Evolution of Directional Wave Spectra in the Marginal Ice Zone: A New Model Tested with Legacy Data

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ABSTRACT
Field experimental data from a 1980s program in the Greenland Sea investigating the evolution of directional wave spectra in the marginal ice zone are reanalyzed and compared with the predictions of a new, phase-resolving, three-dimensional model describing the two-dimensional scattering of the waves by the vast number of ice floes that are normally present. The model is augmented with a dissipative term to account for the nonconservative processes affecting wave propagation. Observations reported in the experimental study are used to reproduce the ice conditions and wave forcing during the experiments. It is found that scattering alone underestimates the attenuation experienced by the waves during their passage through the ice field. With dissipation, however, the model can replicate the observed attenuation for most frequencies in the swell regime. Model predictions and observations of directional spreading are in agreement for short to midrange wave periods, where the wave field quickly becomes isotropic. For larger wave periods, little spreading can be seen in the model predictions, in contrast to the isotropic or near-isotropic seas reported in the experimental study. The discrepancy is conjectured to be a consequence of the inaccurate characterization of the ice conditions in the model and experimental errors.

1. Introduction

Over 30 years of passive microwave radiometry data collected by several satellites demonstrate that sea ice morphology is changing. In the Arctic, we have experienced more than a 55% decrease in summer sea ice extent during that period (Meier et al. 2013; Jeffries et al. 2013), and there have been reductions in the thickness too (Kwok and Rothrock 2009; Wadhams et al. 2011). In contrast, the Southern Ocean has experienced a modest increase in maximum sea ice extent along with greater variability in its spatial distribution around the Antarctic continent (Simpkins et al. 2013). Compelling evidence suggests that global climate change is responsible in both cases, noting that the physical processes causing the observed effects are different between the two polar oceans.

The primary agent causing metamorphosis of the sea ice during the Arctic summer is the positive ice–albedo feedback effect. An accompanying trending upward of wind and wave intensity (Young et al. 2011; Thomson and Rogers 2014; Thomson et al. 2016) further assists sea ice attrition, as ocean waves propagating through fields of sea ice can fracture the ice floes (see, e.g., Squire et al. 1995; Squire 2007, 2011), enhancing their melting in the summer and aiding freezing in the winter. These latter effects have been identified as potential contributors to the observed sea ice extent trends in the Arctic (Thomson and Rogers 2014) and Southern Oceans (Kohout et al. 2014). Granting that the deleterious contribution from penetrating ocean waves is most pronounced within, say, 100 km of the ice edge, the transformation of the summer Arctic to being more like a marginal ice zone (MIZ; usually defined as being the region of ice receptive to open-ocean processes) signifies an increasingly pivotal role for harmful ocean waves that is fueling contemporary polar oceanographic research such as the U.S. Office of Naval Research (ONR) initiative acknowledged later.

Ocean waves entering an MIZ reduce in amplitude as they propagate farther into the ice field, with the rate of attenuation being directly related to the wave frequency...
as observed during multiple in situ field experiments (Squire and Moore 1980; Wadhams et al. 1988; Meylan et al. 2014). In the swell regime, long waves may travel tens or even hundreds of kilometers into the sea ice, while shorter waves are rapidly extinguished, with the result that the sea’s representative power spectrum becomes narrower, that is, the MIZ acts as a low-pass filter for ocean waves. This behavior is also seen in high-resolution synthetic aperture radar (SAR) imagery, which can frequently reveal the presence of ocean waves as they propagate through ice fields composed of discrete floes such as the MIZ. An excellent illustration of the phenomenon is provided in a recent paper by Gebhardt et al. (2016), who show TerraSAR-X imagery off east Greenland that conclusively demonstrates how the short and long components of swell within the MIZ evolve with penetration into the ice cover. Ardhuin et al. (2015), on the other hand, use SAR wave mode data from Sentinel-1A to show how the wave amplitude can be extracted under certain circumstances and consequently ice-induced wave attenuation can be retrieved.

Attenuation of waves in the MIZ occurs as a result of two physical phenomena: (i) scattering, which is a conservative process that redistributes wave energy in all directions, and (ii) dissipative mechanisms, which can include overwashing near the ice edge and inelastic collisions and breakup of the ice floes. The relative precedence of scattering and dissipative processes in attenuating wave energy in the MIZ remains unclear and is the focus of much ongoing research. The two processes have different effects on the evolution of the directional properties of the wave field in ice-covered oceans; however, scattering tends to broaden the directional spread, while dissipation does not. Recently, Sutherland and Gascard (2016) used airborne scanning lidar observations in the Arctic Ocean to extract the directional wavenumber spectra in the MIZ and found that short waves broaden quickly, while long waves experience little spreading, suggesting scattering and dissipation are dominant at these respective ends of the frequency spectrum.

The findings of Sutherland and Gascard (2016) mirror those of Wadhams et al. (1986, hereinafter WSEP86), who conducted directional wave measurements in the MIZ using wave buoys. The dataset was collected during the Marginal Ice Zone Experiment (MIZEX) in the Greenland Sea, the only in situ experiment to have measured the directional properties of waves in the MIZ to our knowledge. Here, we reanalyze these observations of wave energy attenuation and directional spreading by comparing them with the predictions of the phase-resolving, three-dimensional model of ocean wave interactions with large arrays of sea ice floes, recently developed by the authors [see Montiel et al. (2016, hereinafter MSB16)]. The model of MSB16 is constructed on the basis that two-dimensional scattering by random arrays of ice floes governs the evolution of wave energy and directionality through the MIZ. We augment the original model here by including a simple parameterization of the aggregated effect of all dissipative processes.

As regards two-dimensional wave scattering by large arrays of ice floes, past phase-resolving models have imposed conditions that are impracticable in nature, such as periodic repetitions of a floe or groups of floes (e.g., Bennetts et al. 2010). Although this approach can reproduce observed attenuation rates of wave energy in the MIZ for midrange wave periods, the imposed periodicity restricts the directional spectrum of the wave field to a small number of directions so that spreading cannot be observed. Alternatively, based on early work by Masson and LeBlond (1989), Perrie and Hu (1996, 1997) report a spectral, phase-averaged model that invokes the energy balance equation $\partial S/\partial t + c_g \cdot \nabla S = \Sigma$ to describe the evolution of the directional spectrum $S$ in an ice-covered ocean, where $\omega$ is the radian frequency and $\tau$ is the direction. In the energy balance equation, $c_g$ is the group velocity vector, and $\Sigma$ symbolizes a sum of source terms made up of energy input from the atmosphere, dissipation from wave breaking, nonlinear interactions between spectral components, dissipation losses arising because of the ice, and other dissipative effects such as bottom friction. Neglecting nonlinear interactions and local wave generation, Meylan and Masson (2006) show that this equation is equivalent to a linear Boltzmann equation describing wave energy transport in random media where the waves undergo both scattering and dissipation, as occurs in the MIZ. This latter work is an extension of the earlier study by Meylan et al. (1997) who considered the stationary spatial evolution of a wave spectrum in the MIZ.

