A New Snow Density Parameterization for Land Data Initialization

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ABSTRACT

Snow initialization is crucial for weather and seasonal prediction, but the National Centers for Environmental Prediction (NCEP) operational models have been found to produce too little snow water equivalent, partly because they assume a constant and unrealistically low snow density for the snowpack. One possible solution is to use the snow density formulation from the Noah land model used in NCEP operational forecast models. While this solution is better than the constant density assumption, the seasonal evolution of snow density in Noah is still found to be unrealistic, through the evaluation of both the offline Noah model output and the Noah snow density formulation itself. A physically based snow density parameterization is then developed, which performs considerably better than the Noah parameterization based on the measurements from the SNOTEL network over the western United States and Alaska. It also performs better than the snow density schemes used in three other models. This parameterization could be easily implemented in NCEP operational snow initialization. With the consideration of up to 10 snow layers, this parameterization can also be applied to multilayer snowpack initiation or to estimate snow water equivalent from in situ and airborne snow depth measurements.

1. Introduction

Snow has a strong influence on land–atmosphere interactions because of its much higher albedo and different thermal properties than snow-free land. It is also important hydrologically, in some areas accounting for a majority of water resources, and affecting soil moisture and vegetation well after snowmelt. Dawson et al. (2016) have shown that initialized snow depth (SD) in regional and global operational forecast models at the National Centers for Environmental Prediction (NCEP) is too shallow, especially in mountainous regions, regardless of model grid size (discussed further in section 4). In these models, initialized SWE is even worse because it is obtained by multiplying initialized SD by unrealistically low globally and temporally constant snow densities for thick, well-developed snowpacks: 100 g cm\(^{-3}\) for the Global Forecast System (GFS; Environmental Modeling Center 2003) and Climate Forecast System (CFS; Saha et al. 2014) and 0.200 g cm\(^{-3}\) for the North American Mesoscale Forecast System (NAM; Janjić and Gall 2012) model. Recognizing the wide use of NCEP operational forecasting products (including snow) in the United States and worldwide, there is a need to improve the model snow initializations.

A logical option is to use the Noah snow density formulation, because Noah is the land surface model (LSM) used in the GFS, CFS, and NAM. However, we need to first evaluate how realistic the temporal evolution of snow density produced by Noah is under different environmental conditions. Specifically, we evaluate Noah as used in the North American Land Data Assimilation System (NLDAS; Cosgrove et al. 2003) against observations from the National Resources Conservation Service (NRCS) Snowpack Telemetry (SNOTEL) network. In NLDAS, there is no assimilation of snow data from an external source (such as the method for some operational NCEP initializations), and so the evolution of snow density is reflective of that predicted by Noah. While snow density is more spatially consistent than SWE or snow depth, the scale mismatch between the SNOTEL observations and the NLDAS grid boxes introduces some uncertainty. Therefore, we also test the Noah snow density formulation forced directly with the SNOTEL observations and the NLDAS grid boxes introduces some uncertainty. Therefore, we also test the Noah snow density parameterization for Land Data Initialization.
NLDAS for comparison: the Variable Infiltration Capacity (VIC) model (Wood et al. 1997) and Sacramento Soil Moisture Accounting (SAC-SMA) model coupled with the SNOW-17 model (Burnash et al. 1973; Anderson 1976), as well as that produced by the Snow Data Assimilation System (SNODAS; Barrett 2003).

Another option is to use an external snow density parameterization that is appropriate for predicting snow density from given values of SWE or SD. There are many such methods available to predict snow density in this way, from empirical models that estimate density range in complexity from linear regressions (e.g., Elder et al. 1991; Marchand and Killingtveit 2004; Lundberg et al. 2006; Jonas et al. 2009) to the inclusion of multiple predictor variables (e.g., SD, day of year, elevation) and lookup tables (e.g., Jonas et al. 2009; Sturm et al. 2010; McCreight and Small 2014). Avanzi et al. (2015a) compared 18 empirical density models to 10 SNOTEL observations and found a general overestimation of density and an increase in error with increasing elevation. The use of predictor variables instead of physical processes in the aforementioned models especially limits their applicability for daily data assimilation. Recently, a physically based density model has been developed (e.g., Avanzi et al. 2015b) to predict hourly SWE from SD. It has been shown to perform well for a few locations, but it has not demonstrated the transferability of the site-specific parameterization to global scales. Following these approaches, our second goal is to develop a new, physically based snow density parameterization for regional and global operational snow data initialization.

2. Snow data and snow density parameterizations

a. Snow data

This study uses daily observations of SWE and SD from the NRCS SNOTEL network (locations shown in Fig. 1). Data from water years (WYs) 2012–14 are used, where each WY begins on 1 October of the previous year. Quality control is performed on the daily data to remove spurious observations, as in Dawson et al. (2016). Additional preprocessing not performed in Dawson et al. (2016) for the SNOTEL data includes 1) replacement of SD and SWE values that produce density over 0.500 g cm\(^{-3}\) with a missing value, as reliable values above this are rare; 2) setting SWE and SD observations to zero if either the SWE or SD value is zero; 3) removing sites that have zero days of nonzero SWE in a respective WY (to avoid using sites that only report zero SWE that would artificially reduce calculated error for the parameterization); and 4) removing sites with more than six consecutive days of missing daily 2-m mean air temperature \(T_{2m}\).

