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Influence of the channel bed slope on Shannon, Tsallis, and Renyi entropy parameters

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ABSTRACT

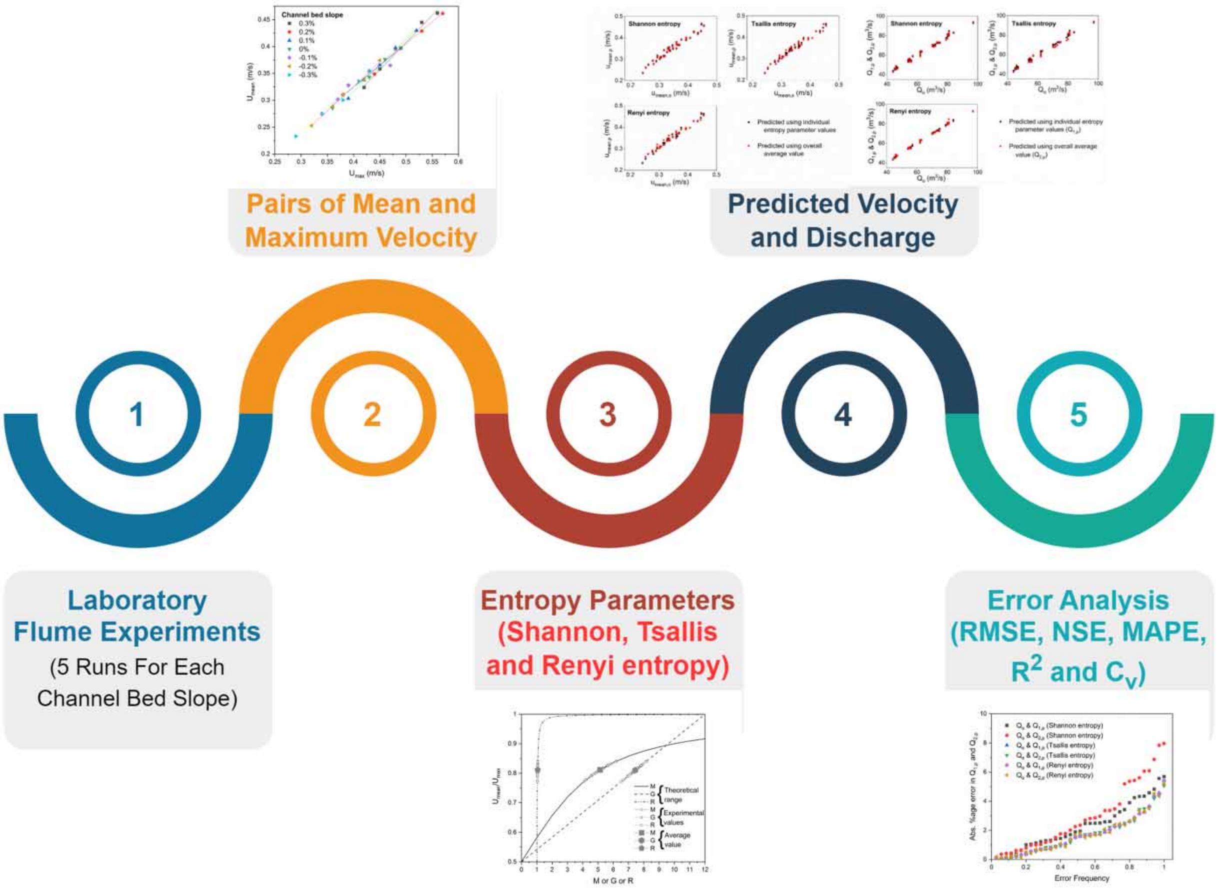
Velocity distribution plays a fundamental role in understanding the hydrodynamics of open-channel flow. Among a multitude of approaches, the entropy-based approach holds great promise in achieving a reasonable characterisation of the velocity distribution. In entropy-based methods, the distribution depends on a key parameter, known as the entropy parameter (a function of the time-averaged mean velocity and maximum velocity), that relates to channel characteristics, such as channel roughness and channel bed slopes. The entropy parameter was regarded as constant for lack of experimental evidence, which would otherwise demonstrate if it had any correlation with channel properties. A series of experiments were conducted to collect velocity data in the laboratory flume for seven different values of the channel bed slope. The experimental data analysis revealed dissimilar fluctuations in entropy parameter values with varying bed slopes, with the lowest coefficient of variation in Renyi's (~0.5%) and the highest in Shannon's case (~10%). Performance evaluation of the predicted results substantiated good accuracy for all three entropies with the best results of Renyi entropy and lent strong support for using a constant (overall average) value of the entropy parameter for a specific channel cross-section rather than separate values for each channel bed slope.

Key words: bed slope, channel cross-section, entropy parameter, Renyi entropy, Shannon entropy, Tsallis entropy

HIGHLIGHTS

- Verification of the influence of the channel bed slope on entropy parameters.
- Velocity observations for mild, horizontal, and adverse channel bed slopes.
- Shannon, Tsallis, and Renyi entropy-based velocity distributions.
- Statistical and experimental evidence supporting the constant nature of all the entropy parameters.
- Modified equation to estimate mean and maximum velocity ratio in terms of the Renyi entropy parameter.

GRAPHICAL ABSTRACT



1. INTRODUCTION

Velocity distribution and streamflow measurement information are critical in hydraulic engineering and flow hydrodynamics. Discharge measurements are needed for water availability analysis, reservoir operation, flood forecasting, and the design of hydraulic structures. There are numerous streamflow estimation techniques, but they involve sophisticated apparatus and high operational costs. The prerequisites for flow estimation are the time-averaged flow velocity and the channel's cross-sectional dimensions, which can be obtained using channel bathymetry surveys or innovative techniques such as the Acoustic Doppler Current Profiler (Simpson 2001). For velocity, non-contact measurements, such as surface velocity radar (Ferro 2003; Welber *et al.* 2015), are preferred to the traditional contact methodologies (Chiu & Chen 2003; Chiu *et al.* 2005). Both types of measurements yield the velocity distribution for the whole cross-section. The velocity distribution describes the spatial distribution of the cross-sectional time-averaged velocity, i.e., the longitudinal component in a transverse channel cross-section, excluding the turbulent part that randomly varies with time (Chiu & Chiou 1986). The mean velocity component is conventionally considered deterministic without any uncertainty in the classical methods, such as the logarithmic law (Prandtl-von Karman universal velocity distribution law) and power law.

