The Saturation of Fluid Turbulence in Breaking Laboratory Waves and Implications for Whitecaps

GRANT B. DEANE, M. DALE STOKES, AND ADRIAN H. CALLAGHAN
Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California

(Manuscript received 11 September 2014, in final form 3 December 2015)

ABSTRACT

Measurements of energy dissipated in breaking laboratory waves, averaged over time and space and directly visualized with a bioluminescent technique, are presented. These data show that the energy dissipated in the crest of the breaking waves is constrained: average turbulence intensity within the crest saturates at around 0.5–1.2 W kg\(^{-1}\), whereas breaking crest volume scales with wave energy lost. These results are consistent with laboratory and field observations of the Hinze scale, which is the radius of the largest bubble entrained within a breaking crest that is stabilized against turbulent fragmentation. The Hinze scale depends on turbulence intensity but lies in the restricted range 0.7–1.7 mm over more than two orders of magnitude variation in underlying unbroken wave energy. The results have important implications for understanding the energetics of breaking waves in the field, the injection of turbulence into the upper ocean, and air–sea exchange processes in wind-driven seas.

1. Introduction

During wind-driven conditions, turbulence from breaking ocean waves enhances the transport of mass and momentum across the air–sea interface (Melville 1996) and controls the numbers and sizes of bubbles in the upper-ocean boundary layer (Garrett et al. 2000; Deane and Stokes 2002), which has important implications for gas exchange (Keeling 1993), marine aerosol production (Lewis and Schwartz 2004), and ocean albedo (Frouin et al. 1996). Measurements of turbulence during wind-forced conditions are greatly complicated by wind, wind-driven waves, and wave breaking. This is especially true for measurements within the upper 50 cm or so of the water column, where turbulence is expected to be largest. Studies using noninvasive remote sensing techniques and near-surface profiling instrumentation and methodologies have only recently begun to shed light on this near-surface region in the field (e.g., Thomson 2012; Sutherland and Melville 2013, 2015). Consequently, few open-ocean datasets of turbulence measurements within the near-surface wave boundary layer exist. Even more challenging is making accurate measurements of turbulence inside actively breaking waves at sea due to the complex bubble-laden two-phase flow and the highly intermittent and random nature of breaking. Therefore, while imperfect in design and nature, lake and laboratory studies remain an important and valuable approach to better understand turbulent processes inside breaking crests.

Recent studies have produced a consistent picture of surface boundary layer turbulence, away from breaking wave crests. At the sea surface, turbulence and air are injected by spilling and plunging breakers to variable (and presently poorly characterized) depths (Gerbi et al. 2009). Downward advection, rather than the direct injection of turbulence from whitecaps, enhances turbulence in the adjacent, deeper, wave-affected surface layer (Stips et al. 2005). At even greater depths, wind-driven turbulence production and dissipation largely follows the law of the wall scaling (Agrawal et al. 1992).

We are not the first to study energy dissipation and air entrainment in breaking laboratory waves. For example, laser-based imaging of turbulence beneath focused wave packets has been reported by Rapp and Melville (1990), Melville et al. (2002), and Drazen and Melville (2009). These well-cited studies have helped quantify the coherent and turbulent motions induced by laboratory
breaking waves and have important implications for the field. However, measurements of fluid turbulence using laser-based digital particle imaging velocimetry (DPIV) are limited to regions outside the region aerated by bubbles. As Rapp and Melville (1990) state: “Here, spikes due to bubbles crossing the measurement point, which contribute to the variance (noise) in the signal, are generally larger than 10%, and it is not possible to resolve the accuracy of this measure of turbulence right at breaking in the bubbly flow. After several wave periods, however, the turbulent measurements are very reliable.” Watanabe et al. (2006) also note that DPIV measurements are limited to nonaerated or diluted bubble flow. These limitations are a function of the large, geometrical, scattering cross section presented by bubbles in high air fraction plumes and have consequences for the high-frequency Doppler sonar systems used to probe whitecaps (see the appendix). Consequently, these earlier studies report the intensity, distribution, and decay of wave-induced turbulence one or more wave periods after the onset of breaking and buoyancy-driven removal of large bubbles and high air fractions from the two-phase flow. The data reported here were taken within the highly aerated flow created during the acoustically active phase of breaking and thus extend the measurement of turbulence to an earlier and more energetic phase of wave breaking.

The acoustically active phase of breaking (Deane and Stokes 2002) denotes to the phase of whitecap evolution during which the two-phase flow created by the breaking wave radiates sound. The dominant sources of noise are newly formed bubbles (Banner and Cato 1988), so the acoustically active period is limited to the duration of bubble entrainment and breakup, corresponding to the initial period of the greatest fluid turbulence. It is this phase of wave breaking that determines the initial bubble size distribution created by the wave and its distribution throughout the wave breaking layer. As we will show, turbulent kinetic energy in a breaking wave crest during the acoustically active phase can be in excess of 1 W kg$^{-1}$, which is three orders of magnitude or more than average levels in the wave breaking layer (Gemmrich and Farmer 1999; Stips et al. 2005; Gerbi et al. 2009). The high levels of turbulent kinetic energy and air fraction encountered in breaking crests during the acoustically active phase significantly complicate measurements of fluid flow. Techniques based on the scattering of acoustic waves, such as high-frequency backscatter Doppler sonars, for example, work outside of actively breaking crests but have not been definitively shown to work within them. The model calculations for Doppler sonar performance in the appendix show these instruments face extreme challenges within high air fraction plumes.

There have been laboratory studies to quantify air fraction in the highly aerated flows that occur during the acoustically active phase of breaking using non-acoustic techniques (e.g., Cox and Shin 2003; Lamarre and Melville 1991, 1994; Lim et al. 2015). Lamarre and Melville (1991) used an electrical conductivity probe to map the spatial and temporal distributions of air entrained by laboratory breakers and demonstrated high air fractions (up to 0.6) in the flow and a large fraction of energy dissipated expended in suspending bubbles (30%–50%). Blenkinsopp and Chaplin (2007) conducted a similar study and also reported high air fractions but a reduced estimate for energy expended to suspend bubbles. There have been laboratory measurements of bubble size distributions during the acoustically active phase of breaking using optical fiber probes (Rojas and Loewen 2007; Blenkinsopp and Chaplin 2010) and photographic techniques (Loewen et al. 1996; Deane and Stokes 2002; Leifer and de Leeuw 2006).

Notwithstanding these studies, little is known about spatial and temporal distributions of turbulent kinetic energy during the acoustically active phase of wave breaking, during which turbulent dissipation rates can exceed 1 W kg$^{-1}$ (Stokes et al. 2004) and air is actively entrained and fragmented into bubbles. A series of laboratory experiments have been conducted to directly visualize fluid shear stress from the moment of initial breaking through to the end of active air entrainment. We have created spatial maps of time-varying fluid shear stress using the flash response of bioluminescent dinoflagellates as in situ sensors (Stokes et al. 2004; Deane and Stokes 2005). When combined with bulk measurements of time- and space-averaged dissipation and bubble size distributions, these data have enabled us to quantify turbulent kinetic energy within breaking laboratory wave crests as a function of underlying wave energy using two independent but complementary approaches.

2. Methods

  a. Experimental setup

The experiments were conducted in a glass-walled wave flume (33 m long, 0.5 m wide, and 0.6-m water depth) filled with natural seawater pumped 300 m offshore from La Jolla Shores Beach. Wave packets of varying scale and slope were generated at one end of the flume using a computer-controlled wave paddle using the method described by Longuet-Higgins.
(1974). The packet slope at the wave break point is adjusted by varying the amplitude of individual, pseudoharmonic wave components, which provides a degree of control over the wave breaking characteristics. Increasing the spectral slope of the wave packet from 0.3 to 0.5 varies the breaker type from spilling to plunging. Four distinct packet types were investigated (see Table 1). When adjusted for the two-dimensional constraints of the channel, these breaking waves overlap in scale with whitecaps in the approximate size range 0.2–0.8 m² (see Fig. 10 in Callaghan et al. 2013).

