

## Discharge modeling and characteristic analysis of semi-circular side weir based on the soft computing method

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### ABSTRACT

In this study, a support vector machine (SVM) and three optimization algorithms are used to develop a discharge coefficient ( $C_d$ ) prediction model for the semi-circular side weir (SCSW). After that, we derived the input and output parameters of the model by dimensionless analysis as the ratio of the flow depth at the weir crest point upstream to the diameter ( $h_1/D$ ), the ratio of main channel width to diameter ( $B/D$ ), the ratio of side weir height to diameter ( $P/D$ ), upstream of side weir Froude number ( $F_r$ ), and  $C_d$ . The sensitivity coefficients for dimensionless parameters to  $C_d$  were calculated based on Sobol's method. The research shows that SVM and Genetic Algorithm (GA-SVM) have high prediction accuracy and generalization ability; the average error and maximum error were 0.08 and 2.47%, respectively, which were about 95.72 and 60.86% lower compared with the traditional empirical model. The first-order sensitivity coefficients  $S_1$  and global sensitivity coefficients  $S_j$  of  $h_1/D$ ,  $B/D$ ,  $P/D$ , and  $F_r$  were 0.35, 0.07, 0.13, and 0.02; 0.63, 0.25, 0.30, and 0.32, respectively.  $h_1/D$  has a significant effect on  $C_d$ . In particular, when  $h_1/D < 0.24$  and  $0.48 < F_r < 0.58$ ,  $0.67 < F_r < 0.72$ , the discharge capacity of the SCSW is relatively large.

**Key words:** dimensionless parameters, discharge characteristics, intelligent model, semi-circular side weir, Sobol's method

### HIGHLIGHTS

- We developed an effective and high-accuracy model for predicting the  $C_d$  of SCSW.
- The importance of dimensionless parameters on  $C_d$  was quantified by Sobol's method.
- It explored the flow characteristics of semi-circular side weir.

## 1. INTRODUCTION

As one of the most common diversion structures, side weirs are used for flow control, drainage networks, irrigation, and wastewater channels (Zahiri *et al.* 2013). In recent years, with the change in extreme weather and a significant increase in storm floods, side weirs have been used as common equipment in sewer networks and irrigation systems to divert excess water flow from channels to other channels (Uyumaz *et al.* 2014). Semi-circular labyrinth side weirs are widely used due to their long overflow front length, stable overflow structure, and facilitation of sediment removal. Also, semi-circular side weir (SCSW) flow as a spatially variable flow has more parameters affecting the discharge coefficient ( $C_d$ ). Therefore, it is important to accurately evaluate the influence and variation law of different factors on the  $C_d$  for the design and operation of this structure.

At present, most scholars mainly use traditional empirical methods to check the discharge capacity of SCSWs. Haghshenas & Vatankhah (2021) proposed discharge calculation equations for SCSW, in which the mean and maximum errors of the best model were 1.87 and 6.31%, respectively. Mamand & Raheem (2018) used SPSS software to fit the empirical equation of the SCSW, and the coefficients of determination ( $R^2$ ) in the form of multivariate linear regression and multivariate power regression were 0.8498 and 0.8584, respectively. Khalili & Honar (2017) gave the calculation equation of the  $C_d$  of the SCSW by using the model experiments and dimensional analysis. The research shows that the  $C_d$  of the SCSW was higher than that of the rectangular side weir. However, the discharge is affected by the plane position of the weir sill, the

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shape of the weir, the upstream and downstream flow conditions, and different flow resistances generated, resulting in different expressions of the  $C_d$ , which are not convenient for users. Also, the discharge coefficients were determined according to empirical equations, which were limited by certain datasets, effective parameter interactions, high uncertainty, numerous assumptions, and other defects (Tao *et al.* 2022), resulting in insufficient mining of physical properties among parameters and limited calculation accuracy.

In recent years, many scholars have attempted to use soft computing techniques for solving the problems of large calculations and inconvenient use of empirical equations (Haghbin & Sharafati 2022; Shen *et al.* 2022; Gharehbaghi *et al.* 2023; Parsaie *et al.* 2023; Seyedian *et al.* 2023; Yarahmadi *et al.* 2023). Jamei *et al.* (2021) developed three linear models for predicting the  $C_d$  of the triangular side orifices. The research shows that the intelligent model can accurately evaluate the discharge capacity of the side orifices under free-flow conditions. Tao *et al.* (2022) used three machine learning models to estimate the  $C_d$  prediction models of the gate under free-flow and submerged-flow conditions. The results show that the model has higher accuracy for the free-flow condition. Ismael *et al.* (2021) used neural network technology for predicting the  $C_d$  of inclined cylindrical weirs with different diameters; the root mean square error (RMSE) of the radial basis function network model was reduced by 9 and 41% compared with the cascade-forward neural network and the back-propagation neural network (BPNN) in the testing stage, respectively. However, with the wide application of intelligent models in weir flow, it has been gradually discovered that this technology has problems such as overfitting and easily falling into local optimum. Therefore, researchers began to try to optimize the hyperparameters of the model through optimization algorithms to derive the best model parameters to improve the forecast accuracy and stability of the model. For example, Haghbin *et al.* (2022) developed a hybrid data-driven approach to evaluate the  $C_d$  of step spillways, and the optimized model improved the performance index to 86.13%. Pradeep & Samui (2022) used a neural network technology hybrid optimization algorithm to predict rock strain, and the results showed that the optimized model was better than other single models in the training and testing phases. Chen *et al.* (2022) aimed to predict the discharge coefficient of streamlined weirs, and the results showed that the hybrid deep data-driven algorithms provide more accurate results than the classical ones. Simsek *et al.* (2023) used the artificial neural network (ANN) to predict the discharge coefficient of trapezoidal broad-crested weir; the study results showed that the Froude number significantly increases the performance of the models in estimating  $C_d$  values, and the ANN method was more successful in determining  $C_d$  than other methods. Balouchi & Rakhshandehroo (2018) used the soft computing models to evaluate the discharge coefficient for combined weir-gate, and multilayer perceptron was considered superior; it had better statistical indices of RMSE, mean absolute error (MAE), and  $R^2$  (0.027, 0.022, and 0.984, respectively).

