CORRESPONDENCE

Comments on "On the Steadiness of Separating Meandering Currents"

DORON NOF,* VOLODYMYR ZHARKOV,[+] WILTON ARRUDA,[#] THIERRY PICHEVIN,[@]
STEPHEN VAN GORDER,[&] AND NATHAN PALDOR**

*Department of Earth, Oceans and Atmospheric Science, and Geophysical Fluid Dynamics Institute,
The Florida State University, Tallahassee, Florida
[+]Center for Ocean–Atmospheric Prediction Studies, The Florida State University, Tallahassee, Florida
[#]Department of Earth, Oceans and Atmospheric Science, The Florida State University, Tallahassee, Florida,
and Instituto de Matemática, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil
[@]Ecoles de Saint-Cyr Coetquidan, Guer, France
[&]Department of Earth, Oceans and Atmospheric Science, The Florida State University, Tallahassee, Florida
**Fredy and Nadine Herrmann Institute of Earth Sciences, The Hebrew University of Jerusalem, Jerusalem, Israel

ABSTRACT

Using integration constraints and scale analysis, van Leeuwen and De Ruijter focused on the steady aspect of the downstream flow in the momentum imbalance articles of Nof and Pichevin appearing in the 1990s and later on. They correctly pointed out that when the steady downstream flow is exactly geostrophic then it must obey the additional downstream (critical) condition $u^2 = g'h$ (where u is the speed, g' is the reduced gravity, and h is the thickness). They then further argue that this additional condition provides "a strong limitation on the generality of their results." These results for steady flows have been incorrectly generalized by the typical reader to eddy generating unsteady flows, which was the focus of Nof and Pichevin.

The current authors argue that, although the van Leeuwen and De Ruijter condition of $u^2 = g'h$ is valid for a purely geostrophic and steady flow downstream, it is inapplicable even for the steady aspect of the Nof and Pichevin solutions because the assumption of a purely geostrophic flow (i.e., $fu = -g'h_Y$ and $v = 0$) was never made at any downstream cross section in Nof and Pichevin. Instead, the familiar assumption of a cross-stream geostrophic balance in a boundary current, which is slowly varying in the downstream direction, as well as time, has been made (i.e., $fu \simeq -g'h_Y$, $v \ll u$, and small $\partial/\partial t$ but nonzero). Perhaps the current authors originally were not as clear about that as they should have been, but this implies that the basic state around which van Leeuwen and De Ruijter expanded their steady Taylor series does not exist in Nof and Pichevin; consequently, their expansion fails to say anything about both the time-dependent and the time-independent Nof and Pichevin. In the current authors' view, the "strong limitation" that they allude to does not exist.

---

1. Introduction

The easiest way to introduce the Nof and Pichevin momentum imbalance idea is via a northward outflow problem (e.g., Pichevin and Nof 1997; Nof 2005) rather than via a retroflection problem, which is more difficult to understand. (Ironically, this is actually the order that we did the original work, but the sometimes treacherous road of getting submitted articles to appear reversed the order in which they were published.) In an attempt to make this note at least semi-self-contained, we reproduce below a few figures and equations from the earlier Nof and Pichevin articles.

Consider the hypothetical steady northward (inviscid) reduced gravity outflow situation shown in Fig. 1. The integrated momentum flux along the coast (obtained by an integration of the x momentum equation along the contour ABCDA) is

$$\int_0^L (hu^2 + g'h^2/2 - f\psi)\, dy = 0, \qquad (1)$$

Corresponding author address: Doron Nof, Department of Earth, Oceans and Atmospheric Science, The Florida State University, 117 N. Woodward Ave., Tallahassee, FL 32306.
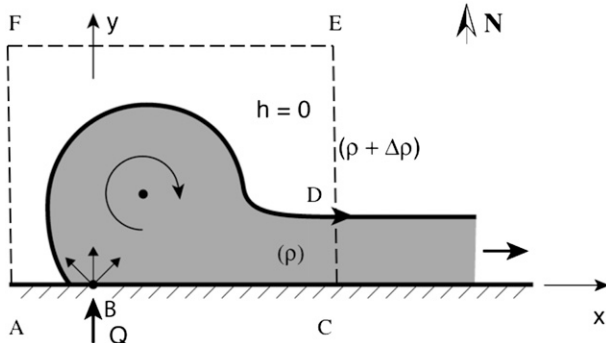E-mail: nof@ocean.fsu.edu