The dispersive focusing method used here allows breaking waves to be generated in a very controlled and repeatable manner in the laboratory. As described in the introduction, many previous laboratory studies have employed similar techniques, and a wealth of information on energy dissipation, momentum flux, and air entrainment has been gained. In the open ocean, there are a variety of different routes that may lead to deep-water wave breaking such as energy convergence in nonlinear wave groups, modulational instability, wave–wave interaction, wave–current interaction, and wave focusing (e.g., Melville 1996; Banner and Peirson 2007; Babanin 2011; Plant 2012). All of these mechanisms leading to instability are further modulated by the presence of wind shear stress. Therefore, while the particular route to breaking employed here may not be representative of all potential mechanisms in the open ocean, it provides a practical solution to generating breaking waves and allowing the resulting two-phase flow to be studied in detail. It is important to note that while laboratory studies have indicated that the evolution of spectral energy within a broadband wave packet after breaking may be dependent on the route to breaking (e.g., Meza et al. 2000; Babanin et al. 2010), there is no conclusive evidence at this time that the gross evolution of the resulting two-phase flow is significantly affected by the route to breaking. That is to say, once the turbulent two-phase flow has been generated, it is unknown if it retains a memory of its particular route to breaking, unlike the spectral energy distribution. This will remain an open question until reliable field measurements of the interior of actively breaking whitecaps are available [see the appendix for a discussion of the Doppler sonar measurements of Gemmrich (2010)]. While every effort should be made to replicate fully three-dimensional wind-driven breaking waves experimentally, this is not always possible given technical constraints, and the focusing method employed here and by those who precede us continues to provide valuable information on turbulent flow, air entrainment, and energy dissipation in breaking waves.

Two separate studies were designed to examine the bulk and finescale features of flows within breaking wave crests: bulk experiments and finescale experiments. The bulk experiments quantified the volume of, and mean dissipation rate within, a breaking crest by monitoring the crest through the sidewall of a glass flume with a high-speed video camera mounted on a two-dimensional robotic tracking system that was programmed to follow the wave centroid. The finescale experiments yielded spatial maps of time-varying dissipation within a breaking wave crest through the analysis of light from the stimulated flash response of the bioluminescent dinoflagellate Lingulodinium polyedrum, which were seeded into the flume and imaged with an intensified camera (Latz and Rohr 1999; Stokes et al. 2004).

The equipment configuration was as follows: Resistance wire wave staffs were mounted in the center line of the channel and 1 m upstream and downstream of the wave break point to provide a time series of surface elevation, which were used to compute the energy lost through breaking. Cross-channel fluctuations were not quantified, but casual observation did not reveal the presence of any strong cross-channel instability or variability. The staffs were cleaned at regular intervals and calibrated at the beginning of each experiment day.

### Table 1. Summary of packet characteristics. See text for an explanation of values.

<table>
<thead>
<tr>
<th>Wave packet</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy density $E$ (J m(^{-1}))</td>
<td>156</td>
<td>84</td>
<td>48</td>
<td>99</td>
</tr>
<tr>
<td>Potential slope $\Theta$</td>
<td>0.47</td>
<td>0.38</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>Amplitude (cm; peak–trough)</td>
<td>23</td>
<td>16</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>Energy dissipated $\Delta E$ (J m(^{-1}))</td>
<td>48</td>
<td>23</td>
<td>9.2</td>
<td>14</td>
</tr>
<tr>
<td>Turbulence dissipation $\Delta E$ (J m(^{-1}))</td>
<td>44</td>
<td>17</td>
<td>13</td>
<td>1.5</td>
</tr>
<tr>
<td>Mean cross-sectional area $A_0$ (cm²)</td>
<td>950</td>
<td>420</td>
<td>360</td>
<td>230</td>
</tr>
<tr>
<td>Mean, average $\overline{\alpha}$ (W kg(^{-1}))</td>
<td>1.0</td>
<td>1.1</td>
<td>0.51</td>
<td>0.30</td>
</tr>
<tr>
<td>Air fraction $\alpha$</td>
<td>0.046</td>
<td>0.036</td>
<td>0.031</td>
<td>0.053</td>
</tr>
<tr>
<td>Hinze scale from $n(a)$: $a_{H,0}$ (mm)</td>
<td>1.3 ± 0.3</td>
<td>1.5 ± 0.3</td>
<td>1.4 ± 0.3</td>
<td>1.6 ± 0.3</td>
</tr>
<tr>
<td>Hinze scale from $x$: $a_{H,x}$ (mm)</td>
<td>1.4 ± 0.2</td>
<td>1.3 ± 0.1</td>
<td>1.3 ± 0.1</td>
<td>2.4 ± 0.1</td>
</tr>
</tbody>
</table>
Breaking waves were imaged through the channel sidewall with high-speed (IDT MotionproX3) and intensified cameras (Roper Scientific Cascade 512b) attached to the robotic system. A translucent partition, placed lengthwise along the channel and 13 cm from the glass sidewall, provided a defined volume for bubble image analysis. When augmented with spanwise vertical partitions at the upstream and downstream ends, it also provided an enclosed volume for seeding the channel with dinoflagellates, which were used to image fluid shear stress. This procedure worked well for plunging breakers, which have a well-defined vortex structure during breaking but was less successful for the gently spilling breaker (packet D) that was distributed along the wave flume and could not be tracked everywhere simultaneously. Consequently, it is likely that estimates of bioluminescence stimulation for this spatially extended packet were underestimated. Seawater surface tension during the bulk dissipation experiments was measured with a Kruss tensiometer and found to be 0.070 N m\(^{-1}\) (±0.0005 N m\(^{-1}\)). Surface tension observations during the bioluminescence experiments are not available, but historical measurements of surface tension for seawater pumped from La Jolla Shores Beach suggest a nominal value in the range 0.070–0.072 N m\(^{-1}\).

b. Estimating total energy lost due to breaking

The energy density of wave packets is calculated from

\[
E = \rho g \int_0^L \eta^2(x) \, dx, \tag{1}
\]

where \(x\) is distance along the wave channel centerline in the direction of wave motion, \(\rho\) is water density, \(g\) is the acceleration due to gravity, the packet disturbance lies in the range 0 \(\leq x \leq L\), and the surface elevation as a function of distance along the channel \(\eta(x)\) was computed from the surface elevation time series using pseudoharmonic analysis. One wave staff was placed centrally at each channel location, so cross-channel fluctuations in the wave packet properties were not measured. After breaking, cross-channel fluctuations in water elevation downstream of the breaking region must be expected, but we assume these to be essentially random in time and space and therefore do not expect such fluctuations to introduce a significant systematic bias when calculating the energy density of the wave packet following Eq. (1) above. The total energy lost from the wave packet due to breaking is estimated from

\[
\Delta E_T = \rho g \int_0^L (\eta_U^2 - \eta_D^2) \, dx, \tag{2}
\]

where \(\eta_U\) and \(\eta_D\) respectively are the surface elevation of the packet measured upstream and downstream of wave breaking.

c. Estimating turbulence dissipation from bioluminescence

Fluid shear stress was visualized and quantified using the flash response of the dinoflagellate \(L.\) polyedrum. This 30-\(\mu\)m scale unicellular microorganism produces a 50-ms flash of light when stimulated and can be used to image and quantify fluid shear stress within breaking wave crests, provided certain conditions are met (Stokes et al. 2004): cells must be uniformly distributed throughout the imaging volume, and cell concentration must be measured before every event imaged. Each flash of \(L.\) polyedrum produces an order of \(10^8\)–\(10^9\) photons of light, making a single cell a very dim light source, and an intensified camera and low background light levels were required for imaging. Stokes et al. (2004) used bioluminescence to estimate levels of turbulent dissipation in a breaking wave, and the measurements presented here are based on similar principles.