However, the prediction model needs to meet the requirements of high accuracy and stability due to the large discharge and complex physical parameters of the SCSW. According to the current literature, research shows that a high-precision SCSW  $C_d$  prediction model has not been developed yet. Therefore, it is important to develop an accurate and stable prediction model for the  $C_d$  of SCSW in this study. In addition, there is also great interest in the interaction characteristics between model inputs and outputs. Zhang *et al.* (2013) used Sobol's method to analyze the sensitivity of potential hydrological processes under different hydrological models and climatic conditions. Nossent *et al.* (2011) successfully applied the Sobol sensitivity method to the prioritization of input parameters of complex environmental models. However, most scholars pay more attention to the stability and accuracy of the weir flow prediction model, and the interactions and variation relationships between input parameters and discharge coefficients have not been explored in depth. Hence, this paper not only establishes the discharge coefficient prediction model for the SCSW but also provides a new method for the accurate calculation of the discharge of the structure. More importantly, based on predecessors, the influence of dimensionless parameters on the discharge coefficient is quantified, and this study fills the research gap in this area.

In summary, this study aims to systematically evaluate the effects of the hydraulic parameters of SCSWs on the  $C_d$ . First, the particle swarm optimization (PSO) algorithm, genetic algorithm (GA), and sparrow search algorithm (SSA) are used to optimize the hyperparameters  $c$  and  $\gamma$  of the support vector machine (SVM) and establish three different models for predicting the  $C_d$  of SCSWs. Then, the accuracy and generalization ability of the intelligent and traditional empirical models are compared using different performance indexes. On this basis, Sobol's method is used to explore the interaction and change process between hydraulic parameters and  $C_d$  and analyze the change law of hydraulic parameters and  $C_d$ . The sensitivity of different hydraulic parameters to  $C_d$  is quantified to provide an essential reference basis for the design and promotion of SCSWs.

## 2. DATA AND MODELS

### 2.1. Experimental data

In this study, the dataset was obtained from Haghshenas & Vatankhah (2021). The experimental device consisted mainly of a pumping station with a recirculation system that provided a horizontal rectangular channel 12 m long, 0.25 m wide, and 0.5 m deep. The weir upstream discharge  $Q_1$  and side weir discharge  $Q_w$  were determined by triangular and rectangular weirs, respectively, and through the accuracy of  $\pm 0.5\%$  electromagnetic flowmeter calibration. The SCSW was installed on the main rectangular channel wall 6 m away from the inlet, and the downstream and upstream water depths of the weir were measured at the centerline of the main channel using a point gauge with an accuracy of 0.1 mm. The SCSWs were made of 10-mm-thick plexiglass sheets with a crest thickness of 1 mm, and the plan layout is shown in Figure 1. Three different weir heights ( $P = 5, 10, \text{ and } 15 \text{ cm}$ ; the weir crest height,  $P$ , varied from 5 to 15 cm for each value of weir diameter) and three different weir diameters ( $D = 25, 30, \text{ and } 40 \text{ cm}$ ) were measured in laboratory measurements.  $Q_1$  varied from 14.7 to 42.1 L/s,  $Q_w$  varied from 3.0 to 25.8 L/s, and the diverted discharge ratio was  $0.14 \leq Q_w/Q_1 \leq 0.73$ . A total of 155 runs were carried out under free-flow conditions, and the data characteristics are shown in Table 1.

### 2.2. Dimensional analysis

As can be seen from Figure 1, the variables that may affect the SCSW discharge include the following:  $h_1$  ( $h_1 = y_1 - p$ ) is the depth of flow relative to the crest point of the upstream weir,  $D$  is the side weir diameter,  $P$  is the side weir crest height,  $V_1$  is the mean velocity at upstream of the side weir,  $y_1$  is the flow depth at the upstream end of the side weir,  $B$  is main channel width,  $\rho$  is water density,  $\mu$  is water viscosity, and  $g$  is gravitational acceleration. The discharge of SCSW can be expressed as Equation (1).

$$Q_w = f_1(h_1, B, D, P, V_1, y_1, \rho, \mu, g) \quad (1)$$

According to the Buckingham- $\pi$  theorem, the above parameters were dimensionally analyzed (Haghshenas & Vatankhah 2021; Saffar *et al.* 2021), and  $D, g, \rho$  were used as three independent variables; the dimensionless parameters that affect  $C_d$  can be expressed as Equation (2).

$$C_d = f_2\left(\frac{h_1}{D}, \frac{B}{D}, \frac{P}{D}, F_r = \frac{V_1}{\sqrt{gy_1}}, Re = \frac{\rho DV_1}{\mu}\right) \quad (2)$$

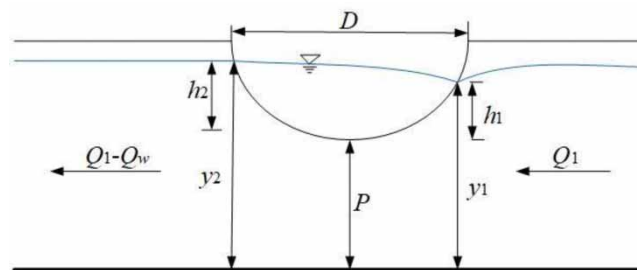


Figure 1 | Plane structure of SCSW.

