Analysis of Vortex Dynamics of Lateral Circulation in a Straight Tidal Estuary*

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(Manuscript received 2 October 2013, in final form 25 July 2014)

ABSTRACT

The dynamics associated with lateral circulation in a tidally driven estuarine channel is analyzed on the basis of streamwise vorticity. Without rotational effects, differential advection and diffusive boundary mixing produce two counterrotating vortices (in the cross-channel section) whose strength and sense of circulation may change during a tidal cycle. The streamwise vorticity equation is determined by a balance between baroclinic forcing and turbulent diffusion, which explains the flood–ebb asymmetry of the lateral circulation. Analysis of the lateral salinity gradient shows that differential advection is the main driver of lateral flows, but boundary mixing can also be an important contributor in stratified estuaries. The strength of lateral circulation decreases with increasing stratification. With rotational effects, the lateral Ekman forcing in the bottom boundary layer drives a one-cell lateral circulation that switches its sense of rotation over the tidal cycle. The vorticity budget analysis reveals a three-way balance among the tilting of planetary vorticity by the vertical shear in the along-channel current, baroclinic forcing, and turbulent diffusion. The structure and magnitude of the lateral circulation change with the width of the estuary, expressed nondimensionally as the Kelvin number Ke. This lateral circulation features two counterrotating vortices in narrow estuaries, one vortex filling up the entire cross section in estuaries of intermediate widths and one vortex confined to the left side (looking into the estuary) in wide estuaries. The magnitude of the streamwise vorticity increases rapidly with Ke, reaches a maximum at Ke ~ 0.5, and decreases slightly in wide estuaries subject to strong rotational control.

1. Introduction

Lateral or transverse circulation in estuaries has received increasing attention because advection by the lateral flow appears to play a leading role in driving estuarine exchange flows. Idealized and realistic modeling investigations have demonstrated that lateral advection is of first-order importance in the along-channel momentum balance (Lerczak and Geyer 2004; Scully et al. 2009; Cheng et al. 2009). Burchard et al. (2011), Burchard and Schuttelkaars (2012), and Cheng et al. (2010) decomposed the estuarine residual circulation into contributions from processes such as gravitational circulation, tidal straining (or asymmetric tidal mixing) circulation, and advectively driven circulation. They found that lateral advection can be a major driving force for the estuarine circulation in some estuaries. Lateral circulation can also affect sediment dynamics (Geyer et al. 1998; Huijts et al. 2008; Chatelet et al. 2009).
2006; Fugate et al. 2007). Geyer et al. (1998), for example, observed a turbidity maximum zone skewed toward the west side of the Hudson River estuary. Such cross-channel variations were explained by the convergence of lateral flows (Ralston et al. 2012).

The dynamics of lateral circulation is also relevant in its own right as it is associated with coherent structures in turbulent channel flows. Several mechanisms have been proposed to explain the generation of lateral circulation in estuaries where stratification effects are influential to the dynamics. In a straight estuarine channel, the driving mechanisms for lateral flows include differential advection of along-channel density gradients (Nunes and Simpson 1985; Lerczak and Geyer 2004), boundary mixing on a sloping bottom (Wunsch 1970; Phillips 1970; Chen and Sanford 2009), Coriolis forcing (Johnson and Ohlsen 1994; Ott and Garrett 1998; Scully et al. 2009; Huijts et al. 2011), and interactions between barotropic tides and cross-channel variations in bathymetry (Li and O’Donnell 1997; Li and Valle-Levinson 1999; Valle-Levinson et al. 2000a). Using a numerical model of an idealized estuarine channel without rotational effects, Lerczak and Geyer (2004) demonstrated that two counterrotating lateral circulation cells are driven by differential advection and cross-channel density gradients and exhibit flood–ebb and spring–neap variability.

In stratified estuaries, Chen and Sanford (2009) showed that boundary mixing over sloping flanks of an estuarine channel can drive a pair of counterrotating lateral circulation cells that persist over the flood–ebb tidal cycle. In estuaries influenced by rotation, the typical cross-channel momentum balance is geostrophic (Pritchard 1956), although lateral advection and friction can be influential (e.g., Valle-Levinson et al. 2000b). Lateral flows can be driven by the ageostrophic along-channel tidal current, cross-channel advective acceleration, and vertical stress divergence (Kalkwijk and Booij 1986; Ott and Garrett 1998; Lerczak and Geyer 2004; Scully et al. 2009). In a study of the Hudson River estuary, Scully et al. (2009) showed that tidal advection by lateral Ekman transport generates one-cell lateral circulation that switches its sense of rotation over a flood–ebb tidal cycle, as found in an analytic model by Huijts et al. (2009). Most of the previous studies on lateral circulations have focused on the cross-channel momentum equation given by

$$\frac{\partial u}{\partial t} = -fu - \frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial}{\partial z} \left( K_y \frac{\partial u}{\partial z} \right),$$

(1)

where (u, v, and w) are the velocity components in the along-channel (x −), cross-channel (y −), and vertical (z −) directions. The positive x axis is pointed upstream (westward), the positive y axis is pointed to the left (looking into the estuary, southward), and the positive z axis is pointed upward (see Fig. 1). The leading-order
momentum balance in the cross-channel direction is the geostrophic balance. However, it is the ageostrophic contribution, or the breakdown of the geostrophic balance, that gives rise to lateral circulation.