After the completion of the aforementioned quality controls, SNOTEL stations with more than 75 missing
SWE or SD values in a respective WY are deemed unreliable and removed (which accounts for 3.5% of the SNOTEL dataset). In addition to analyzing the entire dataset, we analyze a subset of data from sites that are in the conterminous United States (CONUS; i.e., excluding sites in Alaska). The amount of all SNOTEL sites utilized in this study is 673, 688, and 698 for WYs 2012, 2013, and 2014, respectively (643, 657, and 668 of them are in CONUS).

The SNOTEL point measurements are used to evaluate the snow densities produced by the Noah LSM (as implemented in NLDAS) in addition to two other NLDAS LSMs, SAC-SMA and VIC, and also SNODAS (developed by the NWS National Operational Hydrologic Remote Sensing Center). Note that the MOSAIC LSM (Koster and Suarez 1992; also included in NLDAS) is not included in this study because of the assumption of constant snow density in the NLDAS implementation. The NLDAS models are evaluated on a 0.125° × 0.125° latitude–longitude grid, and SNODAS is evaluated on a 30-arc-s latitude–longitude grid. The snow density formulations in all of these products are described in the next section.

We take two approaches to demonstrate the robustness of our results with respect to the scale mismatch between the points (i.e., SNOTEL data) and NLDAS and SNODAS grid boxes: first, we evaluate the Noah snow density parameterization (forced directly by the in situ daily SWE and T_{2m} data) with the SNOTEL data. We also evaluate all of the products with snow density estimates obtained from our observationally based gridded SWE and SD dataset (Dawson et al. 2016). Aggregation of point observations to larger spatial scales removes small-scale variability that gridded model products are unable to represent (Dawson et al. 2016). For this dataset, point measurements of SWE (from SNOTEL stations) and SD [from SNOTEL and NWS Cooperative Observer Program (COOP) stations] are upscaled to gridded data by first applying piecewise regressions of the SWE and SD data, binned into elevation bands, onto a 0.01° × 0.01° digital elevation map to obtain a first guess. Next, residuals between observations and the first guess are interpolated using optimal interpolation and added to the first guess for a final analysis. The interpolation takes into account the vertical and horizontal distances of the observations to limit the influence of relatively high-elevation SNOTEL data on lower elevations and low-elevation COOP data on higher elevations (Dawson et al. 2016). The SNOTEL point measurements are also used in the development of our new snow density parameterization (described below).

b. Snow density parameterizations

The Noah LSM (Ek et al. 2003) is the focus of this study because of its implementation in the GFS, CFS, and NAM forecast models, and its density formulations are provided in appendix A.

Snow quantities in the Noah LSM are represented as a single layer. Compaction and overburden are combined into a single iterative process that is a simplified version of the SNOW-17 model. The effect of melt–refreeze cycles on density is accounted for by adjusting snow density with an estimated amount of snowpack liquid water.

SAC-SMA uses a single snow layer (handled by the SNOW-17 model) with 12 model parameters and one hourly time step. SNOW-17 tracks the snowpack heat deficit from sublimation and air temperature changes. Snow density is calculated based on the formulation of Koren et al. (1999). The density formulation was found to underestimate density for locations with deep snow and overestimate density for very cold locations; however, averaged across all climates, the results were adequate (Anderson 2006). Snowpack liquid water is tracked by SNOW-17, and the removal of column water is empirically estimated from lysimeter data as a lag time based on a ratio of total column ice and excess liquid water. The snow density is then adjusted to account for melt–refreeze processes.

The VIC LSM has a quasi-two-layer snowpack with a thin surface layer for energy balance calculations [similar to Anderson (2006); Andreadis et al. 2009] and the rest of the snowpack as the second layer. The LSM includes an energy balance snowpack model, an canopy interception scheme, and a snow redistribution model. The model utilizes elevation bands (to account for orographic effects) as well as three calibration parameters. The density formulation estimates the density of new snow (estimated with T_{2m}) and combines settling and overburden into one equation similar to Anderson (2006). Settling is estimated by mean snowpack temperature and density-dependent parameters (adjustment due to snowpack liquid water is accounted for by doubling one parameter). Overburden is estimated as half of the new SWE plus a fraction of the previous total SWE divided by the snow viscosity. Then the change of density is the sum of these two rates, one for settling and the other for overburden.

SNODAS assimilates a variety of data (i.e., in situ, remotely sensed, and airborne observations) into a multilayer snow model [based on the Snow Thermal (SN THERM) model; Jordan 1991] that is forced by weather forecast model output. The SNTHERM model operates similarly to VIC in that settling and compaction are calculated as two rates and added together. The total change in density accounts for destructive metamorphism (settling and compaction), constructive metamorphism (taken as a function of snowpack water vapor), and snowpack liquid water (Jordan 1991).
Besides Noah, VIC, and SAC-SMA (all available from NLDAS) and SNODAS (produced by the National Operational Hydrologic Remote Sensing Center), there are many other LSMs, but the comprehensive evaluation of snow densities from all LSMs goes beyond the scope of this study.