Comparisons with two WSEP86 experiments are made using the observed, open-water directional spectrum and typical floe sizes to parameterize the incident spectrum and the MIZ geometry in the model. The Meylan et al. (1997) model was shown to reproduce the attenuation and directional spreading of short waves reasonably well, but significantly underestimated these properties of the spectrum for midrange and long waves. The authors attributed this discrepancy to the incomplete description of the ice cover reported by WSEP86. Perrie and Hu (1996) also affirm good qualitative agreement with the general behavior of the data of WSEP86 using the energy balance equation, although no attempt was made to reproduce the conditions of the experiments in their work.
The key restrictions of the energy balance equation in regards to the present study are (i) it does not resolve wave phases and (ii) the scattering kernel describing the angular redistribution of scattered wave energy is determined by the properties of a solitary floe. In contrast, the model of MSB16 resolves wave coherence, and the MIZ is composed of randomly positioned circular ice floes with sizes selected from an empirical floe size distribution (FSD). Ensembles of simulations for random realizations of the array are then used to predict the evolution of the phaseless directional spectral through the MIZ. In that paper, a case study was conducted that showed the model predicts realistic attenuation rates and directional spreading that each depends on wave frequency. Here, we perform comprehensive quantitative comparisons with WSEP86, using information about the ice cover and incident spectrum reported therein to parameterize the MSB16 model. We further assess the influence of adding dissipative effects in the model on fitting predictions to observations.

2. The Wadhams et al. (1986) field experiment

Toward the end of the MIZEX program, which took place in the 1980s, the Scott Polar Research Institute and the Institute of Oceanographic Sciences (IOS), United Kingdom, collaborated on an in situ field experiment conducted aboard the sealer Motor Ship (M/S) Kvithjørn to investigate how the directional spectrum of waves alters with distance from the ice edge (WSEP86).

Two instruments were utilized (each fabricated in house and antecedent to later commercial buoys): a 1.2-m-diameter IOS pitch–roll buoy deployed in the open water (water station) and a heave–tilt sensor positioned on ice floes (ice floe station). Each device measured the vertical acceleration and the slope in two perpendicular directions, allowing the directional measurements they provide are used to parameterize the MSB16 model. We further assess the influence of adding dissipative effects in the model on fitting predictions to observations.

The pitch–roll buoy data stream was sampled to 12-bit resolution at 2 Hz through an antialiasing filter; the heave–tilt sensor was digitized manually to better than 1% accuracy, recognizing that tiltmeter calibrations are particularly sensitive to temperature, so accuracy would be about 3% as opposed to the 0.1-arc-s resolution specified by the manufacturer.

Five field experiments are reported in WSEP86; four were done outside and/or within MIZs (runs A–D) and one was conducted on each side of an ice edge band detached from the main ice field [see Wadhams (1983) for a definition of an ice edge band]. Data were collected for 34 min, one station at a time, in runs going into or out of the ice edge (or band). Descriptions of the ice conditions during each experiment are provided by WSEP86 (see section 4 herein, where they are elaborated).

In this study, we reanalyze run A, run B, and the band experiment (described below), which provide measurements of the effects of the ice cover on the evolution of the directional wave spectrum. During run C, a single ice floe station was deployed just 1.2 km from the ice edge, too close to detect an appreciable influence of the ice cover on the incoming waves. Run D was carried out with five open-water stations to analyze the wave reflection properties of the MIZ. Although these runs are not considered in the comparison study reported here, the directional measurements they provide are used later to assess the influence of the reflected wave components on directional spreading near the ice edge. WSEP86 provide helpful sketches of the five experiments in their Figs. 1–5 based on visual and radar observations made from the ship.

a. Run A, 8–9 July

The first experiment was a run going into the MIZ that involved five open-ocean wave measurements at −37.1, −27.8, −9.6, −6.3, and −0.7 km outside the ice edge and two wave measurements within the ice located at 4.1- and 7.2-km penetration. Instrument problems arose in this experiment, with the result that only three sites provided directional information, namely, the open-water station located 0.7 km off the ice edge (station 0903) and both stations in the MIZ. The measurements conducted 4.1 km into the MIZ (station 0904) were collected with both an ice floe station and an open-water station in a polyna. It is the only site throughout the dataset where both instruments were deployed next to each other and so is helpful to test consistency between the two sensor packages. Specifically, the influence of the seakeeping characteristics, that is, the response amplitude operators, of the ice floe on the measured energy density of the wave field will be...
discussed below in Fig. 4. The recording station farther into the MIZ (station 0905) was an ice floe station.

b. Run B, 12–13 July

This was an out of ice run with four ice floe stations at 22.5-, 17.8-, 11.2-, and 5.6-km penetration and five open-water stations −5.2, −18.0, −21.2, −40.3, and −73.0 km outside of the ice edge. For the two ice stations at penetrations of 5.6 and 11.2 km (station 1301 and 1203, respectively), the heave–tilt sensor was inoperative, so that a heave sensor, which could not provide directional data, was deployed instead. The other two ice stations 17.8 and 22.5 km into the MIZ (station 1202 and 1201, respectively) provided directional data.

c. Band experiment, 11 July

Three open-water stations were deployed in the vicinity of a narrow band of ice that had detached from the main ice edge; two on the windward side, about 2 km apart (stations 1101 and 1103), and one on the band’s leeward side (station 1102). Unfortunately, the windward side station farthest from the band (1103) provided unusable data, as the buoy developed a fault that was only discovered during the analysis stage. Full directional data are provided by the other two stations, however, which are located on each side of the band.

3. Theoretical model

We model the propagation of a monochromatic directional wave spectrum through an MIZ, described by a large array of compliant ice floes floating on an infinitely wide ocean with finite depth $h$. The evolution of wave energy and directionality in the MIZ is governed by scattering from the constituent ice floes. To account for the wave energy lost in the MIZ, we further introduce a phenomenological parameterization of the dissipative processes.

The MIZ is constructed as an array of $N$ adjacent slabs of width $L$, each containing $O(100)$ circular ice floes of different sizes with randomly selected positions, as shown in Fig. 1. We use Cartesian coordinates $(x, y)$ to locate points in the horizontal ocean surface and $z$, positively oriented upward, to describe the vertical position. We approximate the ice edge by a straight line, for example, $x = 0$ without loss of generality. This is considered reasonable on a scale of $O(10)$ km, based on the sketches of the experiment provided by WSEP86 (see Figs. 1, 2, and 5 therein). The idealized circular floe shape is chosen to simplify the numerical procedure in solving the underlying scattering problem. We conjecture that the effect of randomizing the array over floe size and position on the evolution of the wave spectrum through the MIZ dominates over the influence of floe shape and that circular floes are therefore a reasonable approximation. Floe radii are prescribed to obey a power-law FSD, as empirically measured in many field observations (see, e.g., Rothrock and Thorndike 1984; Toyota et al. 2006, 2011). The FSD reported for the band experiment in WSEP86 was found to fit a power-law distribution, supporting the validity of this assumption in our model. We further assume that all floes have constant thickness $D$, which is reasonable for a MIZ formed by the fragmentation of a uniform and continuous ice cover.