In a recent numerical modeling study, Li and Li (2012) used a new approach to study the lateral circulation driven by along-channel winds. They conducted a diagnostic analysis of the equation for the streamwise (along-channel) vorticity defined as \( \omega_x = \partial \omega / \partial y - \partial u / \partial z \). In an observational study, Collignon and Stacey (2012) also used the streamwise vorticity equation to analyze the dynamics of lateral circulation across a channel–shoal interface in south San Francisco Bay. In a straight estuarine channel where \( \partial / \partial x = 0 \), the vortex stretching and tilting terms are zero. If the Boussinesq approximation is made in the horizontal momentum equations and the hydrostatic assumption is made in the vertical momentum equation, the equation for the streamwise vorticity \( \omega_x = -\partial u / \partial z \) is simplified to

\[
\frac{d\omega_x}{dt} = \int \frac{\partial u}{\partial z} - g \beta \frac{\partial S}{\partial y} + \frac{\partial^2 \omega_x}{\partial y^2} (K_V \omega_x) + \frac{\partial^2 \omega_x}{\partial z^2} (K_H \omega_x)
\]  

(2)

in which \( f \) is the Coriolis parameter, \( g \) is the gravitational acceleration (9.8 m s\(^{-2}\)), \( \beta \) is the saline contraction coefficient \([-7.7 \times 10^{-4} \text{ (kg g}^{-1})\]), and \( K_H \) and \( K_V \) are the eddy viscosity in the horizontal and vertical directions, respectively. In Eq. (2), \( K_H \) is assumed to be a constant. On the rhs of Eq. (2), the first term represents the tilting of planetary vorticity by the vertical shear in the along-channel flow (Pedlosky 1986, p. 189), the second term is the baroclinicity caused by the lateral salinity/density gradient, and the third and fourth terms are the vertical and horizontal diffusion, respectively. The horizontal diffusion term is typically two orders of magnitude smaller than the vertical diffusion term since \( \partial^2 \omega_x / \partial y^2 \ll \partial^2 \omega_x / \partial z^2 \).

The streamwise vorticity equation provides a concise description of the dynamics of the lateral circulation in a tidally driven estuary. The term \( f \partial u / \partial z \) represents the generation of streamwise vorticity by lateral Ekman flow. The lateral baroclinic forcing \( -g \beta \partial S / \partial y \) can be generated either by differential advection (Lerczak and Geyer 2004) or by diffusive mixing along sloping boundaries (Chen and Sanford 2009). This study investigates the dynamics of streamwise vorticity [Eq. (2)] in an idealized estuary to seek insights into the generation and dissipation mechanisms of the lateral circulation. Numerical experiments are conducted in the idealized estuary under different tidal and river flow forcing conditions, with and without rotational effects. The paper is structured as follows: In section 2, the model configuration and numerical experiments are described. Section 3 studies the vortex dynamics of lateral circulation in nonrotating estuaries and examines the transition from differential advection to boundary mixing as stratification increases. Section 4 studies the vortex dynamics in rotating estuaries and examines how the lateral circulation changes with the estuarine width.

2. Model description and numerical experiments

The Regional Ocean Modeling System (ROMS) is used to configure a model for the idealized straight estuarine channel. ROMS is a free-surface, hydrostatic, primitive-equations ocean model that uses stretched, terrain-following vertical coordinates and orthogonal curvilinear horizontal coordinates on an Arakawa C grid (Shchepetkin and McWilliams 2005, 2009a, 2009b; Haidvogel et al. 2008). The model domain is designed as an estuary–shelf system (Fig. 1), following Hetland and Geyer (2004) and Cheng et al. (2009, 2011). The estuary part of the model domain is a straight channel that is 500 km long and does not have slope in the along-channel direction. The cross-channel section features a trapezoidal shape with a maximum (thalweg) depth of 15 m and a minimum depth of 5 m. The width of the channel is 3 km in the model runs reported in section 3 but varies from 750 m to 18 km in those shown in section 4. The continental shelf is 80 km wide and has a fixed cross-shelf slope of 0.05%. The total model domain has 240 grid cells in the east–west direction, 80 grid cells in the north–south direction, and 20 layers in the vertical direction. The estuary has 200 grid cells in the along-channel direction and 31 grid cells in the cross-channel direction. The along-channel grid size increases exponentially from the estuary mouth (50 m) to its head (12 km), providing a highly resolved estuarine region near the mouth. In most of the numerical experiments conducted, the estuary length, considered here as the region influenced by salinity, is less than 150 km. A longer channel (500 km) is used to damp out tides before they reach the upstream boundary. The cross-channel grid in the estuary is uniformly distributed and the vertical layers are uniformly discretized.
The model is forced by tides at the offshore (eastern) open boundary and by river flow at the upstream (western) end of the estuarine channel. At the upstream boundary, a momentum boundary condition is imposed on the depth-averaged velocity. The inflowing river water is prescribed to have zero salinity. To simplify, temperature is uniform everywhere and does not change with time. We have excluded model runs that showed runaway stratification or nonsteady solutions.

Table 1. Summary of numerical experiments in which $U_r$ is the tidal current speed (m s$^{-1}$) at the open boundary, $U_o$ is the river flow velocity (cm s$^{-1}$), $f$ is the Coriolis parameter, $W$ is width of the estuary (km), $Fr_r$ is the river Froude number, $M$ is the estuarine mixing number, $Ke$ is the Kelvin number, $|\Omega_j|$ is the streamwise vorticity magnitude ($10^{-3}$ s$^{-1}$), and $|\beta S/dy|$ and $|f du/dz|$ are the baroclinicity and Ekman forcing terms (multiplied by $10^{-4}$) in the streamwise vorticity equation.