Clearly, traditional laws have limited accuracy, as they are effective for wide-open channels only, i.e., channels with longitudinal velocity variation and maximum velocity occurring at the free surface (Chiu 1988). In such channels, the secondary component of velocity (transverse component) is not effective, whereas in narrow channels, velocity varies in longitudinal as well as in transverse directions (two-dimensional (2D) variation), and maximum velocity occurs below the top surface because of the significant secondary component (Sarma *et al.* 1983; Nezu & Rodi 1986). Hence, Prandtl-von Karman's equation is unsatisfactory for narrow channels and leads to erroneous values near the channel bed and in the vicinity of the free surface. Likewise, a power law is also limited in similar situations (Karim & Kennedy 1987). Given the drawbacks of traditional approaches and the uncertainties associated with natural and human-made factors affecting the velocity of river flow, the entropy-based concept was utilised in a number of studies and was found promising (Chiu & Said 1995; Moramarco *et al.* 2004, 2008, 2011, 2013, 2017; Singh 2013, 2014; Ammari & Remini 2010; Bechle & Wu 2014; Greco & Mirauda 2014; Greco *et al.* 2014; Termini & Moramarco 2017; Luo *et al.* 2018; Roushangar *et al.* 2018, 2020; Mirauda & Russo 2019; Vyas *et al.* 2020; Singh & Khosa 2022, 2023).

Treating the time-averaged velocity probabilistically and using Shannon's entropy (Shannon 1948), a velocity probability distribution was derived (Chiu 1987, 1988) by employing the Principle of Maximum Entropy (POME) (Jaynes 1957). The obtained entropy-based velocity equation was also of logarithmic nature but different from the von Karma–Prandtl equation. Furthermore, Chiu (1989, 1991) derived one-dimensional (1D) and 2D entropy-based velocity distributions and developed a relationship between cross-sectional mean and maximum velocity in terms of the Shannon entropy parameter (M). The parameter M is an essential measure of information about the channel characteristics, such as bed form variations, bathymetry, channel bed slope, and roughness. This parameter M can be derived experimentally from the pairs of u_{mean} and $u_{\max}$ measured at a channel section (Moramarco *et al.* 2004). The relation between mean velocity (u_{mean}) and maximum velocity ($u_{\max}$) is helpful in studying several parameters, like momentum and energy coefficients, the location of mean velocity, and maximum velocity. Many a time, all these parameters were neglected because they involved tedious calculations but were easily correlated to the entropy parameter (M). The M parameter was investigated for the reaches of the Mississippi River, and no fluctuations were observed for the straight reaches, but the bends along the river led to different M values (Xia 1997). Similar parameter values were reported by the other studies (Ardiclioglu *et al.* 2005; Ammari & Remini 2010).

Also, the findings of the significant work by Moramarco & Singh (2010) based on Manning's equation and dip-modified logarithmic law proved the independence of the Shannon entropy parameter from the water surface slope (S_{f}) for the particular case of uniform flow conditions. In that case, the channel bed slope was equivalent to the water surface slope (S_{f}). Greco & Mirauda (2014) developed the relationship between the entropy parameter M and the relative submergence and verified the same through experimental data. The parameter M was regarded as constant for the specific portion of the river investigated or at any particular channel cross-section (Moramarco *et al.* 2004). However, the constant nature of the entropy parameter has not been addressed experimentally in the literature. Despite the decent accuracy provided by the Shannon entropy-based approach, many other forms of entropy were also explored in recent years to derive the velocity distribution and thereby overcome the shortcomings of the previous entropy concepts, such as Tsallis entropy (Tsallis 1988), Renyi entropy (Renyi 1961), and fractional entropy (Wang 2003). Both Tsallis and Renyi entropies-based velocity distributions involve the use of respective entropy parameters, which are also considered in this study.

The paper's objective and the novel part are twofold. Firstly, laboratory experiments were performed at different channel bed slopes, and the velocity data were collected. Secondly, to address the entropy parameter's constant nature for the varying channel characteristics using the experimentally collected data in contrast to the previous entropy-based studies, which surmised it to be constant. As an original contribution, the three entropies were selected (Shannon entropy, Tsallis entropy and Renyi entropy) at once in the present study, and the channel bed slope was selected among the numerous channel characteristics that can influence the entropy parameter's behaviour. Hence, to study the influence of the channel bed slope on the entropy parameter, a large set of experiments were carried out in a laboratory flume to collect the velocity observations. The flume was equipped with a machine-operated channel bed slope-varying apparatus. Interestingly, the three different types of bed slopes (such as mild, horizontal, and adverse) were considered in the present study. Velocity data were recorded for seven different channel bed slope values with the help of the current meter. Then, velocity data processing furnished the entropy parameter values for all the entropy types considered, thereby predicting the velocity and discharge using each entropy. Next, the statistical analysis of the error in the prediction was conducted using four metrics: RMSE, Nash–Sutcliffe coefficient (NSE), mean absolute percentage error (MAPE), and correlation coefficient (R^2).

2. THEORETICAL BACKGROUND

Considering the time-averaged velocity (u) as a random variable, Chiu (1987) derived the probability density function (PDF) of velocity $f(u)$ and the cumulative distribution function (CDF), $F(u)$ (Equation (1)). He also derived 1D and 2D forms of the CDF, depending on the channel and flow characteristics (Chiu 1989). The 2D flow variation accounted for the influence of secondary currents, which allowed the position of maximum velocity to occur at a depth below the free surface, i.e., the dip phenomenon was observed. By contrast, the 1D case neglects the secondary currents and assumes the velocity to increase monotonically with zero value (at the channel bottom) to a maximum value occurring at the free surface.

$$F(u) = \int_0^y f(u(y))dy = \begin{cases} \frac{y}{D}; & 1D \\ \frac{y}{D-h} exp\left(1 - \frac{y}{D-h}\right); & 2D \end{cases} \tag{1}$$

where y is the elevation measured from the bed level, D is the total water depth, and h is the dip (the location of maximum velocity from the free surface). The steps to obtain the entropy-based velocity distribution are: (1) define the respective entropy; (2) define the appropriate constraints; (3) maximise entropy using POME and derive the velocity PDF; (4) define entropy parameter; (5) derive the generalised velocity distribution in terms of defined entropy parameter (Singh 2013). The entropic method is suitable for all types of flow conditions, like the steady, unsteady, uniform, and non-uniform flow for wide and narrow channels (Moramarco & Singh 2010; Singh 2019).

