

From disparity to depth: How to make a grating and a plaid appear in the same depth plane

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Even though binocular disparity is a very well-studied cue to depth, the function relating disparity and perceived depth has been characterized only for the case of horizontal disparities. We sought to determine the general relationship between disparity and depth for a particular set of stimuli. The horizontal disparity direction is a special case, albeit an especially important one. Non-horizontal disparities arise from a number of sources under natural viewing condition. Moreover, they are implicit in patterns that are one-dimensional, such as gratings, lines, and edges, and in one-dimensional components of two-dimensional patterns, where a stereo matching direction is not well-defined. What function describes perceived depth in these cases? To find out, we measured the phase disparities that produced depth matches between a reference stimulus and a test stimulus. The reference stimulus was two-dimensional, a plaid; the test stimulus was one-dimensional, a grating. We find that horizontal disparity is no more important than other disparity directions in determining depth matches between these two stimuli. As a result, a grating and a plaid appear equal in depth when their horizontal disparities are, in general, unequal. Depth matches are well predicted by a simple disparity vector calculation; they survive changes in component parameters that conserve these vector quantities. The disparity vector rule also describes how the disparities of 1-D components might contribute to the perceived depth of 2-D stimuli.

Keywords: stereopsis, depth perception, disparity, horizontal disparity, stereo matching, intersection-of-constraints

Citation: Chai, Y., & Farell, B. (2009). From disparity to depth: How to make a grating and a plaid appear in the same depth plane. *Journal of Vision*, 9(10):3, 1–19, <http://journalofvision.org/9/10/3/>, doi:10.1167/9.10.3.

Introduction

Keen stereoscopic vision relies on relative disparities, the difference between the disparities of two stimuli whose depth is being judged. Yet, while we have rather solid models of disparity processing (e.g., Ohzawa, DeAngelis, & Freeman, 1990; Zhu & Qian, 1996), these are models of absolute, not relative, disparity processing. As a result, these models do not yield direct predictions of human stereo abilities as measured psychophysically. This has not been regarded as much of a drawback; once estimates of absolute disparities are at hand, relative disparity is thought to be only a simple calculation away. A differencing operation is all that is needed (Westheimer, 1979). But is this disparity difference calculated in the same way for all stimuli? And how is the resulting signal used to judge relative depth? We study these questions here.

Current models bring only position or phase—‘where’ parameters—into the calculation of absolute disparity. And only absolute disparities enter into the differencing operation that yields relative disparities. Therefore, relative-disparity calculations, and the stereo abilities that make use of them, should depend only on ‘where’ parameters and should be independent of ‘what’ the stimuli are. Indeed, with few exceptions (e.g., Marr &

Poggio, 1979), the specific position-invariant spatial, temporal, and chromatic parameters that determine stimulus identity have been treated as relevant to the stereo performance only to the extent that they allow position or phase estimates of sufficient range and precision for the stereo task at hand. In other words, the disparity value D is the same regardless of the stimulus from which it came.

However, recent studies have shown *what* stimuli are can strongly affect stereoacuity. Specifically, stereoacuity depends on how similar two stimuli are in spatial frequency and orientation (Farell, 2003a, 2006). These are effects of *inter-stimulus* similarity. They are not limitations imposed by the range or precision of absolute disparity signals of individual stimuli. Rather, they are limitations on the relative disparity signal that can be recovered between two stimuli.

Stereoacuity is best for stimuli whose orientations are the same. A 15° or 20° difference between the orientations of sinusoidal gratings can double the relative disparity threshold measured in the absence of an orientation difference (Farell, 2006). In this study we examine the suprathreshold domain, to see how relative orientation (or relative disparity direction, which is linked to it), affects perceived stereoscopic depth. The stimuli we use are gratings and plaids. We ask the question, What disparities are required for these two stimuli to appear at the same depth? The traditional expectation is that, to a first

approximation, equal horizontal disparities yield the perception of equal depth. Our expectation is that relational properties between the two stimuli will have to be considered as well. Our aim is to find out what these properties are.

Stereo plaids and stereo gratings

Stereo plaids—binocularly viewed superimposed sinusoids of different orientation—have an interesting and useful property: The component gratings (provided they're similar in spatial frequency and are static or drifting within a range of similar speeds) form a plaid that appears in a single depth plane (Adelson & Movshon, 1984; Delicato & Qian, 2005; Farell, 1998, 2003b; Farell & Li, 2004; also see Calabro & Vaina, 2006). Even when their disparities are quite different in magnitude or opposite in polarity, the gratings cohere in depth, becoming a single perceptual object rather than separate transparent objects segregated in depth. This is the stereo counterpart of coherently moving plaids (Adelson & Movshon, 1982).

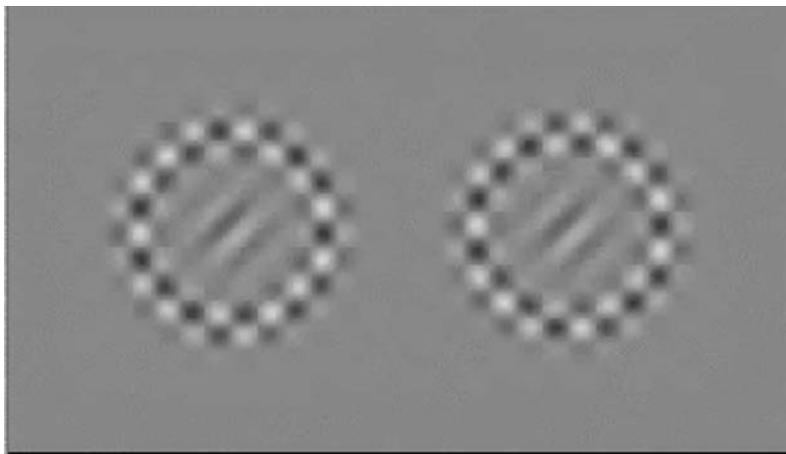
If we superimpose two sinusoids, one having a phase disparity of, say, 30° and the other having a disparity of zero, we will see a coherent plaid at a single depth, but where in depth? We could specify the plaid's depth relative to that of another stimulus presented at the same time. However, if this stimulus is oriented, the relative depth perceived between the two stimuli can vary when the stimulus orientation changes (Farell, 2006). Thus, a plaid and a grating that appear in the same depth plane can appear in separate depth planes simply by rotating the grating, without changing its disparity. [Movie 1](#) shows an example.

A grating, being one-dimensional, has an ill-defined stereo-matching direction. From psychophysical (e.g.,

Arditi, 1982; Farell, 1998; Morgan & Castet, 1997) and physiological evidence (e.g., Anzai, Ohzawa, & Freeman, 1999; Cumming & DeAngelis, 2001), the stereo matching direction of a 1-D stimulus is usually taken to be perpendicular to its orientation. [Other evidence, however, suggests a horizontal matching direction (Cumming, 2002), and the disparity of terminators and occluding contours might add a bias or override the influence of grating disparity altogether (e.g., Anderson, 1994; Nakayama, 1996; van Dam & van Ee, 2004).]

Perpendicular disparity confounds disparity direction and stimulus orientation. Plaids and other 2-D patterns avoid this; regardless of their orientations, the 1-D components of a 2-D pattern can be assigned disparities that give the pattern as a whole any desired disparity direction across the full range of 360° .¹ However, the particular plaids used in Farell's (2006) study did not exploit this property. There, the plaid's components were orthogonally oriented and constrained in their disparities. The result was a confounding of the plaid's disparity direction and its orientation content, just as in the case of individual gratings. Thus, the finding of that study—that perceived depth depends jointly on relative component orientation and relative component disparity between the two stimuli—is consistent with two hypotheses. It is consistent with the hypothesis that a depth match is seen when the two stimuli have *components* with both similar orientations and disparity magnitudes. And it is consistent with the hypothesis that a depth match is seen when the two *patterns* have similar disparity magnitudes and directions, independent of the disparities of their components. Thus, either relative component orientation or relative stimulus disparity direction, or both, might be effective mediators of relative stereoscopic depth.

In this study we deconfound component orientation and pattern disparity direction. We used an annular plaid, functioning as a reference stimulus, and a central test



Movie 1. Perceived relative depth varies with changes in orientation without changes in disparity. The two images can be fused either convergently or divergently. The effect occurs also when the display is static and the presentation brief (150 ms).

grating—as in [Movie 1](#), but static. Our aim is to determine whether the relative depth perceived between these two stimuli is best explained in terms of the relative disparity of their parallel components or the relative disparity of the two patterns considered as wholes. Following the conventions used in studies of plaid motion, we refer to these plaid disparity parameters as component disparities and pattern disparities, respectively; the latter is equivalent to the disparity of the two-dimensional features of the plaid and equals the disparity given by the intersection-of-constraints calculation. For 1-D stimuli like gratings, there is no distinction between component and pattern disparities. In order to dissociate the plaid's component and pattern disparities, we independently varied the orientations and the disparity magnitudes of the plaid's components. To investigate perceived relative depth, we varied the orientation of the test grating with respect to the orientation of the plaid's components and the direction of the plaid's pattern disparity.

Though a number of studies have measured stereoacuity as a function of the orientation of 1-D stimuli (Blake, Camisa, & Antoinetti, 1976; Ebenholtz & Walchli, 1965; Farell, 2003b, 2006; Farell & Ahuja, 1996; Morgan & Castet, 1997; Ogle, 1955), few have measured perceived depth. Those that have measured perceived depth have been explicit in assuming that the stereo matching direction is an intrinsic property of the stimulus and that the horizontal component of the stereo match determines the perceived depth of the stimulus (van Dam & van Ee, 2004; van Ee & Schor, 2000). However, it has not been shown that knowledge about the matching direction gained independently from perceived depth judgments would allow us to accurately predict these judgments.

In three experiments, we take up in turn several hypotheses about the disparity cues that are used in establishing a perceptual depth match between a plaid and grating. We find that the depth match depends on the relationship between the disparity vectors of the two stimuli; the horizontal stimulus disparities themselves are mostly irrelevant. Depth matches between pairs of 2-D stimuli will be taken up in a separate paper.

Methods

The general methods, described here, were common to all experiments. We will take up methodological variations peculiar to individual experiments in the sections dealing with those experiments.

Stimuli

Stimuli were luminance grating and plaid patches. Gratings appeared as centers of center-surround stimulus pairs; plaids appeared as surrounds. The center stimulus

consisted of a two-dimensional Gaussian-windowed patch; the carrier was a single sinusoid. The envelope of the central patch had horizontal and vertical standard deviations of 0.53° of visual angle. The envelope of the surrounding annulus was Gaussian along the radial direction with a standard deviation of 0.34° . Gaussians were truncated at $\pm 2\sqrt{2}\sigma$. The peaks of the center and surround Gaussian envelopes were separated by a distance of 2° visual angle. Spatial frequencies of the sinusoidal carriers were 2 cycles/degree (c/d) in most experiments; 1 c/d (and in a few cases, 3 c/d) patterns were examined in several experiments and produced data similar to those reported here.

Stimuli were presented on luminance-calibrated monitors driven by a Macintosh G5 computer via attenuators that provided monochromatic resolution of approximately 12 bits (Pelli & Zhang, 1991). Michelson contrast of gratings and individual plaid components was 0.1. Presentation duration was either 150 ms or 1 s, with abrupt onsets and offsets. The background luminance was 21 cd/m^2 , which was also the patterns' mean luminance. Observers used a chin rest and viewed the displays with natural pupils in a moderately lit room. Stimuli generation and presentation were controlled by a Matlab (Mathworks, Inc.) program incorporating elements of Psychophysical Toolbox software (Brainard, 1997; Pelli, 1997).