| Run | $U_r$ | $U_o$ | $f$ | $W$ | $Fr_r$ | $M$ | $Ke$ | $|\Omega_j|$ | $|\beta S/dy|$ | $|f du/dz|$ |
|-----|------|------|-----|-----|-------|-----|------|-------------|-------------|-------------|
| 1   | 1    | 2    | 0   | 3   | 0.012 | 1.24 | 0    | 10.5        | 4.77        | 0           |
| 2   | 1    | 3    | 0   | 3   | 0.018 | 1.22 | 0    | 9.7         | 4.98        | 0           |
| 3   | 1    | 6    | 0   | 3   | 0.037 | 1.20 | 0    | 5.5         | 5.53        | 0           |
| 4   | 1    | 9    | 0   | 3   | 0.055 | 1.11 | 0    | 4.0         | 6.19        | 0           |
| 5   | 0.6  | 3    | 0   | 3   | 0.018 | 0.74 | 0    | 1.5         | 3.2         | 0           |
| 6   | 0.6  | 6    | 0   | 3   | 0.037 | 0.72 | 0    | 1.3         | 2.83        | 0           |
| 7   | 0.6  | 9    | 0   | 3   | 0.055 | 0.70 | 0    | 1.2         | 2.81        | 0           |
| 8   | 1    | 2    | $10^{-4}$ | 0.75 | 0.012 | 1.00 | 0.046 | 1.4         | 8.10        | 5.91        |
| 9   | 1    | 2    | $10^{-4}$ | 1.5 | 0.012 | 0.97 | 0.092 | 3.4         | 6.54        | 5.66        |
| 10  | 1    | 2    | $10^{-4}$ | 3   | 0.012 | 1.07 | 0.18  | 9.40        | 3.38        | 3.76        |
| 11  | 1    | 2    | $10^{-4}$ | 6   | 0.012 | 1.01 | 0.37  | 19.1        | 5.04        | 4.08        |
| 12  | 1    | 2    | $10^{-4}$ | 9   | 0.012 | 0.96 | 0.55  | 26.3        | 2.42        | 5.57        |
| 13  | 1    | 2    | $10^{-4}$ | 12  | 0.012 | 0.97 | 0.73  | 22.6        | 3.24        | 5.24        |
| 14  | 1    | 2    | $10^{-4}$ | 15  | 0.012 | 0.97 | 0.92  | 21.9        | 3.72        | 5.16        |
| 15  | 1    | 2    | $10^{-4}$ | 18  | 0.012 | 0.97 | 1.10  | 20.2        | 3.81        | 4.93        |

The model is initialized with no flow, a flat sea surface, and a uniform salinity of 35 g kg$^{-1}$. To simplify, temperature is uniform everywhere and does not change with time. Using a typical tidal current amplitude of 1 m s$^{-1}$ and a sectionally averaged river flow of 0.03 m s$^{-1}$, the model is run for 60 days to obtain an initial salinity distribution in the model domain. This salinity distribution is then used as the initial salinity condition in each numerical experiment. Results obtained at the quasi steady state (after 60 or so more days of integration) are used for the analysis of lateral circulation. During this analysis period, tidally averaged circulation and stratification do not change with time. The model results were organized in terms of the tidal current speed $U_r$, river Froude number $Fr_r$, and the estuarine mixing parameter $M$. The term $Fr_r$ is the net velocity due to river flow scaled by the maximum possible frontal propagation speed and is defined as

$$Fr_r = \frac{Ur}{(g\beta S_{ocean}H)^{1/2}},$$

where $Ur$ is the river flow velocity, $S_{ocean}$ is the oceanic salinity, and $H$ is the mean water depth. When the thalweg depth $H_{max}$ is used, $Fr_r$ in Eq. (3) is multiplied by a factor of 0.82 to account for the difference between the mean and maximum depths. The mixing parameter $M$ quantifies the effectiveness of tidal mixing for a stratified estuary and is defined as

$$M^2 = \frac{C_d U_r^2}{\alpha N_0 H^2},$$

where $C_d$ is the drag coefficient.
where \( C_d \) is a bottom drag coefficient, \( U_T \) is the depth-averaged tidal current speed, \( \omega \) is the tidal frequency, and \( N_0 = (\beta g S_{\text{ocean}}/H)^{1/2} \) is the buoyancy frequency for maximum top-to-bottom salinity variation in an estuary.

The effects of the earth’s rotation on the lateral circulation in estuaries are characterized with the Kelvin number, given by

\[
Ke = \frac{fB}{\sqrt{g' h_S}},
\]

where \( B \) is the estuary width, \( g' \) is the reduced gravity determined by the density difference between the upper and lower layers, and \( h_S \) is the mean depth of the upper layer (Li and Li 2012). Following the approach of Geyer and MacCready (2014), we calculate Ke using the external parameters with \( g' = g S_{\text{ocean}} \) and \( h_S = H \). It represents the ratio of the estuary width to the maximum possible internal Rossby radius of deformation. It is smaller than the value of Ke calculated using \( g' \) between the upper and lower layers in the estuary and the mean depth \( h_S \) of the upper layer (Li and Li 2012).

3. Lateral circulation driven by differential advection and boundary mixing

In this section, the Coriolis force is switched off in the model. The lateral circulation may be generated by differential advection and/or diffusive boundary mixing and is investigated using the streamwise vorticity approach. One case of a weakly stratified estuary and one case of a stratified estuary will be studied in detail, followed by an exploration over a range of stratification conditions.