Linear water wave theory is used to describe the water motion, under the assumption of small wave amplitude compared with the wavelength. Furthermore, assuming time harmonic conditions, the velocity field in the water domain is given by $(\nabla, \partial_t) \Re \{ (\omega i / \omega) \phi(x, y, z) e^{-i \omega t} \}$, where $\nabla = (\partial_x, \partial_y)$ and $g \approx 9.81 \text{ m s}^{-2}$ is the acceleration due to gravity. Throughout the water domain $\Omega$, the complex-valued (reduced) potential satisfies Laplace’s equation

$$ \nabla^2 \phi + \partial_z^2 \phi = 0, \quad (x, y, z) \in \Omega. \quad (1) $$

On the seabed, $z = -h$, the no-flow condition $\partial_z \phi = 0$ holds, and on the ice-free ocean surface boundary $z = 0$, we have the free-surface condition $g \partial_z \phi = \omega^2 \phi$. 

![Fig. 1. Schematic of the geometry in the horizontal plane showing the planar coordinate system, $z$ pointing vertical upward (after MSB16). Each slab boundary is located at $x = nL$ ($n = 0, \ldots, N$), with the ice edge being the line $x = 0$.](http://journals.ametsoc.org/doi/pdf/10.1175/JPO-D-16-0118.1)
Sea ice floes typically have horizontal dimensions that are much larger than their thickness and are observed to deform approximately elastically under wave action (see Squire et al. 1995). Moreover, assuming small vertical deformations compared to the thickness, the Kirchhoff–Love thin, elastic plate theory (Love 1944) can be used to model the wave-induced motion of the ice floes. Here, we augment the plate model with a velocity-dependent damping term following Robinson and Palmer (1990) to account for wave energy dissipation in the MIZ. By integrating dissipative effects in the ice plate equation, we ensure that damping increases with the sea ice concentration, which is reasonable assuming that wave energy dissipation is mainly caused by the presence of the ice cover.

On the underside of an ice floe located at \( z = -d \), where \( d = \rho D/\rho_0 \) is the Archimedean draft with the density of sea ice \( \rho \approx 922.5 \text{ kg m}^{-3} \) and the density of seawater \( \rho_0 \approx 1025 \text{ kg m}^{-3} \), the potential satisfies

\[
(F \nabla^4 + \rho_0 g - d \rho_0 \omega^2 - i \omega \gamma) \partial_z \phi = \rho_0 \omega^2 \phi, \tag{2}
\]

where \( F \) is the flexural rigidity of the ice plate and \( \gamma \) is the viscosity parameter. The flexural rigidity can be expressed as \( F = ED^3/12(1 - \nu^2) \), where \( E \approx 6 \text{ GPa} \) is the Young’s modulus and \( \nu \approx 0.3 \) denotes Poisson’s ratio.

These prescribed values of the elastic parameters are commonly used to characterize the effective flexural behavior of sea ice floes (e.g., Timco and Weeks 2010). The damping term in (2) is included to parameterize the combined effect of all dissipative processes on the attenuation of wave energy in the MIZ. As such, the viscosity \( \gamma \) is not a measurable quantity associated with a particular physical process but one that needs to be calibrated to fit data as discussed in section 5.

To complete the ice floe model, we impose free edge conditions and restrict the horizontal motion. For a floe of radius \( a \), these conditions are given by

\[
\partial_r \psi = 0, \quad r = a, \quad z = -d \tag{3}
\]

and

\[
\partial_r \phi = 0, \quad r = a, \quad -d < z < 0, \tag{4}
\]

respectively, where \( \partial_r = \partial / \partial r \) maps the plate’s displacement to the bending moment, \( \partial_z = \partial / \partial z \) maps the displacement to the vertical shear stress, and \((r, \theta)\) are polar coordinates with the origin at the center of the floe. We have also introduced \( \nabla_{\theta, \phi} = (\partial_{\theta}, \partial_{\phi}/r) \).

We define the multidirectional wave field \( \phi_{\text{in}}(x) \) incident on the ice edge \( x = 0 \). It is defined by a superposition of plane waves traveling toward the MIZ with amplitudes continuously depending on the angle of incidence \( \tau \), such that \( \tau = 0 \) denotes the positive \( x \) direction. It is expressed as

\[
\phi_{\text{in}}(x) = \xi(z) \int_{-\pi/2}^{\pi/2} A(\tau) e^{ik(x \cos \tau + y \sin \tau)} \, d\tau, \tag{5}
\]

where \( x = (x, y, z) \) and \( A(\tau) \) is the prescribed directional amplitude spectrum describing the angular distribution of incident wave amplitudes at the ice edge. The function \( \xi(z) = \cosh k(z + h) / \cosh kh \) describes the vertical motion in the open-water domain. The wave-number \( k \) is the only positive real root of the open-water dispersion relation

\[
kg \tanh kh = \omega^2, \tag{6}
\]

which is associated with open-water-traveling wave modes. Equation (6) also admits an infinite number of imaginary roots, corresponding to evanescent wave modes, generated upon scattering by the ice floes. Their contribution decays exponentially, however, and MSB16 show that their influence on the evolution of wave energy through the MIZ is insignificant in the regime of interest in the present study. In the model described here, we therefore neglect these evanescent modes throughout.

In response to the forcing field \( \phi_{\text{in}} \), reflected and transmitted components are generated from scattering by the MIZ. These components may be expressed as

\[
\phi_{\text{R}}(x) = \xi(z) \int_{-\pi/2}^{\pi/2} A_{\text{R}}(\tau) e^{ik(x \cos \tau + y \sin \tau)} \, d\tau, \tag{7}
\]

for \( x \leq 0 \), and

\[
\phi_{\text{T}}(x) = \xi(z) \int_{-\pi/2}^{\pi/2} A_{\text{T}}(\tau) e^{ik(x - NL \cos \tau + y \sin \tau)} \, d\tau, \tag{8}
\]

for \( x \geq NL \). The angular functions \( A_{\text{R}}(\tau) \) and \( A_{\text{T}}(\tau) \) are, respectively, the unknown reflected and transmitted spectra, which characterize the angular distribution of reflected and transmitted wave amplitudes at \( x = 0 \) and \( x = NL \). In the far field, the potential is then given by \( \phi = -\phi_{\text{in}} + \phi_{\text{R}} \) as \( x \to -\infty \), and \( \phi \sim \phi_{\text{T}} \) as \( x \to \infty \).