Stimuli were centered on two 21 in. monitors, one for each eye. Monitor refresh rate was 75 Hz. Stimuli were viewed through a front-silvered mirror stereoscope. The optical distance was 1.25 m. A fixation stimulus could function as an unwanted reference stimulus; therefore, none was provided and the center-surround patterns were presented on an otherwise blank screen. The contours nearest these patterns were the edges of the monitors' screens, which were located approximately 3.8° vertically and 5.8° horizontally from the outer visible limit of the annulus. The screens' vertical contours and the terminations of its horizontal contours could be viewed only monocularly due to vertical occluders positioned close ($<3 \text{ cm}$) to the eyes. One occluder was on the right side of the right eye, blocking the view of right edge of the right-hand screen; the other occluder did the same for the left edge of the left-hand screen. Because of the occluders' proximity, their retinal images were blurred, preventing fusion with the monitor's edges and preventing either occluders or edges from functioning as binocular reference stimuli.

The sinusoidal carriers of the annular plaid patch had a fixed disparity throughout a run of trials. (An exception, with multiple disparity directions, occurred in a control condition within [Experiment 1](#).) The sinusoid in the central test patch had a disparity that varied from trial to trial according to a constant stimulus procedure, as described below. The absolute phases of all sinusoids were independently randomized (identically in the left and right eyes) on each trial to eliminate potential monocular cues.

The disparities of the Gaussian contrast envelopes were fixed at zero; the only non-zero disparities were interocular

phase shifts of the carriers. As a result, the disparity direction of the envelope was equally different from the disparity direction of all of the carriers, regardless of what those directions were; this neutral status can be obtained only by giving envelopes a disparity uncorrelated with carrier disparities. The zero disparity of the envelopes also allows modulations of depth matches to be attributed unambiguously to the modulation of carrier disparities, which is the effect we are interested in measuring. If carrier and envelope disparities were correlated, observers might have based their depth judgments on either one.

Procedure

We obtained psychometric functions for perceived depth by varying the disparity of the central test grating from trial to trial. The disparity of the surrounding plaid was constant within a block of trials, as were all other stimulus parameters except the absolute phases of the sinusoidal carriers. The variable grating disparities were drawn in random order from a set of six pre-selected values; each of these constant-stimulus values was selected the same number of times across trials. Their magnitudes were chosen on the basis of preliminary data to provide a roughly symmetrical bracketing of the 50% point of the psychometric function.

The observer's task was to judge the center grating as having appeared 'near' or 'far' (that is, closer to the observer versus more distant) relative to the surrounding plaid. Observers initiated each trial with a click of the computer's mouse after fixating between and aligning two vertical nonius lines. The nonius lines vanished shortly before stimulus onset. A brief 'bing' sounded the onset of the stimulus presentation; observers signaled their decisions by clicking 'Near' and 'Far' buttons that appeared on-screen shortly after termination of the stimulus. Observers were not informed about the stimulus or stimulus components whose disparity varied across trials. No feedback was provided about the correctness of responses, for these responses report a subjective perceptual judgment.

Before data collection, observers were given sufficient practice to stabilize performance. Data were gathered in runs of 60 trials, each preceded by 6–8 warm-up trials. At least 3 runs contributed to each data point. Psychometric functions were fit by Weibull or cumulative Gaussian functions (Wichmann & Hill, 2001a, 2001b). The point of subjective equality (PSE), the disparity of the test grating that yielded the 50% point on the psychometric function, defined the perceptual depth match between the grating and the plaid.

Observers

Eight different observers contributed to the data presented here across all experiments. From 2 to 5 of

these eight observers ran in each experiment. Most of the observers were experienced in psychophysical tasks only through exposure to the experiments reported here and were naïve as to the purposes of the experiments. The exceptions were the authors, who participated in [Experiment 3](#). All the observers had normal acuity (in some cases with corrective optics) and normal stereo vision. All observers gave their informed written consent before participating in the experiments, the protocol for which was approved by the Institutional Review Board of Syracuse University.

Experiment 1

We first investigated the hypothesis that the relative depth perceived between two stimuli depends on relative component disparity, not on relative pattern disparity. In particular, we hypothesized that a perceptual depth match between a grating and a plaid would occur when the grating had the same disparity as the plaid's component that was parallel to the grating. The plaid was made of a pair of sinusoidal components (spatial frequency = 2 c/d) oriented symmetrically at angles of 60° and 120°, 52.5° and 127.5°, 45° and 135°, 37.5° and 142.5°, or 30° and 150° (where 0° is horizontal and positive angles increase counter-clockwise). We identify these plaids by their 'plaid angle' (60°, 75°, 90°, 105° and 120°, respectively), the difference between the component orientations; monocular examples are shown in [Figure 1](#). In this experiment, one of the plaid's components had a disparity of zero and the other had a phase disparity of +5°, +15°, or +30°. These disparities gave the plaid an overall disparity

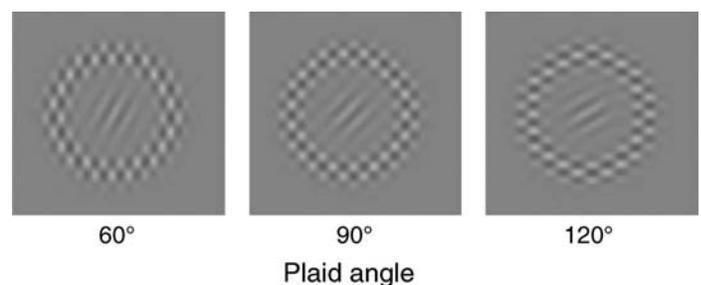


Figure 1. Monocular examples of the stimuli used in [Experiment 1](#); contrast is higher than in the experiment. The annular reference plaids were composed of two symmetrically oriented sinusoids. Plaid angle is the angular difference between the two components and is bisected by the vertical. The phase disparity of one component was 5°, 15°, or 30°, and that of the other was 0°. The central test grating was parallel to the plaid component with zero disparity (R_0 conditions) or the component with non-zero disparity (R_+ conditions). The grating disparity varied from trial to trial; the task was to classify the grating as 'near' or 'far' relative to the plaid.

direction, in the intersection-of-constraints sense, that was parallel to the zero-disparity component (Delicato & Qian, 2005; Farell, 1998). The horizontal component of the plaid's disparity was positive, giving it an expected 'far' depth, that is, behind the plane of the screen, which had a disparity of zero at nonius alignment.

The test grating had the same orientation as one of the plaid's components, either the component with positive disparity ('R₊' gratings) or the component with zero disparity ('R₀' gratings). We hypothesized that the disparity of the R₊ grating yielding a depth match with the plaid would vary directly with the disparity of the plaid's component parallel to the grating (that is, +5°, +15°, or +30° phase disparity). The depth match between the R₀ grating and the plaid should occur at a grating disparity of zero regardless of the plaid's disparity. This is because the plaid's component parallel to the R₀ grating had a disparity of zero. For both R₊ and R₀ conditions, the depth match was hypothesized to be independent of plaid angle. The reason is that variations in plaid angle change

the pattern disparity (both in magnitude and direction) but do not change the component disparities or the parallel orientations of test grating and plaid component. To preview, the results agreed with expectations under the parallel-components hypothesis; subsequent experiments, however, led us to reevaluate, and finally to reject, this hypothesis.

Results

R₊ conditions

The test grating was parallel to the plaid's positive-disparity component in the R₊ conditions; the disparity of the other component was zero. The mean PSEs for these conditions appear in Figure 2. Here the disparities of the grating that yielded depth matches with the plaid are

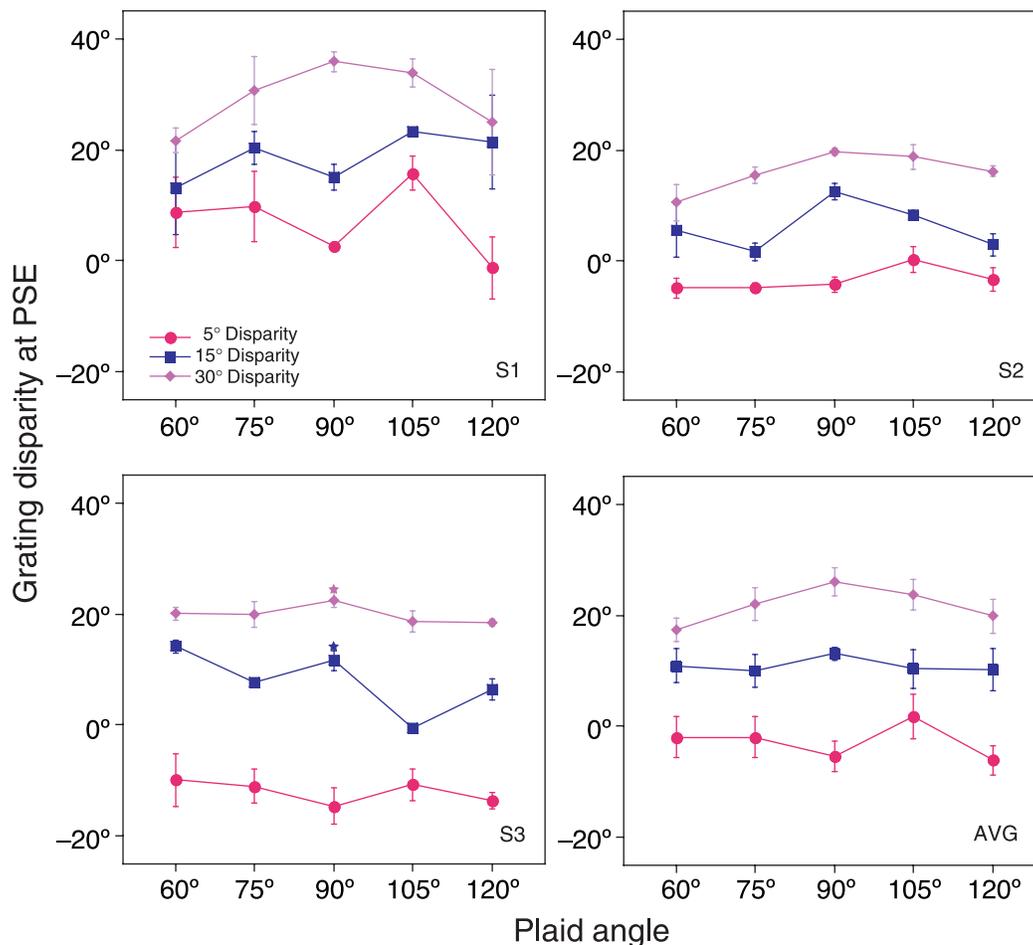


Figure 2. Depth-matching disparities for the R₊ conditions of Experiment 1, plotted as a function of plaid angle. Data points are grating phase disparities that yield PSEs for the depths of the center grating and the surrounding plaid for 3 observers. PSE values averaged across observers are also shown. In R₊ conditions the grating was parallel to the plaid component with non-zero (5°, 15°, 30°) disparity, which is the parameter in these plots. Starred data points for Observer S3 show PSEs when plaid disparity varied across trials (see text). Error bars are ± 1 SEM.

plotted as a function of plaid angle. Data for the three phase disparities of the plaid's non-zero component (5° , 15° , or 30°) appear separately. To match the perceived depth of the plaid, the test grating required a greater disparity as the component disparity increased, and for each of the three component disparities the perceived depth was approximately the same regardless of the plaid angle. Data for individual observers also appear in Figure 2.