2.1. Shannon entropy-based velocity distribution

Shannon entropy (Shannon 1948) can be viewed as the average information content in a random variable or information. Considering velocity (u) as a continuous random variable, the Shannon entropy can be stated by Equation (2) as

$$H = -\int_0^{u_{\max}} f(u) \ln [f(u)]du \tag{2}$$

Using Shannon entropy, Chiu (1988) derived the entropy-based velocity distribution in terms of the CDF and Shannon entropy parameter (M). The entropy parameter (M) is seen as a critical measure of information regarding the cross-sectional channel characteristics, such as the channel bed slope, bed form variations, and channel bathymetry. Depending on the study, the desired 1D or 2D simplification of the CDF can be employed to get the exact velocity distribution expression from the general velocity distribution relation (Equation (3)):

$$u = \frac{u_{\max}}{M} \ln [1 + (e^M - 1)F(u)] \tag{3}$$

Also, a relationship between cross-sectional mean velocity and maximum velocity was developed as a function of a single variable, i.e., entropy parameter (M) given by Equation (4). The historical datasets of pairs of u_{mean} and $u_{\max}$ for any particular channel cross-section would furnish the entropy parameter's numerical value using Equation (4) and the theoretical range of M can be obtained mathematically as $(0, \infty)$

$$\frac{u_{\text{mean}}}{u_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} = \Phi(M) \tag{4}$$

where $\Phi(M) =$ the state equilibrium constant, a function of M only.

2.2. Tsallis entropy-based velocity distribution

Tsallis (Tsallis 1988) postulated a generalisation of the Boltzmann–Gibbs–Shannon (BGS) entropy and gave a new expression (Equation (5)) with a conventional positive constant, k (to maintain unit consistency, which is generally taken as unity) and entropic index (m). Here, the entropic index can be any real number and describes the degree of nonlinearity associated with the system. In nature, most real systems, especially the hydrological processes, show nonlinear behaviour, which gives an edge to Tsallis entropy over Shannon entropy.

$$H = \frac{k}{m-1}\left(1 - \int_0^{u_{\max}} [f(u)]^m du\right) = \frac{k}{m-1}\int_0^{u_{\max}} f(u)\{1 - [f(u)]^{m-1}\}du \tag{5}$$

Also, the limiting case of Tsallis entropy with m approaching unity (and k equal to the unity) yields Shannon entropy (Equation (2)) (Singh 2016). Tsallis entropy-based velocity distribution was obtained (using the same generalised mathematical approach as employed in the case of Shannon entropy) in terms of the generalised CDF and Tsallis entropy parameter, G (Luo & Singh 2011). The obtained velocity distribution was validated using experimental and field flow data and led to improved results, declaring it advantageous over the Shannon entropy-based concept (Cui & Singh 2013). In most of the studies involving the Tsallis entropy, parameter (m) is fixed as a constant value (here, $m=2$) for simplification, as it does not significantly influence the calculations.

$$\frac{u}{u_{\max}}=\frac{2}{G}\left[\frac{(4-G)^2}{16}+G\cdot F(u)\right]^{1/2}-\frac{4-G}{2G} \tag{6}$$

Furthermore, a linear relationship between the Tsallis entropy parameter (G) and the mean and maximum velocity ratio is established (Equation (7)), which succinctly explains channel and flow characteristics. Mathematically, parameter G has a theoretical range from 0 to 12. However, in actual practice, its interval will be narrowed down as the maximum velocity is almost 25–50% in excess of the mean velocity (Singh 2016).

$$\frac{u_{\text{mean}}}{u_{\max}}=\frac{G+12}{24} \tag{7}$$

2.3. Renyi entropy-based velocity distribution

Renyi postulated another generalisation for entropy in terms of a parameter α and called it the entropy of order α (where $\alpha>0$ and $\alpha\neq 1$) is given by Equation (8) (Renyi 1961). Shannon entropy can be deduced as the limiting case ($\alpha\to 1$) of the Renyi entropy:

$$H_\alpha=\frac{1}{1-\alpha}\ln\left(\int_0^{u_{\max}}[f(u)]^\alpha du\right) \tag{8}$$

Numerous studies, as listed above, have been conducted using the Shannon and Tsallis entropies, whereas Renyi entropy has not been used widely. Hence, it is interesting to explore the same. Employing the same generalised mathematical framework, the Renyi entropy-based velocity distribution was obtained in terms of the generalised CDF, Renyi entropy parameter (R), and parameter (α) given by Equation (9) (Kumbhakar & Ghoshal 2016, 2017).

$$\frac{u}{u_{\max}}=\left(\frac{R}{R-1}\right)-\left\{\left(\frac{R}{R-1}\right)^{\frac{\alpha}{\alpha-1}}+\left[\left(\frac{1}{R-1}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{R}{R-1}\right)^{\frac{\alpha}{\alpha-1}}\right]F(u)\right\}^{\frac{\alpha-1}{\alpha}} \tag{9}$$

Similarly, the Renyi entropy parameter (R) was related to the ratio of mean and maximum velocities (Equation (10)). Mathematically, parameter (R) has a very narrow range of significance (1, 1.3) that affects the mean and maximum velocity ratio as compared to other entropy parameters. Furthermore, for the sake of simplification, the judicious value of a parameter (α) without creating any significant errors was considered as $\alpha=0.99$ (which is a close approximation for the limiting case leading to the Shannon entropy). The selected value provided good accuracy compared to the Shannon and Tsallis entropy-based approaches (Kumbhakar & Ghoshal 2016).