Quantitative estimates of fluid shear stress imaged with bioluminescence require a uniformly distributed and known concentration of cells within the breaking volume for every imaged event. This was achieved by gently pouring cultured populations of dinoflagellates into the breaking volume and stirring with a hand paddle for tens of seconds before generating a wave packet. The homogeneity of the resulting mixture was verified by imaging grid-stimulated bioluminescence before the mixing protocols were finalized. The spanwise barriers were removed a few seconds before the arrival of a wave packet. Water samples from the enclosed volume were pumped into a sensor that measured the cell concentration before and after each breaking event. A dark environment was created by tenting the working section of the tank and conducting the experiments at night. The very different lighting conditions for imaging bubble size distributions and bioluminescence required these measurements to be taken from different event ensembles. However, intercomparison is still possible because of the high degree of reproducibility of the experimental method (Deane and Stokes 2002).

The flash response of \(L.\) polyedrum is probabilistic, follows Poisson statistics, and is a nonlinear function of the fluid shear stress \(\tau\) (Deane and Stokes 2005). The flash rate \(\lambda(\tau)\) within a population of organisms subjected to constant shear stress is given by (Deane and Stokes 2005)
A quantitative method for analyzing images of bioluminescence is described in detail in Stokes et al. (2004) but essentially works as follows: The key step is to determine the cell flash rate—the number of cells that flash per second over the camera integration time $\Omega$—within the volume of water imaged by a single pixel of the camera. Each image records the flash response of $C_0 \Delta V$ organisms, where $C_0$ is the cell concentration and $\Delta V$ is the sample volume, in terms of scaled photon counts $p_{ij}$ from the photo detector at position $(i, j)$ within the camera. If the total detector counts for a single cell flash $\Gamma_0$ are known, then the flash rate within a selected region is given by [Eq. (17) in Stokes et al. 2004]

$$\lambda = \frac{1}{\Omega C_0 \Delta V} \frac{\sum_{\text{selected region}} p_{ij}}{\Gamma_0}.$$  \hspace{1cm} (4)

The quantity $\Gamma_0$ can be estimated from images of flashes from single organisms. Given an isolated flash that follows an identifiable path in the image, the photo detector counts for a single flash are

$$\Gamma_0 = \sum_{\text{cell path}} p_{ij}.$$  \hspace{1cm} (5)

In practice, multiple isolated flashes are identified and analyzed to provide a mean value for this quantity. Equations (4) and (5) together allow images of bioluminescence to be converted to images of cell flash rate.

The scattering and attenuation of bioluminescent flashes by bubbles and the stimulation of bioluminescence by rising bubbles complicate the image analysis and need to be treated. Experiments to quantify the stimulation potential of rising bubbles on the dinoflagellate _L. polyedrum_ have been reported (Deane et al. 2015). A full description of this topic lies outside the scope of this article, but a brief discussion follows. Populations of cells were stimulated by isolated bubbles with radii in the range 0.3–3 mm and clouds of bubbles with radii in the range 0.06–10 mm and air fractions comparable with those encountered in the laboratory breaking waves. The stimulation efficiency of isolated bubbles was found to be greatest for bubbles greater than 1 mm in radius, and bubbles smaller than $O(0.3)$ mm did not stimulate flashing. An empirical model of bubble stimulation for clouds of bubbles based on the isolated bubble observations successfully reproduced the observed stimulation from bubble clouds. These modeling efforts formed the basis for estimates of the stimulation potential of the bubble clouds formed by the four breaking packet types, using the observed bubble distributions. The net result was that the rising bubbles stimulated the dinoflagellates at a rate equivalent to an average shear stress of approximately $1.7 \text{ N m}^{-2}$.

The scattering of bioluminescence by the bubbles surrounding the dinoflagellates also needs to be measured and accounted for. A series of experiments were run in the glass-walled wave flume to quantify this effect. A small, spherical, and diffuse incandescent light source was placed in the breaking crest region, and its intensity was imaged before and during a breaking event. The experiment was repeated with the light source moved through a horizontal array of locations in the up- and downstream direction and across the channel width. An analysis of the intensity time series, as a function of position within the cloud, leads to the conclusion that the overall effect of scattering was to decrease the average bioluminescence intensity by approximately 40%.

The overall scheme was to take each unprocessed bioluminescence image, multiply the pixel counts by 1.6 to accommodate bubble scattering effects, and then use Eqs. (3) and (4) to compute the fluid shear stress. A shear stress of $1.7 \text{ N m}^{-2}$ was then subtracted to account for the effects of bubble stimulation. Any negative shear stress levels were set to zero. The estimates of shear stress were converted into a fluid dissipation rate using the formula

$$\varepsilon = \frac{1}{\mu \rho} \tau^2,$$  \hspace{1cm} (6)

where $\mu$ and $\rho$, respectively, are the dynamic viscosity and density of seawater, and $\tau$ is the estimate of shear stress.

Equation (6) represents a departure from standard methods used to estimate the turbulence dissipation rate. Because turbulent energy dissipation occurs on the Kolmogorov microscale $\ell_k = (\mu^2/\rho)^{1/4}$, which is typically much smaller than the scale of the sensors employed to measure fluid shear, estimating the dissipation rate usually requires assumptions to be made about the wavenumber spectrum of the turbulent velocity fluctuations. However, the dinoflagellates employed in this study were roughly $30 \mu\text{m}$ in scale, which is equivalent to the Kolmogorov microscale in isotropic, homogeneous turbulence with a dissipation rate of approximately $1 \text{ W kg}^{-1}$. That is, the length scale of the sensor is of the same order of magnitude as the dissipation microscale for these experiments, and Eq. (6) can be used to compute the dissipation rate from viscosity directly without...
3. Time- and space-averaged turbulent dissipation rate

a. Mathematical formulation

The dominant sinks of energy in breaking waves are the dissipation of turbulent kinetic energy and the suspension of bubbles; secondary effects such as the reradiation of high-frequency waves and the generation of underwater noise can be neglected (Melville 1996). Spilling breakers may lose up to 10% of their underlying wave energy to fluid turbulence and bubble suspension, rising to 25% or more for plunging breakers (Rapp and Melville 1990). Wave breaking, whether spilling or plunging, is assumed to consist of a succession of vortex injection events. An example of a spilling breaker with this kind of structure is shown in Fig. 1a, with the foam strips corresponding to the surface expression of distinct vortices. Each vortex creates a line of two-phase flow consisting of bubbles and fluid turbulence, which we refer to as a bubble plume. The flow is assumed to be homogeneous along a vortex line in a plane parallel to the surface but dependent on depth below the surface \( z \) and time after initial vortex injection \( t \). The geometry for a single vortex is shown in Fig. 1b. The assumption that the plume is a rectangular prism is a simplification that reduces the number of coordinate variables required for the analysis without the loss of any significant generality; the analysis presented below could be generalized to a range and depth-dependent plume geometry.