PSEs varied systematically with the magnitude of the plaid's component disparity. However, at matching depths, the disparities of the grating and the parallel component of the plaid were generally unequal. The grating disparity was smaller. For component disparities of $+5^\circ$, $+15^\circ$, and $+30^\circ$, the parallel test grating appeared at the same depth as the plaid when its disparity was $-2.7^\circ \pm 5.9^\circ$, $10.9^\circ \pm 6.7^\circ$, and $21.8^\circ \pm 5.6^\circ$ (mean \pm *SD*), respectively. Thus, the central grating appeared at a farther distance than a plaid with the same component disparity. The size of this offset ($6.7^\circ \pm 6.1^\circ$ [mean \pm *SD*]) shows no obvious relation to the size of the plaid's component disparity or to its pattern disparity. Therefore, it is best regarded as a bias induced by the center-surround

arrangement of the stimuli rather than a consequence of processing the carrier disparities.

R_0 conditions

In the R_0 conditions the test grating had the same orientation as the plaid's zero-disparity component. PSEs for these conditions, shown in Figure 3, differed markedly from those for the R_+ conditions. Observers perceived depth matches between plaids and R_0 test gratings when the phase disparity of the grating was approximately 0° . Varying the disparity magnitude of the plaid's positive-disparity component had small effects on PSEs. This contrasts with the robust effect of the plaid's component disparity magnitude in R_+ conditions. Note that R_0 and R_+ test gratings do not differ in the disparity magnitudes that give them the same horizontal disparity as the reference plaids. But R_0 and R_+ test gratings required quite different disparity magnitudes in order to match the depth of the same set of reference plaids (Figure 2 vs. Figure 3). So, relative horizontal disparity does not predict depth matches between gratings and plaids.

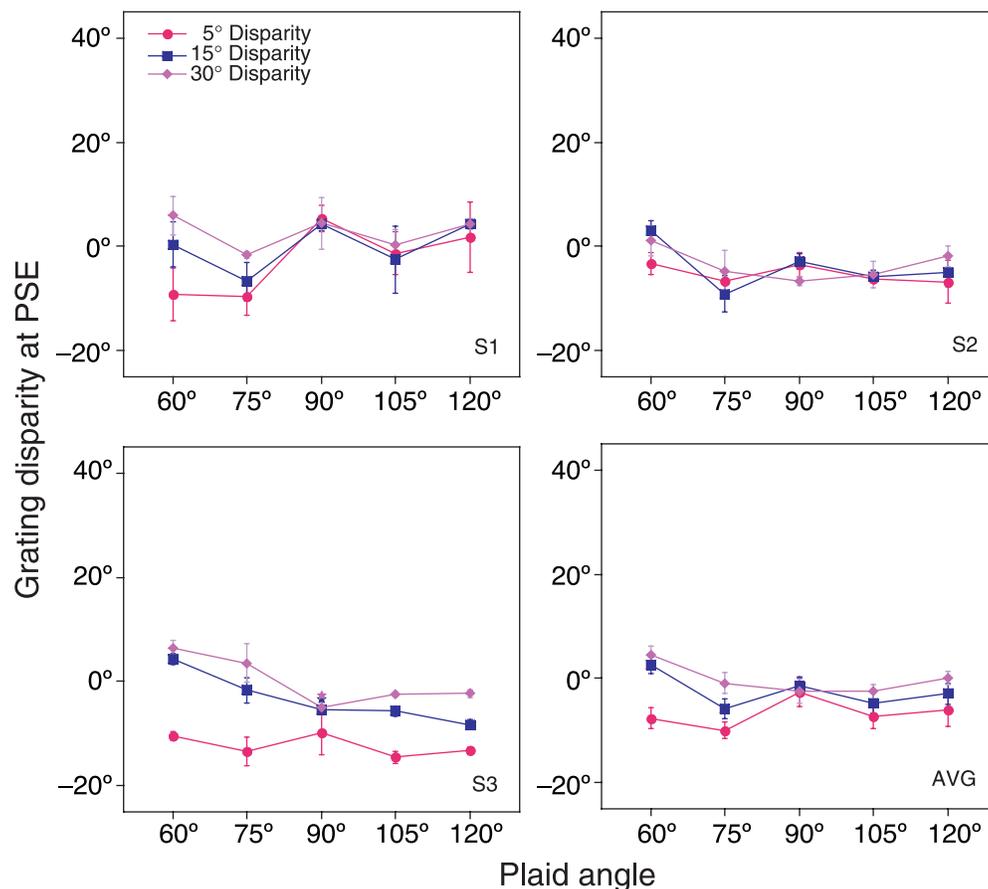


Figure 3. Depth-matching disparities for the R_0 conditions of Experiment 1. For these data, the test grating was parallel to the plaid component with zero disparity. Observers and plotting format are the same as those of Figure 2.

If the perceived depth separation between two the stimuli depended only on the relative disparity of components that had the same orientation in both stimuli, the perceived depth functions of Figures 2 and 3 should be flat. To a first approximation, they are flat. In addition, PSE values should be proportional to the disparity of the plaid component parallel to the test grating; that is, 0° in R_0 conditions and 5° , 15° , or 30° in R_+ conditions. Figures 2 and 3 show this, too, to be the case. However, both R_0 and R_+ conditions also show minor effects of plaid angle; as discussed below, either component disparities or pattern disparities might be the source of these influences. These effects, evident in Figures 2 and 3, contributed to statistically significant interactions between reference grating type, R_+ or R_0 , with component disparity ($F(2, 16) = 107.7$), plaid angle ($F(4, 32) = 8.1$), and both component disparity and plaid angle ($F(8, 64) = 2.8$), as determined by ANOVA, with all $ps < 0.01$, except the last ($p = 0.011$). The main effect of test grating type, R_+ vs. R_0 , ($F(1, 8) = 76.7$), was also significant ($p < 0.01$).

We obtained results similar to those of Figures 2 and 3 when we halved, to 1 c/d, the spatial frequency of the grating and the plaid components. Results were also similar when we extended the stimulus duration to 1000 ms, but the longer duration slightly reduced the gain of the depth matching function. This function relates the disparity magnitudes of the grating and the plaid's component at the depth match. Combining R_+ data across plaid angles and observers, this function was quite linear with a near-unity slope (0.96) at the 150 ms duration. At the 1 s duration, however, the slope was reduced to 0.79 (Figure 4). A proportionately similar reduction occurred for the R_0 conditions. One possibility is that contrast envelopes, having zero disparity, influenced the perceived depth of the carrier at the long duration. The annulus has lengthy edges; perhaps at long durations eye movements take the edges to retinal areas that can readily extract the envelope's zero disparity. This is admittedly without the support of precedents—while McKee, Vergheze, Ma-Wyatt, & Petrov (2007) found a time-varying effect of envelope disparity on the wallpaper illusion, this effect dissipated, rather than increased, over time—yet it's the only idea we've come up with.

Either component disparity or pattern disparity might have mediated the modest effect, seen in Figures 2 and 3, that plaid angle had on depth matches in this experiment. Changing the plaid angle has multiple side-effects: It changes the orientation difference between the non-parallel plaid component and the test grating; it changes the pattern disparity in both magnitude and direction; it changes the horizontal and vertical components of disparity, and does so inversely for the grating and the plaid. One or more of these factors could influence perceived depth in both the R_0 and the R_+ conditions.

Despite these various potential influences of plaid angle (or because they cancel), they show little in the way of a systematic effect. Thus, if we relegate these influences to

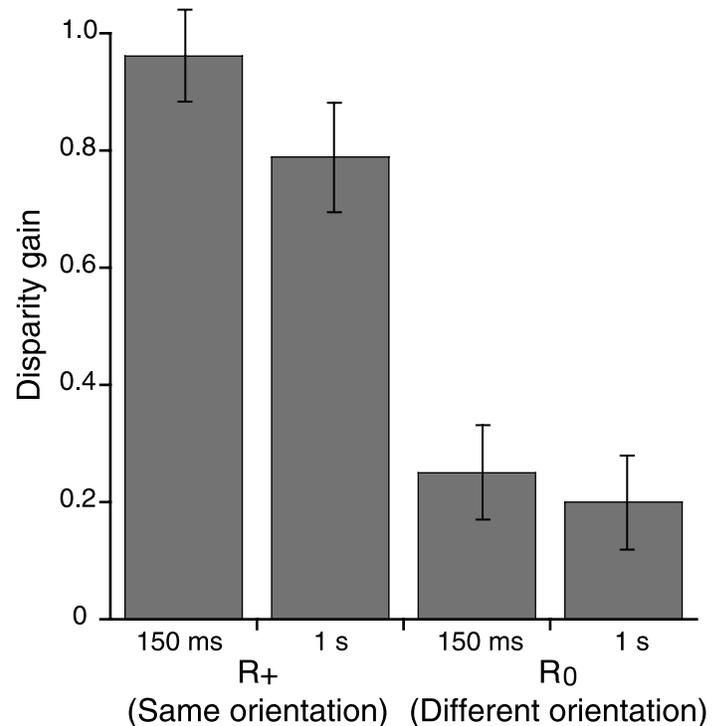


Figure 4. Disparity gain for Experiment 1 as a function of stimulus duration and condition. Gain was measured as the ratio of the test grating's disparity and the plaid's non-zero component disparity at the PSE. Gain is near 1.0 for short (150 ms) presentations in the R_+ conditions, slightly less for long (1 s) presentations, and much less in the R_0 conditions. Error bars are ± 1 SEM.

secondary status, the results of this first experiment are consistent with the hypothesis that the relative disparities of parallel stimulus components, not the relative pattern disparities, determine the perceived depth between the plaid and the grating. This interpretation is consistent with stereoacuity data, because component disparity thresholds limit the observer's sensitivity to pattern disparity (Farell, 2003b; see also van Ee, Anderson, & Farid, 2001) and because relative orientation limits the observer's sensitivity to grating disparity (Farell, 2006). However, as we show below, an alternative interpretation that rejects a contribution of the plaid's components to perceived depth can account for the data of this experiment and of others, without challenging the conclusions of the stereoacuity studies.

Experiment 2

Results of Experiment 1 are consistent with the notion that the relative disparity of components that are parallel across the two stimuli determines the perceived depth separation between the stimuli. If this is valid, then the depth perception of stimuli without parallel components

should be either degraded or based on signals other than the relative disparities of parallel components. We tested this in [Experiment 2](#) using vertical test gratings and symmetrical plaids as stimuli. Both of the plaids' components had the same phase disparity, D_c (15° or 30°); the plaid angle, α , varied between 60° and 105° (we found that observers had difficulty fusing plaids with a plaid angle of 120° under some disparity conditions). Component disparity and plaid angle jointly control the pattern disparity magnitude:

$$D_p = D_c / \cos(\alpha/2). \quad (1)$$

Thus, the stimuli of this experiment differed from those of [Experiment 1](#) in two ways: The test grating differed in orientation from both of the plaid's components, and, because both plaid components had the same disparity, the disparity direction of the plaid was the same as the perpendicular disparity direction of the test grating; both were horizontal. In this experiment the stimulus duration was 150 ms for one observer and 1 s for three others. Other experimental details were as in [Experiment 1](#).