Recognizing the deficiencies of the above snow density parameterizations (see section 3), we also develop a simple, physically based snow density (SNODEN) parameterization for operational snow initialization driven by daily $T_{2m}$ and SWE data. Physical processes include temperature-based aging (destructive metamorphism), overburden (the pressure caused by overlying snowpack), and an estimation of liquid water (due to melt) within the snowpack. Snow densification by wind is implicitly accounted for through the classification of observations into snow classes (Sturm et al. 1995; also see discussion in appendix B). For simplicity, SNODEN does not account for the constructive metamorphism that changes the vertical mass distribution but has no effect on the bulk density (Anderson 2006). All of these physical processes are ignored when a spatially and temporally constant snow density is utilized for SWE initialization in NAM, GFS, and CFS.

State variables include snow density and SWE for individual snow layers, as well as total snowpack liquid water (including SWE and melted water in the snowpack). Every day, SNODEN progresses through four steps: 1) adjustment of density from the previous day, 2) density adjustments based on SWE increments (i.e., positive or negative), 3) density adjustment due to $T_{2m}$, and 4) final adjustment due to density changes from steps 1 to 3. The steps are executed within a framework representing a snowpack of up to 10 static layers (bins) based on density. Density within each bin is weighted by the associated SWE and summed to estimate daily bulk density for comparison to observations. Further details (including all model equations) are given in appendix B.

3. Results

a. Evaluation of snow density from Noah

To determine whether Noah produces density accurately enough for use for snow initialization in NCEP’s operational models, we first compare Noah snow density from NLDAS (which we call Noah$_{N}$) to the snow density observed at SNOTEL stations. For the comparison between Noah$_{N}$ and the SNOTEL density data, daily Noah$_{N}$ SWE and SD data are subset by the closest grid cell to each CONUS SNOTEL site in Fig. 1 (bulk density is calculated as SWE/SD). To ensure that each Noah$_{N}$ grid box is only compared to a single SNOTEL observation, instances where a Noah$_{N}$ grid box contained more than one SNOTEL are removed (which reduced the dataset by 10%).

In general, Noah$_{N}$ underestimates snow density in the early snow season and overestimates snow density in the late snow season. Figure 2 shows the temporal evolution of the median snow density (considering all SNOTEL locations) within each hydroclimatic class in the CONUS. Usually, early season underestimates are followed by an unrealistically rapid increase in density during March (beginning at approximately day 150, Fig. 2) to a maximum value (0.400 g cm$^{-3}$). Thereafter, snow density remains close to this value until snow disappears. Table 1 (CONUS values are in brackets) shows that the mean absolute error (MAE) between Noah$_{N}$ and SNOTEL snow density ranges from 0.032 to 0.044 g cm$^{-3}$ while the relative MAE (calculated as the MAE divided by the mean of the 50th percentile of observed density; rMAE) ranges from 9.2% to 15.3% for CONUS snow classes (Table 1).

Obviously, there is a scale mismatch between the point measurements and gridded snow density from Noah$_{N}$. Here we address this issue from two perspectives. First, we use our newly developed daily gridded SWE and SD datasets (Dawson et al. 2016) averaged over the six $2^\circ \times 2^\circ$ shaded boxes (shown in Fig. 1) to evaluate the Noah$_{N}$ daily products over the same boxes. Figures S1 and S2 in the supplemental material show that area-averaged SWE and SD from Noah$_{N}$ are generally underestimated compared to our upscaled SWE and SD data, though this underestimate is less than that of the NCEP operational snow initialization products evaluated in Dawson et al. (2016). Furthermore, the ratios between Noah$_{N}$ SD and the upscaled SD data are larger than the corresponding ratios between Noah$_{N}$ SWE and the upscaled SWE data (Table 2). For instance, the mean SWE ratio averaged across all six boxes and all three WYs for Noah$_{N}$ is 0.33 while the SD ratio is 0.44 (Table 2). Because the ratio of SWE/SD gives the bulk snow density, these results indicate that the Noah$_{N}$ density is too low for these areas in Table 2.

Another way to avoid the scale mismatch is to compare the SNOTEL snow density (point) measurements with those produced by the Noah snow density formulation (Noah$_{N}$) directly computed from the SNOTEL daily SWE and $T_{2m}$ (point) measurements based on the equations in appendix A. To test the Noah formulation based on the SNOTEL SWE and $T_{2m}$ data only, we make two approximations. First, $T_{2m}$ is used as a proxy for the mean daily snowpack temperature to calculate the densification due to settling and overburden. Second, Noah adjusts density due to snowmelt every time step (set to 1 h for NLDAS) with an addition of liquid
water totaling 13% of SWE per day if the snow surface
temperature is above freezing [Eq. (A5)]. In our test
here, the estimated duration of above-freezing temper-
atures is set to 4 h if the daily $T_{2m}$ is above freezing.
Without this change, Noah$_F$ would unrealistically in-
crease snow density to a maximum value (0.400 g cm$^{-3}$) within the first few months of the WY.

