic algorithms.

4.2.2. Comparison of adaptability

As we all know, discrete step sizes are the key factors to affect the optimization results for the DDPA algorithm; the smaller the discrete step size, the higher the accuracy with the higher the time cost. As the step size increases, the accuracy gradually decreases. But it is easy for us to find a suitable step size according to the characteristics of the decision variables.

Many factors affect the optimization results of the heuristic algorithm, such as the characteristics of the algorithm itself, the size of the population, the control parameters, the form of the constraints, the number of variables, etc.

Figure 6 shows the convergence curves of all heuristic algorithms. It presents different iteration processes because each algorithm searches differently. In addition, the No-Free-Lunch (NFL) has proved that no algorithm can solve all optimization problems. From Figure 6, it can be observed that the SCA and GSA algorithms are unable to adapt to the reservoir optimization problem studied in this work. Of course, it cannot deny that they show excellent performance in other optimization problems. Other algorithms may be applicable, but these still have many problems to overcome to obtain the optimal solution.

Population size is also an important factor for optimizing performance. Figure 7 shows the optimization results of different population sizes. The figure shows that increasing the population can improve the optimization accuracy within a range. But

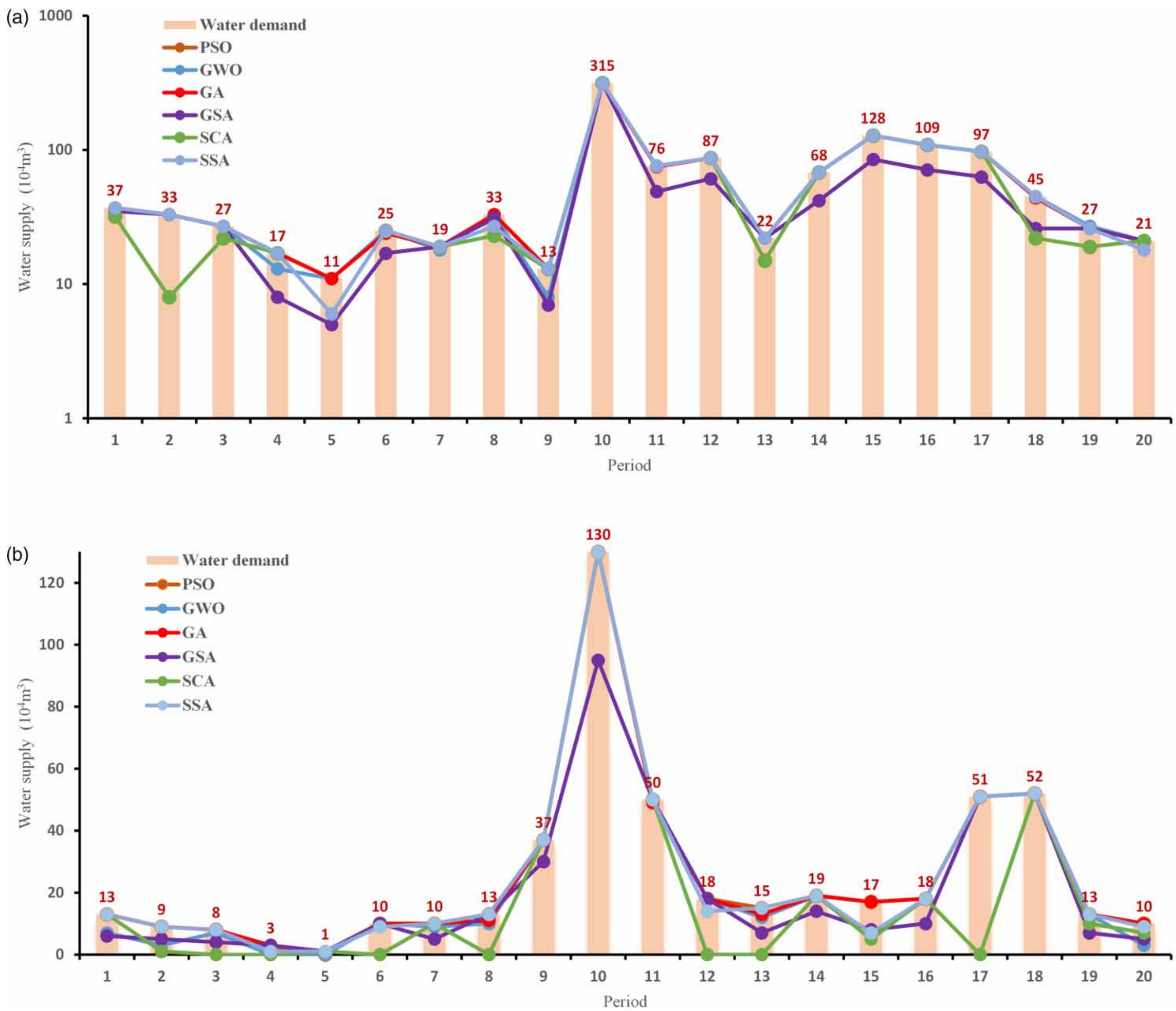


Figure 5 | Water supply of all heuristic algorithms.

Table 3 | Volumetric reliability and vulnerability obtained with different algorithms

Reservoir	Algorithm	Reliability (%)	Vulnerability (%)	Formula
SH	DDPA	100	0	
	PSO	100	0	$\text{Reliability} = \frac{\sum_{t=1}^{T}(X_{i,t}/YS_{i,t})}{T}$
	GWO	96.6	38.5	
	GA	99.4	4	
	GSA	77.5	54.5	
	SCA	77.5	100	$\text{Vulnerability} = \max_{t=1}^{T}\left\{1 - \frac{X_{i,t}}{YS_{i,t}}\right\}$
	SSA	95.6	45.4	
HWB	DDPA	100	0	
	PSO	93.3	100	
	GWO	77.6	100	
	GA	98.5	15.4	
	GSA	74.1	53.8	
	SCA	54.4	100	
	SSA	91.6	66.7	

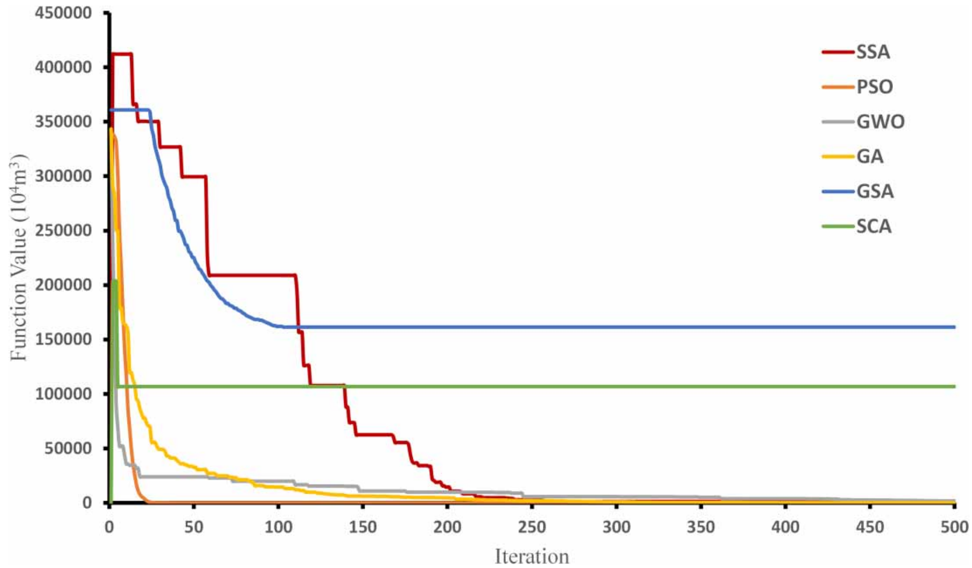


Figure 6 | Convergence curve of heuristic algorithms.