Results

If stereo depth is mediated by similarly oriented components, then component disparity should have a diminishing effect on the PSE as plaid angle increases. This is because the larger the plaid angle, the greater the orientation difference between the plaid's components and the test grating. As seen in [Figure 5a](#), the results from the four observers do not show this effect. The disparity of the grating at the depth match increased with plaid angle for both component disparities, 15° and 30° , and increased in proportion to these component disparities while generally exceeding them in magnitude.

PSEs increased with plaid angle at a rate approximating that given by [Equation 1](#). Individual observers differed somewhat in the absolute grating disparities that yielded a depth match; this seems to reflect a static bias to see the central grating as offset in the near or far direction. However, all observers show similar modulation of depth-matching disparities with variation in the plaid angle and the plaid's component disparities; overall, the gain of this modulation was close to 1.0 ([Figure 5b](#)).

The plaid's disparity direction in this experiment was horizontal; so, too, was the grating's perpendicular disparity. To see whether the effect of plaid angle shown in [Figure 5](#) is specific to horizontal disparities, we repeated the measurements after rotating the stimuli. The rotation angle was 30° counter-clockwise. After rotation, the test grating was no longer vertical and the reference plaid was no longer symmetrical. However, the grating's perpendicular disparity still paralleled the plaid's

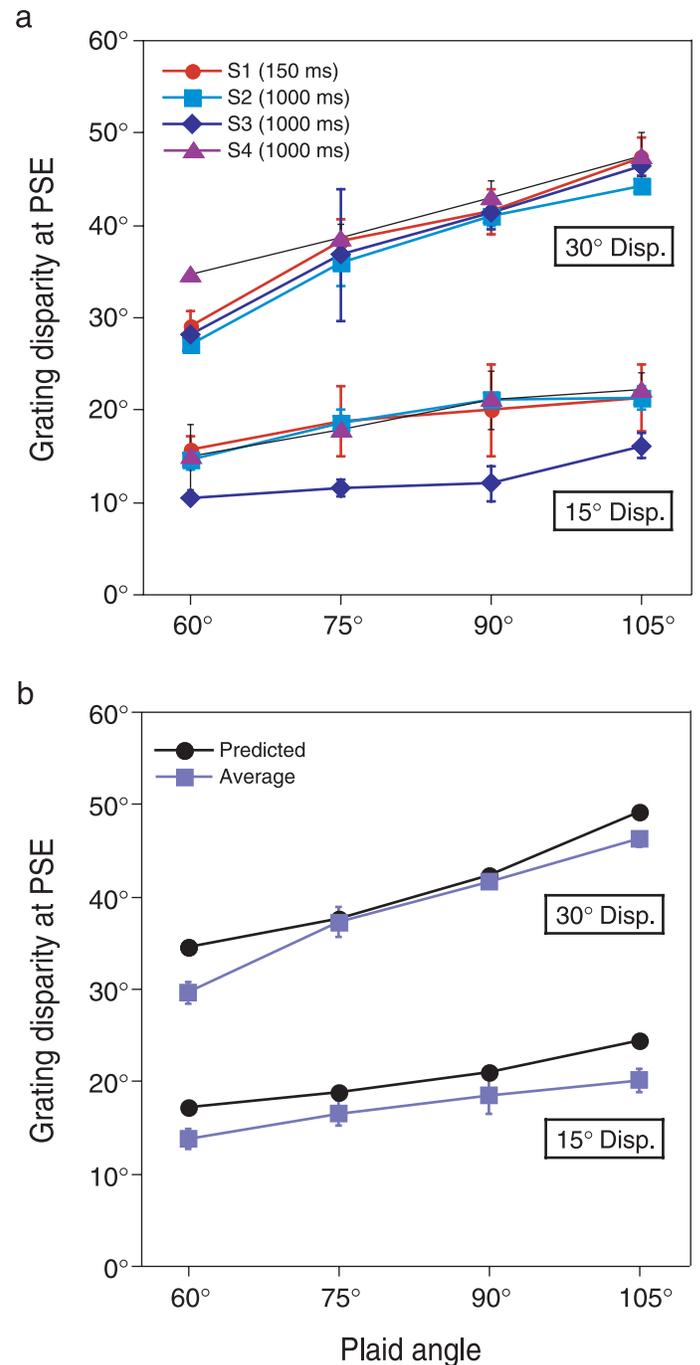


Figure 5. Depth-matching disparities for [Experiment 2](#). PSEs are shown as a function of plaid angle for 15° and 30° component phase disparities. Stimulus duration was 1 s for 3 of the 4 observers and 150 ms for the other. Predicted values are the grating phase disparities whose horizontal extents equal the plaid disparities given by [Equation 1](#). (a) Individual observer's data; (b) mean data. Error bars = 1 SEM.

disparity. The resulting PSEs (not shown) were the same as in [Figure 5](#) in all essential respects. Thus, a strictly horizontal disparity direction is not required to obtain depth-matching functions that vary with pattern disparity. However, [Experiment 1](#) indicated that component

disparities, not pattern disparities, determined depth matches. What is unclear, then, is how to reconcile the results of Experiments 1 and 2.

The data of Experiment 1 are consistent with the hypothesis that the relative disparity of parallel components determines perceived depth. The data of Experiment 2 are consistent with the hypothesis that parallel components are not needed for relative disparity to modulate perceived depth. The difference between these results might be resolved by postulating the use of the more reliable of two disparity signals for calculating relative depth; one signal—relative component disparities—is more reliable when component orientations are shared across stimuli, and the other—relative pattern disparities—is more reliable when no shared orientation exists. Or, more parsimoniously, the divergent results of the two experiments can be resolved by considering disparity direction, as we do next.

Experiment 3

In this experiment we compared depth matches measured when the grating's perpendicular disparity direction was

parallel to the disparity of the plaid and when it was not parallel. We made this comparison when the grating had the same orientation as a plaid component and when it did not. These manipulations pit those of the previous two experiments against each other. We do this in order to test an alternative hypothesis that, as will be discussed below, is consistent with the data of both of the previous experiments. The hypothesis is that the grating and the plaid appear at the same depth match when they have the same disparity magnitude in the same direction—provided one selects the proper direction for comparing the magnitudes. Which direction this is will be detailed later, once the data from Experiment 3 are at hand.

The plaid was made of 2 c/d sinusoids oriented at 45° and 90° or at 135° and 90° . The 90° -orientation component had a phase disparity of 21.2° ; the 45° - and 135° -orientation components had a phase disparity of either 0° or 30° . This gave the plaid a spatial disparity whose magnitude was constant at 2.5 minutes of visual angle and whose direction was either $+45^\circ$ or -45° . Independent of the plaid parameters, the central reference grating had an orientation of 45° , 90° , or 135° ; its disparity varied from trial to trial in the manner used in the two previous experiments. Observers viewed the stimulus pairs for 150 ms and judged the center to be

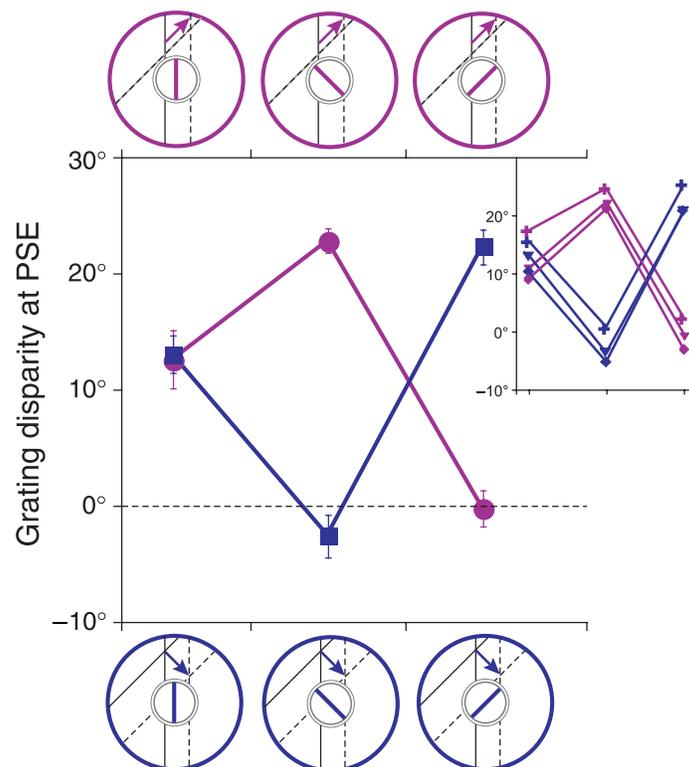


Figure 6. Depth-matching disparities for Experiment 3. PSEs appear as a function of the disparity direction of the reference plaid and the orientation of the test grating. The plaid's disparity vector is given by the arrow connecting left- and right-eye images (solid and dashed lines) in stimulus diagrams above and below the graph; test grating orientation is shown by the central line in these diagrams. Vertically aligned data points share test orientation and differ by 90° in disparity direction, which is color-coded. Averages of 3 observers' data appear in the main plot; inset shows individual observer data. Error bars are ± 1 SEM.

‘near’ or ‘far’ relative to the surround. Within blocks of trials the only variation was in the grating disparity and the absolute phase of the sinusoids. Remaining details of stimuli and procedure followed those of previous experiments. The six basic stimulus conditions of this experiment (represented by 45°/90° plaids) are shown schematically above and below the plot in [Figure 6](#).

Results

Across the six conditions of the experiment, the plaid and the grating appeared at the same depth when the grating phase disparity, averaged across observers, had one of three disparities: -3° , $+13^\circ$, or $+23^\circ$ ([Figure 6](#)). Oblique test gratings oriented parallel to the plaid’s disparity vector had a depth-matching disparity of approximately zero (-3° ; see color-coded stimulus diagrams in [Figure 6](#)); those oriented perpendicular to this vector had a depth-matching disparity of $+23^\circ$. These values were essentially the same whether the grating’s orientation was parallel to a plaid component or differed, by at least 45° , from any plaid component. Vertical reference gratings required a disparity of $+13^\circ$, on average, for a depth match. For each observer, the three values differed significantly from each other ($p < 0.01$, Bonferroni t -tests).

Consistent with results of [Experiment 1](#), horizontal disparity did not predict perceived depth in this experiment. The horizontal-disparity prediction is that PSEs in [Figure 6](#) would be the same whether the oblique test gratings (represented in center of stimulus diagrams) were oriented parallel or orthogonal to the plaid’s disparity vector (arrows in diagrams). But these PSEs were not at all the same; the lowest PSEs in [Figure 6](#) are from the parallel gratings and the highest are from the orthogonal gratings. Nor—unlike [Experiment 1](#)—does the relative disparity of commonly oriented components predict the data (compare the stimuli yielding the two highest PSEs). However, if we accept the existence of a bias to see the center as ‘far’ relative to the surround (by the equivalent of somewhere between 3 to 8 degrees of phase disparity), then we can account for the data of [Figure 6](#) by considering the disparity vectors of the plaid and the grating. We can do this by considering how the plaid’s disparity vector, $[D_p, \phi]$, projects onto the grating’s perpendicular disparity axis. This projection gives the disparity the grating would have if it had been a component of the plaid:

$$D_g = D_p \sin(\theta - \phi), \quad (2)$$

where D_g is the grating’s depth-matching disparity and θ is its orientation ($0^\circ \leq \theta < 180^\circ$ and $0^\circ \leq |\phi| \leq 180^\circ$).² This says, in effect, that a stimulus disparity direction, not the horizontal meridian, defines a reference axis along which

the projected disparity magnitudes of the plaid and the grating are compared when judging their relative depths. This disparity direction can be taken to be that either of the plaid or of the grating (Chai & Farell, 2008).³

Generality

We tested this projection hypothesis using additional grating orientations, plaid angles, and plaid component disparities. Plaid components had orientations of 60° and 120° or 30° and 150° . In each of these cases, both components had disparities of 15° (giving the plaid a horizontal disparity direction) or the two component disparities were 15° and 0° (giving the plaid a disparity direction of $+30^\circ$, -30° , $+60^\circ$, or -60°). Test grating orientations ranged between 90° and 165° when the plaid had a horizontal disparity and between 45° and 135° when the plaid’s disparity direction was $\pm 30^\circ$ or $\pm 60^\circ$.