FIG. 1. A schematic diagram of the hypothetical (Northern Hemisphere) steady configuration shown by Nof and Pichevin to be impossible on a β plane (adapted from Nof 2005). This is because the along-shore momentum flux of the slowly varying downstream boundary current, which is pushing westward, is not balanced. As a result, eddies are periodically shed on the left-hand side (Fig. 2).

where u is the speed along the coast, h is the thickness, f is the Coriolis parameter, ψ is the streamfunction, $g' = g\Delta\rho/\rho$ (where $\Delta\rho$, ρ are the density difference/density of the layers and $\Delta\rho/\rho \ll 1$), and L is the width of the boundary current downstream at CD (i.e., $y = L$ when $h = 0$). Note that symbols and abbreviations are defined in both the text and the appendix. Here, L, which is a weak function of x, depends on the outflow potential vorticity and is on the order of the Rossby radius,

$$R_D = (2g'Q)^{1/4}/f^{3/4},$$

where Q is the outflow's volume flux (equal to $g'H^2/2f$, where H is a depth scale). Assuming steadiness, one dimensionality (i.e., $v \ll u$ but nonzero), and geostrophy in the cross-stream direction at CD (but allowing the flow to vary on a much longer downstream length scale; see, e.g., Charney 1955) and neglecting terms $\sim O(\beta R_D/f_0) \sim (0.01)$, we get

$$\int_0^L hu^2 \, dy = 0. \qquad (2)$$

Obviously, (2) cannot be satisfied so there cannot be a steady outflow of the kind pictured in Fig. 1. This is what Nof and Pichevin called the paradox. Aside from the momentum constraint discussed here, there may also be a yet-unknown hydraulic constraint involving a condition similar to the familiar criticality of uniform flow alluded to by van Leeuwen and De Ruijter (2009, hereafter VL-DR) ($u^2 = g'h$). This would ensure the establishment of stationary waves downstream because their propagation tendency will be arrested by the advection. Whether such an additional condition exists is of no consequence to the more general time-dependent Nof and Pichevin
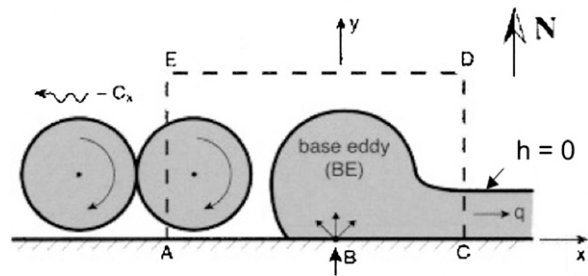

FIG. 2. Schematic diagram showing the Nof and Pichevin resolution of the momentum imbalance paradox. The "wiggly" arrow denotes migration. Because of the imbalance shown in Fig. 1, anticyclonic eddies are generated on the left-hand side (looking offshore). Through β these eddies are forced to propagate to the left. Nof and Pichevin obtained their analytical solution by equating the momentum flux through EA to the momentum flux through CD. The base eddy, which is the eddy in contact with the source, should be distinguished from the already detached eddies downstream.

problem discussed below. (Note that the above hypothetically steady problem is defined here as problem 1, whereas the more general time-dependent problem will be defined, in a moment, as problem 2.)