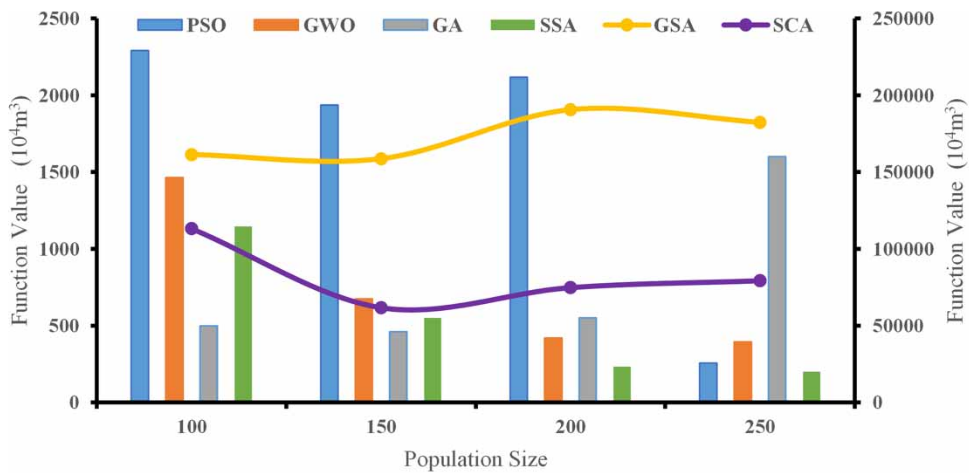


Figure 7 | Optimization results for different population sizes.

blindly increasing the population number will not only increase the computational complexity but also reduce the optimization performance. The performance of the algorithm is closely related to the population size and the other parameters.

Therefore, the applicability of the heuristic algorithm requires specific analysis of specific problems and a lot of time to select reasonable parameters. The selection of parameters itself is an optimization process. So, the DDPA algorithm is more adaptable.

4.2.3. Comparison of efficiency

The DDPA algorithm is a function optimization method, so it only needs to be run once to get the final result. But the heuristic algorithm is a random search method, and the results of each run vary greatly. Taking PSO as an example, the objective function value of the best result in 30 runs is 88×10^4 m^3, but the worst is $13{,}465 \times 10^4$ m^3. The gap between the two is huge. Therefore, it needs to run multiple times to obtain a better result for the heuristic algorithm. Figure 8 shows the difference in results of all heuristic algorithms run independently 30 times through a box plot.

So, compared with DDPA, the heuristic algorithm takes more time to obtain the optimal solution. The DDPA can balance the contradiction between accuracy and efficiency.

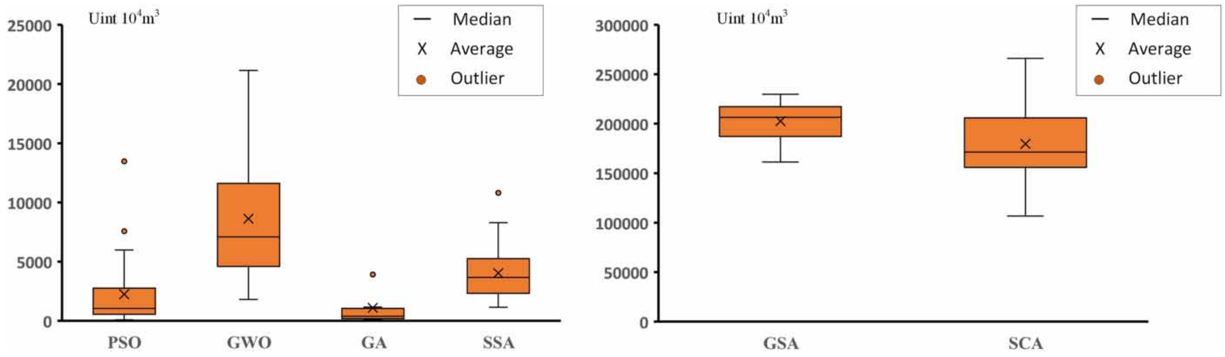


Figure 8 | Box plot of heuristic algorithms in 30 runs.

5. CONCLUSIONS

In this paper, according to the characteristics of optimal water resource scheduling of the reservoir, a complex mathematical model was proposed for the optimal allocation of water resources of the series reservoir with abundant water resources but the uneven allocation of time and space. And a decomposition and dynamic programming aggregation (DDPA) method was proposed. The comparison between DDPA and six heuristic algorithms was discussed from optimality, adaptability, and efficiency. The results demonstrated that (1) the DDPA has high optimization accuracy and still has a better global search ability for multi-variable and multi-dimensional optimization problems. (2) Implementing the DDPA is more adaptable as it needs fewer initial parameters than the heuristic algorithm. (3) It is more straightforward and accurate to check the satisfaction of constraints, such as logical judgments and equality conditions, in the DDPA compared to the heuristic algorithm. (4) The DDPA method could effectively solve the problem of balancing the accuracy of the optimization results and calculation time.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

CONFLICT OF INTEREST

The authors declare there is no conflict.

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