Data for the resulting 20 stimulus conditions appear in [Figure 7](#). This plot shows observed PSEs as a function of predicted PSEs based on [Equation 2](#). For both observers, the relation between predicted and observed PSEs was strong for each of the 3 plaid disparity directions ($r^2 > 0.954$ for 5 of the 6 correlations, and $r^2 = 0.866$ for the sixth, for which the disparity direction was horizontal and the observer was S4; all $ps < 0.01$ by F -test). The best-fitting linear function has a slope of 0.76 for one observer and 0.72 for the other. The data show the previously observed bias to see the center as farther than expected from its disparity, especially at large disparities, that accounts for the less-than-unity slope of these functions.

One can see from [Figure 7](#) that the disparity projection expressed in [Equation 2](#) applies quite generally, describing reasonably well the depth matches observed for a variety of stimulus conditions. It applies when the plaid has components with equal disparities and with unequal disparities, when the plaid’s components are sampled from a wide range of orientations, and whether the plaid’s disparity has a horizontal direction or a direction as far as 60° from horizontal; it also applies when the test grating has an orientation drawn from a great range, from vertical to 15° of horizontal.

The projection predictions also handle the data of the [Experiments 1](#) and [2](#). In [Experiment 1](#), the projection of the plaid’s disparity vector onto the grating’s perpendicular disparity axis is independent of plaid angle in both the R_+ and R_0 conditions. It is equal to the disparity of the plaid’s component that is parallel to the grating in the R_+ condition and is equal to zero in the R_0 condition. In [Experiment 2](#), the plaid and the grating had the same disparity direction. Thus, the projection prediction is that their disparity magnitudes along this common direction should be equal at the depth match. The data of both experiments are in good agreement with these predictions.

[Equation 2](#) can be described by the disparity-vector projection illustrated in [Figures 8a–8c](#). Here, the plaid’s

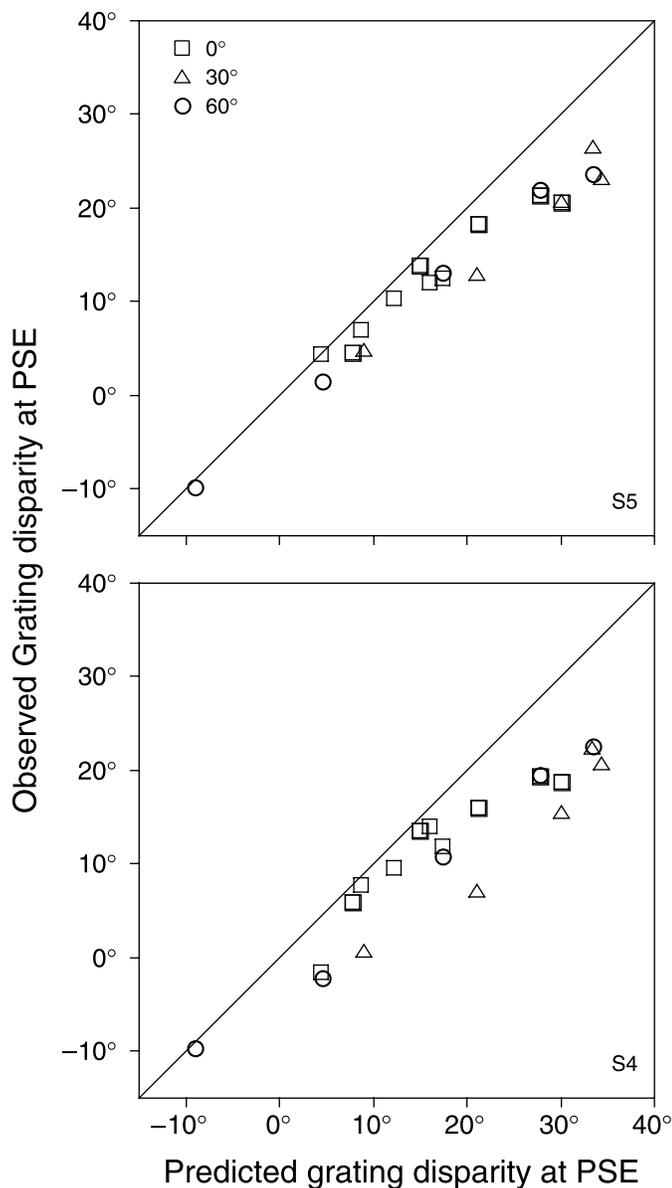


Figure 7. Depth-matching disparities plotted against disparity projection predictions for two observers. The reference plaid had a disparity direction of 0°, 30°, or 60°. The orientation of the test grating was sampled from a wide range (see text). Predictions are from Equation 2 in the text.

disparity vector (violet arrow) is superimposed on the grating (solid line). This vector is projected (dashed line) upon the grating’s perpendicular disparity axis (thin line). Equation 2 predicts a depth match when the grating’s disparity (blue arrow) equals this projection. Note in the figure (panel b) that a depth match that is predicted (and observed, see lower left points in Figure 7) between a plaid with positive horizontal disparity and a grating with negative horizontal disparity. Such a prediction arises only for non-horizontal plaid disparities. Specifically, the prediction holds when the grating is oriented between

the plaid’s disparity vector and the horizontal. Thus, among the plaid and grating pairs that are seen at the same depth are those that, when viewed individually relative to an isotropic zero-disparity reference stimulus, appear not only at different depths, but also at different depth polarities: The plaid appears behind the reference stimulus and the grating appears in front of it (Farell, Chai, & Fernandez, 2009). One can find a kin of this phenomenon in ‘depth-reversed’ plaids, where the superposition of one grating with zero-disparity, for example, and another with positive (‘far’) disparity, produces a negative-disparity (‘near’) plaid (Farell, 1998).

Horizontal orientations and vertical disparities

We next tested Equation 2 as it applies to horizontal stimulus orientations. Two of three predictions were supported and the other was rejected. First, we created

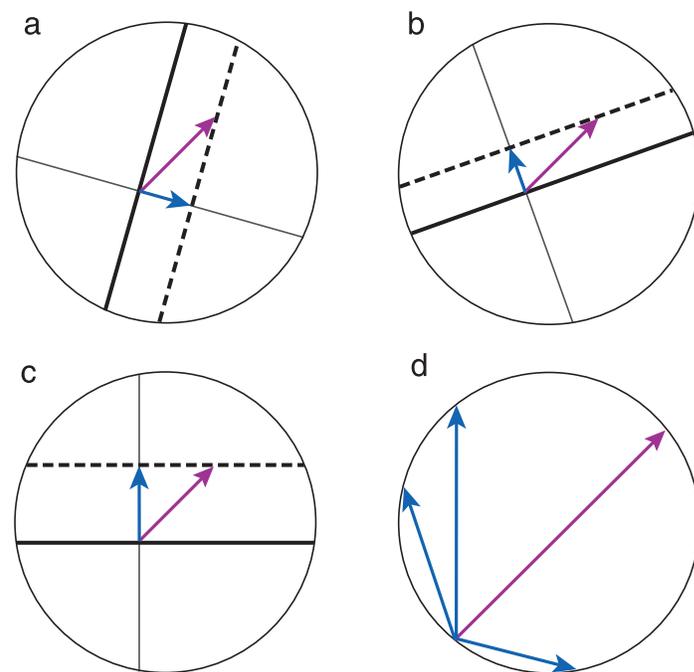


Figure 8. Sketches of predicted depth-matching grating disparities. (a–c) Thick solid and broken lines depict the left and right eyes’ views of gratings with various orientations. The blue arrow depicts the perpendicular grating disparity that is predicted to yield a depth match with a plaid having the disparity vector shown by the violet arrow. The plaid’s disparity vector is the same in all three cases. The grating’s disparity vector has a horizontal component with the same sign as the plaid’s disparity (a), the opposite sign (b), or a value of zero, in the case of the vertical disparity vector of a horizontal grating (c). The general prediction for depth-matching disparities defines a circle bisected by the reference disparity vector (d).

plaids in which one component was horizontal and we determined whether Equation 2 accurately predicted depth matches between these plaids and non-horizontal gratings. Plaid components with orientations of 0° and 45° , for example, will generate a pattern disparity direction of $+45^\circ$ or -135° when the 45° component has zero disparity and the 0° component has non-zero disparity. The question is whether the horizontal component of such a plaid contributes less effectively than components of other orientations to visually useful pattern disparities.

Plaid component orientations were 0° and either 45° or 135° . One of these components had a non-zero disparity. When this was the horizontal component, the plaid had an oblique disparity direction; when it was the non-horizontal component, the plaid had a horizontal disparity direction. With the test grating orientation set to 45° , 135° , or 90° , we measured PSEs with the methods previously used; stimulus duration was 150 ms. Figure 9 shows the results of plotting PSEs for these stimulus pairs against the values predicted by Equation 2. Data were averaged over 3 observers, whose results were similar. There was a clear modulation of grating's depth-matching disparity with variation in the disparity of the plaid's horizontal component (blue squares in Figure 9). PSEs approximated

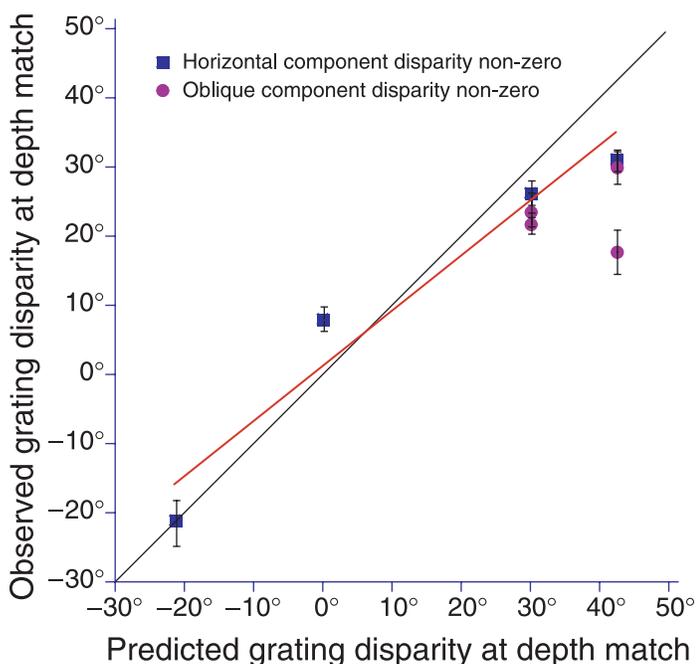


Figure 9. Grating disparities yielding depth matches with reference plaids having horizontal components. Means PSEs for 3 observers are plotted against projection predictions given by Equation 2. The orientation of the test grating was 90° , 45° , or 135° . The disparity direction of the reference plaid was 45° or 135° when the horizontal component had non-zero disparity (square symbols) and 0° when the oblique component had non-zero disparity (circular symbols). Red line gives the linear fit to the data plotted with square symbols. Error bars are ± 1 mean SEM.

those measured when the plaids had non-horizontal components (Figure 7) or horizontal components with zero disparity (Figure 9, circles).