The total energy loss due to wave breaking \( \Delta E_T \) within the volume of a vortex is partitioned into three terms:

\[
\Delta E_T = E_{e, \text{form}} + E_{e, \text{decay}} + E_{\text{PE}},
\]

where \( E_{e, \text{form}} \) is energy dissipated through fluid turbulence during the acoustically active phase of the plume evolution, \( E_{e, \text{decay}} \) is energy dissipated through fluid turbulence during the decay phase of the plume, and \( E_{\text{PE}} \) is the potential energy of the submerged bubbles at the end of the acoustically active phase. The cross-sectional area of the bubble plume viewed from above (the area of the foam strip) is \( A(t) \), \( e(t, z) \) is the time- and depth-dependent turbulent dissipation rate (W kg\(^{-1}\)), \( \alpha(t, z) \) is the time- and depth-dependent air fraction, \( z_p(t) \) is the time-dependent plume penetration depth, \( W \) is the length of the vortex in the along-crest direction, and \( \tau_A \) is the duration of the acoustically active period (see Fig. 1b).

The energy dissipated through fluid turbulence during the acoustically active phase can be expressed as

\[
E_{e, \text{form}} = \rho \int_0^{\tau_A} \int_0^{z_p(t)} A(t) e(t, z) \left[ 1 - \alpha(t, z) \right] dz \; dt,
\]

where \( \rho \) is the water density. The inner integral over depth yields the power dissipated by fluid turbulence in the fluid phase of the plume as a function of time, and the term \( 1 - \alpha(t, z) \) allows for the exclusion of regions of the plume containing air. The integral of this quantity over time, multiplied by fluid density, yields the total energy dissipated by turbulence in the fluid phase of the plume over the acoustically active period.

We need to express Eq. (8) in terms of time- and depth-averaged variables, which can be accomplished through repeated application of the second mean value theorem for integrals. Substitution of the depth-averaged turbulent dissipation
\[
\bar{e}(t) = \frac{1}{\bar{z}_p(t)} \int_0^{\bar{z}_p(t)} e(t, z) \, dz \tag{9}
\]
and depth-averaged, dissipation-weighted air fraction
\[
\bar{\alpha}_e(t) = \frac{\int_0^{\bar{z}_p(t)} e(t, z) \alpha(t, z) \, dz}{\int_0^{\bar{z}_p(t)} e(t, z) \, dz} \tag{10}
\]
into Eq. (8), and noting that \( A(t) \) is a function of time only, yields
\[
E_{e, \text{form}} = \rho \int_0^{\tau_A} V(t) \bar{e}(t) [1 - \bar{\alpha}_e(t)] \, dt, \tag{11}
\]
where \( V(t) = A(t) \bar{z}_p(t) \) is the time-varying volume of the bubble plume. Equation (11) has a simple, physical interpretation. The energy dissipated by turbulence during the acoustically active phase of plume formation is equal to the time integral of the depth-averaged power dissipated by the turbulence \( \bar{e} \), multiplied by the volume of the plume, and adjusted by the factor \( 1 - \bar{\alpha}_e \), which compensates for the volume of the plume taken up by bubbles. According to Eq. (10), air fraction at depths where levels of turbulent dissipation are high is more important to changes in dissipation due to the exclusion of water volume by air than air fraction at depths where turbulent dissipation levels are low.

The second mean theorem for integrals can be used again to allow \( E_{e, \text{form}} \) to be expressed in terms of variables averaged over both depth and time. We define the time- and space-averaged dissipation and air fraction to be
\[
\bar{\bar{e}} = \frac{1}{\bar{\tau}_A} \int_0^{\bar{\tau}_A} V(t) \bar{e}(t) \, dt = \frac{1}{\bar{\tau}_A} \int_0^{\bar{\tau}_A} V(t) \bar{e}(t) \, dt, \tag{12}
\]
where
\[
\bar{\tau}_A = \frac{1}{\tau_A} \int_0^{\tau_A} V(t) \, dt, \tag{13}
\]
and
\[
\bar{\bar{\alpha}}_e = \frac{1}{\bar{\tau}_A} \int_0^{\bar{\tau}_A} V(t) \bar{e}(t) \bar{\alpha}_e(t) \, dt = \frac{1}{\bar{\tau}_A} \bar{e} \int_0^{\bar{\tau}_A} V(t) \bar{\alpha}_e(t) \, dt. \tag{14}
\]
With these definitions, Eq. (8) can be expressed in terms of the averaged variables:
\[
E_{e, \text{form}} = \rho \bar{\tau}_A \bar{\bar{e}} \bar{\bar{\alpha}}_e (1 - \bar{\bar{\alpha}}_e). \tag{15}
\]
The energy used to suspend bubbles \( E_{PE} \) can be estimated by calculating the potential energy of bubbles entrained in the plume toward the end of the acoustically active phase, when turbulence is still intense enough to suspend and fragment bubbles. This energy is given by
\[
E_{PE} = g \rho A(\tau_A) \int_0^{\tau_A} z \alpha(\tau_A, z) \, dz. \tag{16}
\]
Defining the depth-averaged air fraction to be
\[
\bar{\alpha}_z(t) = \frac{1}{\int_0^{\tau_A} z \, dz} \int_0^{\tau_A} z \alpha(t, z) \, dz = \frac{2}{\bar{\tau}_A} \int_0^{\bar{\tau}_A} \bar{\alpha}(t, z) \, dz, \tag{17}
\]
the potential energy at the end of the acoustically active phase \( t = \tau_A \) can be written as
\[
E_{PE} = \frac{1}{2} g \rho \bar{\tau}_A^2 \bar{\alpha}_z(\tau_A) = \frac{1}{2} g \rho V(\tau_A) \bar{\tau}_A \bar{\alpha}_z, \tag{18}
\]
where the volume of the plume at the end of the acoustically active phase is given by \( V(\tau_A) = W A(\tau_A) \), where \( W \) is the along-crest length of the vortex. Adding Eqs. (15) and (18), and neglecting the contribution of turbulent kinetic energy during the decay phase of the plume, yields an approximation for the energy dissipated by the plume in terms of space- and time-averaged variables:
\[
\Delta E_r \simeq \rho (1 - \bar{\bar{\alpha}}_e) \bar{\bar{\bar{e}}} \bar{\bar{\alpha}}_z + \frac{1}{2} g \rho \bar{\bar{\alpha}}_z V(\tau_A) \bar{\tau}_A. \tag{19}
\]
If we further assume that the along-crest length \( W \) of the line vortex does not change during the acoustically active phase, then the energy dissipated per meter of the wave crest is given by
\[
\Delta E = \Delta E_r / W \simeq \rho (1 - \bar{\bar{\alpha}}_e) \bar{\bar{\bar{e}}} \bar{\bar{\alpha}}_z + \frac{1}{2} g \rho \bar{\bar{\alpha}}_z P(\tau_A) \bar{\tau}_A, \tag{20}
\]
where \( P(t) \) is the plume cross-sectional area in a plane normal to the water surface and parallel with the direction of wave motion (i.e., the plume cross section as seen through the glass sidewall of the flume), and the time-averaged plume cross section is given by
\[
\bar{P} = \frac{1}{\bar{\tau}_A} \int_0^{\bar{\tau}_A} P(t) \, dt. \tag{21}
\]
Early in the experiments, attempts were made to simultaneously image the flash response of bioluminescent dinoflagellates and air entrainment by fitting the two imaging video systems with light filters (blue to image the bioluminescence and red to image the bubble plumes). The extremely high sensitivity required to image the bioluminescence during the high levels of light required to illuminate the bubble plumes resulted in cross talk between the two systems and made simultaneous measurements impossible. Consequently, we lack the simultaneous measurements of fluid shear stress and air fraction to compute the time- and depth-averaged, dissipation-weighted air fraction $\bar{\alpha}$, given by Eq. (14). However, we can simplify the problem by assuming that the air fraction $\alpha(t, z) = \bar{\alpha}_0(t)$ is constant with depth at any given time. In this case, Eq. (20) simplifies to

$$\Delta E = \rho(1 - \bar{\alpha}_0)\bar{\tau}_A + \frac{1}{2} gp\bar{\alpha}_0(\tau_A)P(\tau_A)z_p(\tau_A). \quad (22)$$

where

$$\bar{\alpha}_0 = \frac{1}{\tau_A} \int_0^{\tau_A} \alpha_0(t) \, dt. \quad (23)$$