a. Differential advection

First we present a case study of a weakly stratified estuary (run 2 in Table 1) in which lateral circulation is driven by differential advection. Figure 2 shows two snapshots of the lateral and vertical velocity components, streamwise vorticity, along-channel flow, and salinity distributions at a midestuary section during a tidal cycle. A pair of counterrotating vortices develops during the flood tide; the left vortex (looking into the estuary)
rotates in the clockwise direction, while the right vortex rotates in the counterclockwise direction. The distribution of the streamwise vorticity captures the essential characteristics of the two vortices. The vortices reach their maximum strength at about 2 h after the maximum flood (Figs. 2a,b). The lateral circulation redistributes the salinity field in the cross-channel section: upwelling on the shallow shoals lifts isopycnals, whereas downwelling in the center channel depresses isopycnals, creating a saddle shape in the isohalines (Fig. 2c). During the ebb tide, the velocity difference between the center channel and side boundaries produces lateral salinity differences that result in upwelling in the center channel and downwelling near the two side boundaries. The left vortex rotates in the counterclockwise direction, while the right vortex rotates in the clockwise direction. Initially this pair of vortices is confined to a near-surface region in the middle of the estuarine channel and sits above a pair of decaying vortices generated during the flood tide. At 1 h after the slack ebb tide, however, the vortices occupy the full width of the estuarine channel with the maximum intensity near the sea surface (Figs. 2d–f). The vortices are stronger in flood than in ebb. When averaged over a tidal cycle, the lateral circulation shows a pair of weak counterrotating vortices (Fig. 2g). The tidally averaged along-channel flow shows the expected two-layer exchange in the channel with outflow over shoals and at the surface (Fig. 2h, e.g., Valle-Levinson et al. 2000b). The averaged salinity shows stratified water above a bottom layer over the sloping bathymetry (Fig. 2i). The vortex dynamics of the lateral circulation is analyzed by averaging the equation of the streamwise vorticity over a control volume. Analysis is done in the volume-averaged streamwise vorticity over a control volume. Analysis is done in the equation of the streamwise vorticity (Figs. 3a,b). The lateral circulation redistributes the salinity field in the cross-channel section: upwelling on the shallow shoals lifts isopycnals, whereas downwelling in the center channel depresses isopycnals, creating a saddle shape in the isohalines (Fig. 2c). During the ebb tide, the velocity difference between the center channel and side boundaries produces lateral salinity differences that result in upwelling in the center channel and downwelling near the two side boundaries. The left vortex rotates in the counterclockwise direction, while the right vortex rotates in the clockwise direction. Initially this pair of vortices is confined to a near-surface region in the middle of the estuarine channel and sits above a pair of decaying vortices generated during the flood tide. At 1 h after the slack ebb tide, however, the vortices occupy the full width of the estuarine channel with the maximum intensity near the sea surface (Figs. 2d–f). The vortices are stronger in flood than in ebb. When averaged over a tidal cycle, the lateral circulation shows a pair of weak counterrotating vortices (Fig. 2g). The tidally averaged along-channel flow shows the expected two-layer exchange in the channel with outflow over shoals and at the surface (Fig. 2h, e.g., Valle-Levinson et al. 2000b). The averaged salinity shows stratified water above a bottom layer over the sloping bathymetry (Fig. 2i).

To determine whether the lateral baroclinic forcing is generated by differential advection or diffusive boundary mixing, we adopt the approach of Chen and Sanford (2009) by differentiating the salt balance equation with respect to the cross-channel direction \( \gamma \):

\[
\frac{\partial S_y}{\partial t} = \frac{\partial (uS_x)}{\partial y} - \frac{\partial (uS_y)}{\partial y} - \frac{\partial (wS_z)}{\partial y} + \frac{\partial^2 (K_{yz} \frac{\partial S}{\partial z})}{\partial y \partial z} ,
\]

where \( K_{yz} \) is the eddy diffusivity. The first term (i) is the rate of change in lateral salinity gradient, the second term (ii) is the differential advection of the longitudinal salinity gradient by lateral shear in the along-channel current, the third term (iii) is the advection by the lateral circulation, and the fourth term (iv) is the lateral variations in the vertical diffusive salt flux gradient, which is associated with boundary mixing. Equation (6) is averaged over the left half of the estuarine channel in the control volume and the values of terms (ii) and (iv) are compared (Fig. 4a). Term (ii) is positive during the flood tide and negative during the ebb tide. Term (iv) represents lateral variations in the vertical diffusive salt flux gradient and is associated with mixing over the sloping boundary. This salt diffusion term is smaller on flood.
than on ebb because the bottom mixed layer is thicker during the flood. Figure 4a shows that \(-\frac{\partial (uS_x)}{\partial y}\) is markedly larger than \(-\frac{\partial (uS_x)}{\partial y}\) during the ebb. This explains the flood–ebb asymmetry of the baroclinic forcing in the vorticity equation (Fig. 3c). The lateral circulation redistributes the salinity field in the cross-channel section; the term has an opposite sign to \(-\frac{\partial (uS_x)}{\partial y}\) during most of the tidal cycle. Hence, there is a self-imposed limit in the growth of \(\frac{\partial S_y}{\partial t}\), which is a manifestation for the nonlinear negative feedback on the lateral circulation dynamics. The differential advection of the longitudinal salinity gradient between the center channel and shallow shoals generates the lateral salinity gradient that drives the lateral circulation. As soon as the lateral circulation is developed, it acts to equalize this lateral salinity difference so that the lateral circulation does not grow indefinitely.