Two of the observers contributing to Figure 9 also contributed depth-match data under conditions that were identical except that the reference plaids had a component oriented at 90° rather than 0° ; these data were previously reported (Figure 6). When these components—oriented at 90° in one plaid and at 0° in the other—had the equal disparity magnitudes, their respective plaids had identical pattern disparities. The slopes of the linear fits to the data, when plotted as in Figure 9, were similar for these two reference plaid types: 0.72 and 0.78 for one observer and 0.79 and 0.78 for the other (data from Figures 9 and 6, respectively). Thus horizontal components appear to contribute as effectively to visually useful pattern disparities as do vertical components. This result agrees with our prior evidence that the magnitude and direction of the pattern disparity are the only parameters of the plaid that affect the PSEs. Component orientations and disparities have no effect independent of their contribution to pattern disparity.

The second prediction concerns strictly vertical plaid disparities. According to Equation 2, a vertical grating with zero disparity should match the depth of a vertical-disparity plaid; this is consistent with the notion that a uniform vertical disparity is not a depth cue. But an oblique grating should have a non-zero disparity at the depth match. We tested this using plaids with components oriented at 45° and 135° . The two components had phase disparities that were equal in magnitude (7.5° or 15°) and opposite in sign. For each of the two disparity magnitudes, both combinations of signs and orientations were used, producing plaids with disparity directions of 90° and 270° . Test grating orientation was 45° or 135° . Within a block of trials, each of the two gratings was paired with plaids having vertical disparities of equal magnitudes and opposite directions. We presented these stimulus pairs for 150 ms and used our standard procedure of varying the gratings' disparities from trial to trial on two observers. At a depth match with any of the four plaids, gratings oriented at 45° and 135° should have disparities with opposed signs and with magnitudes equal those of the plaid's components. This is very close to what we observed. The signs were opposite and the magnitudes at the depth match, predicted to be 7.5° and 15° , averaged 6.8° and 19.0° , respectively. Thus, a strictly vertical disparity of a reference stimulus controls the relative depth of a 1-D stimulus, as was previously shown by Ito (2005). Quantitatively, the depth match between the two stimuli follows Equation 2.

The third prediction is that Equation 2 will apply when the test grating is horizontal (see panel c in Figure 8). We used reference plaids both with and without horizontal components and having both oblique and horizontal disparity directions. We collected data from three observers (those who contributed to Figure 9). The PSEs for all

three observers were hard to evaluate because of the shallowness of the slopes of the psychometric functions when the test grating was horizontal. Their slopes were, on average, less than one-seventh the value of those for non-horizontal test gratings. Thus, varying the disparity of a horizontal test grating produced no more than a weak change in the relative depth of the grating and the plaid (and this might be only because the grating wasn't coded as strictly horizontal). Yet, according to Equation 2, the change should have been the same as for test gratings with other orientations. Thus, while the disparity of a horizontal 1-D component contributes to the perceived depth of the pattern of which it is a part (Figure 9), the disparity of a lone horizontal 1-D stimuli appears not to enter into the calculation of stereo depth. In light of Figure 8c, one could use this finding to argue that for 1-D stimuli the perpendicular disparity direction is the functional disparity direction.

Discussion

We investigated perceptual depth matches between 1-D and 2-D stimuli. A plaid with a particular disparity magnitude and direction served as the 2-D reference stimulus. Paired with the plaid was a grating with a variable disparity, which served as the 1-D test stimulus. Humans can readily judge the relative depth of these stimulus pairs, though subjectively some pairs are more difficult to judge than others.

Horizontal disparity is the traditional metric for predicting what depth they will see. Extrapolating from disparity threshold data, however, we expected that relative orientation would be an important predictive variable. We hypothesized that the relative disparity of parallel stimulus components would be the best predictor of depth matches between the grating and the plaid; components most similar in orientation should have the greatest influence on perceived depth. We initially accepted this component-disparity hypothesis over an alternative pattern-disparity hypothesis, based on results of Experiment 1. We found evidence challenging this conclusion in Experiment 2. A new hypothesis, consistent with the data of the first and the second experiments, found support in Experiment 3. The results of Experiment 3 falsified the parallel-components hypothesis supported by Experiment 1 and quantified the disparity-vector effect underlying the results of Experiment 2.

Depth matches, according to our evidence, are seen when the two stimulus disparities project with approximately equal magnitude onto the grating's perpendicular disparity axis. This disparity-projection calculation applies to depth matches between 1-D and 2-D stimuli. It cannot be extended to pairs of 2-D stimuli, which we consider in a subsequent paper.

The relative depth of 1-D and 2-D patterns

The projection algorithm for predicting depth matches between 1-D and a 2-D stimuli (Equation 2) is a variant of the intersection-of-constraints construction (see Figure 8d), familiar from coherent motion studies (Adelson & Movshon, 1982). Intersection of constraints is also useful in describing coherent depth, where superimposed gratings with different disparities form a plaid seen in a single depth plane (Delicato & Qian, 2005; Farell, 1998; Farell & Li, 2004). In motion and depth coherence, the constraints operate on spatially co-extensive components and describe how they combine into a single object. As used here, the constraints operate between spatially separate stimuli and serve to calculate relational properties between them.

Equation 2 describes how depth-matching disparities vary with the orientation of the grating relative to the disparity vector of the plaid. With one exception, our data show little evidence that absolute orientation or absolute disparity direction affects depth matches independently of their contribution to the disparity-projection calculation of Equation 2 (the case of horizontal test gratings is that exception.)⁴ This does not mean that absolute orientation and disparity direction do not affect perceived depth, only that they do not make a major contribution to depth matches, where perceived depth separation is zero. There is also no evidence, aside from the horizontal-grating exception, that the disparity projection calculation applies only over a limited range of absolute disparity directions. Therefore, disparity encoding must vary with the magnitude and polarity of both vertical and horizontal disparity components—the information available, up to a global rotation, in a 2-space vector code for both 1-D and 2-D stimuli (for a recent discussion of this issue, see Serrano-Pedraza & Read, 2009).

Equation 2 provides a decent fit to the data from all the conditions of our study. Figure 10 plots all the PSEs (excluding data from horizontal test gratings) against the disparity projection prediction. Each data point is the mean of one condition from Experiments 1, 2, 3, or associated experiments, averaged across observers. Grating orientation ranged from vertical to 15° of horizontal across these conditions. The plaid disparity direction ranged from horizontal to 30° of vertical (except for 4 points, where the direction was vertical). Relative to the plaid disparity direction, grating orientation ranged from parallel through perpendicular. Presentation duration ranged from 150 ms to 1 s, and observers ranged from the highly experienced and informed to the inexperienced and naïve. The red line in Figure 10 is the best linear fit; it has a slope of 0.90 and an r^2 value of 0.834.

For 1-D stimuli, depth percepts are relational, not intrinsic, functions of disparity. Two gratings with identical disparity magnitudes—including gratings with identical horizontal disparities—will generally appear at different depths relative to a reference stimulus; and two

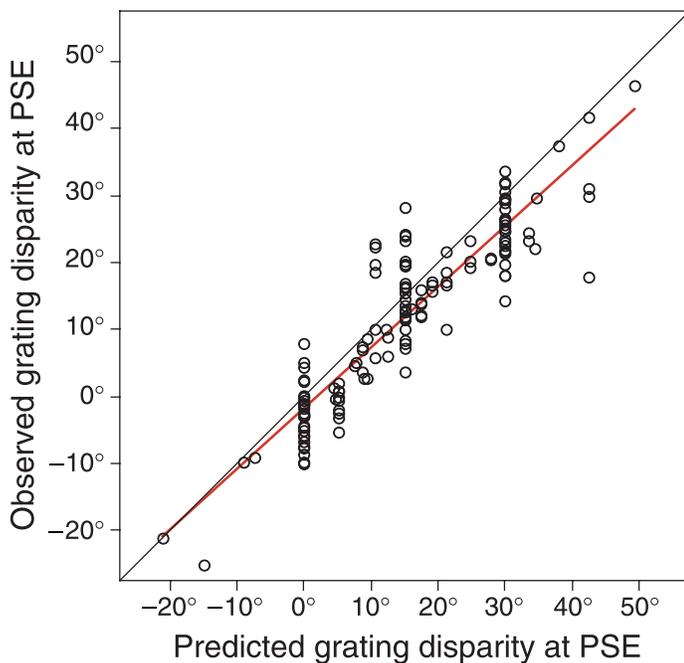


Figure 10. PSEs for all non-horizontal test-grating conditions plotted against projection predictions. Each data point is the mean across observers for one condition. The equality line appears in black; the best linear fit, with slope of 0.90, is shown in red.

gratings that appear at the same depth as one reference stimulus will not generally appear at the same depth relative to a different reference stimulus (Farell et al., 2009). The depths will vary with the disparity vectors of both the gratings and the reference stimuli. In effect, the disparity vector of one stimulus defines a reference axis for comparing the disparity magnitudes of the two stimuli (Figure 8; cf. Glennerster, McKee, & Birch, 2002; Petrov & Glennerster, 2006).

Could we perhaps choose a different direction along which to compare the disparities? Could we perhaps choose the horizontal direction? In the case of gratings, and presumably of other 1-D stimuli, we cannot. Directions, including the horizontal direction, that are not tied to the disparity vectors of the stimuli, and thus not represented in Equation 2, will not work. As an example, suppose we create a plaid with, say, a positive disparity—a ‘far’ plaid—whose a disparity direction is $+45^\circ$ (other oblique directions would work as well). And suppose we create a grating with a disparity initially set to zero and ask an observer to adjust the disparity of the grating so that the two stimuli appear in the same depth plane. If the grating’s orientation is greater than 45° (but not too close to 180° , of course), the observer would turn the adjustment knob one way to reach a depth match, giving the grating a positive disparity. If the grating’s orientation is less than 45° (but not too close to 0°), the observer would turn the knob the other way, giving the grating a negative

disparity (see lower left points in Figures 7 and 9). The horizontal disparities of the gratings would have different magnitudes and opposite signs whether we measured disparities horizontally or measured the horizontal components of perpendicular disparities. What makes these different disparities equivalent is that both are equal to the projection of the plaid’s disparity vector onto the gratings’ perpendicular axes (Figures 8a and 8b). If the two gratings with these depth-matching disparities were superimposed, the resulting plaid would have the same disparity as the reference plaid. Yet if the gratings, each with the disparity set for the depth match, were compared to a zero-disparity reference stimulus, one would appear in front of it and the other behind it (Farell et al., 2009). Both gratings are capable of taking on a horizontal disparity equal to that of the plaid, but if they did so, no more than one of them would be seen at the same depth as the plaid. Thus, horizontal disparity in itself is not useful in the general case as a predictor of the perceived depth between a grating and a plaid.

Previous studies of the depth of 1-D stimuli

The aperture problem in stereo concerns the interocular matching direction for 1-D stimuli. Discussion of this issue has rested on two assumptions: that a 1-D stimulus has an intrinsic matching direction and that stereo threshold and perceived depth measures can reveal it (Morgan & Castet, 1997; van Dam & van Ee, 2004; van Ee & Schor, 2000). Though reasonable, these assumptions have not been put to the test. Our results show that a grating with a particular disparity does not have an intrinsic perceived depth, regardless of the direction along which one measures the disparity. This would seem to invalidate both assumptions. Nevertheless, we can compare the data of other perceived-depth studies with ours, whatever the interpretational differences.