Solving Eq. (22) for the time- and space-averaged dissipation rate $\bar{\alpha}$ yields

$$\bar{\alpha} = \frac{\Delta E - \frac{1}{2} gp\bar{\alpha}_0(\tau_A)P(\tau_A)z_p(\tau_A)}{\rho(1 - \bar{\alpha}_0)\bar{\tau}_A}. \quad (24)$$

### b. Measured time- and space-averaged dissipation

Equation (24) provides a means of estimating the time- and depth-averaged dissipation $\bar{\alpha}$ of fluid turbulence within the actively breaking crest in terms of observables. The total packet energy loss $\Delta E$ is estimated by analyzing the change in energy content of the pre- and postbreaking wave packets measured with the upstream and downstream wire wave height gauges. The plume vertical cross section at the end of the acoustically active phase $P(\tau_A)$ and plume penetration depth $z_p(\tau_A)$ are estimated from the heightened albedo of the aerated region imaged with a high-speed video camera of the breaking wave crest, which was taken through the side of the glass-walled flume. The plume cross-sectional area at the end of the acoustically active phase was estimated to be the maximum plume cross-section area. The time of the maximum plume cross-section area and the end of the acoustically active phase was generally within $\frac{1}{2}s$, making this an acceptable estimate. The duration of the acoustically active phase $\tau_A$ is found by determining the time at which the breaking wave stops radiating underwater noise. The bubble size distribution and air fraction is estimated through a computer-aided analysis of high-resolution photographs of bubbles in the wave crest taken near the end of the acoustically active phase.

The bulk experiments produced the estimates of space- and time-averaged dissipation $\bar{\alpha}$ and plume vertical cross-sectional area $P(\tau_A)$, shown in Fig. 2. Data points are denoted by symbols (wave gauge and camera data denoted by filled circles and the dinoflagellate flash response denoted by open symbols), and the gray lines show least-mean-square regression curves. The vertical error bars on the bulk dissipation estimates were calculated as follows: The energy dissipated during the decay phase of plume evolution can be introduced into Eq. (24) by assuming that it is some unknown fraction $\beta$ of the energy dissipated during the acoustically active phase:

$$\Delta E_T = (1 + \beta)E_{\text{form}} + E_{\text{PE}}. \quad (25)$$

With this assumption, Eq. (24) becomes

$$\bar{\alpha} = \frac{\Delta E - \frac{1}{2} gp\bar{\alpha}_0(\tau_A)P(\tau_A)z_p(\tau_A)}{(1 + \beta)\rho(1 - \bar{\alpha}_0)\bar{\tau}_A}. \quad (26)$$

The factor $\beta$ allows an estimate of the error introduced into the average dissipation: $\beta = 1$ corresponds to an equal partition of energy between the acoustically active phase and the decay phase, whereas $\beta = 0$ corresponds to all dissipation occurring in the acoustically active phase. Equation (26) has been used to estimate the error in observed dissipation by choosing various combinations of $\bar{\alpha}_0$ and $\beta$. The error bars in the bulk estimates of average dissipation in Fig. 2 were generated with the choices $\beta = 1$ and $\bar{\alpha}_0 = 0.15$ and $\beta = 0$ and $\bar{\alpha}_0 = 0.01$. These are reasonable choices to invest the minimum and maximum amount of energy, respectively, into fluid turbulence during the acoustically active phase. The four open symbols in Fig. 2a show dissipation rates obtained using the dinoflagellate flash response. The error bars in the bioluminescent data were estimated by combining the standard errors from run-to-run variability and uncertainty introduced through light scattering from bubbles. The combined data are scattered through the range $0.34$–$1.55W\text{kg}^{-1}$ with a mean value of $(0.69 \pm 0.28)W\text{kg}^{-1}$. The equation for the least-mean-square trend line, computed using both bulk and dinoflagellate data points, is $\bar{\alpha} = 0.68(\pm0.06)$ with $R^2 = 0.0002(\pm0.012)\Delta E\text{Wkg}^{-1}$ with $R^2 = 0.00005$, showing essentially no systematic dependence of time- and space-averaged turbulence dissipation on energy lost by the packet. The dissipation values measured using
bioluminescence have a mean value of \((0.8 \pm 0.4) \text{ W kg}^{-1}\)
consistent with the combined data.

The combined data and regression curve in Fig. 2a show that time- and space-averaged dissipation rates are essentially independent of the total wave energy lost over the range of scales investigated. In contrast, increases in energy dissipation due to the increasing wave slope are accommodated by increased plume cross-sectional area, which scales linearly with the wave energy lost in Fig. 2b. The least-mean-square trend line for the plume cross-sectional area is

\[
P(t_A) = 0.003(\pm 0.001)\Delta E + 0.010(\pm 0.015) \text{ m}^2 \quad \text{with} \quad r^2 = 0.853.
\]

In these experiments, increasing the wave energy dissipation by either increasing wave scale or wave slope results in more two-phase flow entrained by the breaking crest but leaves the space- and time-averaged dissipation within that flow unchanged within measurement error. We are calling this effect “turbulence saturation.” A least-mean-square analysis of the dissipation rates obtained just from the bioluminescence do show an increase in average dissipation of 17 mW kg\(^{-1}\) (W m\(^{-1}\))\(^{-1}\) of wave energy dissipated, so we
do not discount the possibility that space- and time-averaged fluid turbulence can increase with increasing packet energy loss, although even the bioluminescence data show those increases to be modest.

c. Spatial distributions of turbulence dissipation

Spatial maps of the time-averaged dissipation rate were obtained from the bioluminescence experiments, shown in Fig. 3. Maps for three plunging breakers, A–C, reveal a core of dissipation centered on the primary entrained vortex with secondary vortex entrainment visible in maps B and C. A well-defined vortex structure is not evident for the spilling breaker (map D), but this may be partly due to the difficulties of tracking the spilling wave crest. Dissipation rates are relatively uniform and independent of the wave scale outside of the vortices. Although there is evidence of dissipation increasing within the primary vortex with increasing wave scale, values do not increase in proportion to wave energy lost and actually attain their highest values in plunging breaker B.

Figure 3e shows the probability density functions of dissipation rate in the shortest spatial and time scales studied (1 cm and 50 ms). Plunging breakers A–C show remarkably similar distributions, with an approximately $e^{-4/3}$ scaling for dissipation rates in the range 1–5 W kg$^{-1}$. The dissipation rate rolls off increasingly rapidly beyond 5 W kg$^{-1}$ and precipitously beyond 10 W kg$^{-1}$. The gently spilling packet D rolls off much earlier, at around 2 W kg$^{-1}$.

The data presented in Figs. 2 and 3 provide the evidence for what we are calling turbulence saturation. As the wave scale increases, more energy is lost to breaking, and this additional energy is associated with an increase in plume cross-sectional area rather than an increase in fluid shear stress within the plume. A similar trend can be found in the laboratory study of Lamarre and Melville (1991), who report a relatively constant value of the wave energy dissipated normalized by plume volume and wave scale across three different breaker types and measured after the end of the acoustically active period.