Within the MIZ, the wave field at each slab boundary \( x = nL \) \((n = 0, \ldots, N)\) is expressed as the coherent superposition of left-traveling and right-traveling directional wave fields, that is, \( \phi(x) = \phi_{\text{nL}}^{(+)}(x) + \phi_{\text{nL}}^{(-)}(x) \), where

\[
\phi_{\text{nL}}^{(\pm)}(x) = \xi(z) \int_{-\pi/2}^{\pi/2} A_{\text{nL}}^{(\pm)}(\tau) e^{ik(x - NL \cos \tau + y \sin \tau)} \, d\tau. \tag{9}
\]

The \( A_{\text{nL}}^{(\pm)}(\tau) \) represent rightward (+) and leftward (−) angular amplitude functions, and we refer to them as
directional spectra. The approximation comes from neglecting the evanescent components of the directional wave field, arising from decomposing a circular wave mode into plane waves (see MSB16 for more details). It should be noted that the incident, reflected, and transmitted spectra are \( A_n^\pm(\tau) = A_n^\pm(\tau) \) and \( A_n^\pm(\tau) = A_n^\pm(\tau) \), respectively, and that \( A_n^\pm(\tau) = 0 \), as we do not consider left-traveling wave forcing on the MIZ.

A numerical method was developed by MSB16 to compute the directional spectra \( A_n^\pm(\tau) \), \( n = 0, \ldots, N \), and the reader is referred to the original paper for details of the method. The method fully resolves wave interactions in the MIZ, that is, wave scattering by a floe is influenced by the scattering from all the other flocs in the MIZ, not just those within the same slab.

We describe the evolution of the forward propagating directional wave field through the MIZ using the phaseless directional energy density function

\[
S_n^+(\tau) = |A_n^+(\tau)|^2, \quad (10)
\]

for \( n = 0, \ldots, N \) and \( \tau \in [-\pi/2, \pi/2] \). This function characterizes the angular distribution of forward-propagating wave energy at \( x = nL \), so that the total forward energy is

\[
E_n^+ = \int_{-\pi/2}^{\pi/2} S_n^+(\tau) \, d\tau. \quad (11)
\]

We do not consider backward-propagating wave components, which are underestimated due to the finite extent of the MIZ in the \( x \) direction (see MSB16).

Our model allows us to quantify the directional spreading of the forward-propagating wave field through the MIZ. It is measured by the directional spread (expressed in radians) defined at \( x = nL \) for \( n = 0, \ldots, N \) as

\[
\sigma_n = \sqrt{2(1 - r_n)}, \quad (12)
\]

where

\[
r_n^2 = \left( \int_{-\pi/2}^{\pi/2} \hat{S}_n^+(\tau) \cos \tau \, d\tau \right)^2 + \left( \int_{-\pi/2}^{\pi/2} \hat{S}_n^+(\tau) \sin \tau \, d\tau \right)^2, \quad (13)
\]

with \( \hat{S}_n^+(\tau) = S_n^+(\tau)/E_n^+ \). To ensure consistency with MSB16, this definition of the directional spread is slightly modified from that of WSEP86, which considers energy density functions over the full range of directions, that is, \([−\pi, \pi]\). A simple multiplicative factor that links \( \sigma_n \) and

the directional spread used by WSEP86 will be introduced in section 4a.

4. Parameterization of the theoretical model

We parameterize the theoretical model to reproduce the wave and ice conditions present during run A, run B, and the band experiment described in section 2 as closely as possible. We fix the water depth to be \( h = 200 \text{ m} \) in all our simulations, which is sufficiently large to approximate deep-water conditions in the range of wave periods considered here, that is, \( T = 2\pi/\omega = 6–15 \text{ s} \).

a. Incident directional spectrum

The forcing field is parameterized by assuming an incident energy density function of the form

\[
S_0^+(\omega, \tau) = E_0^+(\omega)S_0^+(\omega, \tau), \quad (14)
\]

where \( E_0^+(\omega) \) is the incident wave energy. The directional spreading function \( S_0^+(\omega, \tau) \) is further parameterized using a modified version of the commonly used cosine power \( 2\delta \) distribution proposed by Longuet-Higgins et al. (1963). It is given by

\[
\hat{S}_0^+(\omega, \tau) = W \frac{2^{2\delta-1}(s+1)}{\pi(2s+1)} \cos \frac{1}{2} (\tau - \tau_0), \quad (15)
\]

defined over the reduced range of directions \([−\pi/2, \pi/2]\), as opposed to the range \([−\pi, \pi]\) considered by Longuet-Higgins et al. (1963). This truncated distribution is an assumption of our model aimed at (i) parameterizing the forcing field with a sea state incident on the MIZ (taking the full range of directions would also parameterize the reflected components) and (ii) capturing common features of open-ocean sea states in an idealized manner (the cosine \( 2\delta \) distribution is widely used to fit open-ocean directional data). The full directional distribution of the sea state at the time of the experiments is not consistently provided by WSEP86.

The weighting factor \( W = W(\omega, \tau) \) in (15) is chosen such that

\[
\int_{-\pi/2}^{\pi/2} \hat{S}_0^+(\omega, \tau) \, d\tau = 1
\]

to account for the fact that the incident spectrum is defined over the reduced range of directions. For narrowly spread incident spectra, we have \( W \approx 1 \). It should be noted that for the truncated distribution used here, the directional spread must be computed numerically, while in the more general case where the range is \([−\pi, \pi]\), the directional spread can be calculated analytically in terms of the constant \( s \).
For each frequency $\omega$, the incident wave energy $E_0^i(\omega)$ is assumed to be constant, so that local wave energy generation and the evolution of the forcing term over space and time are neglected. In the swell regime considered here, it is a reasonable assumption as wave generation is a result of distant storms so that the wave field is expected to be almost uniform, up to a phase change. In addition, WSEP86 did not report significant variations in the wave conditions during the experiments, which could potentially challenge the time-harmonicity assumption. The energy density spectrum of the closest open-water station to the ice edge, which is station 0903, 1302, and 1101 for run A, run B, and the band experiment, respectively, is used to set $E_0^i(\omega)$. For these three experiments, the spectra are extracted directly from Figs. 15, 11, and 17c of WSEP86.

The parameter $\tau_0 = \tau_0(\omega)$ introduced in (15) is the mean wave direction and represents the peak of the function $S_0^i(\omega, \tau)$ for each frequency $\omega$. The directional spreading function $S_0^i(\omega, \tau)$ also depends on the parameter $s = s(\omega)$, which controls the directional spreading of the forcing field, with narrower seas corresponding to larger values of $s$.