Table 1 | Statistics of data characteristics

Statistical parameters	$B/D$	$P/D$	$h_1/D$	$F_r$	$C_d$
Maximum	1	0.6	0.469	0.815	0.780
Minimum	0.625	0.125	0.156	0.174	0.565
Mean	0.799	0.299	0.304	0.433	0.663
Middle quartile	0.833	0.250	0.305	0.420	0.652
SD	0.155	0.136	0.083	0.153	0.056

where  $F_r$  is the Froude number upstream of the side weir and  $R_e$  is the Reynolds number. For side weirs, the flow of water is usually turbulent, and the influence of dynamic viscosity ( $R_e$ ) on the hydraulic characteristics of flow is negligible (Norouzi *et al.* 2020); the dimensionless parameters affecting the  $C_d$  can be expressed as Equation (3).

$$C_d = f_5\left(\frac{h_1}{D}, \frac{B}{D}, \frac{P}{D}, F_r\right) \quad (3)$$

### 2.3. Support vector machine

In this study, the dataset is small, the sample uncertainty is high, and the sample parameters are highly nonlinear. Therefore, a suitable large-scale, fast, and robust model is selected. Meanwhile, SVM is a powerful supervised learning technique that can provide reliable and robust predictions (Najafzadeh & Oliveto 2020). Considering that the PSO and GA belong to the traditional swarm intelligence algorithm, and the SSA belongs to the new swarm intelligence algorithm by using the same dataset to compare the hyperparameter changes between the three algorithms, the stability and reliability of the model can be better determined.

SVM is a classification technique proposed by Vapnik based on the statistical learning theory and structural risk minimization (SRM) (Cortes & Vapnik 1995) and is now widely used for high-accuracy prediction due to its advantages in solving nonlinear problems (Ahmad *et al.* 2014; Parsaie *et al.* 2019; Najafzadeh & Niazmardi 2021; Parsaie *et al.* 2021). Its purpose is to generate a decision boundary between two classes, which is called a hyperplane, and the separating hyperplane is determined by the orthogonal vector  $w$  and the bias  $b$ . Its direction is as far away from the nearest data points in each class, and these nearest points are called support vectors (Huang *et al.* 2018); its model structure is shown in Figure 2. The solution to the nonlinear problem can be achieved by mapping the data to a higher dimensional feature space with the help of kernel functions (Najafzadeh *et al.* 2016). There are two very important parameters  $C$  and  $\gamma$  in the SVM model, and the parameter  $C$  represents the penalty. The value of  $C$  affects the prediction accuracy and the value of  $\gamma$  affects the partitioning of the feature space; the parameter  $\gamma$  has a greater impact on the results than the penalty factor  $C$ . Therefore, to obtain suitable  $C$  and  $\gamma$ , three optimization algorithms are used in this study to optimize the values of  $C$  and  $\gamma$  globally to obtain the best performance prediction model for the  $C_d$  of the SCSW. The optimization process is shown in Figure 3.

### 2.4. SVM and PSO

PSO is a classic population search algorithm, and its calculation equation is as follows (Huang & Dun 2008; Ardjani *et al.* 2010; Cuong-Le *et al.* 2022):

$$v_{i+1} = \omega v_i + c_1 r_1 (p_{\text{best}} - x_i) + c_2 r_2 (g_{\text{best}} - x_i) \quad (4)$$

$$x_{i+1} = x_i + v_{i+1} \quad (5)$$

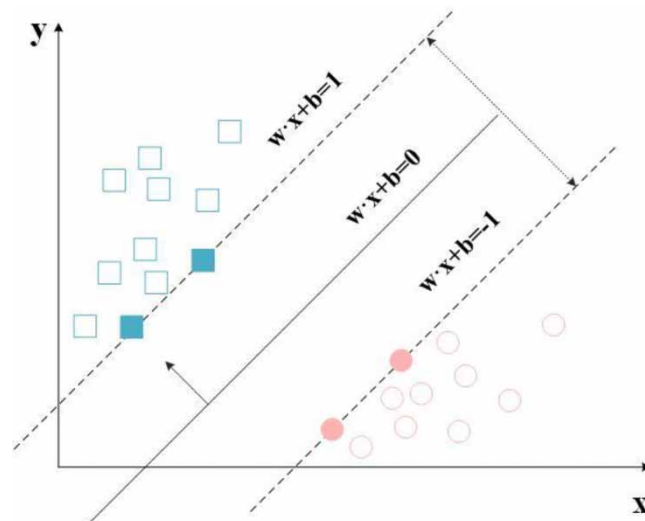
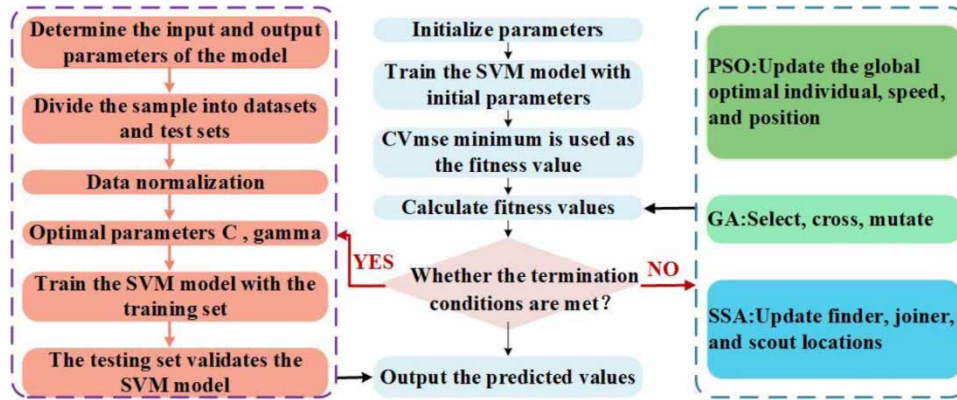


Figure 2 | SVM plan structure diagram.