### b. Boundary mixing

A stratified estuary is studied next (run 6 in Table 1). Distributions of along-channel velocity, salinity, cross-channel and vertical velocity vectors, and streamwise vorticity at a midestuary cross section are shown at two
A scrutiny of the streamwise vorticity equation provides more information. Figure 6b shows the time series of the volume-averaged streamwise vorticity ($\vec{\omega}_x$ left and $\vec{\omega}_x$ right) for the left and right half of the estuarine channel in the control volume. A comparison between Figs. 3b and 6b shows that the streamwise vorticity is one order of magnitude smaller in the stratified case than in the weakly stratified case. The term $\vec{\omega}_x$ left stays positive during most of the tidal cycle (except for a short period around maximum ebb) so that the lateral circulation cell remains rotating clockwise. However, the vorticity magnitudes fluctuate substantially over a tidal cycle: $\vec{\omega}_x$ left reaches the maximum at $\sim 2$ h after the maximum flood and a secondary maximum at $\sim 3$ h after the maximum ebb. The lateral circulation is shut down briefly at $\sim 1$ h before the maximum ebb. This temporal evolution of the streamwise vorticity indicates a dominant balance/competition between baroclinic forcing and turbulent diffusion (Fig. 6c). The temporal change term in the vorticity equation is weak, reflecting smaller and slower changes of $\vec{\omega}_x$ left over a tidal cycle than in the weakly stratified case (cf. Figs. 6 and 3). The baroclinic pressure gradient is generated by the upslope flow along the sloping boundary. The upslope flow tilts the flat isohalines upward, which creates a baroclinic force that drives flow from the boundary to the interior, as suggested in Garrett et al. (1993). Turbulent diffusion generated in the bottom mixed layer spins down the lateral circulation cell. Both baroclinic forcing and turbulent diffusion show two peaks over a tidal cycle: a larger peak near the maximum flood and a smaller peak near the maximum ebb. The maxima of the baroclinic forcing lead the maxima of $\vec{\omega}_x$ left by 2–3 h because $d\vec{\omega}_x$ left$/dt$ is determined by small imbalances between the baroclinic forcing and turbulent diffusion. Also, it takes time for the vortices to spin up to their maximum strength.

To quantify the roles of differential advection and boundary mixing in generating the lateral baroclinic forcing in run 6, a diagnostic analysis of Eq. (6) is carried out for the left half of the estuarine channel in the control volume. As shown in Fig. 4b, the lateral variation in the vertical diffusive salt flux gradient

$$\frac{\partial^2 S}{\partial y \partial z}$$

associated with boundary mixing is now comparable to differential advection of the longitudinal salinity gradient by lateral shear $-\partial (\mu S_x)/\partial y$. It is not surprising that the maximum streamwise vorticity is reached after the maximum flood when both terms add to the generation of positive values of $\vec{\omega}_x$ left. The terms
2y/C18

\( \frac{\partial^2}{\partial y \partial z} \left( K \frac{\partial S}{\partial z} \right) \)

and \(-\frac{\partial (uS_x)}{\partial y}\) are of opposite signs during the ebb, resulting in weaker baroclinic forcing and smaller \(\omega_{x\text{left}}\).

The lateral advection term

\[ \frac{\partial (uS_x)}{\partial y} - \frac{\partial (wS_z)}{\partial y} \]

is the feedback of the lateral circulation itself on the salinity distribution in the cross-channel section. It can be interpreted as a convergence or divergence term that strengthens or weakens the lateral salinity gradient during different phases of a tidal cycle. It becomes the largest term in Eq. (6) in run 6 because \(\partial S/\partial z\) and \(\partial S/\partial y\) are larger than those in run 2 even though the velocities \(\nu\) and \(w\) are smaller. As shown in Fig. 4b, the volume-integrated budget of Eq. (6) is well balanced, despite being a high-order numerical analysis.

c. Transition

The above two runs illustrated that differential advection is the dominant driver of the lateral circulation under weak stratification, while diffusive boundary mixing becomes an important player under strong stratification. Additional numerical runs were conducted to examine the relative roles of differential advection and diffusive boundary mixing over a range of stratification conditions. Cases consisted of two offshore tidal current amplitudes (1 and 0.6 m s\(^{-1}\)) and several river flow velocities (2 to 9 cm s\(^{-1}\)).

As a measure of the overall strength of the lateral circulation, the absolute vorticity magnitude \(A^{-1} \iint |\omega_x| dA\) is averaged over one-half of the estuarine cross section (to cover one of the two counterrotating vortices) and then averaged over the estuarine segment, that is, \((AL_x)^{-1} \int (\iint |\omega_x| dA) \ dx\), where \(L_x\) is the length of the control volume used in the analysis of the vorticity budget. The tidal average of this vorticity magnitude is given by
where $T$ is the tidal period. The values of $|\Omega_x|$ for the seven nonrotational runs are shown in Table 1. The term $|\Omega_x|$ is much larger at the tidal speed of 1 m s$^{-1}$ than at 0.6 m s$^{-1}$. At a fixed tidal velocity, however, $|\Omega_x|$ decreases as the river flow increases. The strength of lateral circulation thus depends on both tidal forcing and river flow (and hence stratification). To separate the two effects, values of $|\Omega_x|$ are normalized by a circulation metric $\frac{1}{L_c} \int \left( \int \omega_x |dA \right) dx \right) dt$, (7)

where $\omega_x$ is the frictional velocity calculated from the tidal average of the absolute magnitude of the bottom stress (averaged over the bottom of the control volume), and $L_c$ represents the perimeter of the vortex. The velocity $u^*$ is chosen as a characteristic velocity scale because the tidal current is the ultimate driving force for the lateral circulation. Figure 7a shows how the normalized streamwise vorticity $|\Omega_x|/\int_{L_c} u^* dl$ varies with $Fr_f$. It decreases more rapidly with $Fr_f$ at the tidal speed of 1 m s$^{-1}$ than at 0.6 m s$^{-1}$. This suggests a dependence of $|\Omega_x|/\int_{L_c} u^* dl$ on $M$, as shown in Fig. 7b. These limited numerical experiments are not designed for constructing a regime diagram of $|\Omega_x|$ in the $Fr_f-M$ parameter space or develop a general theory for the lateral circulation since real estuaries feature complex and varied geometry that is quite different from the idealized estuary. Nevertheless, they serve an important purpose to illustrate how the vortex dynamics of the lateral circulation changes as stratification increases.