We extract information from WSEP86 to estimate the parameters $\tau_0$ and $s$ of the forcing field. The incident swell direction for stations 0903, 1302, and 1101 at the peak frequency is provided by WSEP86 relative to north. Figures 1, 2, and 5 in WSEP86 provide radar observations of the ice edge, which we approximate by a straight line to determine its orientation relative to north. This allows us to estimate the mean wave direction $\tau_0$ with respect to the ice edge. For runs A and B, the dependence of $\tau_0$ on frequency is not provided, so we assume it is constant for all frequencies considered here. For the band experiment, $\tau_0(\omega)$ is shown in Fig. 17a of WSEP86 and is seen to be almost constant over the frequency range of interest. The values of $\tau_0$ (degrees) extracted from WSEP86 are summarized in Table 1.

To estimate the spreading parameter $s$ of the forcing field, we use measurements of the directional spread $\sigma_0$ at stations 0903, 1302, and 1101. These measurements include the contributions of the incident and reflected fields. To remove the influence of the reflected field, we crudely subtract $10^\circ$ from the values reported in WSEP86, recognizing that the directional spread measured at the wave stations farthest from the ice edge, where reflected waves are negligible, is approximately $10^\circ$ lower than that observed close to the ice edge. This effect can be seen for runs B, C, and D. As previously explained, the definition of the directional spread in WSEP86 differs slightly from that used in our model, so an adjustment is required. For an isotropic forward-propagating wave field, that is, $S_0^i(\tau) = 1/\pi$, we find that $
abla$.

### b. Marginal ice zone

Several quantities are required to parameterize the MIZ: namely, a representative ice floe thickness $D$, concentration $c$, and the MIZ’s extent in the $y$ direction, $H_{MIZ}$. FSD is described by a floe radius $a$, selected from a bounded power-law distribution with probability density function $(a/a_{\min})^{-\kappa}$ for $a_{\min} \leq a \leq a_{\max}$, where $\kappa$ is a constant parameter and $a_{\min}$ $(a_{\max})$ is the minimum (maximum) floe radius. Following MSB16, we further introduce a critical minimum radius $a_{\crit}$ such that floes with radius $a < a_{\crit}$ have negligible effects on the scattering properties of the array and can therefore be neglected while solving the wave interaction problem. MSB16 found that choosing $a_{\crit}$ to be similar to the wavelength is reasonable.

### Table 1. Parameterization of the incident directional spectrum.

<table>
<thead>
<tr>
<th>$T$ (s)</th>
<th>$\tau_0$ (°)</th>
<th>$\sigma_0$ (°)</th>
<th>$s$</th>
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</tbody>
</table>

The spread $\theta_2$ is only given at the peak wave frequency for runs A and B, while Fig. 17b in WSEP86 provides the directional spread over the frequency range of interest. The spreading parameter $s$ is obtained by solving (12) numerically (using MATLAB’s built-in root finder), where $\sigma_0$ is provided by (16) and the directional energy density function is given by (15). The values of $\sigma_0$ and the parameter $s$ are listed in Table 1.
The MIZ is composed of multiple “zones” of uniform concentration and FSD to allow variations of these parameters with distance from the ice edge, as reported by WSEP86. Each zone of width $W$ is then clustered into $Z_N$ slabs. An algorithm was proposed by MSB16 to generate a random array of circular floes in each slab with the FSD considered here. After removing the floes with radius $a < a_{\text{crit}}$, the number of floes in each slab is denoted by $N_f$. The parameters of the MIZ for run A, run B, and the band experiment are summarized in Table 2.

### Table 2. Parameterization of the MIZ morphology.

<table>
<thead>
<tr>
<th></th>
<th>$H_{\text{MIZ}}$ (km)</th>
<th>$D$ (m)</th>
<th>$W$ (km)</th>
<th>$c$</th>
<th>$Z_N$</th>
<th>$a_{\text{min}}$ (m)</th>
<th>$a_{\text{max}}$ (m)</th>
<th>$a_{\text{crit}}$ (m)</th>
<th>$\kappa$</th>
<th>$N_f$</th>
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</thead>
<tbody>
<tr>
<td><strong>Run A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>20</td>
<td>2.5</td>
<td>5</td>
<td>0.4</td>
<td>20</td>
<td>10</td>
<td>100</td>
<td>50</td>
<td>2</td>
<td>89</td>
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<tr>
<td>Zone 2</td>
<td>—</td>
<td>—</td>
<td>15</td>
<td>0.8</td>
<td>45</td>
<td>—</td>
<td>150</td>
<td>—</td>
<td>—</td>
<td>196</td>
</tr>
<tr>
<td><strong>Run B</strong></td>
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<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>20</td>
<td>3</td>
<td>3</td>
<td>0.8</td>
<td>10</td>
<td>10</td>
<td>80</td>
<td>50</td>
<td>2</td>
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<tr>
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<td>—</td>
<td>6</td>
<td>0.4</td>
<td>20</td>
<td>—</td>
<td>100</td>
<td>—</td>
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<td>Zone 4</td>
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<td>—</td>
<td>200</td>
<td>—</td>
<td>—</td>
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<tr>
<td>Band experiment</td>
<td>6.7</td>
<td>2</td>
<td>0.67</td>
<td>0.8</td>
<td>3</td>
<td>10</td>
<td>50</td>
<td>30</td>
<td>2.4</td>
<td>175</td>
</tr>
</tbody>
</table>

1) **RUN A**

The ice conditions were reported by WSEP86 in the vicinity of the two stations 0904 and 0905, respectively located at penetrations of 4.1 and 7.2 km. Specifically, the ice pack was described as “loose,” that is, of low concentration, at station 0904, with “typical” floe thicknesses of 2.5 m and diameters of 250 m. At station 0905, WSEP86 characterized the ice morphology as “close pack,” that is, of high concentration. No information about the floe size was reported at this station. Also the FSD was not analyzed for this experiment.

In our model, we reproduce the change in ice concentration by constructing an MIZ composed of two zones of 5- and 15-km extent in the $x$ direction having concentrations of 0.4 and 0.8, respectively. We also account for an increase in the floe size with distance from the ice edge (Squire et al. 1995), setting the maximum floe radius to be 100 and 150 m in zones 1 and 2, respectively. In addition, we set the critical radius to be $a_{\text{crit}} = 50$ m, the validity of which was shown in MSB16 for a similar FSD, and $\kappa = 2$, which is a typical value for real FSDs [see, e.g., Rothrock and Thorndike (1984), who reported values between 1.7 and 2.5 from field data analysis]. A single realization of the MIZ parameterized to reproduce the conditions of run A is shown in Fig. 2a.