Two previous studies have measured the perceived depth of non-vertical 1-D stimuli (the vertical case is of little interest with regard to the stereo aperture problem). Both studies compared the perceived depths of a line and a disk (van Dam & van Ee, 2004; van Ee & Schor, 2000). Behind each study was the assumption that the magnitude of the horizontal component of disparity determines perceived depth. (The assumption has also been invoked to account for the perceived tilt of quasi-1-D stimuli, oblique line segments, in the ‘induced effect’; Arditi, Kaufman, & Movshon, 1981; Arditi, 1982). The horizontal disparity of the line was constant, while the disparity of the disk varied from trial to trial. van Dam & van Ee (2004) found that the line and the disk appeared at the same depth when their horizontal disparities were approximately equal, provided the line’s ends were effectively obscured. This agrees with our data, because the disparity of their disk was strictly horizontal. The disk disparity used in van Ee

& Schor's (2000) study was more complicated; it could take on any of 12 directions. To be reasonably comparable to our study, their method would have had to measure for each of these directions the disparity magnitude that resulted in a depth match. But van Ee and Schor's disk disparities were restricted to coincide with the disparity constraint line defined by an orientation of 45° and a horizontal disparity of 15 arcmin (R. van Ee, personal communication, 2008). Thus the disk was presented at a single disparity magnitude at each of the 12 disparity directions.

van Ee & Schor (2000) reported that the task was impossible at a line orientation of 20° . For other orientations (35° to 90°), they found that the disk's horizontal disparity component at the depth match was somewhat less than the line's, ranging between approximately 12 to 14 arcmin (versus 15' for the line). The lower end of this range came from the most nearly horizontal of the line's orientations (35°). Our results predict a horizontal disparity of 15 arcmin for all cases (except for the case of the 45° line). This is because a 15 arcmin horizontal disparity is equivalent to the projected disparity magnitude. For the 45° line, all the disk's presented disparity vectors should lead to a depth match, for they were constructed to be consistent with the disparity constraint line of the 45° target line. So, we have no prediction for this condition (but see below).

van Ee and Schor interpreted their result by suggesting that the disparity direction of the line can be calculated by combining all possible matching directions. The horizontal component of the resultant disparity determines perceived depth. They also suggested that the range of vertical matches is limited, so in practice only a subset of possible disparity directions contributes to the effective direction. The limited vertical range imparts a horizontal bias to this effective direction; a range limited to about 10 arcmin gave a good fit to the data.

This is a reasonable picture. Indeed, the high spatial-frequency components of the line should limit the range of vertical disparities, just as they impose a limit on horizontal disparity thresholds (Farell, Li, & McKee, 2004). But it is a picture that should be applied to the disk as well as the line: The disk's vertical disparity component should have affected its perceived depth (as it affected the perceived depth of the disks in the study of Friedman, Kaye, and Richards (1978)). If the perceived depths of the disks and the line are not identically modulated by their vertical disparity components, then the disparity-projection hypothesis would have a foothold for predicting depth matches in the 45° -line condition, a condition that otherwise seems problematic (see two paragraphs back). The disk's vertical disparity might have affected its perceived depth in other disparity-direction conditions, as well.

In any case, the results of van Ee and Schor (2000), and those of van Dam and van Ee (2004), are reasonably

consistent with ours, despite the different stimuli used. However, their assumption that the horizontal disparity of a 1-D stimulus determines its perceived relative depth seems invalidated by our data and is not required by theirs.

Psychometric slopes

Perceived depth equality is the focus of our study, with the 50%-point on the psychometric function providing the data. The slope of the psychometric function gives a measure of disparity gain or depth resolution, which is largely independent of the issue of depth equality. However, we examined the slopes for all the conditions of our experiments; on a few points, which we summarize here, they speak to, and reinforce, conclusions drawn from the PSE data already presented.

Slope values generally fell within a fairly narrow range despite the broad range of stimulus disparity and orientation values. With one exception, there was little evidence of a systematic variation of slope values across stimulus conditions. This exception occurred at long (1 s) stimulus durations. Slopes were no different across the three applicable target orientations (45° , 90° , 135°) and the two stimulus durations (150 ms, 1 s), except for vertical test gratings at long durations. Here, psychometric functions were steeper by a factor of almost two ($F(1, 42) = 30.43$, $p < 0.001$). As noted earlier, there was no analogous effect of duration on PSEs. Vergence eye movements are a possible source of the difference. We did not record eye movements and so cannot test for interactions between vergence changes and stimulus orientation. We do have some preliminary evidence that eye movements in themselves do not affect the outcome of [Experiment 1](#) (where, however, the gratings were non-vertical). In an unreported variation of that experiment, we instructed observers either to change fixation from the center to the surround approximately midway through 1-second trials or to maintain fixation on the center. They reported the instructions easy to comply with. We found no significant difference between the two conditions in slopes or in PSEs, which closely resembled the PSEs of [Figures 2 and 3](#).

The lack of systematic variation among slope values in other test-grating cases might seem surprising, for two reasons: Stereoacuity, at least for 2-D stimuli, is usually highest when disparities are horizontal (Farell, 2003b; Morgan & Castet, 1997; Stevenson & Schor, 1997) and near the horopter (e.g., Badcock & Schor, 1985; Blakemore, 1970; Ogle, 1953). However, there are limits to both these generalities. Most importantly, stereoacuity, rather than varying with position relative to the horopter, can vary relative to a stimulus-defined reference frame (Glennerster et al., 2002; Petrov & Glennerster, 2006; Westheimer, 1979).

This explains both the narrow range of slope values and why a large change in absolute disparity need not be more

detectable than a small change. Suppose that we have two differently oriented gratings, each at a perceptual depth match with the same reference stimulus. Their disparities would be different (see, e.g., Figure 6). Consequently, if we now reduce each of these grating disparities by the same phase angle (as might occur from one trial to the next under a constant-stimulus procedure), the absolute disparities of the two gratings would decrease by different amounts—different by a factor of 3 in the case of gratings oriented 18° and 68° relative to the reference disparity direction, for example. The steepness of the psychometric function does not vary systematically across test-grating conditions, so these very different changes in absolute disparity should be equally detectable. This may seem counter-intuitive, but only if zero disparity is regarded as special. It is zero disparity, as an origin on the disparity axis, that converts a given numerical change in disparity into a large change in the absolute disparity of one grating and a small change in that of another. But absolute disparities are relevant only if the horopter is brought to bear on depth discriminations. People might instead discriminate depth directly on the basis of relative disparity; they might, as the data presented here suggest, use one stimulus, rather than the horopter, as a reference frame for judging another stimulus. In that case, sensitivity should be independent of the absolute disparities of the stimuli (and even independent of whether the disparities have the same or different signs). This is in line with the psychometric slope data.

Envelope disparities

In our experiments, the disparities of contrast envelopes were uncorrelated with the disparities of the carriers. The envelopes added a constant zero-disparity signal to the display. Because of this, the envelopes could have functioned as reference stimuli, allowing observers to side-step a direct comparison of grating and plaid depths. That is, observers could have compared the grating with its envelope to get one depth estimate, compared the plaid with its envelope to get another estimate, and then compared these two estimates. In this case, however, depth matches might vary with parameters of the grating and the plaid, but they would do so independently rather than interactively as was observed. So, envelopes were evidently not used to mediate depth comparisons. Given our interest, this is a good thing.

Our interest was to study the effects of grating and plaid parameters on perceived depth. We think that studying these effects with stimuli in which envelope and carrier disparities are correlated would be problematic, for two general reasons. First, one would have to determine the appropriate direction and magnitude for the envelope disparities. They should be selected so that they correlated equally with those of the disparities of the gratings and the

plaids. But because of the aperture problem, correlating anything with the direction or magnitude of a grating's disparity is a rather uncertain proposition. And if this problem were overcome, then a second one would arise: The aim of the study—to characterize how perceived depth varies with carrier parameters—would be jeopardized, for it would be difficult to discount the possibility that perceived depth had varied with the correlated envelope disparities instead.

By decorrelating envelope and carrier disparities, we by-pass these concerns. These disparities have distinct signatures. If observers had judged envelope disparities, then psychometric functions would have been flat, which they were not. Contrariwise, if carrier disparities determined the PSEs measured here, then the effect of the plaid's disparity magnitude on PSE would be expected to be linear, which it very nearly is (Figures 4, 5, and 10).

Of course, the uncorrelated disparities used here are unlike the disparities of natural objects; the textures and edges of rigid objects have physically coupled depths. Visually, however, decorrelation occurs frequently in natural viewing conditions because of occlusion. Occlusion leads to aperture viewing, where a surface is framed by the boundaries of other surfaces. Texture and edge disparities, being properties of different objects, are then partially or even thoroughly dissociated, in the manner of the carrier and envelope disparities of the stimuli used here.

Ito's (2005) displays

Ito (2005) created a display that showed a combined effect of orientation and vertical disparity on perceived depth. The displays consisted of dots with vertical disparity interspersed among oblique lines with zero disparity. The lines appeared in front of or behind the dots, depending on whether their orientation was left-oblique or right-oblique. The effect occurred when the stimulus presentation was short as well as long, so did not depend on eye movements. The orientation-contingent depth was not seen when the dots' disparity direction was horizontal.

To explain these phenomena, Ito suggested that a neural shift cancelled the vertical disparity of the dots. This neural shift accomplishing what a vertical-disparity-cancelling eye movement would do, given a sufficiently long stimulus presentation. Its side-effect is to create a vertical offset in the nominally zero-disparity lines. The direction of subsequent stereo-matching of the lines contains a horizontal component, the polarity of which differs in sign between left-oblique or right-oblique lines. This accounts for the depth difference.

This explanation, however, is specific to vertical dot disparities; it treats horizontal disparities as the domain of a separate processing system. It should fail when the cancellation of vertical disparity does not result in a stereo

match, as when both horizontal and vertical disparity components are present. Without a stereo-match signal, the shifting mechanism would not know when it had nulled the vertical component of the disparity. A different signal (for example, one available from *a priori* knowledge of the vertical disparity) would have to control it.

But if some such signal were available in most of our conditions, it would seem to be lacking in the variable-reference-disparity control condition of [Experiment 1](#) and in our test of predictions for vertical-disparity plaids paired with oblique gratings, in which the reference disparity was also uncertain (see page 11). In any case, disparity projection calculation provides an alternative account. It straightforwardly predicts Ito's (2005) results for both vertical and horizontal dot disparities. It would also apply to intermediate dot disparity directions, whatever their combination of horizontal and vertical components. In addition, it avoids the necessary corollary of the shift hypothesis, made explicit in Ito's account and rejected by our data, that the perceived depth of a 1-D stimulus depends on its horizontal disparity.

Conclusions

Two patterns with very different disparities can appear at the same depth. Observers see stereo depth matches between 1-D and 2-D stimuli (a grating and a plaid in our experiments) when the disparity vectors of both stimuli project with equal magnitudes onto the grating's perpendicular disparity axis. Depth matches between these stimuli are not predicted by horizontal disparities; in fact, it is possible for a 1-D stimulus and a 2-D stimulus to match in depth when their horizontal disparities have opposite signs.