The greater part of the data in Fig. 2 is derived from the bulk estimates of dissipation rate, which provide a depth and time average as previously described [see Eq. (26)]. However, more detailed information about the spatial distribution of the turbulence intensity and its overall variability is provided by the bioluminescence data in Fig. 3. The probability density distribution functions in Fig. 3e show that dissipation rates exceeding 10 W kg$^{-1}$ lie in the extreme tail of the distributions. Integrating the probability density functions provided estimates of the 50th and 95th percentiles of total dissipation: 50% of the energy resided below 1.5, 2.1, 2.1, and 1.0 W kg$^{-1}$ for events A through D, respectively, and 95% of the energy resided below 5.6, 6.7, 6.2, and 1.5 W kg$^{-1}$ for the same events.
d. Comparison with field observations

To demonstrate that our laboratory study captures relevant oceanic scales, we have plotted scaled dissipation rate versus scaled depth along with data from lake and open-ocean studies in Fig. 4. Depth in Fig. 4 is scaled as \((z_0 - z)/H_s\), where \(z_0\) is the roughness length of the ocean surface, and \(H_s\) is the significant wave height. These two variables have been calculated according to

\[
H_s = \frac{0.2 U_{10}^2}{g}, \tag{27}
\]

where \(g\) is the acceleration due to gravity and \(U_{10}\) is the 10-m height wind speed, and

\[
z_0 = \frac{\alpha u_*^2}{g}, \tag{28}
\]

where \(\alpha = 14000\) is a dimensionless constant and \(u_*\) is the water friction velocity. The water friction velocity is calculated in terms of the wind drag coefficient \(C_D\) using

\[
u_* = \left(\frac{\rho a C_D U_{10}^2}{\rho}ight)^{1/2}, \tag{29}
\]

where \(\rho\) is the density of air, and the drag coefficient is given by

\[
C_D = 0.001(0.75 + 0.067 U_{10}). \tag{30}
\]

See the caption of Fig. 4 and appendix A in Deane et al. (2013) for further details on the scaling calculations. We note that different definitions of depth have been adopted in some of the studies plotted in Fig. 4, with some authors using depth below the mean sea surface while others use instantaneous depth. These differences will introduce additional scatter between the various datasets, particularly for data near the sea surface. Data from this laboratory study are shown as red diamonds in the upper right. The two lines of symbols correspond to assumed winds speeds of 6 (top symbols) and 12 m s\(^{-1}\) (bottom symbols). These wind speeds yield roughness scales in the range 0.06–0.31 m, which brackets the experimental plume penetration depths (Fig. 3). Blue and gray symbols, respectively, designate data from the ocean and lakes. The black squares are data adapted from Gemmrich (2010), where the gray level within each square represents the probability of observing a scaled energy dissipation (darker color indicates higher probability). Gemmrich’s observations were scaled using a wind speed of 11 ms\(^{-1}\), a significant wave height of 0.3 m, and a surface roughness depth \(z_0 = 0.44H_s\) [for this choice of \(z_0\), see Fig. 14 and subsequent text in Gemmrich and Farmer (1999)]. The observations were taken to be at 0.025 m, which is \(\frac{1}{2}\) of the depth over which Gemmrich’s data were averaged. If our comments on the role of bubbles in Gemmrich’s data are correct (see the appendix), then the last six boxes in this sequence are biased by the motions of large bubbles and do not extend beyond the base of bubble plumes. The three annotated blue dots are measurements provided by Gemmrich and Farmer (2004). They are particularly noteworthy because the wave breaking conditions were known at the time of the measurements. The points labeled “background,” “residual,” and “breaking” correspond to dissipation, respectively, in background conditions, shortly after a breaking event, and during a breaking event.

The data from this laboratory study lie well outside the bounds of the field data. This is because the new laboratory data described here have been scaled to correspond to the space- and time-averaged dissipation rates expected in actively breaking whitecaps, which represent an upper limit for fluid turbulence near the sea surface. The only field measurements of this quantity
that we are aware of are those of Gemmrich (2010), and, for reasons explained in the appendix, his report of dissipation rates in breaking wave crests in excess of 200 W kg\(^{-1}\) should be viewed cautiously until the performance of high-frequency Doppler sonars within actively breaking, air entraining waves is verified. The gray line labeled \(e_{\text{max}}\) is a parameterization of expected maximum, scaled dissipation rate due to Gerbi et al. (2009). Remarkably, this parameterization agrees well with the data from our laboratory experiments, although Gerbi et al. gave no indication that they expected their parameterization to apply to this physical regime.

4. The bubble Hinze scale

a. Background on the bubble Hinze scale

Fluid turbulence during active breaking fragments bubbles larger than the Hinze scale:

\[
a_H = 2^{-8/5} e^{-2/5} (\sigma W_{\text{ec}}/\rho)^{3/5},
\]

where \(\sigma\) is surface tension and \(W_{\text{ec}} = 4.7\) is the critical Weber number. Bubbles smaller than this scale are stabilized to fragmentation by surface tension forces, leading to a characteristic decrease in the power-law scaling of the size distribution from \(-10/3\) to \(-3/2\), with the break point occurring at the Hinze scale (Garrett et al. 2000; Deane and Stokes 2002; Blenkinsopp and Chaplin 2010).

b. Bubble size distribution analysis

Two fields of view with approximately 70- and 170-\(\mu\)m per pixel resolution were used to resolve bubbles in the size range 0.1–10 mm. Typical images taken at the end of the acoustically active period are shown in Fig. 5. Similar images were analyzed by manually sizing bubbles using image processing software (Deane and Stokes 1999; Stokes and Deane 1999). Between six and nine images from each of the four packets were analyzed, each image containing between 500 and 1200 bubbles. Equivalent spherical bubble radius was estimated from the measured bubble cross-sectional area. The sampling volume was estimated from the image scale and depth of field and was limited to the region of each image that actually contained bubbles. Following oceanographic convention, the bubble distributions \(n(a)\) are presented as a function of bubble radius \(a\) and have units of bubbles per cubic meter per micrometer radius increment.

Figure 5 shows bubble size distributions measured at the end of active breaking during the bioluminescence experiments. All the distributions in Fig. 5 show a rolloff for bubbles smaller than approximately 0.3 mm. The rolloff is not consistent with an analysis of the recordings of wave noise, which show the emission of sound up to at least 50 kHz corresponding to bubbles approximately 0.065 mm in radius. Deane and Stokes (2010) have shown that the power-law scaling of radiated sound with frequency is consistent with a bubble density power-law scaling of \(a^{-3/2}\) for these small bubbles (see Fig. 12 in Deane and Stokes 2010). We do not know if the decrease in observed bubble density for these small bubbles was due to limited resolution in the optical system or if the small bubbles were distributed heterogeneously throughout the wave crest (e.g., near the water surface) and therefore not counted. In the absence of further information, the discrepancy between the optical bubble measurements and the acoustic recordings remains unexplained.

c. Observations of bubble Hinze scale from bubble size distribution analysis

The Hinze scales for the bubble size distributions in Fig. 5 have been estimated as follows: An algorithm was devised whereby the vertical offset of a power law of fixed slope was systematically varied, and the number of observations falling within a prescribed vertical interval of the line was counted. The best-fit offset was chosen to be that which resulted in the greatest number of data
packets A–D varied by over a factor of 5 (48, 23, 9.2, and 1.6 mm, respectively, for packets A–D, with a standard deviation of 1.3, 1.5, 1.4, and 0.7 mm for a reliable estimate of their density. The analysis was limited to bubbles in the size range 0.3–5 mm. Smaller bubbles were excluded from the analysis on the grounds that the measurements may not be reliable (see discussion above). Bubbles larger than 5-mm radius were excluded because the limited number of images analyzed provided too few bubbles for a reliable estimate of their density.