2) **RUN B**

At the location of the first ice floe station 1201, located 22.5 km from the ice edge and deployed on a floe of diameter 350 m, WSEP86 reported a “heavy pack” morphology. The second station 1202 was on a 200-m-diameter floe in a “similar pack,” 17.8 km from the edge. The third station 1203, 11.2 km into the MIZ, was on a 3-m-thick floe of diameter 72 m within a “lighter pack.” At 5.6 km from the edge, the fourth ice station 1301 laid on a “heavily rotted floe” in a region with concentration 0.1. At the ice edge, WSEP86 reported the presence of a “compact” ice cover with a concentration of 0.8, which resembles an ice edge band detaching from the ice pack.

This information is integrated into a parameterization of the MIZ by defining four zones with concentration 0.8, 0.1, 0.4, and 0.8, respectively, corresponding to (i) the detaching ice edge band, (ii) the low concentrated region where 1301 is located, (iii) the light pack region in the vicinity of 1203, and (iv) the dense pack farther into the MIZ. As no information is given on the FSD of the MIZ during run B, we fix $a_{\text{min}} = 10$ m, $a_{\text{crit}} = 50$ m, and $\kappa = 2$ for all the zones, as in run A. We account, however, for an increase in the maximum floe size with distance from the ice edge, which can be inferred from the size of the floes used for the ice stations. We set $a_{\text{max}} = 80$ m in zones 1 and 2, 100 m in zone 3, and 200 m in zone 4. An example of a model-generated MIZ for run B is depicted in Fig. 2b.

3) **BAND EXPERIMENT**

Between the two water stations 1101 and 1102, deployed windward and leeward of the band, respectively, the width of the band was reported in WSEP86 to be approximately 670 m. The typical floe diameter and thickness observed was 40 and 2–3 m. As the ship traversed the band, photographs of the ice were taken, allowing the WSEP86 authors to describe a more detailed FSD. Specifically, a binned FSD is provided in Table 7 of WSEP86, in which the proportion of the area covered by floes of diameter 12.5, 25, 35, 55, and 100 m is 0.2, 0.3, 0.3, 0.1, and 0.1, respectively, relative to the total ice-covered area.

We model the ice edge band by an array of width $W = 0.67$ km and length $H_{\text{MIZ}} = 6.7$ km. The binned...
FSD described in WSEP86 was fitted to the power-law distribution with a coefficient of determination \( R^2 = 0.9572 \), allowing us to extract the parameter \( k = 2.4 \). We further set \( a_{\text{max}} = 50 \text{ m} \) (in alignment with WSEP86), \( a_{\text{min}} = 10 \text{ m} \), \( a_{\text{crit}} = 30 \text{ m} \), and \( D = 2 \text{ m} \). Although WSEP86 does not report the concentration of the band, it is well established that ice edge bands are normally highly concentrated regions (e.g., Martin et al. 1983). Therefore, we set \( c = 0.8 \) in the band. A simulated band used for our simulations is shown in Fig. 2c.

c. Simulations

In all the simulations, the incident directional spectrum is expressed as \( A^\text{inc}(\tau) = \sqrt{S_0^\text{inc}(\tau)} \), where \( S_0^\text{inc}(\tau) \) is given by (14), so that waves incident at any angle have the same phase (chosen to be 0 here without loss of generality). This approach differs from that used by MSB16, who defined the incident directional spectrum with its phase varying randomly with the incident angle \( \tau \). Here, we assume that the incident directional swell at a given frequency has been generated from a single source, so that the multiple angles of incidence are in phase.

Randomness is incorporated in the model when generating the MIZ. The algorithm used to generate a random array in a slab parameterized from the power-law FSD described in section 4b was devised by MSB16 (see appendix B therein). The MIZ is then formed by stacking random realizations of the slabs.

We conduct Monte Carlo simulations to estimate the mean energy \( E_n^+ \) and directional spread \( \sigma_n \), where \( n = 0, \ldots, N \) for each parameterized experiment. MSB16 showed that the sensitivity of \( E_n^+ \) and \( \sigma_n \) to random variations in the arrangement of ice floes is small, so that only a small number of simulations need to be performed. Here, we average \( E_n^+ \) and \( \sigma_n \), \( n = 0, \ldots, N \), over 10 simulations (i.e., random realizations of the MIZ) and compute the standard error of the mean to estimate the associated uncertainty.

5. Results

a. Run A

In Fig. 3, the model’s predictions are compared with the wave energy density recorded by the sensors at stations 0904 and 0905 over the range of wave periods \( T = 6-15 \text{ s} \). At station 0904, spectra generated for both the water and ice floe stations are displayed. Although the two spectra are qualitatively similar, the wave energy estimated from buoy measurements is one to two orders of magnitude larger than that obtained from on-ice measurements. WSEP86 suggest that this difference is due to the small heave amplitude relative to that of the surrounding free surface for a floe of diameter approximately equal to 250 m. In the analysis of the data, no gain factor correction was applied to the ice floe...
station measurements to account for the seakeeping characteristic of the floe.

The model predictions are given by the total forward energy $E_n^1$, defined in (11), after $n = 16$ and 26 slabs, that is, at the approximate location of stations 0904 and 0905 relative to the ice edge. For station 0904, we observe that the scattering-only simulations (i.e., without dissipation) overestimate the energy recorded at the water station for wave periods $T$, $14$ s. Longer waves experience very little attenuation and have a similar energy to that of the incident field. The agreement is worse for $T$, $8$ s, for which the discrepancy exceeds one order of magnitude and gets better as the wave period increases. This comparison suggests that the scattering process alone does not explain the observed attenuation of wave energy in run A.

At the location of the ice floe station 0905, the discrepancy between the scattering-only model predictions and the observed energy is larger than one order of magnitude for $9 \leq T \leq 15$ s, and it is over two orders of magnitude for smaller wave periods. This is similar to the discrepancies observed for station 0904 when comparing predictions with observations from the ice floe station, suggesting such measurements underestimate the open-water wave energy.

We ran the scattering model for values of the viscosity parameter $\gamma = 200$, 400, and 600 Pa s m$^{-1}$ in order to test the sensitivity of model predictions with respect to dissipation. For station 0904, the energy spectrum predicted with $\gamma = 200$ Pa s m$^{-1}$ agrees well with the observed water station spectrum in the range of wave periods $8 \leq T \leq 13$ s. For longer waves, the dissipation is overestimated, although the scattering-only prediction shows a good agreement as discussed earlier. For $T \sim 8$–10 s, the spectra predicted with $\gamma = 400$ and 600 Pa s m$^{-1}$ agree well with the observations. Comparing model predictions with the ice station spectrum, we observe that taking a value of $\gamma = 600$ Pa s m$^{-1}$ provides a reasonably good agreement with the observed spectrum for $T \geq 8$ s. For shorter waves, the observed energy was much smaller and the model could not reproduce such values, as the effect of further increasing $\gamma$ reduces at higher frequencies. For station 0905, the model is able to replicate the observed ice station spectrum reasonably well for $\gamma = 400$ and 600 Pa s m$^{-1}$ for $T \geq 8$ s.