**Figure 3** | Process diagram of optimization algorithm.

where  $x_i$  is the position of each particle,  $v_i$  is the velocity of each particle,  $p_{best}$  is the particle optimal value,  $g_{best}$  is the global optimal value,  $r_1$  and  $r_2$  are random numbers between 0 and 1, and  $c_1$  and  $c_2$  are acceleration factors.

The particle swarm regards the two parameters of  $C$  and  $\gamma$  of the SVM as two particle swarms and first sets the parameters of population size and iteration number for population and velocity initialization, inputs the randomly generated  $C$  and  $\gamma$  into the SVM model for training; the mean square error of model cross-validation ( $CV_{mse}$ ) is used as the model fitness function, the minimum fitness of the particle represents the optimal particle position at this time, and the optimization algorithm ends when the iteration number meets the set value.

## 2.5. SVM and GA

The GA is an adaptive optimization method with a global search function that uses random search to efficiently guide the parameter space to encode each individual. The key technology of the algorithm consists of five elements: encoding of parameters, initialization of the population, calculation of the fitness function, layout of genetic operations, and control of the parameter arrangement (Li & Kong 2014). Therefore, through continuous evolution from generation to generation, an optimally adapted individual can eventually be obtained. It has the advantages of global optimality, implicit parallelism, high stability, and wide availability (Li & Kong 2014; Guan *et al.* 2021).

The basic steps of the GA:

- (1) *Encoding*: The GA represents the solution data in the solution space as genotypic string structure data in the genetic space before searching, and the different combinations of these string structure data constitute the different points.
- (2) *Initial population generation*:  $N$  initial string structure data are randomly generated, each string structure data is called an individual,  $N$  individuals form a population, and the GA uses these  $N$  string structure data as initial points to start evolution.
- (3) *Adaptability evaluation*: Adaptability indicates the strengths and weaknesses of individuals or solutions. The fitness function is defined in different ways for different problems.

Finally, the optimal solution is obtained by three basic operations: selection, crossover, and variation.

## 2.6. SVM and SSA

The SSA is a new intelligent optimization algorithm that simulates the foraging and anti-predation behavior of sparrows (Xue & Shen 2020; Yan *et al.* 2022). At present, it has been widely used in related fields. Throughout the foraging process, there are three behaviors: discoverer, joiner, and alerter. Among them, the identities of the discoverer and joiner are changed dynamically. The location update of the discoverer is shown in Equation (6).

$$X_{ij}^{t+1} = \begin{cases} X_{ij}^t \exp\left(-\frac{i}{\alpha C_{\max}}\right), & R_2 < ST \\ X_{ij}^t + QL, & R_2 \geq ST \end{cases} \quad (6)$$



where  $X_{ij}^t$  is the position of the  $i$ th individual in the region dimension after the  $t$ th iteration in the sparrow population;  $a$  is a uniform random number,  $a \in (0,1)$ ;  $C_{\max}$  is the maximum number of iterations;  $R_2$  is a random number with a warning value of  $[0,1]$ ; ST is the security threshold with an interval of  $[0.5, 1]$ ;  $Q$  is a random number and obeys the standard normal distribution; and  $L$  is a  $1 \times d$  dimensional matrix. The location update of the joiner is shown in Equation (7).

$$X_{ij}^{t+1} = \begin{cases} Q \exp\left(-\frac{X_w - X_{ij}^t}{i^2}\right) & i > n/2 \\ X_p^{t+1} + |X_{ij}^t - X_p^{t+1}|A^+L, & i \leq n/2 \end{cases} \quad (7)$$

where  $X_p^{t+1}$  is the best position of the discoverer in the  $t + 1$  iteration;  $X_w$  is the worst position in the current sparrow population;  $A^+ = A^T(AA^T)^{-1}$ ,  $A$  is  $1 \times d$  matrix with element 1 or  $-1$ .  $n$  is the population size. When  $i > n/2$ , the  $i$ th joiner with low fitness is not fed and needs to be foraged; conversely, when  $i \leq n/2$ , the joiner will forage near the optimal position. The location update of the alerter is shown in Equation (8).

$$X_{ij}^{t+1} = \begin{cases} X_b^t + \beta|X_{ij}^t - X_b^t|, & f_i > f_g \\ X_{ij}^t + K \left( \frac{|X_{ij}^t - X_w^t|}{(f_i - f_w) + \varepsilon} \right), & f_i = f_g \end{cases} \quad (8)$$

where  $X_b^t$  is the global optimum position;  $\beta$  is a random number that conforms to the standard normal distribution.  $K$  is a uniform random number,  $K \in [-1,1]$ ;  $f_i$ ,  $f_w$ , and  $f_g$  are the current position fitness value, the worst position fitness value, and the optimal position fitness value, respectively;  $\varepsilon$  is the minimum constant to the denominator is not 0.  $f_i > f_g$  indicates that sparrows are at the edge of the population and are vulnerable to attack;  $f_i = f_g$  indicates that sparrows are in the middle of the population, warning of danger, and adjust the search strategy in time to avoid attacks.