$$\frac{u_{\text{mean}}}{u_{\max}}=\frac{\alpha}{1-\alpha}\frac{\left(1-R^{\frac{\alpha}{1-\alpha}}\right)^{-1}}{(\mathrm{R}-1)^{\frac{\alpha}{1-\alpha}}}\left\{\frac{\alpha-1}{2\alpha-1}\left[\left(\frac{1}{R-1}\right)^{\frac{2\alpha-1}{\alpha-1}}-\left(\frac{R}{R-1}\right)^{\frac{2\alpha-1}{\alpha-1}}\right]+\frac{\alpha-1}{\alpha}\frac{R}{1-\mathrm{R}}\left[\left(\frac{1}{R-1}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{R}{R-1}\right)^{\frac{\alpha}{\alpha-1}}\right]\right\} \tag{10}$$

Owing to the difficulty of working with the tedious expression (Equation (10)), a new simplified form (Equation (11)) was developed to streamline the originally given relationship into a more straightforward form, significantly reducing the computational efforts. The coinciding graphs (Figure 1) of both (original and modified) relations justify the simplification performed. Hence, the modified equation was used to calculate the Renyi entropy parameter, and the same was utilised for checking the effect of channel bed slopes and discharge analysis.

$$\frac{u_{\text{mean}}}{u_{\max}} = \frac{R}{R-1}\left[1 + \frac{\alpha}{2\alpha-1}\frac{\left(1 - R^{\left(\frac{2\alpha-1}{\alpha-1}\right)}\right)}{\left(R^{\left(\frac{2\alpha-1}{\alpha-1}\right)} - R\right)}\right] \tag{11}$$

3. MATERIALS AND METHODS

The experiments for collecting the velocity data were performed on a tilting rectangular experimental flume having 7.5 metres long straight reach and 0.309 metres wide at the Watershed Hydrology Laboratory of the Department of Hydrology, Indian Institute of Technology Roorkee (India). The flume is fitted with a smoothly flexible inclination mechanism capable of adjusting the channel bed slope from −0.5 to +2.5%. For the present study, the velocity profiling was done for the varying channel bed slopes, and consideration was given to seven distinct slopes (ranging from −0.3 to +0.3% at an interval of 0.1%), keeping all other channel characteristics constant. For each bed slope value, five different discharge runs were considered. Velocity observations were made at a particular flume cross-section located at 4.5 m from upstream). The selected cross-section was divided into evenly spaced verticals at 2 cm intervals along the width, resulting in grid-points where velocity observations were recorded (Figure 2). The depth (y) was measured from the channel bottom, whereas the transverse location (x) of the verticals was measured from the centre-line. Also, the tailgate height in the outlet element was fixed for the entire experiment to ensure that the flow profiles were unaffected by the tailwater conditions.

Furthermore, the sufficient point velocities were measured using the velocity meter (Model – MiniAir 20) on all the selected verticals. The measuring instrument was stabilised at all locations for 30–40 s to get accurate estimates. Before each recorded observation, the inclination of the channel bed was meticulously set and stabilised. Furthermore, the five discharge rates for each value of the channel bed slope were considered for recording different sets of velocity data at the pre-defined channel cross-section. Multiple measurements for the same discharge rate were taken to ensure precision and reliability. The

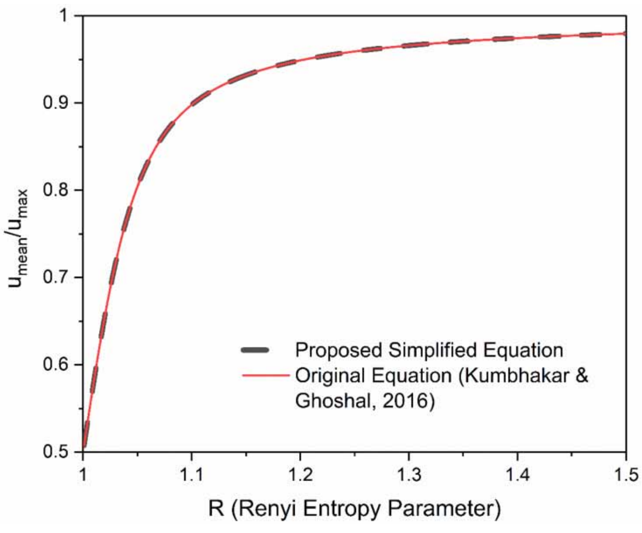


Figure 1 | Renyi entropy parameter (R) using the original and modified equations.

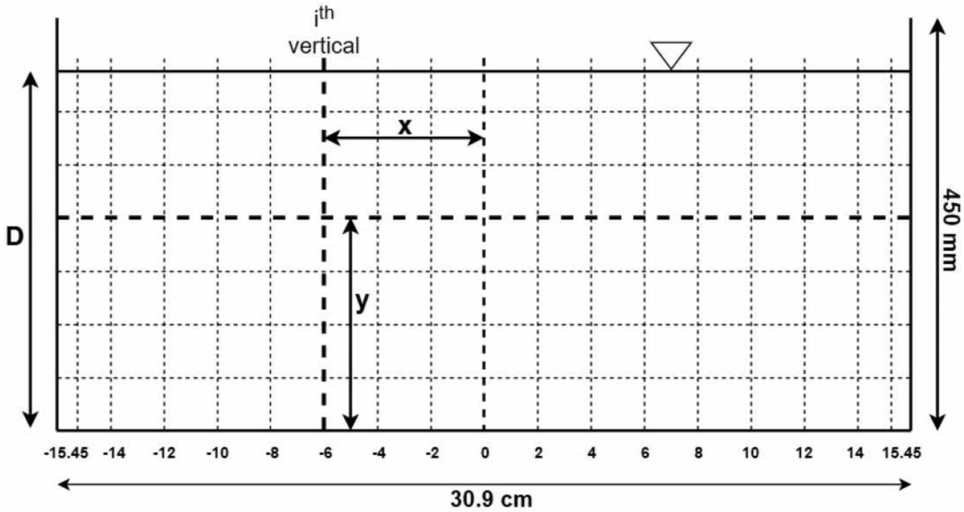


Figure 2 | Schematic diagram of the selected flow cross-section indicating measurement grid-points.

maximum velocity was directly obtained from the cross-sectional velocity observations, whereas the mean velocity was calculated from the discharge calculations using flow and channel characteristics discharge and cross-sectional flow area. Table 1 summarises the main flow characteristics of experimental data. The obtained mean, and maximum velocity pairs were plotted in Figure 3. Finally, the collected data at the designated cross-section were used to analyse the flow behaviour and to check the influence of the channel bed slope on the entropy parameters.