The directional spread at stations 0904 and 0905 is analyzed in Fig. 4. Unfortunately, no quantitative values of the directional spread are given in WSEP86 (p. 369), who only reported “the wave field to be isotropic over the entire range of frequencies present in the incoming sea.” Theoretical predictions of the directional spread $\sigma_n$ for $\gamma = 0$, 200, 400, and 600 Pa s m$^{-1}$ are therefore only displayed in Fig. 4.

As expected, we observe an increase of the predicted directional spread between stations 0904 and 0905 (located deeper in the MIZ) for all cases considered. For the scattering-only case, the spread remains almost constant with a value of 36°–37° and 38°–39° at station 0904 and 0905, respectively, for wave periods $T \leq 10$ s, while it decreases rapidly for increasing periods beyond 10 s. This effect was also observed by MSB16, who attributed the transition between these two regimes to the

![Fig. 3. Comparison of the total forward energy spectra predicted by the model and observed wave energy density spectra during run A at stations (a) 0904 and (b) 0905 in the range of wave periods $T = 6$–15 s. The observed spectra at each station are extracted from Fig. 15 of WSEP86 and are shown as dashed black lines (water station) and solid black lines (ice floe station). The predicted energy spectra are displayed for values of the viscosity parameter $\gamma = 0$, 200, 400, and 600 Pa s m$^{-1}$ (blue, red, green, and magenta lines, respectively).]
wavelength becoming larger than the maximum floe size. The same conclusion can be reached here for station 0904, which is located in zone 1 where the maximum floe diameter is 200 m, corresponding to a wave period of $T \approx 11.3$ s. For station 0905 (located in zone 2), the maximum floe diameter is 300 m, which is the wavelength associated with the period $T \approx 13.8$ s. Although this value is significantly larger than the identified transition at 10 s, note that the relative increase in spreading between stations 0904 and 0905 of waves with periods 11, 12, and 13 s is approximately 21%, 20%, and 13%, respectively, compared to less than 10% for all other periods. This suggests a good correlation exists between directional spreading and the maximum floe size, which can be observed deeper into the MIZ.

Including dissipation in the model generally tends to decrease the directional spreading experienced by the wave field. We also observe that the transition from significant spreading for small wave periods to little spreading for large wave periods becomes smoother as the value of $\gamma$ increases.

The values of directional spread predicted by the model, with and without dissipation, are well below the theoretical value for an isotropic spectrum. While this potentially disagrees with the WSEP86 observations, the authors of that paper did not designate a threshold for isotropy. In run B (discussed subsequently), directional spread values of approximately 40° are reported as isotropic by WSEP86. This suggests that our predictions of spreading for wave periods $T \approx 10$ s, which are close to 40° would be considered isotropic in WSEP86 and therefore in agreement with the data. On the other hand, the much diminished spreading of longer waves predicted by our model cannot be associated with isotropy.

b. Run B

The energy spectrum observed during run B at the ice floe stations 1301, 1203, 1202, and 1201 is depicted in Fig. 5 and compared to our predictions of the total forward energy $E_n^T$ for $n = 19, 37, 55,$ and 65, respectively. Model predictions are shown for the $\gamma = 0$ (scattering only) and $\gamma = 100, 200,$ and 300 Pa s$^{-1}$ (with dissipation). The scattering-only predictions overestimate the energy observed at each ice floe station. For station 1301, the discrepancy is between one and two orders of magnitude, while for the other three stations it exceeds two orders of magnitude for all wave periods considered. The discrepancies observed here are similar to those discussed for run A, when comparing the scattering-only predictions with the ice floe station observations. Consequently, some of the difference can safely be attributed to the relatively modest heave response of the ice floes compared to the amplitude of waves in the neighboring open sea.

We normally obtain a better fit to the observed spectra when dissipation is introduced into the model. For station 1301, we see a reasonable agreement between model predictions and observations for $T < 12$ s, with the spectrum generated by taking $\gamma = 100$ Pa s$^{-1}$, giving the best fit for $7 \leq T \leq 10$ s, while large values are needed for smaller and larger periods. For periods $T \geq 12$ s, we could take $\gamma > 300$ Pa s$^{-1}$ to get a better fit to the data. Qualitatively, the general shape of the energy spectra predicted with dissipation is significantly different

![Figure 4](https://example.com/fig4.png)
from the observed spectrum, so that a single value of $g$ providing a fit over the entire range of wave periods cannot be found. Although an optimized value of $g$ can be found for each frequency, which would result in a good fit for all wave periods, the functional form $g(v)$ would only be applicable to that particular experiment. We prefer investigating the sensitivity of our predictions with respect to the parameterized dissipation model.

Similar behavior is observed for stations 1203, 1202, and 1201, although the agreement worsens with penetration at small wave periods. This is due to the saturation effect of the dissipative model in this frequency range, which was also observed for run A. In particular, increasing $g$ beyond a certain value for short-wave periods does not result in additional wave energy attenuation. As a consequence, our dissipative model is not able to replicate the observed wave energy decay for small wave periods. On the other hand, it is possible to find a value of $g$ for each wave period $T > 8$ s, such that the model fits the observations well.

Figure 6 shows the directional spread predicted by the model at the locations of stations 1301, 1203, 1202, and 1201. The directional spread was also reported by WSEP86 for stations 1202 and 1201, but at the peak period only, that is, $T \approx 15.2$ and 15.6 s, respectively. These values are given in Table 5 of WSEP86, and we apply the correction factor 0.6 justified in section 4a to compare them with the directional spread defined in (13) for a forward-propagating spectrum.

The behavior of the spread for the scattering-only predictions is as expected, with small wave periods experiencing significantly more spreading than large wave periods. The spread increases for all wave periods with distance from the ice edge, and it reaches near isotropy for $T \approx 11$ s at station 1202. Farther into the ice cover (station 1201), the model predicts a directional spread that is slightly larger than the theoretical value for isotropy in this regime. Analyzing the evolution of the shape of the directional spectra through the MIZ (not displayed here), we find that beyond the point at which isotropy is reached, the wave energy of small and mid-range angular components keeps attenuating, while that of large angular components does not, resulting in a distorted spectrum with spread larger than the theoretical.
value of isotropy. This is a consequence of the finite extent of the simulated MIZ in the $y$ direction. We then interpret such values as characterizing an isotropic directional spectrum. For wave periods larger than 11 s, we observe a steep decline of the spreading, which can be associated with the maximum floe size in the MIZ as discussed for run A.