## 2.7. Sobol's sensitivity analysis method

The Sobol method (Sobol 1990), as a global sensitivity analysis method based on variance decomposition, obtained the importance of the input parameters on the output results by calculating the first-order sensitivity and the global sensitivity of the input parameters (Lu et al. 2018). The objective function  $f(x)$  of the model is decomposed as the sum of  $2^p$  increasing terms:

$$f(x_1, x_2, \dots, x_n) = f_0 + \sum_{i=1}^n f_i(x_i) + \sum_{i=1}^n \sum_{j=i+1}^n f_{ij}(x_i, x_j) + \dots + f_{i_1, \dots, i_n}(x_{i_1}, \dots, x_{i_n}) \quad (9)$$

where  $f_0$  represents the constant in the objective function, and each integral variable in the formula is 0, then the expression is

$$\int_0^1 f_{i_1, i_2, \dots, i_s}(x_{i_1}, x_{i_2}, \dots, x_{i_s}) dx_{i_k} = 0 \quad (10)$$

$$\text{where } 1 \leq i_1 < \dots < i_s \leq n, \quad 1 \leq k \leq n \quad (11)$$

$$\text{Total variance: } V(Y) = \int f^2(x) dx - f_0^2 \quad (12)$$

$$\text{Partial variance: } V_{i_1, i_2, \dots, i_s} = \int f^2_{i_1, i_2, \dots, i_s} dx_{i_1} dx_{i_2} \dots dx_{i_s} \quad (13)$$

$$\text{First-order sensitivity coefficient: } S_i = \frac{V_i}{V(Y)} \quad (14)$$

$$\text{Global sensitivity coefficient: } S_{Ti} = 1 - \frac{V_{\sim i}}{V(Y)} \quad (15)$$

where  $V(Y)$  represents the sum of the parameters on the output results of the model objective function  $f(x)$ ;  $V_{i_1, i_2, \dots, i_s}$

represents the influence of the interaction of the parameter combination on the model output results.  $V_i$  represents the influence of the  $i$ th parameter on the output result of the model objective function  $f(x)$ ; and  $V_{\sim i}$  represents the sum of the variance caused by all parameters except the  $i$ th parameter.

## 2.8. Evaluation index

This study used several statistical methods to evaluate model performance. The parameters are RMSE, correlation coefficient ( $R$ ), mean absolute percentage error (MAPE), standard deviation (SD), scatter index (SI), developed discrepancy ratio (DDR), bias coefficient (Bias). Methods are defined in the following equations.

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^N (O_i - P_i)^2}{N}} \quad (16)$$

$$\text{MAPE} = \frac{1}{N} \sum_{i=1}^N \left| \frac{O_i - P_i}{O_i} \right| \times 100 \quad (17)$$

$$R = \frac{\sum_{i=1}^N (O_i - \bar{O}_i)(P_i - \bar{P}_i)}{\sqrt{\sum_{i=1}^N (O_i - \bar{O}_i)^2 \sum_{i=1}^N (P_i - \bar{P}_i)^2}} \quad (18)$$

$$\text{SD} = \sqrt{\frac{1}{N} \sum_{i=1}^N (O_i - \bar{O}_i)^2} \quad (19)$$

$$\text{SI} = \frac{\text{RMSE}}{O_a} \quad (20)$$

$$\text{DDR} = \frac{P_i}{O_i} - 1 \quad (21)$$

$$\text{Bias} = \frac{1}{n} \sum_{i=1}^n (P_i - O_i)^2 \quad (22)$$

where  $O_i$  and  $P_i$  represent the experimental and predicted values of  $C_d$ , respectively, and  $O_a$  and  $P_a$  represent the mean of the experimental and predicted values, respectively.

## 3. RESULTS AND DISCUSSION

### 3.1. Model comparison

In this study, 109 experimental datasets were selected as the training set and the remaining 46 sets were used as the testing set.  $h_1/D$ ,  $B/D$ ,  $P/D$ , and  $F_r$  were used as model inputs and  $C_d$  as model outputs. The global optimization of the hyperparameters  $C$  and  $\gamma$  of SVM was performed by three optimization algorithms, PSO, GA, and SSA; the specific parameter settings of each model are shown in Table 2, and the performance indexes of all models were finally obtained as shown in Tables 3 and 4. When the SVM model is used to calculate the  $C_d$  of the SCSW, the RMSE, MAPE, SD, and  $R$  were 0.047, 0.076, 0.073, and 0.897 in the training phase, respectively. The RMSE, MAPE, SD, and  $R$  were 0.045, 0.072, 0.062, and 0.926 in the testing phase, respectively. The PSO-SVM, GA-SVM, and SSA-SVM are significantly superior in each evaluation index in the training and testing phases than SVM, indicating that all three optimization algorithms can effectively improve the performance of SVM through global optimization search.

Figure 4 shows the scatter plot for the experimental and predicted values for all models. Larger values of  $R$  indicate the better fitting ability of the models, and the closer the predicted and experimental values are to the trend line (1:1). As can be seen from Figure 4, compared with SVM, the  $R$  of PSO-SVM, GA-SVM, and SSA-SVM increased by about 6.65, 9.11, and 7.23% in the training phase, respectively. Also, the  $R$  increased by about 2.83, 4.04, and 2.42% in the testing phase, respectively. It can be seen that among the three optimization models, GA-SVM has better generalization ability and

**Table 2** | Parameter settings of all models

Model	Parameter	Value	c	$\gamma$
PSO-SVM	Particle swarm size	20	0.1	6.72
	Number of iterations	30		
	Inertia factor	0.9		
	Acceleration constants	2		
	Speed range	[-1,1]		
GA-SVM	Population size	20	4.05	4.40
	Number of iterations	30		
	Crossover probability	0.5		
	Mutation probability	0.1		
SSA-SVM	Number of sparrows	20	0.1	4.33
	Number of iterations	30		
	warning value ST	0.6		
	Proportion of discoverers	0.7		
	Proportion of detectors	0.2		

**Table 3** | All model performance indexes in the training stage

Model	RMSE	MAPE (%)	SD	R	SI	Bias
SVM	0.047	0.076	0.073	0.897	0.071	0.0130
PSO-SVM	0.021	0.053	0.043	0.961	0.031	0.0028
GA-SVM	0.014	0.037	0.041	0.987	0.022	0.0008
SSA-SVM	0.019	0.048	0.044	0.967	0.024	0.0031