 Bubble Hinze scales estimated from the bubble size distribution measurements were 1.3, 1.5, 1.4, and 1.6 mm, respectively, for packets A–D, with a standard deviation of ±0.3 mm. Although the total energy lost by packets A–D varied by over a factor of 5 (48, 23, 9.2, and 14 J m⁻¹, respectively), the observed Hinze scale in these laboratory breakers is essentially constant, consistent with the direct observation that turbulence within the wave crest saturates.

d. Calculations of bubble Hinze scale from observed dissipation rates

Hinze scales can be estimated using Eq. (31) and the mean highest 1/3 of values of ε observed at the most energetic point of breaking. The justification for this choice is that it is the turbulence with the highest levels of shear giving rise to the highest pressure fluctuations across the scale of a bubble that result in the fragmentation of the smallest bubbles. The Hinze scale observed at the end of active breaking is a frozen remnant of this process, cut off at the smallest bubble scales by surface tension. Calculations of bubble fragmentation using the turbulence measured in packet A can be found in Deane and Stokes (2010), which presents a model-based derivation of a Hinze scale for packet A, consistent with the value listed in Table 2 using the mean 1/3 highest value of ε method (see Fig. 5b in Deane and Stokes 2010).

The variables in Table 2 are calculated as follows:

- Energy density $E$ is calculated from Eq. (1). Potential slope $\Theta = \sum b_j k_j$ is the maximum slope attainable if the products of wave amplitude $b_j$ and wavenumber $k_j$ are added linearly at the wave break point. Energy dissipated by a packet $\Delta E$ is the difference between packet energy densities up- and downstream of the wave break point. Air fraction $\alpha$ and Hinze scale $a_H^2$ were estimated from the measured bubble size distribution. The Hinze scale from $\epsilon$ was calculated from the greatest 1/3 values of dissipation using Eq. (31). This analysis yields expected Hinze scales of 1.4, 1.3, 1.3, and 2.4 mm for packets A–D, respectively, in good agreement with the observed scales for packets A–C. The calculated Hinze scale for packet D is larger than the observed value, suggesting that the turbulence dissipation may have been underestimated for this event. This possibility is consistent with the early rolloff of the dissipation probability density function in Fig. 3e.

Under the turbulence saturation hypothesis, we expect the bubble Hinze scale to be insensitive to wave energy dissipated over a wide range of wave scales. Collected observations of Hinze scales measured in laboratory, open-ocean, and surfzone breaking waves are given in Table 2. The wave energy dissipated for these various datasets is not measured but is typically 10%–25% of the underlying wave energy density (Rapp and Melville 1990) for laboratory breaking waves, and can be assumed to be in this range or larger for shoaling surfzone waves, but is almost certainly significantly smaller for open-ocean breaking waves. Because the fraction of underlying wave energy dissipated by breaking varies between the laboratory, surfzone, and open ocean, it would be premature to draw strong conclusions about the apparent invariance of the bubble
Hinze scale despite the large variability in underlying wave energy density.

5. Concluding remarks

We have described three independent methods to characterize fluid turbulence in actively breaking wave crests: direct visualization of fluid shear stress using the bioluminescent flash response of marine dinoflagellates, bulk estimates of time- and space-averaged turbulence dissipation, and observations of the bubble Hinze scale. The measurements strongly suggest that time- and space-averaged fluid turbulence in actively breaking laboratory wave crests remains approximately constant, independent of the overall loss of energy from the breaking wave crest. We are calling this phenomenon the “bubble Hinze scale.”

Our observations of turbulence saturation are consistent with the observed bubble Hinze scale and calculations of the Hinze scale from observed probability density distributions of turbulence. Moreover, the few anecdotal measurements of the bubble Hinze scale available from the field are somewhat consistent with the saturation hypothesis, although it must be noted that the field observations of the Hinze scale do show more scatter than the laboratory data. Further indirect evidence of turbulence saturation can be found in Zhao et al. (2014), who made measurements of the oceanic acoustic noise field in tropical cyclones at wind speeds between 10 and 50 m s\(^{-1}\). While the overall power of the acoustic noise increased with increasing wind speed, the shape of the acoustic power spectrum (which is closely linked to the sizes and numbers of bubbles radiating sound during active wave breaking) remained relatively constant and was very similar to the acoustic power spectrum reported in Deane and Stokes (2002). These remarkable observations led the authors to conclude “that Deane and Stokes’s (2002) 10-cm high laboratory breaking waves have the same turbulence levels as the 10-m high breaking waves observed in tropical cyclones.”

The observations of turbulence and the bubble Hinze scale in breaking laboratory waves reported here are inconsistent with Gemmrich’s report of turbulence in the crests of freshwater waves breaking on a lake (Gemmrich 2010). As described in detail in the appendix, our model calculations suggest that these differences are due to a misinterpretation of the Doppler shifts reported by Gemmrich’s 2-MHz backscatter sonar. The twin effects of thin acoustical skin depth in high air fraction bubble plumes and Doppler bias due to the buoyancy of large bubbles dominating the backscattering cross section undermine the operation of high-frequency Doppler sonars pointed at breaking wave crests. Laboratory experiments to characterize the fidelity of such measurements are required before they can be viewed with confidence.

Turbulent, two-phase flows can be categorized by two dimensionless numbers: the ratio of gas to liquid volume \(\alpha\) and the radius of the largest bubble entrained by the flow normalized by the Hinze scale: \(\beta_F = \frac{a_{max}}{\Delta H}\). Studies of gas bubbles fragmenting in fluid turbulence are designed to ensure that bubbles larger than the Hinze scale are present, that is, \(\beta_F > 1\), but typically keep \(\alpha\) low to limit gas phase modification of the turbulence (e.g., Martínez-Bazán et al. 1999). Studies to investigate the effect of gas bubbles on turbulence permit high \(\alpha\) but keep \(\beta_F < 1\) to limit turbulence modification of the gas phase. Neither of these regimes is representative of the high \(\alpha\) and high \(\beta_F\) flows within a breaking wave, where two-way interactions between the gas phase and turbulence certainly occur. Our data suggest that turbulence saturates in this regime in laboratory breaking wave crests and possibly also whitecaps, limiting the range of bubble Hinze scales. Possible explanations for the observation of turbulence saturation in laboratory breaking waves are gas-induced suppression of fluid turbulence or the inertial scaling of plume penetration depth with increasing wave scale and slope favoring increased plume volume rather than turbulence intensity.

Figure 10 in Callaghan et al. (2013) shows the correspondence between the scale of the kinds of laboratory breakers reported here and whitecaps on seas driven with 6–12 m s\(^{-1}\) winds. It is clear that this study captures only the tail end of ocean-relevant breaking scales, and measurements of waves of greater scale, either in the laboratory or the field, are required to see if turbulence saturation extends to larger and more energetic waves.

Our laboratory observations have implications for breaking ocean waves. If the turbulence saturation effect carries over into the field then bubble creation physics and the densities and scale of bubbles in breaking wave crests can be expected to be largely independent of wave scale and slope. A constant Hinze scale has implications for any air–sea property or exchange processes that depend on bubble size, such as the production of marine aerosols, the generation of underwater ambient noise, and changes in ocean albedo due to whitecap foam.

Acknowledgments. We are grateful to Dr. David Farmer for many stimulating discussions on the topic of bubbles and turbulence in breaking waves. Three
anonymous reviewers helped us make significant improvements to the manuscript, and we gratefully acknowledge their efforts. We thank Mr. Paul Harvey and Mr. David Aglietti of the SIO Hydraulics facility for experimental assistance and Dr. James Rohr and Dr. Michael Latz for assistance with the experiments and the culturing of order 10^7 cell marine dinoflagellates. Mr. Pierre-Jean’s assistance in analyzing the bubble images and Mr. James Uyloan’s assistance in conducting the experiments are gratefully acknowledged. We are pleased to acknowledge the financial support of the National Science Foundation, Grants OCE-1061050, OCE-1155123, and OCE-1434866.