Table 1 compares these point values from Noah$_F$ against the SNOTEL observations over CONUS and Alaska (some of which represent different hydroclimatic environments than are found in the CONUS). The MAE (rMAE) between Noah$_F$ and the SNOTEL data ranges from 0.026 to 0.038 g cm$^{-3}$ (7.6%–12.1%; Table 1), which is a reduction from the 0.032–0.044 g cm$^{-3}$ (9.2%–15.3%) range of Noah$_N$ for CONUS snow classes (Table 1, values in brackets). The weighted mean rMAE (rMAE weighted by the number of samples per snow class; bottom row of Table 1) is reduced from 13.2% to 10.4% for Noah$_F$. However, similar to Noah$_N$, Noah$_F$ also under-
estimates density early in the winter and overestimates it later: it has a similar transition to maximum density after snowmelt initiation (Fig. 3).

b. Evaluation of snow density from VIC, SAC-SMA, and SNODAS

All other products (VIC, SAC-SMA, and SNODAS) are evaluated in a similar manner as Noah$_N$. Each product is subset for the closest grid cell to each SNOTEL location and density is estimated as SWE divided by SD. These products are also evaluated using the gridded SWE and SD datasets from Dawson et al. (2016). However, since the density formulations in these products rely on more information besides tem-
perature and SWE (e.g., amount of snowpack water), their density formulations are not directly tested with SNOTEL SWE and $T_{2m}$ data.

Of the additional models tested, SAC-SMA has the lowest errors and SNODAS has the highest errors in the ephemeral and maritime climates (Table 1). By contrast, VIC has the largest errors in the prairie and alpine hydroclimates. In the ephemeral and maritime hydroclimates, SNODAS has larger errors than Noah$_N$, VIC has errors that are similar in magnitude, and SAC-SMA has smaller errors (Table 1). In the prairie and alpine hydroclimates,
TABLE 1. Relative MAE (%; mean of three WYs) displayed as a percentage of the mean of the median observed density within each snow class (Fig. 2). First column values in parentheses are the sample sizes for each snow class used to calculate the weighted mean for all sites in CONUS and Alaska. Brackets indicate values for CONUS only (i.e., without Alaska). Note that bracketed percentages are slightly different for SNODEN because of the removal of duplicates within NLDAS grid boxes. Tundra and taiga classifications do not exist in the NLDAS or SNODAS domain.

<table>
<thead>
<tr>
<th>Snow Class</th>
<th>SNODEN</th>
<th>Noah ( _F ) [Noah ( _N )]</th>
<th>SNODAS</th>
<th>SAC-SMA</th>
<th>VIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tundra (9)</td>
<td>6.5</td>
<td>17.3</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Taiga (28)</td>
<td>8.2</td>
<td>17.7</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Maritime (446 [336])</td>
<td>5.3 [5.1]</td>
<td>7.6 [9.2]</td>
<td>[11.1]</td>
<td>[5.3]</td>
<td>[9.7]</td>
</tr>
<tr>
<td>Ephemeral (182 [152])</td>
<td>5.9 [5.6]</td>
<td>11.1 [9.6]</td>
<td>[16.2]</td>
<td>[9.7]</td>
<td>[8.3]</td>
</tr>
<tr>
<td>Prairie (779 [700])</td>
<td>7.1 [7.1]</td>
<td>12.1 [15.3]</td>
<td>[8.1]</td>
<td>[8.1]</td>
<td>[9.8]</td>
</tr>
<tr>
<td>Alpine (615 [552])</td>
<td>4.3 [4.3]</td>
<td>9.5 [14.6]</td>
<td>[9.9]</td>
<td>[8.3]</td>
<td>[10.6]</td>
</tr>
<tr>
<td>Weighted mean</td>
<td>5.8 [5.7]</td>
<td>10.4 [13.3]</td>
<td>[10.0]</td>
<td>[7.7]</td>
<td>[9.9]</td>
</tr>
</tbody>
</table>

hydroclimates, VIC, SAC-SMA, and SNODAS all have much smaller errors than Noah \( _N \).

Temporally, VIC tends to overestimate density throughout the entire snow season (Fig. 2). This is consistent with the findings of Andreadis et al. (2009). SAC-SMA, on the other hand, underestimates density for the latter half of the snow season in all snow classes, but overestimates density early in the winter in the ephemeral, prairie, and alpine environments. SNODAS usually underestimates density through the course of the snow season in the maritime and ephemeral hydroclimates and toward the end of the snow season in the prairie and alpine environments.

