CORRIGENDUM

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1. Introduction

The paper by Mellor (2003, hereafter M03) generally had a useful approach to the derivation of vertically dependent, coupled wave–current equations, including the final form of the continuity equation [(50)], the momentum equation [(51a)], and the wave energy equation [(52a)]. However, the details of the wave radiation stress, although they agree with the vertically integrated classical result, have since been found to be in error; the error is predominantly related to the treatment of wave pressure. A new paper, Mellor (2011), should be instructional—the basis of Stokes drift is examined—and helpful in understanding some of the corrections to M03 detailed below. The new paper and the corrected M03 are mutually corroborative with Longuet-Higgins and Stewart (1964), Phillips (1977), and Smith (2006).

2. The corrections to M03

All variables are defined as in M03. The list of corrections is as follows:

1) In (22), amend the second line as it is obtained directly from the transformation (19a) for pressure. Thus,

$$\frac{\partial}{\partial t}(s_\zeta u_\alpha) + \frac{\partial}{\partial x_\beta}(s_\zeta u_\alpha u_\beta) + \frac{\partial}{\partial \zeta}(\mathring{\omega} u_\alpha) - \varepsilon_{\alpha\beta z} f_z s_\zeta u_\beta + s_\zeta \frac{\partial p}{\partial x_\alpha} - s_\alpha \frac{\partial p}{\partial \zeta} = -\frac{\partial}{\partial x_\beta}(s_\zeta \langle u'_\alpha u'_\beta \rangle) + \frac{\partial}{\partial x_\zeta}(s_\beta \langle u'_\alpha u'_\beta \rangle) - \frac{\partial}{\partial \zeta}\langle w' u_\alpha \rangle. \tag{22$'$}$$

2) As an aid to comprehension, (24a) and (24b) are repeated; thus,

$$s_\zeta = D + \tilde{s}_\zeta, \tag{24a}$$

$$\tilde{s}_\zeta = kDa \frac{\cosh kD(1+\zeta)}{\sinh kD}, \tag{24b}$$

to which we add

$$s_\alpha = \frac{\partial \hat{\eta}}{\partial x_\alpha} + \zeta \frac{\partial D}{\partial x_\alpha} + \tilde{s}_\alpha, \tag{24c}$$

$$\tilde{s}_\alpha = -ka \frac{\sin kD(1+\zeta)}{\sin kD} \sin\psi. \tag{24d}$$

Note that (24b) and (24d) are derived from $\tilde{w} = \partial \tilde{s}/\partial t$, where $\tilde{w}(-h) = 0$; the latter is manifest in the fact that $\tilde{s}_\alpha(-1) = 0$. Instead of (34f), $\overline{\tilde{s}_\alpha \tilde{p}} = 0$ (because $\overline{\sin\psi \cos\psi} = 0$). Delete (34f) and (51c) and the term $\overline{\tilde{s}_\alpha \tilde{p}}$ everywhere in the paper. However, a similar term in (45)–(47) due to wind-generated pressure (proportional to $\sin\psi$) is retained.

3) Delete $g\tilde{s}$ from the left side of (29b) following the improved interpretation of pressure in Mellor (2011), and delete (34e).

4) In (34b), correct the coefficient of the buoyancy term and, as a consequence of Mellor (2011), add terms so that

$$\hat{p} = -gD\zeta - gD\int^0 (\hat{\rho} - \rho_0)/\rho_0 \, d\zeta - \overline{\tilde{w}^2} + \delta(\zeta)E/2, \tag{34b$'$}$$

where the Dirac delta function $\delta(\zeta) = 0$ for $\zeta \neq 0$ but $D\int_{-1}^{\varepsilon}\delta(\zeta)\,d\zeta = D\int_{-\varepsilon}^{\varepsilon}\delta(\zeta)\,d\zeta = 1$ for vanishingly small ε. The first two terms in (34b′) are conventional (no waves); the last two terms are due to waves.

5) As a result of the above changes, the expression for the wave radiation stress [(34c)] is amended to be

$$S_{\alpha\beta} = D\{\overline{\tilde{u}_\alpha \tilde{u}_\beta} + \delta_{\alpha\beta}[-\overline{\tilde{w}^2} + \delta(\zeta)E/2]\}, \tag{34c$'$}$$

and therefore (51b) is

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DOI: 10.1175/JPO-D-11-032.1

$$S_{\alpha\beta} = kDE\left\{\frac{k_\alpha k_\beta}{k^2}F_{\rm CS}F_{\rm CC} - \delta_{\alpha\beta}\left[F_{\rm SS}F_{\rm SC} - \frac{\delta(\zeta)}{2k}\right]\right\}. \quad (51b')$$

6) In the fifth line following (53), the equation following "furthermore" should be $\hat{u}_\alpha \cong U_\alpha$.