4. RESULTS AND DISCUSSION

4.1. Effect of the channel bed slope

The influence of the channel bed slope on the entropy parameters (Shannon entropy parameter, Tsallis entropy parameter, and Renyi entropy parameter) was evaluated using the relations defined by Equations (4), (7) and (11). The Renyi entropy parameter (R) was calculated using the proposed simplified equation (Equation (11)), which rendered the same results as obtained by the original equation (Equation (10)), confirming the equal accuracy of both the equations as evident from Figure 1. The values of respective entropy parameters were obtained from the mean and maximum velocity pairs. The five runs considered for each bed slope value provided five pairs of the mean and maximum velocities, summing to a total of 35 distinct pairs. Experimental data showed that the mean and maximum velocity ratio varied in the small interval of 0.77–0.84. Furthermore, every velocity pair corresponded to a unique value of the respective entropy parameter. Table 2 summarises the range of the individual entropy parameters and the mean and maximum velocity ratio for the corresponding channel bed slope values.

Furthermore, the average entropy parameter values for each channel bed slope were calculated using the best-fit line slope of the five runs for the respective bed slope (Figure 4). Similarly, an overall average for the channel cross-section, irrespective

Table 1 | Main flow characteristics of experimental data

Channel bed slope (%)	U_{mean} (m/s)	U_{max} (m/s)	Discharge, Q (m^3/h)	Flow depth, D (cm)	Aspect ratio, A_r
−0.30	0.242–0.360	0.29–0.43	45–80	16.7–20.0	1.55–1.85
−0.20	0.253–0.370	0.32–0.45	45–80	16.0–19.5	1.59–1.93
−0.10	0.262–0.360	0.34–0.47	45–80	15.5–20.0	1.55–2.00
0	0.277–0.393	0.36–0.49	45–80	14.6–18.3	1.69–2.12
0.10	0.293–0.434	0.39–0.52	45–85	13.8–17.6	1.76–2.24
0.20	0.306–0.447	0.38–0.57	45–90	13.2–18.1	1.71–2.34
0.30	0.316–0.455	0.42–0.56	45–80	12.8–15.8	1.96–2.41

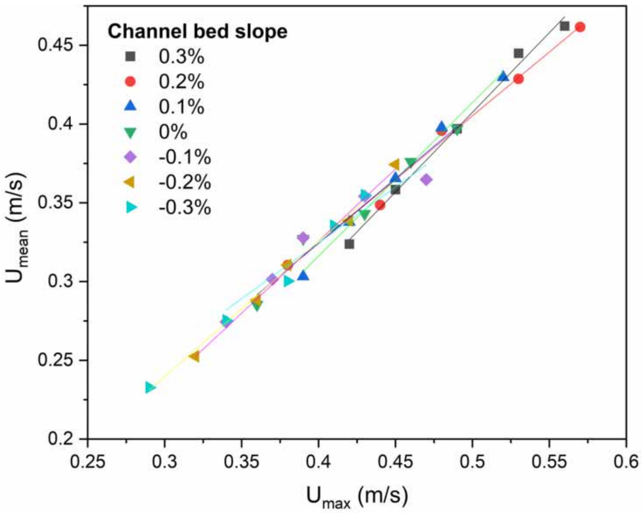


Figure 3 | Mean and maximum velocities observed at different channel bed slopes (%).

Table 2 | Ranges of the considered entropy parameters

Channel bed slope (%)	$\frac{u_{mean}}{u_{max}}$	Shannon entropy parameter (*M*)	Tsallis entropy parameter (*G*)	Renyi entropy parameter (*R*)
−0.30	0.790–0.830	4.5354–5.6004	6.9692–7.8053	1.0461–1.0570
−0.20	0.789–0.832	4.5037–5.8425	6.9400–7.9629	1.0458–1.0595
−0.10	0.776–0.841	6.1921–4.1741	8.1750–6.6267	1.0631–1.0424
0	0.792–0.839	4.5916–6.1261	7.0205–8.1365	1.0467–1.0624
0.10	0.777–0.829	4.2086–5.7232	6.6610–7.8876	1.0427–1.0583
0.20	0.792–0.825	4.5893–5.5763	7.0185–7.7879	1.0466–1.0568
0.30	0.771–0.840	4.0555–6.1474	6.5067–8.1476	1.0412–1.0626

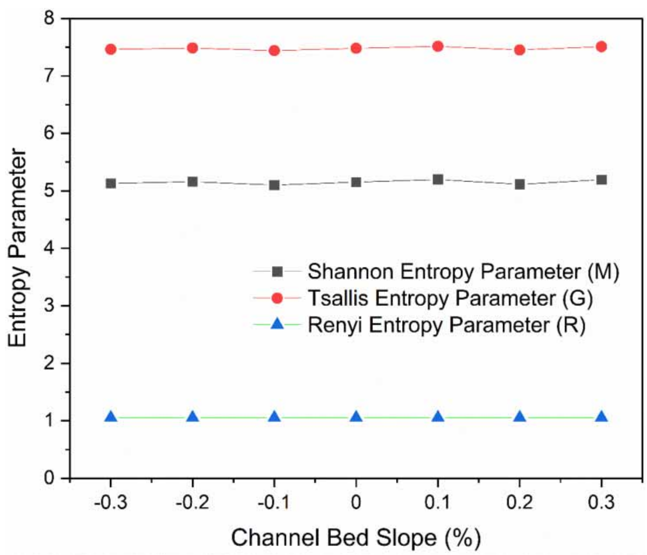


Figure 4 | Average entropy parameter values for all the channel bed slopes (%) under consideration.