To compare the relative roles of differential advection and boundary mixing, the tidal averages of the absolute magnitudes were calculated for the volume-averaged terms (ii) (differential advection denoted as $|\omega_{adv}|$) and (iv) (boundary mixing denoted as $|\omega_{diff}|$) in Eq. (6). The ratio of $|\omega_{diff}|/|\omega_{adv}|$ is plotted against $Fr_f$ in Fig. 7c. The boundary mixing term is about 20%–40% of the differential advection term at $Fr_f = 0.02$ and increases to about 60% at $Fr_f = 0.06$. Overall, the differential advection term is the dominant mechanism generating the lateral circulations in estuarine channels where the rotational effects are negligible. However, as stratification increases, the relative importance of boundary mixing increases.
gets stronger and $Fr_f$ increases, the lateral circulation weakens and boundary mixing increases in importance.

4. Lateral circulation driven by Ekman forcing

The effect of Coriolis accelerations (or Ekman forcing) on the lateral circulation is investigated in this section. The Coriolis parameter $f = 10^{-4} \text{s}^{-1}$ is used as a representative value for a midlatitude estuary. In the model runs, the offshore tidal current speed is set at $U_T = 1 \text{ m s}^{-1}$ and river flow is set at $U_r = 0.02 \text{ m s}^{-1}$, but the width ($W$) of the estuary varies from 750 m to 18 km. The vorticity analysis is conducted in detail for one model run, and then we study how the streamwise vorticity changes as $W$ increases.

a. Ekman forcing

Run 10 ($W = 3 \text{ km}$ and $Ke = 0.18$, see Table 1 for parameter values) is selected for the detailed vorticity analysis. Distributions of along-channel velocity, salinity, cross-channel and vertical velocity vectors, and streamwise vorticity are plotted to show the temporal evolution of lateral circulation at a midestuary cross section during different phases of a tidal cycle (Fig. 8).

During flood, the lateral flow exhibits one-cell circulation that rotates in the clockwise direction (looking into the estuary): the bottom (surface) layer moving toward the left (right). Flood currents are deflected toward the left inside the bottom Ekman layer. Mass conservation requires a return flow in the surface layer directed to the right. This lateral circulation cell advects and rearranges the salinity field in the cross-channel section, as shown in the comparison of the salinity distributions at maximum flood and 1 h later. Near the bottom, water is more saline on the left half of the estuarine channel than on the right half, producing a lateral baroclinic pressure gradient that opposes the bottom current. The clockwise circulation cell continues to strengthen during the flood tide and reaches its strongest rotation at $\sim 1 \text{ h}$ after maximum flood. However, the circulation cell rotates in the counterclockwise direction during the ebb tide. The Coriolis force deflects the ebb current inside the bottom Ekman layer so that the lateral current is directed toward the right. A return flow develops in the surface layer and is directed to the left. The right-moving bottom current advects higher salinity across the sloping bottom on the right side and generates an adverse baroclinic pressure gradient that opposes the lateral bottom currents. Therefore, there is negative feedback between the lateral circulation and lateral baroclinic forcing, which will become evident upon analysis of the streamwise vorticity budget. When averaged over a tidal cycle, the lateral circulation features weak flows owing to the flood–ebb asymmetry in the magnitude and spatial pattern of the streamwise vorticity. The along-channel residual current shows the expected two-layer flow: seaward surface flow leaning toward the left side, while the landward bottom flow favors the right side and occupies the deep channel (e.g., Valle-Levinson et al. 2000b; Valle-Levinson 2008). The salinity field shows stratified water with the isopycnals tilting downward on the southern side, consistent with the earth’s rotation effects, but the isopycnals are less steep than the zero flow interface.

The above analysis suggests that the breakdown of the geostrophic balance within the bottom Ekman layer is the primary mechanism generating the one-cell lateral circulation in rotating systems. However, other factors such as the lateral density gradient and vertical stratification may also play important roles in determining the strength of the lateral circulation. Analysis of the streamwise vorticity equation is carried out to better understand these mechanisms (Fig. 9). The dominant terms in the vorticity budget are the tilting of planetary vorticity by the vertical shear in along-channel flows, the vertical diffusion, and the baroclinicity caused by sloping isopycnals [see Eq. (2)]. Not surprisingly, the sign of the streamwise vorticity (or the sense of the

![Fig. 7. Normalized magnitude of the streamwise vorticity in nonrotating runs (Ke = 0) vs (a) Fr, and (b) M. (c) Ratio of the vertical diffusion to differential advection in the equation for the lateral salinity gradient vs Fr_f.](image-url)
lateral circulation) is set by the tilting of planetary vorticity by the along-channel current. On flood, the along-channel current is positive (upstream) so that its vertical shear generates a positive streamwise vorticity $\omega_{\text{tr}}$, where the overbar stands for the average over the control volume (clockwise lateral circulation, looking into estuary). In contrast, ebb generates negative (downstream) along-channel current shear to generate negative

![Diagram](image-url)
The maximum vorticity is reached about 1 h after the maximum flood, while the minimum vorticity is reached about 1 h before the maximum ebb. Because the vertical shear in the along-channel current is larger on ebb than on flood, the magnitude of \( f \frac{\partial u}{\partial z} \) is larger in ebb. Turbulent diffusion acts to spin down \( \omega_x \) and smooth the spatial gradients in \( \omega_x \). The magnitude of the diffusion term is also larger on ebb than on flood and acts as a counterbalance to the vorticity generation term \( f \frac{\partial u}{\partial z} \).