In another paper (Mellor 2008), (51b′) was derived correctly, but subsequent difficulty was encountered in vertically integrating that paper's Cartesian momentum equations while incorporating vertical boundary conditions to compare with the corresponding vertically integrated equations in Longuet-Higgins and Stewart (1964) and Phillips (1977). In M03, the advantage of the sigma coordinate equations is that the boundary conditions are incorporated in the derivation ab initio as in the derivation of the vertically integrated equations.

The paper by Mellor (2005) is correct after deletion of the term $S_{p\alpha}$ in that paper's (2a), (8), and (12), and after substituting the above (51b′) for (3a).

3. Further details

Contained herein are further details for intrepid readers who may wish to check the derivations of M03 as corrected here.

After inserting s_ζ and s_α from (24a)–(24d) and $p = \hat{p} + \tilde{p}$ into (22′) and phase averaging, one obtains

$$\overline{s_\zeta\frac{\partial p}{\partial x_\alpha}} - \overline{s_\alpha\frac{\partial p}{\partial \zeta}} = D\frac{\partial \hat{p}}{\partial x_\alpha} - \left(\frac{\partial \hat{\eta}}{\partial x_\alpha} + \zeta\frac{\partial D}{\partial x_\alpha}\right)\frac{\partial \hat{p}}{\partial \zeta}.$$

Then use the revised (34b′) in the above equation. After defining $b = g(\hat{\rho} - \rho_0)/\rho_0$ and after considerable algebra,

$$\begin{aligned}\overline{s_\zeta\frac{\partial p}{\partial x_\alpha}} - \overline{s_\alpha\frac{\partial p}{\partial \zeta}} &= gD\frac{\partial \hat{\eta}}{\partial x_\alpha} + D^2\int^0\left(\frac{\partial b}{\partial x_\alpha} - \frac{\zeta}{D}\frac{\partial D}{\partial x_\alpha}\frac{\partial b}{\partial \zeta}\right)d\zeta \\ &\quad + \frac{\partial}{\partial x_\alpha}\left\{D\left[-\overline{\tilde{w}^2} + \delta(\zeta)\frac{E}{2}\right]\right\} \\ &\quad + \frac{\partial \hat{\eta}}{\partial x_\alpha}\frac{\partial \overline{\tilde{w}^2}}{\partial \zeta} + \frac{\partial D}{\partial x_\alpha}\frac{\partial}{\partial \zeta}(\zeta\overline{\tilde{w}^2}).\end{aligned}$$

The first two terms on the right are pressure parts of the conventional, sigma coordinate momentum equation (without waves). The third term is part of the wave radiation stress in (34c′). The fourth term on the right, after integration, was ignored by Phillips but not by Smith (2006); however, Smith subsequently neglected the term on the basis that the mean surface slope is small. The fifth term on the right integrates to zero and therefore does not appear in Phillips (1977) or in Smith (2006). An alternate justification for neglect of the last two terms is as follows: Compare the coefficients of the two terms involving $\partial\hat{\eta}/\partial x_\alpha$; note that $\partial\overline{\tilde{w}^2}/\partial\zeta$ = order$(\overline{\tilde{w}^2}) = gD(ka)^2\tan kD/(kD)$ and therefore is small relative to gD. The same reasoning can be applied to the fourth term on the right, except that, for shallow water, $\partial h/\partial x_\alpha$ must be small (as in most oceanographic applications) as a consequence of using the linear wave functions that invoke the bottom boundary condition $\tilde{w}(-h) = 0$.

4. Summary of M03 equations as amended

- Equation (50) is unchanged.
- Expunge the term containing $\overline{\tilde{s}_\alpha\tilde{p}}$ in (51a) and substitute (51b′) for (51b). Also, divide ζ by D in the buoyancy integral term and change the sign in the definition of b.
- In (52a), expunge the term containing $\overline{\tilde{s}_\alpha\tilde{p}}$; the terms containing turbulence fluxes can be consolidated into a single empirical wave dissipation term.
- The Mellor (2005) paper has an updated version of the turbulence energy equation along with discussion of its interaction with the mean energy and wave energy equations.

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