Nof and Pichevin resolved the paradox by arguing that a chain of eddies (Fig. 2) is formed on the western side of the outflow to compensate for the momentum flux of the jet on the eastern side. This way the momentum flux of the westward moving eddies balances the momentum expressed by (2) via a nonzero term on the right-hand side of (2). Because eddies move westward (due to β) much more slowly than their orbital speed (which is on the same order as the mean flow downstream), they are much larger than R_D (see Nof and Pichevin), and so is the x scale of the downstream current.

Nof and Pichevin then took the above and applied it to the (Southern Hemisphere) retroflection case (Fig. 3) near a coastline with zero slant ($\gamma \to 0$ so that the coast is zonal). Here, there are two currents with their momentum flux pointing westward (because the momentum is proportional to u^2 not u) so the mass flux going into the eddies is larger than in the northward outflow problem and, as a result, the eddies themselves are also larger (see Nof and Pichevin 1996; Pichevin et al. 1999). It is no surprise, therefore, that retroflection eddies are the largest rings in the World Ocean (Olson and Evans 1986; Olson 1991).

We see that the Nof and Pichevin case actually involves two sub-problems. The first is a simple northward outflow problem from a point source on a beta plane or its analogous retroflection along a zonal wall. This problem involves a hypothetical eastward coastal current that is initially assumed to be steady and slowly varying in x. This is referred to as problem 1. We then reject this possibility on the ground that it does not satisfy the
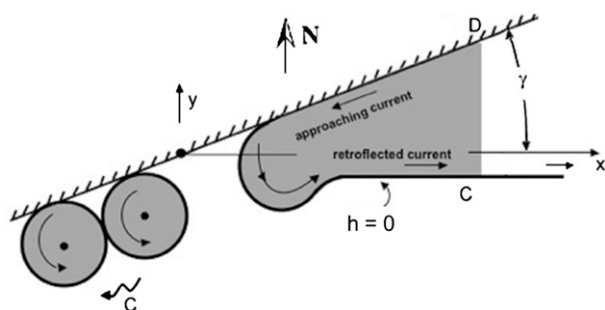
FIG. 3. The retroflection paradox and its resolution (in the Southern Hemisphere) according to Nof and Pichevin (adapted from Pichevin et al. 1999). To simplify the analysis, Nof and Pichevin considered the cases where there is no coastline tilt (i.e., $\gamma \rightarrow 0$). To compensate for the westward momentum flux (or flow force) created by the approaching and retroflected slowly varying boundary currents, westward-propagating anticyclonic rings are generated periodically. In this scenario, the eddies exert an eastward momentum flux analogous to the backward push associated with a firing gun. The wiggly arrow denotes migration.

momentum integral and regard this as a paradox. Nof and Pichevin then address the second problem where the steadiness is relaxed to allow for eddies to periodically form and shed on the west side. This is referred to as problem 2.

VL-DR focused on the steady problem (problem 1) and took a purely geostrophic basic state downstream (i.e., steady, no v), which does not actually exist in either of these two problems (1 and 2). They correctly say that, under these conditions, there is an "additional constraint" to problem 1. This has been incorrectly applied by some readers to problem 2. Recall that problem 2 is an unsteady problem, so steady considerations do not apply. It will be apparent from sections 2 and 3 that we agree that there might be some kind of a control (different, however, from the uniform flow condition derived by VL-DR) to problem 1. By "control," we mean a condition under which the flow (which is varying in x) will support a stationary wave (i.e., the steady advective flow cancels the wave propagation tendency). Because this problem is dismissed as unphysical anyway, we are not sure what is the sense in looking for it. We shall also see that problem 2 cannot possibly have such a constraint because it is unsteady. That is to say, we argue that the VL-DR basic state does not exist in either problem 1 or 2 of Nof and Pichevin and therefore their expansion and conclusions are irrelevant to both cases.