**Table 4** | All model performance indexes in the testing stage

Model	RMSE	MAPE (%)	SD	R	SI	Bias
SVM	0.045	0.072	0.062	0.926	0.069	0.0120
PSO-SVM	0.017	0.016	0.046	0.953	0.026	0.0010
GA-SVM	0.009	0.008	0.043	0.965	0.014	0.0004
SSA-SVM	0.016	0.016	0.047	0.949	0.031	0.0008

prediction stability. Figure 5 shows the Taylor plots for all the models; the longitudinal distance from the origin represents the SD, the purple radial lines indicate the correlation coefficient ( $R$ ), and the green circular arcs show the RMSE. As the circle section expanded, this parameter value increased. Moreover, the SD,  $R$ , and RMSE of the training and testing phases were specified by a single point, and the model closest to the reference point was considered the best model. It can be seen that SVM has the worst prediction effect and GA-SVM has the best prediction result, where PSO-SVM and SSA-SVM have almost the same effect. Therefore, GA-SVM can be used as the optimal intelligent prediction model for the  $C_d$  of SCSW.

### 3.2. Comparison with empirical equations

Figure 6 shows the DDR values for all models, which can be used to evaluate the distribution of errors in detail. It can be seen that the overall error of GA-SVM is small, indicating that the model has high prediction accuracy. Meanwhile, Haghshenas & Vatankhah (2021) used the dimensional analysis technique to fit the SCSW discharge calculation model, and the average error and maximum error of the best model were 1.87 and 6.31%, respectively. The mean and maximum errors of GA-SVM were 0.08 and 2.47%, respectively, which were about 95.72 and 60.86% lower compared to the traditional empirical model, indicating that the intelligent model has higher accuracy in predicting the  $C_d$  of SCSWs. Figure 7 shows the error



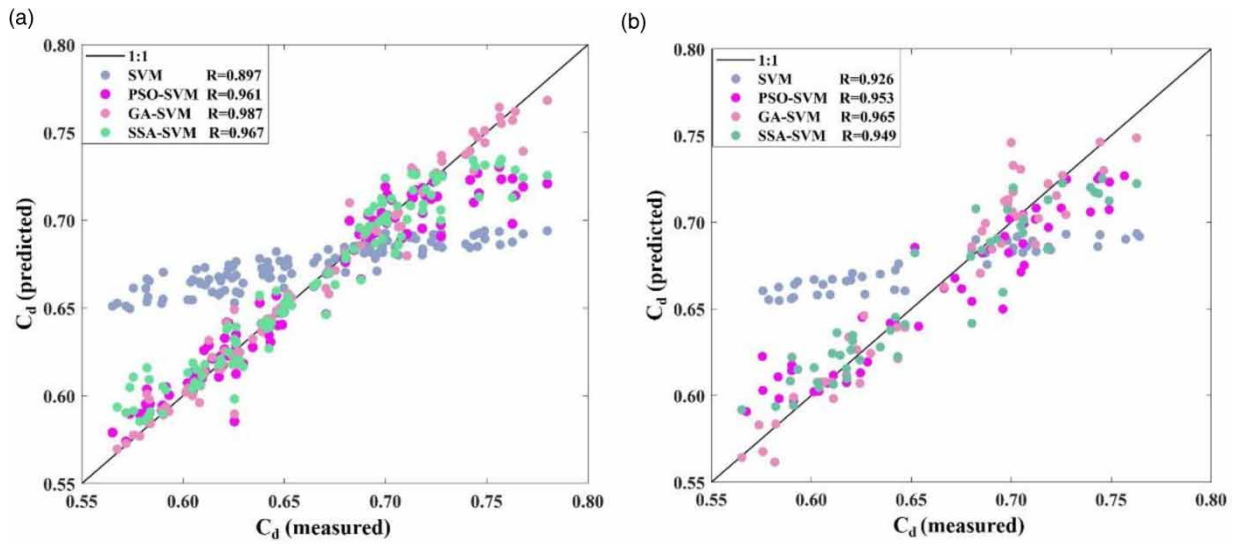


Figure 4 | Scatter plot of experimental and predicted values of  $C_d$ . (a) Training stage and (b) testing stage.

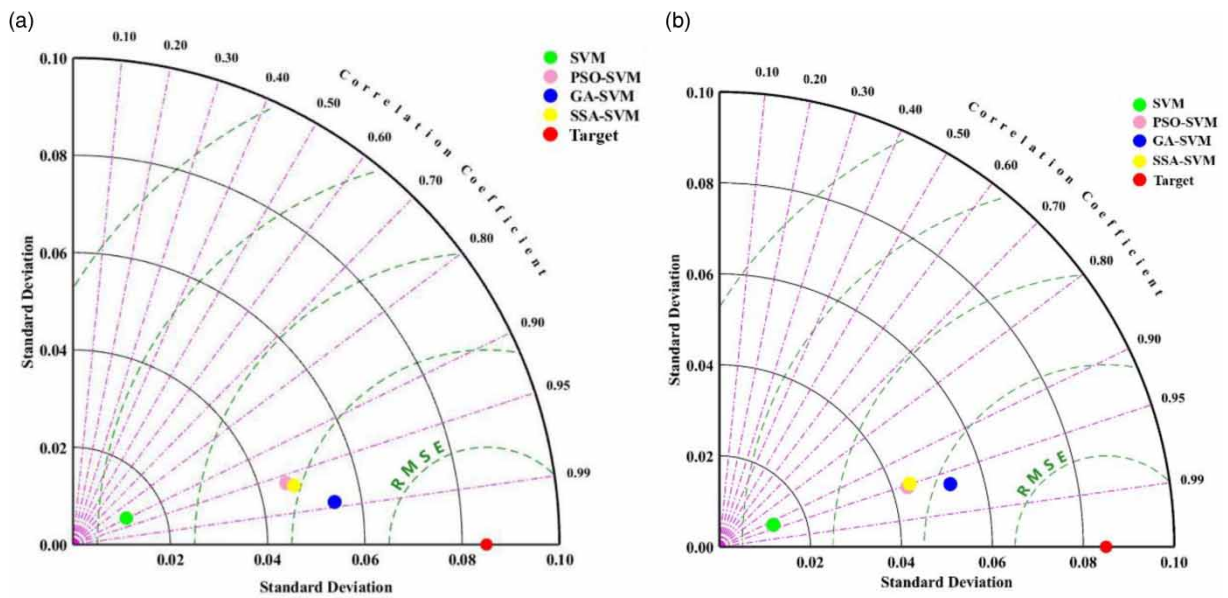


Figure 5 | The Taylor diagram for all models. (a) Training stage and (b) testing stage.