The depth-matching function described here varies with the disparity directions of the stimuli, and for 1-D stimuli, disparity direction varies with orientation. As a result, the function describes depth matches that are not veridical. Nor are they general, for the function cannot be applied to depth matches between 2-D stimuli. Nevertheless, when applied to the 1-D components within a region of space, disparity projection could serve as a veridical and general computational step for segregating the components of one object from those of others and combining these components into 2-D perceptual patterns (Farell, 1998; Patel, Bedell, & Sampat, 2006; Patel et al., 2003).

Acknowledgments

We would like to thank our observers for their effort, endurance, and seriousness, and Julian Fernandez for insights in all phases of the study. We want also to thank

Jenny Read for her thoughtful suggestions. And we thank an anonymous reviewer for the insight that this shortened version of the paper is long enough and for inadvertently allowing us to teach ourselves a lesson.

This research was supported by National Institutes of Health Grant EY012286 (B.F.).

Commercial relationships: none.

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Footnotes

¹A pattern with a 2-D spatial structure can have an ambiguous disparity. It might be periodic along one direction and therefore have multiple correspondence magnitudes, or along multiple dimensions and have multiple correspondence directions and magnitudes (as is the case with plaids). For our purposes, it is in their nearest-neighbor matches that the disparities of 2-D patterns differ importantly from those of 1-D patterns. Nearest-neighbor matches are independent of the monocular spatial structure of 2-D patterns, but they are in the direction normal to the orientation of 1-D patterns. Hence, the nearest-neighbor matches of 2-D patterns, which we use to define the disparity vector of these patterns, can be given an arbitrary direction. To measure the disparity vector of a stimulus, we consider the superposition of the two retinas and measure the magnitude and the right-eye-to-left-eye direction of the offset of nearest corresponding points.

²If orientations are relabeled as $-180^\circ \leq \theta < 0^\circ$, then the angular difference in [Equation 2](#) should be reckoned as $(\phi - \theta)$.

³Equivalently, [Equation 2](#) predicts a depth match when the grating has the same disparity magnitude as the plaid in the direction of the plaid's disparity. Equivalently again, it predicts a depth match when the plaid's disparity vector is consistent with the grating's disparity constraint line, or when $\mathbf{A} \cdot \mathbf{B} = |\mathbf{B}|^2$, where \mathbf{A} is the plaid's disparity vector and \mathbf{B} is the grating's perpendicular disparity vector.

⁴This makes it unlikely that disparity signals from second-order features have a systematic effect on depth matches in our experiments. We checked this by calculating the disparities of second-order features at the sum and difference of the spatial frequencies of the components of our plaids. Their low spatial-frequency content makes these second-order features the most likely to have an influence (Delicato & Qian, 2005). We found no significant relationship between these disparities and those of the test gratings in our PSE data.

References

- Adelson, E. H., & Movshon, J. A. (1982). Phenomenal coherence of moving visual patterns. *Nature*, *300*, 523–525. [PubMed]
- Adelson, E. H., & Movshon, J. A. (1984). Binocular disparity and the computation of two-dimensional motion. *Journal of the Optical Society of America*, *1A*, 1266.
- Anderson, B. L. (1994). The role of partial occlusion in stereopsis. *Nature*, *367*, 365–368. [PubMed]
- Anzai, A., Ohzawa, I., & Freeman, R. D. (1999). Neural mechanisms for processing binocular information: II. Complex cells. *Journal of Neurophysiology*, *82*, 909–924. [PubMed] [Article]
- Arditi, A., Kaufman, L., & Movshon, J. A. (1981). A simple explanation of the induced size effect. *Vision Research*, *21*, 755–764. [PubMed]
- Arditi, A. (1982). The dependence of the induced effect on orientation and a hypothesis concerning disparity computations in general. *Vision Research*, *22*, 247–256. [PubMed]
- Badcock, D. R., & Schor, C. M. (1985). Depth-increment detection function for individual spatial channels. *Journal of the Optical Society of America A, Optics and Image Science*, *2*, 1211–1216. [PubMed]
- Blake, R., Camisa, J., & Antoinetti, D. N. (1976). Binocular depth discrimination depends on orientation. *Perception & Psychophysics*, *20*, 113–118.
- Blakemore, C. (1970). The range and scope of binocular depth discrimination in man. *The Journal of Physiology*, *211*, 599–622. [PubMed] [Article]
- Brainard, D. H. (1997). The psychophysics toolbox. *Spatial Vision*, *10*, 433–436. [PubMed]
- Calabro, F. J., & Vaina, L. M. (2006). Stereo motion transparency processing implements an ecological smoothness constraint. *Perception*, *35*, 1219–1232. [PubMed]
- Chai, Y.-C., & Farell, B. (2008). How does perceived depth depend on disparity direction [Abstract]? *Journal of Vision*, *8(6)*:1088, 1088a, <http://journalofvision.org/8/6/1088/>, doi:10.1167/8.6.1088.
- Cumming, B. G. (2002). An unexpected specialization for horizontal disparity in primate visual cortex. *Nature*, *418*, 633–636. [PubMed]
- Cumming, B. G., & DeAngelis, G. (2001). The physiology of stereopsis. *Annual Review of Neuroscience*, *24*, 203–238. [PubMed]
- Delicato, L. S., & Qian, N. (2005). Is depth perception of stereo plaids predicted by intersection of constraints, vector average or second-order feature? *Vision Research*, *45*, 75–89. [PubMed]
- Ebenholtz, S., & Walchli, R. (1965). Stereoscopic thresholds as a function of head- and object-orientation. *Vision Research*, *5*, 455–461. [PubMed]
- Farell, B. (1998). Two-dimensional matches from one-dimensional stimulus components in human stereopsis. *Nature*, *395*, 689–693. [PubMed]
- Farell, B. (2003a). Relative binocular disparity: Relative to what [Abstract]? *Journal of Vision*, *3(9)*:462, 462a, <http://journalofvision.org/3/9/462/>, doi:10.1167/3.9.462.
- Farell, B. (2003b). Detecting disparity in two-dimensional patterns. *Vision Research*, *43*, 1009–1026. [PubMed]
- Farell, B. (2006). Orientation-specific computation in stereoscopic vision. *Journal of Neuroscience*, *26*, 9098–9106. [PubMed] [Article]
- Farell, B., & Ahuja, S. (1996). Binocular disparity of 1- and 2-D contours. *Investigative Ophthalmology and Visual Science*, *37*, S284.
- Farell, B., Chai, Y.-C., & Fernandez, J. M. (2009). Projected disparity, not horizontal disparity, predicts stereo depth of 1-D patterns. *Vision Research*, *49*, 2209–2216. [PubMed]
- Farell, B., & Li, S. (2004). Seeing depth coherence and transparency. *Journal of Vision*, *4(3)*:8, 209–223, <http://journalofvision.org/4/3/8/>, doi:10.1167/4.3.8. [PubMed] [Article]
- Farell, B., Li, S., & McKee, S. P. (2004). Coarse scales, fine scales, and their interactions in stereo vision. *Journal of Vision*, *4(6)*:8, 488–499, <http://journalofvision.org/4/6/8/>, doi:10.1167/4.6.8. [PubMed] [Article]
- Friedman, R. B., Kaye, M. G., & Richards, W. (1978). Effect of vertical disparity upon stereoscopic depth. *Vision Research*, *18*, 351–352. [PubMed]
- Glennerster, A., McKee, S. P., & Birch, M. D. (2002). Evidence for surface-based processing of binocular disparity. *Current Biology*, *12*, 1–20. [PubMed]
- Ito, H. (2005). Illusory depth perception of oblique lines produced by overlaid vertical disparity. *Vision Research*, *45*, 931–942. [PubMed]
- Marr, D., & Poggio, T. (1979). A computational theory of human stereo vision. *Proceedings of the Royal Society, London B: Biological Sciences*, *204*, 301–328. [PubMed]
- McKee, S. P., Verghese, P., Ma-Wyatt, A., & Petrov, Y. (2007). The wallpaper illusion explained. *Journal of Vision*, *7(14)*:10, 1–11, <http://journalofvision.org/7/14/10/>, doi:10.1167/7.14.10. [PubMed] [Article]
- Morgan, M. J., & Castet, E. (1997). The aperture problem in stereopsis. *Vision Research*, *39*, 2737–2744. [PubMed]

- Nakayama, K. (1996). Binocular visual surface perception. *Proceedings of the National Academy of Sciences of the United States of America*, *93*, 634–639. [PubMed] [Article]
- Ogle, K. N. (1953). Precision and validity of stereoscopic depth perception from double images. *Journal of the Optical Society of America*, *43*, 906–913. [PubMed]
- Ogle, K. N. (1955). Stereopsis and vertical disparity. *Archives of Ophthalmology*, *53*, 495–504. [PubMed]
- Ohzawa, I., DeAngelis, G. C., & Freeman, R. D. (1990). Stereoscopic depth discrimination in the visual cortex: Neurons ideally suited as disparity detectors. *Science*, *249*, 1037–1041. [PubMed]
- Patel, S. S., Bedell, H. E., & Sampat, P. (2006). Pooling signals from vertical and non-vertically orientation-tuned disparity mechanisms in human stereopsis. *Vision Research*, *46*, 1–13. [PubMed]
- Patel, S. S., Ukwade, M. T., Stevenson, S. B., Bedell, H. E., Sampath, V., & Ogmen, H. (2003). Stereoscopic depth perception from oblique phase disparities. *Vision Research*, *43*, 2479–2792. [PubMed]
- Pelli, D. G. (1997). The VideoToolbox software for visual psychophysics: Transforming numbers into movies. *Spatial Vision*, *10*, 437–442. [PubMed]
- Pelli, D. G., & Zhang, L. (1991). Accurate control of contrast on microcomputer displays. *Vision Research*, *31*, 1337–1350. [PubMed]
- Petrov, Y., & Glennerster, A. (2006). Disparity with respect to a local reference plane as a dominant cue for stereoscopic depth relief. *Vision Research*, *46*, 4321–4332. [PubMed]
- Serrano-Pedraza, I., & Read, J. C. A. (2009). Stereo vision requires an explicit encoding of vertical disparity. *Journal of Vision*, *9*(4):3, 1–13, <http://journalofvision.org/9/4/3/>, doi:10.1167/9.4.3.
- Stevenson, S. B., & Schor, C. M. (1997). Human stereo matching is not restricted to epipolar lines. *Vision Research*, *37*, 2717–2723. [PubMed]
- van Dam, L. C. J., & van Ee, R. (2004). Stereoscopic matching and the aperture problem. *Perception*, *33*, 769–787. [PubMed]
- van Ee, R., Anderson, B. L., & Farid, H. (2001). Occlusion junctions do not improve stereoacuity. *Spatial Vision*, *15*, 45–59. [PubMed]
- van Ee, R., & Schor, C. M. (2000). Unconstrained stereoscopic matching of lines. *Vision Research*, *40*, 151–162. [PubMed]
- Westheimer, G. (1979). Cooperative neural processes involved in stereoscopic acuity. *Exp. Brain Research*, *36*, 585–597. [PubMed]
- Wichmann, F. A., & Hill, N. J. (2001). The psychometric function: I. Fitting, sampling, and goodness of fit. *Perception & Psychophysics*, *63*, 1293–1313. [PubMed] [Article]
- Wichmann, F. A., & Hill, N. J. (2001). The psychometric function: II. Bootstrap-based confidence intervals and sampling. *Perception & Psychophysics*, *63*, 1314–1329. [PubMed] [Article]
- Zhu, Y. D., & Qian, N. (1996). Binocular receptive field models, disparity tuning, and characteristic disparity. *Neural Computation*, *8*, 1611–1641. [PubMed]