of the channel bed slope value, was computed using the best-fit line plot of all the datasets comprising the 35 runs for all the channel bed slopes under consideration. The results are plotted in Figure 5, along with the theoretical range of entropy parameters, to determine the exact variation of entropy parameters with the mean and maximum velocity ratio. The coefficient of variation was calculated to check the dispersion of the parameter values. Table 3 summarises the overall average values of the entropy parameters under consideration, along with their standard deviation and coefficient of variation (C_v). Shannon's entropy parameter showed the maximum variation (~10.6%) from the overall average value, whereas Renyi's parameter showed the minimum variation (~0.53%). Between them, the Tsallis entropy parameter depicted about 5.67% variation. Experimental results showed that the variations portrayed by the entropy parameters were proportional to their theoretical range, as the parameter with minimum and maximum variations had the smallest range (1, 1.3) and largest range (0, ∞), respectively.

Figures 4 and 5 illustrate that changes in the channel bed slope do not largely influence the entropy parameter values and the mean and maximum velocity ratio. Interestingly, the results of the present study strongly uphold the findings of Moramarco & Singh (2010) for the particular case of the uniform open-channel flow. By contrast, the influence of the channel bed slope variation reduced significantly as we moved from Shannon's parameter to the Tsallis parameter and finally to Renyi's parameter. In view of experimental findings, it can be concluded that the channel bed slope variation can be treated as of a constant nature and has minimal effect on the overall applicability of the entropy concept due to the slight percentage variation in the entropy parameter values and the ratio of the mean and maximum velocity. Additionally, the entropy parameter's constant nature is justified by the mean velocity and discharge calculations, as discussed in a further section in which these results are used to decide the acceptability of the single average value for the whole channel by performing the correlation and error analysis of the observed and predicted values.

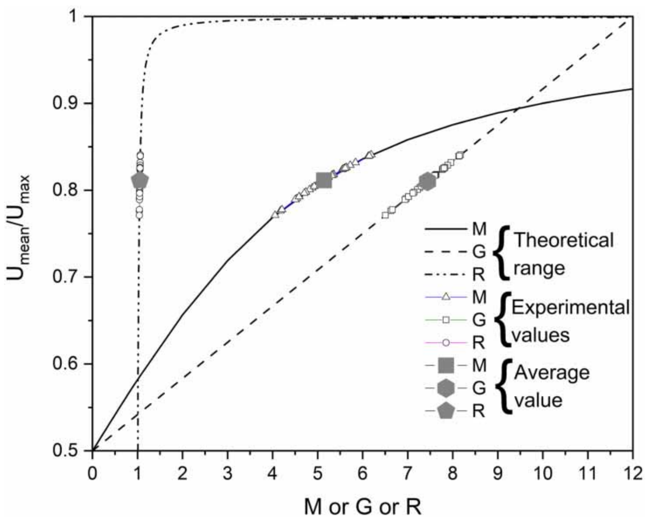


Figure 5 | Experimental data and theoretical range for Shannon entropy parameter (*M*), Tsallis entropy parameter (*G*), and Renyi entropy parameter (*R*).

Table 3 | Overall average values of the entropy parameters

Entropy parameter	Overall average value	Standard deviation	C_v (%)
Shannon entropy parameter (*M*)	5.14058	0.545	10.61
Tsallis entropy parameter (*G*)	7.44323	0.422	5.67
Renyi entropy parameter (*R*)	1.0523	0.006	0.53

4.2. Measure of error

The error of predicted and observed values of the mean velocity and discharge was analysed to justify the applicability of the entropy parameter's single value for the varying bed slope. To draw inferences about the performance, the prominently used metrics were employed, such as root mean squared error (RMSE), NSE, MAPE, and correlation coefficient (R^2). Subscripts ($_{-,o}$) and ($_{-,p}$) were used for observed and predicted values, respectively. The mean velocity was predicted for the corresponding maximum velocity values using Equations (4), (7) and (11) for Shannon entropy, Tsallis entropy, and Renyi entropy, respectively. The input for Equations (4), (7) and (11) were the maximum velocity (which was measured experimentally) and the respective entropy parameter values for each channel bed slope. The predicted mean velocity ($u_{\text{mean},\ p}$) values were plotted against the corresponding observed mean velocity ($u_{\text{mean},\ o}$) values in Figure 6. Furthermore, the correlations between predicted and observed values of mean velocity for all datasets were determined for both cases, i.e., using the individual (the best-fit line slope value for each bed slope) and the overall average value of the entropy parameter (the best-fit line slope value considering all the runs cumulatively). Apart from the correlation coefficient (R^2), the pairs of observed and predicted values (derived using Shannon, Tsallis, and Renyi entropies) were used to determine the RMSE, NSE, and MAPE. The complete statistical analysis results are reported in Table 4. The mean velocity values predicted using all the models were consistent with measured values. There were no substantial discrepancies among the predicted values using the overall average parameter values, as seen from the high values of NSE and R^2 (i.e., near unity). Moreover, the low values of the RMSE and MAPE also confirmed the decent accuracy. All the considered metrics showed minimal variation when the predictions were made using the overall average value instead of the individual parameter values, thus, justifying the acceptability of the single average value of the entropy parameters. However, the Renyi entropy-based estimates appeared more accurate than others, as observed from the error analysis results. Finally, it can be inferred that the predicted mean velocity was accurate and reliable, which can be applied to calculate the flow rate, sediment discharge, and concentration without significant percentage error.