The general tendency for SNODAS and SAC-SMA to underestimate snow density can also be seen in Table 2. Generally, SD ratios (between SD from gridded products and the upscaled SD data) are larger than SWE ratios (between SWE from gridded products and the upscaled SWE data) for SAC-SMA and SNODAS, meaning that over the \( 2^\circ \times 2^\circ \) boxes, snow density is generally underestimated by these models. However, for VIC, these ratios are similar for both SD and SWE (averaging 0.33 across all hydroclimates), meaning that even though SD and SWE are severely underestimated in VIC, on average, over large areas, snow density is relatively consistent with that suggested by the upscaled SWE and SD data.

c. Comparison of our new snow density parameterization with NLDAS and SNODAS

Our snow density parameterization (i.e., SNODEN) uses time series of SNOTEL daily \( T_{2m} \) and SWE to predict the density evolution of up to 10 snow layers. The two parameters (discussed in appendix B) are identical for the CONUS (the area used for evaluation of SNODEN, SNODAS, and the three NLDAS models) and CONUS with additional Alaskan sites (which are used to evaluate Noah \( _F \) and SNODEN).

Over CONUS, Table 1 (bracketed values) shows that the rMAEs for Noah \( _N \), VIC, and SNODAS are the worst, ranging from 8.2% to 15.3% (or from 0.024 to 0.056 g cm\(^{-3} \)) for MAEs). The results are better for SAC-SMA (partly because of its use of the SNOW-17 model), and the performance of SNODEN is the best, with rMAEs ranging from 4.3% to 7.1% (or from 0.012 to 0.020 g cm\(^{-3} \)) for MAEs only. Furthermore, SNODEN performs best in every snow class among the five snow density parameterizations, and its weighted mean rMAE for all classes is less than half of that from Noah \( _N \), demonstrating its superior performance. Over CONUS and Alaska (Table 1), SNODEN also performs better than Noah \( _F \) in each snow class, with its weighted mean rMAE for all classes being 56% of that from Noah \( _F \).

Note that any of these snow density parameterizations in Table 1 would still be much better than the global constant snow density assumptions from NCEP (0.100 and 0.200 g cm\(^{-3} \) for the GFS and NAM, respectively) whose rMAEs would range from 65% to 71% and 31% to 43%, respectively, over CONUS (not shown). Additionally, Fig. 2 clearly shows that observed snow density increases through time and a constant snow density is not adequate. The constant values are outside of the range of median snow density estimated by all other evaluated products for the maritime and ephemeral hydroclimates for most of the winter.

As reflected by the small errors in Table 1, Figs. 2 and 3 show that SNODEN is able to more realistically reproduce the median temporal evolution of density than either Noah \( _N \) or Noah \( _F \). SNODEN is closer to the observed values through much of the snow season than other models (Fig. 2), but it tends to overestimate snow density late in the snow season for all classes except for tundra and taiga (Figs. 2, 3). By contrast, snow densities in the other parameterizations are consistently too high (e.g., VIC), too low (SNODAS), or are too low for part of the snow season and too high for another part (e.g., SAC-SMA, Noah \( _N \), and Noah \( _F \)).

4. Conclusions

Currently, NCEP operational models use spatially and temporally constant values of snow density to...
initialize SWE from SD in the initializations of GFS and CFS. This is unrealistic, and instead, a physically based snow density parameterization should be used when initializing SWE from SD data. An obvious option is to use snow density values from an LSM such as Noah that is implemented in the GFS, CFS, and NAM. However, an evaluation of Noah\textsubscript{N} for grid boxes coincident with SNOTEL point observations reveals an early winter underestimation followed by an unrealistic increase to the maximum allowable density of 0.400 g cm\textsuperscript{-3}.

This poor performance cannot be explained by the scale mismatch between point measurements and Noah\textsubscript{N} gridded values based on two additional tests (between gridded vs gridded values and point vs point values). For instance, when the Noah snow density parameterization (Noah\textsubscript{N}; forced with SNOTEL SWE and temperature data) is directly tested using the SNOTEL point data, snow density estimates are improved, but it still has the same tendency to underestimate snow density early in the snow season and increase snow density too quickly midway through the snow season. Furthermore, bulk snow density is less spatially variable than either SD or SWE (Elder et al. 1998; López-Moreno et al. 2013; Sturm et al. 2010). One of the main factors in snow density evolution is temperature, which is less variable than precipitation that is responsible for much of the spatial variability of SD and SWE. Since temperature in mountainous areas is generally below freezing for much of the winter, the differences in density evolution between point observations and 0.125° × 0.125° NLDAS grid boxes cannot be fully explained by grid size differences (discussed in more detail below). These findings demonstrate that while the snow density formulation included in Noah may be better than the constant density assumption for NCEP snow initialization, it remains deficient, and a better snow density parameterization is still needed.

Other NLDAS models (SAC-SMA and VIC), as well as SNODAS, also offer other viable snow density formulations. Although the temporal evolution of snow density from these products appears more realistic than from Noah, VIC tends to overestimate snow density [which is consistent with the finding of Andreadis et al. (2009)] and SNODAS tends to underestimate snow density, especially in maritime and ephemeral environments. One possible explanation for this deficiency of SNODAS is the assimilation of SD and SWE observations across different scales and platforms (i.e., aircraft data, surface point observations, and remotely sensed data) without using snow density to constrain the assimilation. Furthermore, it might stem from the subjective methods used to update (nudge) the model. SAC-SMA tends to overestimate snow density early in the snow season and underestimate snow density late in the snow season. Note that errors between the median of the observations and each product cannot simply be solved with a correction factor. The variation of these errors with time and location would necessitate a correction factor for each product at each SNOTEL location for optimal performance that ignores issues in the products themselves.