2. VL-DR argument and its relationship to Nof and Pichevin

In their appendix, immediately below (A6), VL-DR correctly argue that, when the flow is zonal, steady, and

purely geostrophic at any downstream cross section, the flow cannot develop meanders and it cannot be attached to either meanders or a retroflection upstream or downstream unless $u^2 = g'h$. [Note that the VL-DR Taylor series analysis is analogous to that used in Killworth (1983); see Pratt and Whitehead (2008) for a discussion of the criticality condition, $u^2 = g'h$.] When the flow and/or its boundaries (a front on the southern side in our case) do vary with both x and t, then neither $v = 0$ nor $\partial/\partial t = 0$ and this criticality condition need not be satisfied.

In Nof and Pichevin, the steady downstream flow is merely assumed to be geostrophic across CD (Figs. 1–3), in agreement with the common assumption made in any slowly varying boundary current (in both x and t). Nowhere has it been assumed, explicitly or implicitly, that $\partial/\partial x \equiv 0$ or $v \equiv 0$ anywhere. It is expected that neither of the two will be identically zero anywhere (except the wall in the outflow problem) because the retroflection eddy is much larger than the downstream current width, but it is not infinitely long. Specifically, it is expected that the downstream current will vary on the same larger scale as the eddy.

As far as Nof and Pichevin unsteady problem (problem 2) is concerned, even when the downstream current contains meanders with an amplitude reaching ⅓ of their length, the geostrophic approximation across CD is still valid on the order of $\sim(1/3)^2 \sim 10\%$ (as our numerical runs confirm; see Fig. 7a in Pichevin et al. 1999). Just to be absolutely sure, we also checked whether any of our numerical solutions happen to satisfy the condition $u^2 = g'h$ or something similar to it, which implies decreasing velocities toward the front ($h \rightarrow 0$). We noted that, on the contrary, in all of our dozens of experiments the streamlines appear to be uniformly spaced, implying that the velocity increases (rather than decreases) toward the front.

3. The mathematical aspect of VL-DR Taylor expansion

From a mathematical viewpoint, the essence of the argument regarding the variability in x presented in section 2 is that there exists a small parameter, v/u, which we will call ε. (Note that we are speaking here about both problems 1 and 2 and that ε here is not the same as that used in Nof and Pichevin, $\beta R_D/f$.) Equation (2) has to hold only to order ε^0, the terms that constitute the "mean state." From this point onward, the problem is clear: One can expand the terms in the mean state to any of their Taylor series order and they will not yield the same effects as those of the order ε terms. Namely, Nof and Pichevin are relying on the next-order terms (and not

on the mean state variables), whereas VL-DR analyze only the mean state variables.

This situation can perhaps be best understood if we examine the example of a long-wave instability (e.g., Killworth et al. 1984): As long as $k = 0$ (where k is the zonal wavenumber that is assumed to be a small parameter), the flow is stable and any analysis based on the functions of this state will not produce anything new. Only when one brings in the next-order terms (in k) the instability occurs and some new unstable features of the flow field emerge. Similarly, in the steady Nof and Pichevin case, as long as one remains in an exact (imaginable) downstream geostrophic regime (i.e., steady, no v), then the VL-DR arguments hold. However, even the slightest deviation from this state will bring in the paradox that Nof and Pichevin are alluding to. This easily explains the $u^2 = g'h$ issue, which Nof and Pichevin, of course, do not get but VL-DR get in their analysis of the mean state.

4. Discussion

In the beginning of their article, VL-DR discuss the mechanism leading to retroflection, noting that Nof and Pichevin attribute it to a momentum imbalance of the kind explained above. VL-DR later state that "although the idea of [Nof and Pichevin (1996)] is appealing, it seems to be contradicted by other studies." VL-DR then list Dijkstra and De Ruijter (2001) and Ou and De Ruijter (1986) as evidence for the contradiction of Nof and Pichevin with previous articles but never spelled out clearly what the contradiction between these steady problems and the Nof and Pichevin unsteady solution really is.