Similarly, correlations between predicted and observed discharge values were plotted. The discharge was estimated for the individual and overall average value of the entropy parameters and labelled as $Q_{1,p}$ and $Q_{2,p}$, respectively. Apart from the mean velocity, channel characteristics, such as total water depth and channel geometry, are prerequisites for discharge estimation (Table 1). In this study, experiments were conducted on the laboratory flume having constant channel width, and the

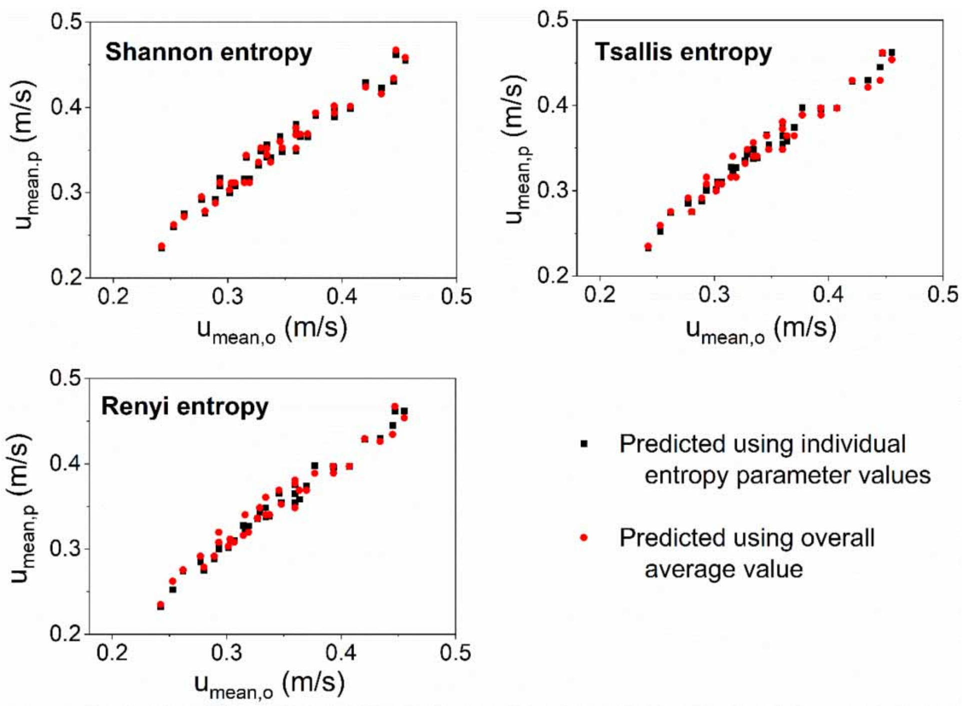


Figure 6 | Observed and predicted mean velocities using individual parameter values and an overall average value for all the datasets.

Table 4 | Statistical analysis of the predicted velocity using the individual and overall average value of the respective entropy parameters

Entropy	Entropy parameter value	RMSE	NSE	MAPE (%)	R^2
Shannon	Individual values	0.012	0.953	2.8	0.9634
	Overall average value	0.012	0.952	2.9	0.9631
Tsallis	Individual values	0.009	0.972	2.3	0.9817
	Overall average value	0.012	0.954	2.9	0.9689
Renyi	Individual values	0.009	0.972	2.3	0.9818
	Overall average value	0.013	0.946	3.1	0.9701

water depth was available for all datasets, which rendered the channel cross-sectional area. Finally, the discharge ($Q_{1,p}$ and $Q_{2,p}$) was computed using the predicted mean velocity and cross-sectional area. The same procedure was adopted to estimate the discharge for all entropy parameters, and the results are plotted in Figure 7. The statistical analysis of the error between the predicted and observed discharge values for all the datasets was determined for both cases, i.e., using the individual and overall average parameter values. The results of the statistical analysis are reported in Table 5.

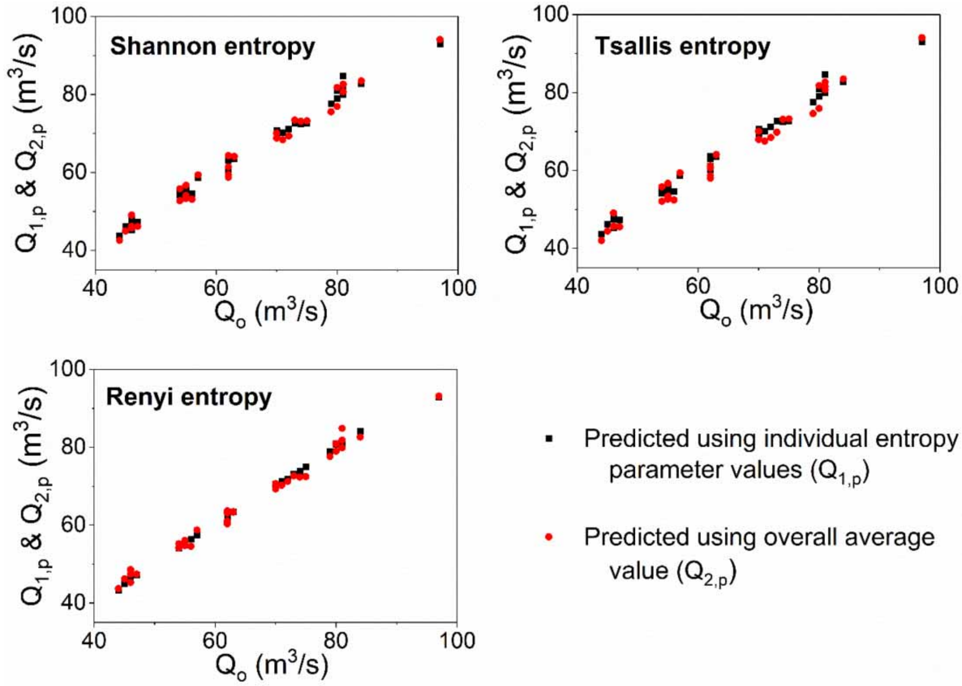


Figure 7 | Comparison between predicted and observed discharge. Q_o = observed discharge, $Q_{1,p}$ = discharge computed by individual entropy parameter values, and $Q_{2,p}$ = discharge computed by the overall average value of entropy parameter.