The NLDAS LSMs perform better compared to the operational NCEP snow initializations evaluated in Dawson et al. (2016). This improvement cannot be contributed solely to the smaller grid spacing of NLDAS (0.125° × 0.125°) compared to NCEP initializations (ranging from 12 km × 12 km to 50 km × 50 km). In Dawson et al. (2016), the Rapid Refresh (RAP) model had a similar spatial grid size to that of NAM. RAP cycled snow quantities from one time step to the next, while NAM used an external source for snow thickness data and a constant density (during the study period). Through time, RAP outperformed NAM by a large margin, which shows that model grid size is a second-order issue compared to the snow data utilized for initialization.

A similar grid size comparison can be made between SNODAS and NLDAS median snow density in Fig. 2. The smaller grid spacing of SNODAS (30 arc s) did not contribute to improved performance of median snow density compared to the relatively coarse NLDAS data (0.125° × 0.125°). SNODAS actually performs consistently worse than NLDAS after day 160 in Fig. 2 for all snow classes. Additionally, Broxton et al. (2016) found that grid size and atmospheric forcing could not account for the deficient SWE in Global Land Data Assimilation System (GLDAS) and reanalysis products (with the main deficiency being too much ablation at near-freezing temperatures).

In this study we have developed a simple, physically based snow density parameterization (SNODEN) that generally does not suffer from these problems. It does not tend to over- or underpredict snow density for much of the snow season (though there is a bit of a divergence

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**Table 2. Ratios of SD (SWE) calculated as the 2° × 2° area-averaged quantities (mean of all three WYs from 1 Dec to 1 Jun) for each product divided by upsampled observations. Note that results over WA are not shown for VIC because of unrealistically high values over glacial areas. Five boxes are shown in Fig. 1.**

<table>
<thead>
<tr>
<th></th>
<th>Noah\textsubscript{N}</th>
<th>SAC-SMA</th>
<th>VIC</th>
<th>SNODAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>0.58 (0.39)</td>
<td>0.64 (0.55)</td>
<td>0.37 (0.36)</td>
<td>0.83 (0.65)</td>
</tr>
<tr>
<td>MT</td>
<td>0.63 (0.53)</td>
<td>0.53 (0.48)</td>
<td>0.35 (0.38)</td>
<td>1.08 (1.00)</td>
</tr>
<tr>
<td>YS</td>
<td>0.46 (0.33)</td>
<td>0.66 (0.59)</td>
<td>0.35 (0.36)</td>
<td>0.97 (0.82)</td>
</tr>
<tr>
<td>WA</td>
<td>0.29 (0.24)</td>
<td>0.26 (0.22)</td>
<td>—</td>
<td>1.03 (0.85)</td>
</tr>
<tr>
<td>ID</td>
<td>0.22 (0.18)</td>
<td>0.27 (0.25)</td>
<td>0.19 (0.20)</td>
<td>0.83 (0.74)</td>
</tr>
<tr>
<td>Mean</td>
<td>0.44 (0.33)</td>
<td>0.57 (0.42)</td>
<td>0.33 (0.33)</td>
<td>0.98 (0.81)</td>
</tr>
</tbody>
</table>
As such, it performs best for each snow class among the five parameterizations against the SNOTEL observations. In particular, the relative MAE of SNODEN is on average just about half of that from the Noah model over CONUS and CONUS plus Alaska. Furthermore, SNODEN also simulates snow density for a multilayer snowpack. This could potentially result in added flexibility if it is desired to initialize a multilayer snowpack. Given that SNODEN is a simple stand-alone snow density parameterization, it can be easily implemented in NCEP operational snow initialization. Better SWE initialization of NCEP’s models could potentially improve forecasts that are run from these initialized states. One concern for initialization is computational time. The estimation of snow density for all observations in this study (391,126 station days of nonzero daily data over a 3-yr period) took approximately 5% longer than Noah on a standard consumer laptop. Note that this estimate is dependent on coding efficiency and implementation and could possibly be reduced. Additionally, SNODEN can potentially provide SWE estimates from in situ SD measurements at COOP stations and from airborne lidar SD measurement.