Ou and De Ruijter (1986) considered different physical systems (i.e., different boundary conditions) than we did, so it is not surprising that the results are not the same (see also Pichevin et al. 2009). We have countless counterexamples (to VL-DR statements) where the numerics support our analytics. One example is displayed in Fig. 8 of Pichevin et al. (1999), and a second is shown in Fig. 8 of Zharkov et al. (2010). In the second example, the red solid and dashed lines (theory) and red diamonds (numerics) correspond to the Nof and Pichevin case ($\gamma = 0$), and those in black color are closely related to VL-DR ($\gamma = 90°$). VL-DR also considered the case of a free meandering steady outflow where $\partial v/\partial x$ is the same order as $\partial u/\partial y$ (holds at the outflow section; see Fig A1. of VL-DR), a situation never considered by Nof and Pichevin.

Finally, we respectfully disagree with the VL-DR statement that "the apparent contradiction between previous work on separating currents, and the more recent work by Nof and Pichevin is solved." We think that VL-DR presented interesting arguments but have not resolved any clearly identifiable problem or contradiction. The basic state around which they expanded their Taylor series simply does not exist in the steady Nof and Pichevin case, so their expansion is irrelevant to Nof and Pichevin and there are no additional constraints.

APPENDIX

Symbols and Abbreviations

u, v	Zonal and meridional speeds in the x and y directions
h	Upper-layer thickness
g'	$g\Delta\rho/\rho$, reduced gravity
f	Coriolis parameter
$L(x, t)$	Downstream current width
Q	Outflow or incoming current volume flux
q	Outgoing current volume flux
R_D	$=(2g'Q)^{1/4}/f^{3/4}$, Rossby radius
ε	Small parameter, v/u
β	Variation of the Coriolis parameter with latitude

REFERENCES

Charney, J., 1955: The Gulf Stream as an inertial boundary layer. *Proc. Natl. Acad. Sci. USA,* **41,** 731–740.

Dijkstra, H. A., and W. P. M. de Ruijter, 2001: On the physics of the Agulhas Current: Steady retroflection regimes. *J. Phys. Oceanogr.,* **31,** 2971–2985.

Killworth, P., 1983: Some thoughts on the thermocline equations. *Ocean Modell.,* **48,** 1–5.

——, N. Paldor, and M. E. Stern, 1984: Wave propagation and growth on a surface front in a two-layer geostrophic current. *J. Mar. Res.,* **42,** 761–785.

Nof, D., 2005: The momentum imbalance paradox revisited. *J. Phys. Oceanogr.,* **35,** 1928–1939.

——, and T. Pichevin, 1996: The retroflection paradox. *J. Phys. Oceanogr.,* **26,** 2344–2358.

Olson, D. B., 1991: Rings in the ocean. *Annu. Rev. Earth Planet. Sci.,* **19,** 283–311.

——, and R. H. Evans, 1986: Rings of the Agulhas Current. *Deep-Sea Res.,* **33,** 27–42.

Ou, H. W., and W. P. M. De Ruijter, 1986: Separation of an inertial boundary current from a curved coastline. *J. Phys. Oceanogr.,* **16,** 280–289.

Pichevin, T., and D. Nof, 1997: The momentum imbalance paradox. *Tellus,* **49,** 298–319.

——, ——, and J. Lutjeharms, 1999: Why are there Agulhas rings? *J. Phys. Oceanogr.,* **29,** 693–707.

——, S. Herbette, and F. Floc'h, 2009: Eddy formation and shedding in a separating boundary current. *J. Phys. Oceanogr.,* **39,** 1921–1934.

Pratt, L., and J. Whitehead, 2008: *Rotating Hydraulics: Nonlinear Topographic Effects in the Ocean and Atmosphere*. Springer, 589 pp.

van Leeuwen, P. J., and W. P. M. De Ruijter, 2009: On the steadiness of separating meandering currents. *J. Phys. Oceanogr.,* **39,** 437–448.

Zharkov, V., D. Nof, and W. Weijer, 2010: Retroflection from a double slanted coastline—A model for the Agulhas leakage variability. *Ocean Sci.,* **6,** 997–1011.