density plot of GA-SVM in the testing phase; 91.31% of the prediction errors were below 2%, and the errors were mainly concentrated at  $C_d = 0.7$ , indicating that GA-SVM has high accuracy and stability in predicting the  $C_d$ .

### 3.3. Quantitative analysis of parameters

From the above analysis, it can be seen that GA-SVM can be used as a prediction model for the  $C_d$  of SCSW. Therefore, the model input parameters were further quantified and analyzed using Sobol's method. As can be seen from Figure 8, the sensitivity coefficients of the dimensionless parameters and the first-order sensitivity coefficients  $S_1$  of  $h_1/D$ ,  $B/D$ ,  $P/D$ , and  $F_T$  were 0.35, 0.07, 0.13, and 0.02, respectively, indicating that when only the influence of a single parameter on  $C_d$  was

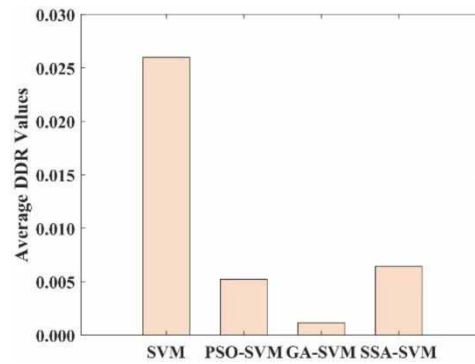


Figure 6 | DDR values for all models.

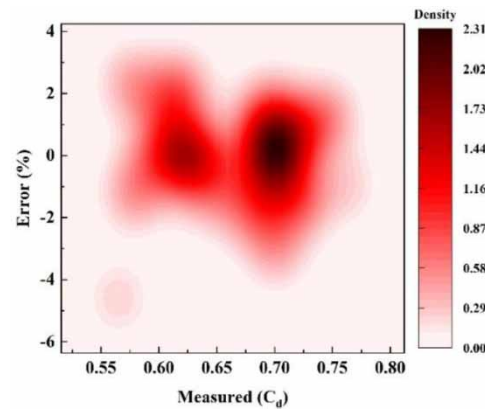


Figure 7 | Error density plot of GA-SVM in the testing phase.

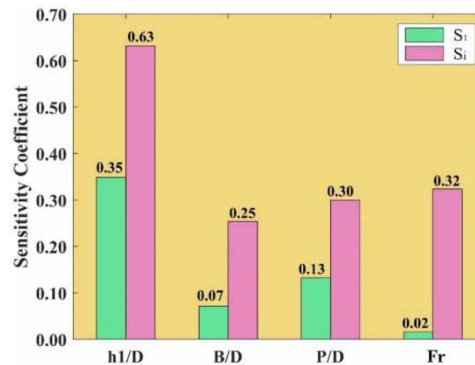


Figure 8 | Sensitivity coefficients of dimensionless parameters.

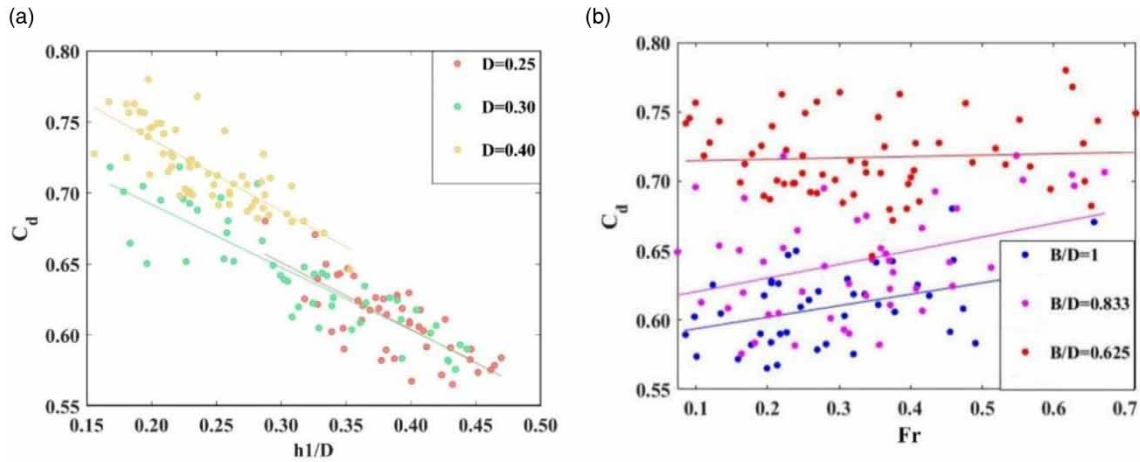
considered, the influence of  $h_1/D$  on  $C_d$  was the largest, followed by  $P/D$ , and  $Fr$  has the least effect on  $C_d$ . When a parameter interacts with other parameters, the global sensitivity coefficients  $S_i$  for  $h_1/D$ ,  $B/D$ ,  $P/D$ , and  $Fr$  were 0.63, 0.25, 0.30, and 0.32, respectively. Also,  $h_1/D$  has the greatest effect on  $C_d$ , indicating that  $h_1/D$  is an important parameter affecting  $C_d$ . However, the effect of  $Fr$  on  $C_d$  after interacting with other parameters was only inferior to  $h_1/D$ , indicating that the ratio of flow velocity to flow depth plays an important role in the assessment of  $C_d$  under the influence of geometric parameters. Therefore,  $h_1/D$  and  $Fr$  should be considered important parameters when assessing the discharge capacity of SCSW.