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APPENDIX A

Noah Snow Density Formulations

Snow accumulation (ablation) is solved with a mass-balanced energy approach (Ek et al. 2003) while snowfall
(SWE\textsubscript{new}) density $\rho_{\text{new}}$ is estimated by 2-m air temperature $T_{2m}$ with the formulation of Gottlib (1980):

$$
\rho_{\text{new}} = \begin{cases} 
0.05 & T_{2m} < -15 \\
0.05 + 0.0017 \times (T_{2m} + 15)^{1.5} & T_{2m} \geq -15 
\end{cases}.
$$

(A1)

New snowfall then adjusts density:

$$
\text{SD}_{\text{new}} = \text{SWE}_{\text{new}} \times \rho_{\text{new}} \quad \text{and} \quad \rho_n = \frac{\text{SD}_n \times \rho_n + \text{SD}_{\text{new}} \times \rho_{\text{new}}}{\text{SD}_n + \text{SD}_{\text{new}}}.
$$

(A2)

Further adjustments to density follow a simplified approach of SNOW-17 whereby snow aging and overburden are combined into a single process based on total SWE ($\text{SWE}_T$) and an overburden parameter $B$:

$$
\rho_n = \rho_n \times \left\{ \left[ e^{(B \times \text{SWE}_T)} - 1 \right] / (B \times \text{SWE}_T) \right\}, \quad (A3)
$$

$$
B = \Delta n \times C_1 \times e^{0.08 \times T - C_2 \times \rho_n}, \quad \text{and} \quad T = (T_{\text{SNOW}} + T_{\text{SOIL}}) / 2. \quad (A4)
$$

The variable $T_{\text{SNOW}}$ is the snow surface temperature and $T_{\text{SOIL}}$ is the soil surface temperature (both °C). Parameters include the fractional increase in density ($C_1$: 0.01 cm\textsuperscript{-1} h\textsuperscript{-1}), a constant from Kojima (1967) ($C_2$: 21 cm\textsuperscript{3} g\textsuperscript{-1}), and model time step $\Delta n$ (h). Snow density is then adjusted to include effects of snowmelt when the snow surface temperature is above freezing:

$$
\rho_n = \rho_n \times (1 - dw) + dw \quad \text{with} \quad dw = 0.13 \times \frac{\Delta n}{24}. \quad (A5)
$$

**APPENDIX B**

**Our New Snow Density Parameterization**

Our snow density (SNODEN) parameterization estimates density and SWE for up to 10 evenly spaced (0.05 g cm\textsuperscript{-3}) intervals layers (bins) ranging from (0.05 to 0.5) g cm\textsuperscript{-3} based on density. The top (bottom) bin holds the least (most) dense snow. State variables (initialized as zero) include density and SWE for each bin and the total snowpack liquid water (including SWE and melted water in the snowpack). The following equations for each step (outlined in section 2) are applied to each bin with a daily time step. Additionally, checks for missing SWE and 2-m air temperature are performed at the beginning of each time step. If SWE is missing, the last nonmissing SWE value is retained to calculate an increment for the next nonmissing SWE value. If temperature is missing, the mean of the previous seven days is used (quality control removes SNOTELs with more than six consecutive days of missing temperature). Note that the initial SNOTEL SWE (day one of each WY) is set to zero.

Because snow density behaves differently in different environments, all snow density comparisons are done separately in different hydroclimate classes. We use the snow classification data of Sturm et al. (1995) to group SNOTEL sites (Fig. 1). Temperature, precipitation, and land-cover data were combined in Sturm et al. (1995) to classify a global 0.5° x 0.5° grid into one of eight categories (water, tundra, taiga, maritime, ephemeral, prairie, alpine, and ice). In particular, the land-cover type for each grid box was used as a proxy for wind speed (low wind for tree cover types and high wind for non–tree cover types).

In step 1, density from the previous day (subscript $n - 1$) is adjusted. First destructive metamorphism is estimated from

$$
\rho_n = \rho_{\text{max}} - (\rho_{\text{max}} - \rho_{n-1}) \times \exp \left[ \frac{1}{f(T_{2m})} \right]. \quad (B1)
$$

This is derived by taking the natural logarithm and differentiating the Brasnett (1999) formulation, but it differs by inclusion of a $T_{2m}$-dependent (°C) e-folding time:

$$
f(T_{2m}) = \begin{cases} 
10 & T_{2m} < -5 \\
4 \times (1 - 0.3 \times T_{2m}) & -5 < T_{2m} < 0 \\
4 & T_{2m} \geq 0
\end{cases}. \quad (B2)
$$

A temperature dependence is used here because Sommerfeld and LaChapelle (1970) explained that snow temperature is proportional to metamorphism rate (i.e., colder snow metamorphoses slower than warmer snow). Equation (B1) is then applied to every density bin below the maximum density attainable through aging $\rho_{\text{max}}$ (g cm\textsuperscript{-3}), which is estimated for each snow class as the mean density (from 1 December to 1 March) of all SNOTEL sites within each respective snow class (for all three WYS).