Table 5 | Statistical analysis of the predicted discharge using the individual and overall average value of the respective entropy parameters

Entropy	Entropy parameter value	RMSE	NSE	MAPE (%)	R^2
Shannon	Individual values	1.535	0.987	2	0.9809
	Overall average value	1.926	0.98	2.6	0.9803
Tsallis	Individual values	1.535	0.987	2	0.9904
	Overall average value	2.306	0.971	3.1	0.9901
Renyi	Individual values	0.801	0.997	0.7	0.9976
	Overall average value	1.534	0.987	2	0.9864

Considering the near-to-unity values of the NSE and R^2, it can be inferred that the predicted discharge values showed a good agreement with the observed ones for all of the entropy types under consideration. The low RSME and MAPE values also confirmed the same. Like mean velocity, the Renyi entropy predicted the discharge with the highest accuracy. Furthermore, as we move from the case of individual values to the overall average parameter value, all the considered metrics depicted minor variations, showing no substantial discrepancies among predicted values using the overall average parameter values. Additionally, the absolute percentage error in the computed discharge (Equation (12)) was quantified to strongly justify the acceptance of the overall average entropy parameter value.

$$\text{Absolute percentage error} = \frac{|Q_o - Q_p|}{Q_o} * 100 \tag{12}$$

For more clarity about the accuracy and efficiency of the considered entropies, the frequency error analysis plot (Figure 8) was made, which also validated the use of the overall average value of entropy parameters. From Figure 8, it can be inferred that Shannon entropy depicted higher percentage errors (up to ∼8%), and Renyi's entropy emerged as a better candidate among all entropies by showcasing minor percentage errors (up to ∼5%).

Starting with the Shannon entropy, the predicted discharge showed a minor variation from the observed discharge when computed using the overall average parameter value instead of the individual values. Also, the absolute percentage error did not increase significantly and remained less than 8% for both cases, and the 80% results had a percentage error of less than 5%. A similar analysis was repeated for the other two entropies (namely, Tsallis and Renyi), and it was found that the absolute percentage error in predicted discharge remained less than 6% for both cases. To summarise, all three entropies did not deviate much when calculations were done using the overall average value instead of the individual entropy parameter values. This paves the way to use a single averaged value of the entropy parameter for the whole channel for any value of channel bed slope, and it also reduces the computations significantly.

Hence, it can be concluded that the entropy parameters are not highly sensitive to the changes in the channel characteristics, especially the channel bed slope, as considered in the present study. The Shannon entropy parameter was initially surmised as a constant entity for a particular channel section (Chiu 1989; Moramarco & Singh 2010). By contrast, the present study provided a piece of experimental evidence to support the constant behaviour (here, overall average value).

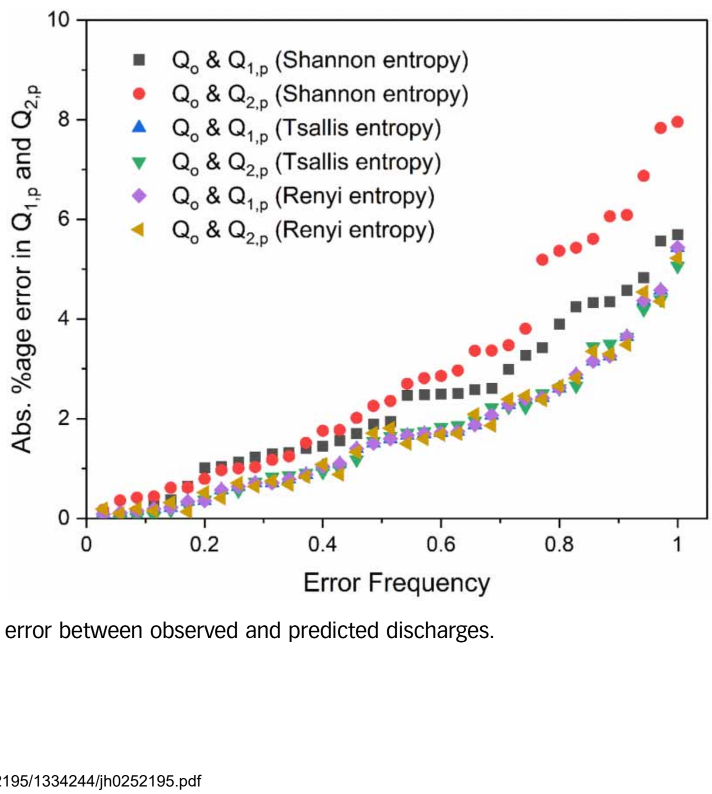


Figure 8 | Absolute percentage error between observed and predicted discharges.

5. CONCLUSION

In totality, the influence of the channel bed slope on the entropy parameters and the entropy-based mean and maximum velocity ratio was evaluated in this study. The key inferences regarding the predicted mean velocity and discharge were outlined as follows.

- Renyi entropy parameter values and overall average value lie in a small interval owing to the narrow range of significance and are confirmed by the minimal standard deviation and C_v value. By contrast, Shannon and Tsallis entropy showed higher standard deviation and C_v value leading to lower accuracy than the Renyi entropy.
- The lengthy and tedious relation of the mean and maximum velocity ratio in terms of the Renyi entropy parameter was modified into a more straightforward form reducing the computations significantly.
- Among all the entropies considered, Renyi entropy outperformed Shannon and Tsallis entropy, as evident from the statistical analysis results of the error between observed and predicted mean velocity and discharge. The absolute percentage error in the discharge calculations comes to be less than 10% in all the cases under consideration. High values of the NSE and R^2, and low values of the RMSE and MAPE also justified the high accuracy of the predicted discharge.
- Most importantly, the changes in the channel bed slope did not significantly affect the entropy parameters, as the absolute percentage error ranged less than 8% in the worst cases. Hence, the entropy parameter can be treated as a constant parameter instead of the surmised entity in the purview of the experimental data for the varying channel bed slopes. Interestingly, this conclusion firmly upholds the findings of Moramarco & Singh (2010) for the uniform open-channel flow.
- Finally, the acceptable results of the statistical analysis of the error pave the way for the convenient application of the overall average entropy parameter value to any field or laboratory velocity data under the varying channel bed slope conditions. Further investigations are, however, needed to generalise the constant nature of the entropy parameter for other cases by performing experiments with different channel and flow characteristics.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

CONFLICT OF INTEREST

The authors declare there is no conflict.

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