### 3.4. Parameter sensitivity analysis

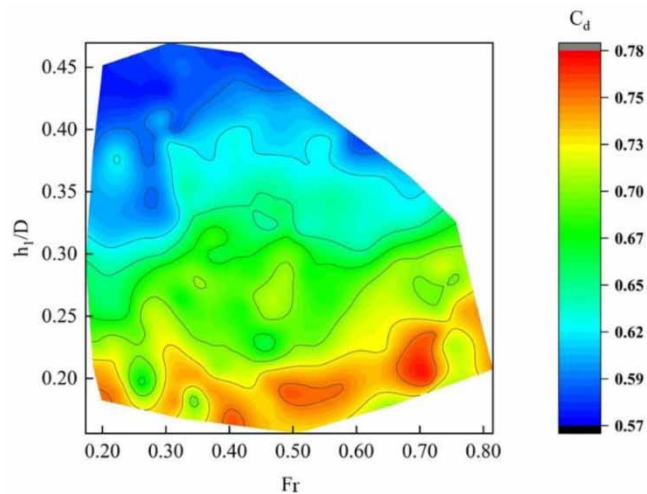
Figure 9 shows the variation relationship between the dimensionless parameters  $h_1/D$ ,  $B/D$ ,  $P/D$ , and  $F_r$  and the predicted  $C_d$  for the SCSW. As can be seen from Figure 9(a),  $C_d$  decreases with the increase of  $h_1/D$ . As  $D$  increases, the trend of  $C_d$  decreases more obviously; where the variation trend of  $D = 0.25$  m and  $D = 0.30$  m is very close. As can be seen from Figure 9(b),  $C_d$  increases with the increase of  $F_r$ , and the trend increases more slowly when  $B/D = 0.625$ . The trend increases more obviously when  $B/D = 0.833$ . For the same  $F_r$ ,  $C_d$  shows a decreasing trend as  $B/D$  increases.

### 3.5. Analysis of discharge characteristics

Figure 10 shows the variation of the most influential input parameters against the predicted  $C_d$ . It can be seen that when  $h_1/D < 0.24$ ,  $0.48 < F_r < 0.58$ , and  $0.67 < F_r < 0.72$ , the  $C_d$  of the SCSW is larger and has a higher discharge capacity of the SCSW at this time. Also, when  $F_r < 0.50$  and  $0.40 < h_1/D < 0.47$ , the  $C_d$  of SCSW is smaller, and the discharge capacity of SCSW is relatively small at this time.



**Figure 9** | Variation of the dimensionless parameters versus predicted discharge coefficient. (a) The variation between  $h_1/D$  and predicted  $C_d$ . (b) The variation between  $F_r$  and predicted  $C_d$ .



**Figure 10** | Variation of the most influential inputs versus predicted  $C_d$ .

#### 4. CONCLUSION

In order to achieve accurate water measurement and reasonable distribution of water resources in small channels, a semi-circular labyrinth side weir is used as an efficient and greater discharge capacity control structure. In this study, PSO-SVM, GA-SVM, and SSA-SVM optimization models were developed based on SVM. Then, Sobol's method was introduced to calculate the sensitivity coefficients of different dimensionless parameters  $h_1/D$ ,  $B/D$ ,  $P/D$ , and  $F_r$  to  $C_d$ . This paper evaluates the effect of different factors on the discharge capacity of SCSW. The parameter variation range of various discharge capacities is proposed, and the variation law between different parameters and  $C_d$  is analyzed. The following conclusions were drawn.

- (1) In the current study, GA-SVM can be used as an efficient and high-accuracy prediction model for the  $C_d$  of SCSW. In the testing phase,  $R = 0.987$ ,  $MAPE = 0.037\%$ ,  $RMSE = 0.014$ ,  $SD = 0.041$ ,  $SI = 0.022$ , and  $Bias = 0.008$ , and 91.31% of the prediction errors were below 2%; the model has high generalization ability, stability, and prediction accuracy, and this model effectively solves the problems of large computational complexity and difficult coefficient correction in traditional empirical models.
- (2) The quantitative analysis showed that the  $S_1$  and  $S_i$  of  $h_1/D$ ,  $B/D$ ,  $P/D$ , and  $F_r$  were 0.35, 0.07, 0.13, and 0.02; and 0.63, 0.25, 0.30, and 0.32, respectively;  $h_1/D$  was the most important parameter affecting  $C_d$ , the effect of  $F_r$  on  $C_d$  after interacting with other parameters was only inferior to  $h_1/D$ , and  $C_d$  decreased as  $h_1/D$  increased. As  $D$  increased,  $C_d$  decreased the greater the trend. As the diameter of the side weir increases, the lateral flow will increase significantly in the subcritical flow regime.
- (3) When  $h_1/D < 0.24$ ,  $0.48 < F_r < 0.58$ , and  $0.67 < F_r < 0.72$ , the  $C_d$  of SCSW is greater. Meanwhile, when  $F_r < 0.50$  and  $0.40 < h_1/D < 0.47$ , the  $C_d$  of the SCSW is relatively small. This can provide an important reference basis for the application of SCSW in practical engineering.

In addition, in this study, the width of the main channel is constant. Therefore, it is necessary to further explore the influence of the width change of the main channel on the discharge coefficient of the SCSW.

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#### DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

#### CONFLICT OF INTEREST

The authors declare there is no conflict.

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