Second, overburden (i.e., the effect of pressure within the snowpack on density) is estimated from

$$
\rho_n = \rho_n + (\text{SWE}_{\text{ob}} \times a), \quad (B3)
$$

where $\text{SWE}_{\text{ob}}$ represents half of the SWE in a respective bin plus SWE from all bins above (i.e., with smaller density). To estimate the value of the parameter $a$, overburden is first empirically estimated by an evaluation of our snow density parameterization without this process. The slope of the linear regression between observed maximum SWE and the average bias (model minus observed) is used to estimate the initial value of $a$ ($5 \times 10^{-7}$ g cm\textsuperscript{-3} mm\textsuperscript{-1} day\textsuperscript{-1}) and is later optimized (discussed below).
In step 2, density is then adjusted based on SNOTEL SWE increments (i.e., positive or negative). First, if the increment is positive, new snow density \( \rho_{\text{new}} \) (g cm\(^{-3}\)) is estimated by the temperature-dependent equation of Gottlib (1980), which is also utilized by the Noah LSM in Eq. (A1):

\[
\rho_{\text{new}} = \begin{cases} 
0.05 & T_{2m} < -15 \\
0.05 + 0.0017 \times (T_{2m} + 15)^{1.5} & T_{2m} \geq -15
\end{cases}
\]

(B4)

Density of new snowfall (and associated SWE) is then added into the appropriate bin. However if the appropriate bin has nonzero density (subscript \( n \)), the new snowfall density (subscript “new”) is added to the appropriate bin to produce an adjusted density:

\[
\rho_n = \frac{\text{SWE}_n + \text{SWE}_{\text{new}}}{\rho_n + \text{SWE}_{\text{new}}/\rho_{\text{new}}},
\]

(B5)

Second, negative increments are assumed to represent sublimation, melt, and wind removal. The absolute value of the negative increment is removed from the topmost nonzero SWE bin. Removal continues with the next (denser) nonzero bin until the increment is satisfied.

In step 3, adjustment of density due to the daily \( T_{2m} \) is then accounted for. If \( T_{2m} \) is above 0°C, a liquid water increment \( \Delta \text{liq} \) (mm) due to snowmelt is obtained by

\[
\Delta \text{liq} = \min(\min(T_{2m}/10, 1) \times b, 0.01 \times \text{SWE}_T),
\]

(B6)

where SWE\(_T\) represents the total SWE, and the melt parameter \( b \) [mm (°C\(^{-1}\) day\(^{-1}\))] is initially estimated as 4 mm (°C\(^{-1}\) day\(^{-1}\)) and later optimized. The total snowpack liquid water liq\(_{n-1}\) (mm) is then updated as liq\(_n\) = min(0.1 \times SWE\(_T\), liq\(_{n-1}\) + \Delta liq) with \( \Delta \text{liq} \) from Eq. (B6) added to the previous total liquid water estimate liq\(_{n-1}\) (mm) and capped at 10% of SWE\(_T\). If liq\(_n\) is equal to 10% of SWE\(_T\), then the increment of water needed to reach 10% of SWE\(_T\) is adjusted by \( \Delta \text{liq} = 0.1 \times \text{SWE}_T - \text{liq}_{n-1} \). The effect of additional liquid water as a fraction of SWE\(_T\) is then added to the snowpack:

\[
\rho_n = \left(1 - \frac{\Delta \text{liq}}{\text{SWE}_T}\right) \times \rho_n + \frac{\Delta \text{liq}}{\text{SWE}_T} \times \rho_{\text{new}},
\]

(B7)

where the density of water \( \rho_w \) is assumed to be 1 g cm\(^{-3}\).

In step 4, density is then redistributed into the proper bin to account for increases in density from steps 1 to 3. If the proper bin is empty, the density and SWE are added to the bin. If the proper bin is nonzero, SWE is added and the adjusted density is calculated by Eq. (B5). If the density in the top (bottom) bin is below (above) the bins lowest (highest) accepted value, the density is set to the lowest (highest) accepted value.

Optimization of parameters \( a \) and \( b \) is performed for each snow class by minimization of weighted mean absolute error (wMAE). Instead of assuming every day carries identical weight, absolute error is multiplied by the number of nonzero observations on the respective day. The sum of the weighted error is divided by the sum of nonzero observations. This limits the influence of late (early) season errors when error may be higher but few observations report nonzero density. The mean of the optimal parameters (obtained by minimization of wMAE) across all snow classes is weighted by the sample sizes for each snow class (6 \( \times 10^{-6} \) g cm\(^{-3}\) mm\(^{-1}\) day\(^{-1}\) for \( a \), and 4 mm (°C\(^{-1}\) day\(^{-1}\)) for \( b \), identical for all snow classes). This is because of the small SNOTEL sample sizes for the tundra and taiga snow classes (9 and 28, respectively) compared to the other snow classes (ranging from 182 to 779).

Additionally, eight sensitivity tests (outlined in Table S1 in the supplemental material) are performed on the SNODEN parameters that are halved, doubled, or remain unchanged. SNODEN is more sensitive to the overburden parameter \( a \) shown by tests 3–5 (6–8) where overburden is halved (doubled; Fig. S3 in the supplemental material). The model is relatively insensitive to changes in the melt parameter \( b \), where tests 1 and 2 only slightly influence density. The relative insensitivity to the melt parameter is likely due to the maximum of 10% liquid water placed on the snowpack. After the snow reaches 10% liquid water content, it remains at 10% and the melt parameter no longer influences density.

REFERENCES