The Ekman-driven lateral circulation in an unstratified channel is controlled by the competition between the tilting and diffusion terms with no influence by baroclinicity. In a stratified estuary, the baroclinic forcing exerts a lateral baroclinic torque \( \overline{g \beta \delta S / \partial y} \) that may oppose the vortex tilting term \( \overline{f \partial u / \partial z} \). The clockwise lateral circulation advects bottom water to the left and produces a negative lateral baroclinic torque, whereas the counterclockwise lateral circulation advects bottom water to the right and produces a positive lateral baroclinic torque. However, there is a phase lag between \( \overline{f \partial u / \partial z} \) and \( \overline{g \beta \delta S / \partial y} \). The term \( \overline{f \partial u / \partial z} \) is set by the vertical shear in the along-channel current and is approximately in phase with the tidal current (Fig. 9c). On the other hand, the lateral baroclinic torque is generated by sloping isopycnals in the cross-channel section and is thus affected by the lateral circulation itself. The minimum of \( \overline{g \beta \delta S / \partial y} \) is reached in early ebb (about hour 7) and 2 h later than the maximum of \( \overline{f \partial u / \partial z} \) (about hour 5). The maximum of \( \overline{g \beta \delta S / \partial y} \) is reached in early flood (about hour 14) and 4 h later than the minimum of \( \overline{f \partial u / \partial z} \) (about hour 10). Hence, the baroclinic torque \( \overline{g \beta \delta S / \partial y} \) may reinforce and weaken the one-cell lateral circulation, depending on the phase of the tidal cycle.

b. Effect of estuary width

A total of eight model runs illustrate how the width of the estuary affects the lateral circulation (runs 8 to 15 in Table 1). First, the structure and strength of lateral circulation is compared in estuaries of four different widths but under identical tidal and river flow forcing conditions. In the narrow estuary with \( W = 750 \text{ m} \) (Ke = 0.046, run 8), two counterrotating vortices occupy the estuarine channel, with divergent flows along the bottom boundary and convergent flows near the surface.
This two-cell circulation pattern is similar to that seen in run 2 (Fig. 2), but the lateral circulation is affected by the Coriolis force. Isopycnals are advected to the left of the estuarine channel by the flood current (Figs. 10b,c). In the estuary with $W = 1.5$ km ($Ke = 0.092$, run 9), the lateral circulation displays a complex spatial structure (Fig. 10d). Two counterrotating vortices can still be identified, but they are not symmetric with respect to the axis of the estuarine channel. The effect of lateral Ekman forcing is evident in the cross-channel salinity distribution (Fig. 10f). In estuaries with larger widths ($W = 3–18$ km, $Ke = 0.18–1.10$), the lateral circulation only features one vortex. In run 11 with $W = 6$ km ($Ke = 0.37$), one vortex occupies the whole cross section (Figs. 10g–i). This vortex switches its sense of rotation as the tide switches from the flood to ebb phase. In the widest estuary with $W = 18$ km ($Ke = 1.10$, run 15), the strong rotational control confines the freshwater plume and the lateral circulation to the left half of the estuarine channel (looking into the estuary, Figs. 10j–l).

Next we compare the streamwise vorticity balance among the model runs (Fig. 11). In the narrow estuary of $W = 750$ m (run 8), the lateral circulation features a pair of counterrotating vortices, but each vortex only briefly switches its sense of rotation during the late ebb (Fig. 11a).

Stratification is relatively strong so that diffusive boundary mixing is a significant factor in the generation of the lateral circulation. In the streamwise vorticity equation for the left vortex, the baroclinic forcing is the largest term (Fig. 11d). The Ekman forcing term $\frac{f \partial u}{\partial z}$ is smaller but still significant. Although the flows are slightly asymmetric to the central axis, the advective flux across the interface between the two vortices is small (not shown).

In run 11 with $W = 6$ km, one lateral circulation cell fills in the entire cross section. This time series of the streamwise vorticity (Fig. 11b) looks similar to that in Fig. 9b, but the phase lag between $\vec{v}_x$ and the barotropic tidal current is different. Maximum $\vec{v}_x$ occurs around the maximum flood, while minimum $\vec{v}_x$ occurs 2 h before the maximum ebb. This is in contrast to run 10 when maximum $\vec{v}_x$ is 1 h after maximum flood and minimum $\vec{v}_x$ is 1 h before maximum ebb. The sign and magnitude of $\vec{v}_x$ are set by the Ekman forcing term $\frac{f \partial u}{\partial z}$, which changes sign from flood to ebb and is larger on ebb. The Ekman forcing term is opposed by the turbulent diffusion, but the baroclinic forcing is an important term in the overall streamwise vorticity balance. As the Ekman forcing advects the isopycnals to the left of the estuarine channel, it sets up an adverse baroclinic forcing that opposes

**FIG. 10.** (a),(d),(g),(j) Snapshots of streamwise vorticity (color image) and cross-channel and vertical velocity vectors (arrows), (b),(e),(h),(k) along-channel velocity, and (c),(f),(i),(l) salinity at a cross-channel section for four estuaries with different widths (looking into the estuary) near the maximum flood: (top to bottom) width = 750 m (run 8), 1.5 km (run 9), 6 km (run 11), and 18 km (run 15). $Ke = 0.046, 0.092, 0.37, and 1.10$ in runs 8, 9, 11, and 15.
it, and vice versa during the ebb tide. Again there is
a phase lag between \( \frac{f}{u} \) and \( \frac{g\beta}{S/y} \).

In the widest estuary of \( W = 18 \) km, the maximum \( \sigma_x \) is
reached near the end of flood, while the minimum \( \sigma_x \) is
reached near the end of ebb (Fig. 11c). The time series of
\( \sigma_x \) is out of phase of the barotropic tidal current. The term
\( \frac{f}{u} \) generates the streamwise vorticity, but it takes 3 h
for \( \sigma_x \) to spin up to the maximum strength (Fig. 11f). The
baroclinic forcing \( \frac{g\beta}{S/y} \) stays positive throughout
the tidal cycle. In this wide estuary, the Coriolis force
exerts a strong control on the salinity field such that the
freshwater plume is confined to the left side of the estu-
arine channel (see Fig. 10l) and the isopycnals are sloping
downward at all times. The laterally forced flows never
interact with the right side of the estuarine channel
(looking into the estuary) and slopes of the isopycnals are
relatively unchanged. Hence, \( \frac{g\beta}{S/y} \) remains positive
over the entire tidal cycle. This is different from the es-
tuaries of intermediate widths in which the baroclinic
forcing switches sign over a tidal cycle (e.g., Figure 11e).