APPENDIX

Absorption and Scattering of High-Frequency Sound by Bubbles in a Breaking Wave Crest

Doppler sonars measure the motion of remote scattering centers using the Doppler shift of sound pulses transmitted by the sonar and reradiated by the scattering center. If information is desired about fluid motion local to the scattering center, then the scatterer must either behave as a Lagrangian tracer or its motion relative to the fluid must be known. The ability of high-frequency Doppler sonars to probe fluid motions in the interior of whitecaps can be studied through model calculations of the absorption of sound within the plume and by estimating the relative contribution of backscatter from large (>0.5-mm radius) bubbles, which do not act as Lagrangian tracers.

The model calculations summarized here are based on three bubble size distributions measured within breaking wave crests in the laboratory (Deane and Stokes 2002) and at sea (Bowyer 2001; Deane and Stokes 2002) and Foldy’s theory for scattering and absorption of sound by an ensemble of scattering centers (Foldy 1945). Air fraction within the whitecap is treated as a free parameter and allowed to vary over the range 0.01–0.6, the upper limit being set by the field observations of Gemmrich and Farmer (1999).

From Foldy (1945), the flux of energy in a plane wave propagating through a plume of bubbles is reduced by the factor $e^{-S_e}$ over the distance $x$, where the extinction cross section per unit volume is given by

$$S_e = \int_{a_{\text{min}}}^{a_{\text{max}}} \sigma_{e,\text{full}}(a, \omega)n(a) \, da,$$

(A1)

where $n(a)$ is the bubble size distribution with units of bubbles per unit volume per unit radius increment, $a$ is the bubble radius, $\omega$ is the center frequency of the pulse, and $\sigma_{e,\text{full}}$ is the asymptotically valid extinction cross section, discussed below. The smallest and largest bubbles in the distributions are taken to be $a_{\text{min}} = 60 \, \mu$m and $a_{\text{max}} = 6 \, mm$, respectively, consistent with the observations of Bowyer (2001) and Deane and Stokes (2002).

The standard expression for the extinction cross section of a bubble is (e.g., Prosperetti 1977, corrected by a factor of 2)

$$\sigma_e = \frac{8\pi\beta a \omega^2}{(\omega_0^2 - \omega^2) + 4\beta_{\text{tot}}^2 \omega^2}, \quad ka < 1,$$

(A2)

where $\beta_{\text{tot}}$ is the bubble dimensional damping constant accounting for viscous, thermal, and acoustic losses; $c$ is the speed of sound in the bubble-free water; $\omega_0$ is the natural frequency of bubble oscillation; and $k = \omega/c$ is the acoustic wavenumber in the bubble-free water. The restriction $ka < 1$ holds if the bubble is small compared to an acoustic wavelength but breaks down for megahertz sonars operating in breaking wave crests, which contain millimeter-scale bubbles. In this case, an asymptotically valid extinction cross section can be developed by forcing it to asymptote to the well-known geometrical limit of $2\pi a^2$ as $ka \to \infty$. A suitable equation with the correct asymptotic behavior is

$$\sigma_{e,\text{full}} = \sigma_e + \frac{(ka)^{\alpha_e}}{\beta_e + (ka)^{\beta_e}} \cdot 2\pi a^2,$$

(A3)

where the constants $\alpha_e = 2.9$ and $\beta_e = 0.12$ are determined by performing a least-mean-square fit between Eq. (A3) and a full scattering theory developed by Anderson (1950) for scattering from a fluid sphere over the range $0.1 \leq ka \leq 10$. Anderson’s model cannot be simply adopted for the problem because it does not account for thermal losses, which are important for bubbles that are scattering sound.

The results of the calculations can be expressed as an acoustical skin depth (Deane 1997), which is normally the length over which the energy in a plane wave is attenuated by a factor of $e^{-1}$. Because sound pulses scattered by bubbles in the interior of a breaking wave crest must make a round trip, from the sonar to the bubble and back again, we define the skin depth here to be $2L_{\text{skin}} = 1/S_e$. Three bubble distributions were chosen for the model calculations: a distribution measured in the ocean at Dingle Harbor, Ireland, reported by Bowyer (2001); a distribution measured 100 miles west of Point Conception, California, reported by Deane and Stokes (2002); and a laboratory distribution from Deane and Stokes (2002) representative of bubble plumes generated by plunging laboratory breaking waves in saltwater. The smallest and largest bubbles in the distributions...
were chosen to be 60 µm and 6 mm, respectively. This is the range of bubble sizes reported by Bowyer (2001), which is also consistent with the oceanic observations of Deane and Stokes (2002).

Figure A1 shows the round trip skin depth for a 2-MHz sonar signal at 2 MHz calculated for the three bubble distributions. The skin depth is 1 cm at an air fraction of \( \alpha = 0.026 \) for the Point Conception distribution and \( \alpha = 0.055 \) for the Dingle Harbor and laboratory distributions. Typical air fractions in the two-phase flow of laboratory breakers during their acoustically active phase are around 0.2 or more, leading to expected round trip skin depths of between 1 and 3 mm at 2 MHz. For reference, a 2-MHz sonar signal scattered by a bubble located 10 cm from the bottom boundary of a bubble plume that has a skin depth of 3 mm will be attenuated in power by the factor \( \exp(\frac{-100}{3}) \approx 3 \times 10^{-15} \), which can be safely assumed to lie below the performance limit of any commercially available sonar. Assuming an air fraction of 0.2 for oceanic bubble plumes is reasonable, given that air fractions as high as 0.6 have been observed in the field (Gemmrich and Farmer 1999).

The round trip skin depth calculations strongly suggest that bubble plumes with air fraction in the range of 0.03–0.06 will present operational challenges for high-frequency Doppler sonar systems. Plumes with air fractions of 0.2 and higher will be essentially acoustically impenetrable. Thus backscatter returns are limited to an \( O(\text{cm}) \) thick shell on the exterior of bubble plumes during the acoustically active phase of wave breaking. The question remains if backscattered acoustic energy from bubbles within this layer can be interpreted in terms of fluid motions. This will be possible if either 1) all the bubbles are moving in the same direction with the same speed, in which case their motion will appear as a constant offset in the sonar Doppler signal and could be processed out, or 2) the bubbles are sufficiently small that their motion due to buoyancy can be neglected in comparison with the motion of the fluid surrounding them. Unfortunately, it is difficult to argue that either of these conditions hold in an actively breaking wave crest. A calculation of the differential backscatter cross section (not shown here) shows that 70% or more of the backscatter signal at 2 MHz comes from bubbles \( O(0.5) \) mm radius and larger. Bubbles of this size have buoyant rise speeds of 15 cm s\(^{-1}\) or more and interact strongly with the intense fluid turbulence within the bubble plume. Thus, the sonar signal that is backscattered from bubbles within an \( O(\text{cm}) \) shell on the plume exterior will not report fluid motions unless the Doppler bias introduced by large, buoyant bubbles is accounted for. Lowering the sonar frequency will help with the bias problem because more backscatter will come from bubbles closer to resonance with the sonar signal, which are smaller and will contribute less to Doppler bias. However, it will exacerbate the absorption problem because more bubbles closer to resonance also increase the extinction cross section, making the skin depth even smaller.

The model calculations presented here have implications for reported field observations of turbulence in actively breaking wave crests (e.g., Gemmrich 2010). The calculations suggest that even if high-frequency backscatter Doppler sonar signals could penetrate through to the interior of actively breaking wave crests—and this seems very doubtful—the returned signal will be contaminated by the motion of bubbles larger than the 0.5-mm radius that dominates the backscatter cross section. Laboratory experiments to resolve these issues and verify the performance of the high-frequency Doppler sonars used to image the interior of actively breaking wave crests are required. In the absence experiments to verify instrument performance, and we are not aware of any at this time, reports of fluid turbulence in breaking wave crests using high-frequency Doppler sonar that do not account for the absorption of acoustic energy or the motion of bubbles within the plume should be treated cautiously.

REFERENCES