Figure 12 summarizes the model results in the rotating
runs. As a measure of the overall strength of the lateral
circulation, the absolute vorticity magnitude is averaged
over the control volume (the left vortex for runs 8–9
and the whole cross section for runs 10–15) and the
tidal cycle to obtain \( \Omega_L \). The normalized vorticity
\( \Omega_L / \int u_0 dl \) initially increases rapidly with increasing
\( Ke \). It reaches a maximum around \( Ke = 0.55 \) (or \( W = 9 \) km) and then decreases gradually at higher values
of \( Ke \). Under the same tidal and river flow conditions
(\( Fr_f = 0.012 \) and \( M = 0.95–1.07 \) in these runs), the
strength of the lateral circulation increases as the width
of the estuary increases. The lateral circulation reaches
the maximum strength at an intermediate width at which
the one-cell vortex still fills up the entire estuarine cross
section. In wider estuaries with \( Ke > 0.7 \), the Coriolis
force restricts the vortex to the left part of the estuary
such that the lateral circulation weakens. This transition
from a width-filling vortex to a partial-filling vortex oc-
curs when the lateral baroclinic torque remains positive
over the entire tidal cycle rather than switching signs
between flood and ebb tides.

5. Conclusions

An approach based on the streamwise vorticity has
been used to interpret the dynamics of lateral circulation

\[ \frac{g\beta}{S/y} \]

FIG. 12. Normalized magnitude of the streamwise vorticity as
a function of \( Ke \).
in a tidally driven estuarine channel under a range of stratification and tidal forcing conditions. Without rotational effects, differential advection of the longitudinal salinity gradient and diffusive mixing over the sloping flanks of the channel are the two major generation mechanisms of the lateral salinity/density gradient and streamwise vorticity. These mechanisms produce two counterrotating vortices whose strength and sense of circulation may change during a tidal cycle: vortices switching sense of rotation under weak stratification and vortices maintaining the same sense of rotation under strong stratification. With rotational effects, the bottom Ekman layer derived from tidal currents triggers a one-cell vortex that switches its sense of rotation over a tidal cycle. These model results are similar to those reported in previous studies (Lerczak and Geyer 2004; Chen and Sanford 2009; Scully et al. 2009). This work contributes to better understanding of the lateral circulation by using the vorticity analysis approach to interpret the dynamic balance. It also provides a unifying framework to explore the transition from differential advection to diffusive boundary mixing in nonrotating systems (as stratification increases) and the transition from two-cell circulation to one-cell circulation in rotating systems (as the width of the estuary increases).

Analysis of the streamwise vorticity budget has yielded new insights into the generation and dissipation mechanisms of the lateral circulation in estuaries. Without rotational effects, the temporal evolution of the streamwise vorticity is determined by a balance between the baroclinic forcing and turbulent diffusion, which explains the flood–ebb asymmetry of the lateral circulation over a tidal cycle and the phase lag between the maximum tidal current and the maximum vorticity. With rotational effects, the temporal evolution of the streamwise vorticity is determined by a three-way balance: the tilting of planetary vorticity by the vertical shear in the along-channel current, the turbulence diffusion, and the baroclinic current. The first generates the streamwise vorticity, but is opposed by the second, and the third shows a delayed response. Because \(\vec f \partial \vec u / \partial \vec z\) is larger on ebb than on flood, the streamwise vorticity is slightly stronger on ebb. This flood–ebb asymmetry is opposite that found in lateral circulation generated by differential advection.

A total of 15 numerical experiments examine the influence of river flow, tidal current, and width of the estuary on the streamwise vorticity. As stratification increases, the magnitude of streamwise vorticity in nonrotating runs decreases and the ratio of diffusive boundary mixing to differential advection increases. This indicates that differential advection is the main driver of lateral flows under no rotation. However, as stratification increases, boundary mixing can also be a relevant lateral flow driver. In rotating runs, both the magnitude and structure of the lateral circulation changes with the width of the estuary. In narrow estuaries where the Coriolis force plays a secondary role, the lateral circulation features a pair of counterrotating vortices. In estuaries of intermediate widths, the lateral circulation features one vortex that fills up the entire estuarine cross section. In wide estuaries, the one-cell vortex is confined to the left side of the estuary (looking into the estuary in the Northern Hemisphere) due to rotational control. The magnitude of the streamwise vorticity increases rapidly as the estuarine width increases, but reaches a maximum at an intermediate value of \(Ke\) and decreases slightly as \(Ke\) increases. This happens as the rotational control limits the strength of the lateral circulation in the widest estuaries.

In the future it would be worthwhile to apply the vorticity analysis to lateral circulations generated in real estuaries. For example, one-cell lateral circulation has been observed in the James River estuary. Its strength varied little over the spring–neap tidal cycle in contrast to what is expected from the numerical experiment in Lerczak and Geyer (2004). A preliminary analysis of the streamwise vorticity budget in the James River shows that increased Ekman forcing \(\vec f \partial \vec u / \partial \vec z\) during spring tides is largely balanced by enhanced turbulent diffusion, such that the net-generating force for streamwise vorticity remains relatively unchanged over a spring–neap cycle. This analysis and model data comparison will be reported in the future.

Acknowledgments. We are grateful to NSF (OCE-0825826, OCE-0825833, and OCE-0825876) for the financial support. We thank three reviewers for their helpful comments.

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