Introducing dissipation in the model, we observe that the wave field spreads faster for small wave periods and slower for midrange to large wave periods compared to the scattering-only case. At station 1301, the transition occurs at $T \approx 8$ s, with increasing values of $\gamma$ being associated with faster spreading before the transition and slower spreading after the transition. At station 1203, the transition occurs at $T \approx 9.5$ s. We cannot clearly see a transition at stations 1202 and 1201 because the directional spread is similar to that of the scattering-only case for all the values of $\gamma$. At the peak wave period, the observed directional spectra at stations 1202 and 1201 are almost isotropic. This result differs significantly from our predictions, which show that for such long waves, namely, $T \approx 15$ s, the wave field experiences almost no spreading. This is a consequence of the wavelength of the forcing field ($\sim 350$ m) being larger than the maximum floe diameter, that is, 300 m, which indicates that the wave field experiences little scattering in the MIZ in this regime.

c. Band experiment

The wave energy of the predicted transmitted wave field through the ice edge band is depicted in Fig. 7a for $\gamma = 0, 200, 400, \text{ and } 600 \text{ Pa s m}^{-1}$ and compared to the...
observed spectrum at station 1102 located on the leeward side of the band. We observe a general good agreement between the scattering-only predictions and observations over the entire range of wave periods. For wave periods larger than 10 s, the transmitted energy of the scattering-only case is almost identical to the incident energy given by the spectrum of station 1101, suggesting the band does not affect wave propagation in this regime. The observed spectrum of station 1102 lies slightly below that of 1101, however, so that adding some level of dissipation in the model provides a good fit. For $T > 10$ s, the directional spread of the scattering-only case is again similar to that of the incident spectrum, supporting our previous assertion that scattering is negligible in this regime. The observed spread shows some variability in this range of wave periods, although it agrees reasonably well with model predictions for $10 \leq T \leq 13$ s. For larger wave periods, the directional spreading observed at station 1102 is enhanced by the band, which is not what would be expected, as we would expect to observe very little spreading or no spreading for such long waves. For wave periods smaller than 10 s, a good qualitative agreement is observed between model predictions and observations. Specifically, the directional spread of station 1102 reaches a minimum at $T \approx 9$ s and then increases for shorter waves. A similar behavior is observed in the model predictions.

6. Discussion and conclusions

A theoretical model of wave attenuation and directional spreading in the MIZ (MSB16) recently developed by the authors has been used to reproduce the only known in situ measurements of directional wave evolution in ice-covered oceans reported in the literature (WSEP86). The original scattering model was augmented with a parameterized viscous term to account for dissipative processes. Three experiments (runs A and B and the band experiment) were reanalyzed for testing the model, and every effort was made to include information provided about the ice and wave conditions during the experiments into the parameterization of the model. Wave energy and directional spread in the range of wave periods $5 \leq T \leq 15$ s were computed at the location of the recording stations and compared with those reported in WSEP86. Key findings are summarized below:

1) In runs A and B, scattering alone underestimates the rate of wave energy attenuation across the frequency range. Including some level of dissipation in the model, however, provides a good fit to the wave energy observations for $T > 7$–8 s by choosing an appropriate value of the viscosity parameter for each wave period. For smaller wave periods, the dissipation model saturates so that wave energy does not
Discrepancies between model predictions and observations are due to a combination of modeling assumptions, limited data for model parameterization, and experimental errors. We enumerate some of the most likely sources of discrepancy:

1) Comparisons between model predictions and observations are generally better when the data were collected using floating pitch–roll buoys (water stations) as opposed to heave–tilt sensors (ice floe stations). When both sensors were deployed at the same location during run A, the energy spectra they recorded showed significant differences, often exceeding one order of magnitude, for example, at small wave periods. Consequently, we argue that any poor agreement seen between our predictions and observations at high frequency can be largely explained by the fact that ice floe station data were used in the comparison, while the model measures wave energy at the free ocean surface.

2) Although a parameterization of dissipative effects was successfully introduced in the model to fit the observations of wave energy for midrange and large wave periods, we showed that it does not perform well for small wave periods, so that an alternative model may need to be considered in this regime to account for dissipative effects. For instance, we could vary the viscosity parameter for different floes depending on the concentration in the neighboring region. In this study, we kept the viscosity constant so that dissipation would scale up with concentration, which is reasonable for dissipative processes such as floe collisions or inelastic bending, but other processes such as overwash or whitewater may be more prevalent in regions of low concentration. Moreover, it is still unclear how to relate the dissipation parameter to observable sea ice and wave conditions, which would inevitably be needed for forecasting purposes. Unfortunately, this outcome is a feature of all current models that attempt to reproduce how ocean waves and fields of sea ice interact, whether these are viscoelastic continuum models (e.g., Mosig et al. 2015), spectral wave models (e.g., Meylan and Masson 2006), or phase-resolving scattering models (e.g., MSB16). Simultaneous measurements of the ice properties and waves within the MIZ for a range of ice conditions using state-of-the-art satellite-borne SAR technology and other promising new sensors (see, e.g., Gebhardt et al. 2016) offer the best chance at calibrating the models.

3) In runs A and B, the observed isotropy in the MIZ for large wave periods differs significantly from our predictions of slow spreading in this regime. Several factors may explain this discrepancy. First, it is possible that the MIZ in our model has been poorly parameterized, as the FSD during these runs was not accurately reported by WSEP86. If this is so, large floes not accounted for in the model could exist with the potential to cause long waves to spread directionally. Second, we analyzed the directional spread of the forward-propagating field only. This may underestimate the directional spreading of the full directional spectrum, as backward-traveling components are likely to propagate isotropically, having been generated from backscattering farther into the MIZ. Third, these observations are actually surprising and inconsistent with SAR and lidar data of waves in ice fields showing little spreading of long waves (see, e.g., Liu et al. 1991; Sutherland and Gascard 2016), which raises the question of the equipment difficulties experienced such as buoys...
malfunctioning or being inappropriately calibrated for the conditions experienced.

4) A number of simplifying assumptions in the model may also cause some discrepancy in the comparisons of predictions with observations, for example, circular floe shape, constant floe thickness, power-law FSD, straight ice edge, and finite extent of the MIZ. While the latter assumption is elemental in the model, others arise because we do not have enough information in the WSEP86 dataset. For instance, WSEP86 reported a nonstatic ice edge when the measurements were conducted, which may affect the validity of the straight ice edge assumption. We do not expect that these approximations significantly affect the results, however, although sensitivity tests beyond the scope of the present paper would be needed to discard them conclusively.

5) The stationary incident wave energy $E_0(\omega)$ assumed to parameterize the forcing field may also be a source of bias. We recognize that this quantity is affected by winds, the ice edge location and the effective fetch over which the waves have developed, so that it should vary according to the energy balance equation discussed in the introduction, in which ice-induced source terms are removed (see, e.g., Young 1999). In addition, the point measurements described in section 2 are imperfect whenever asynchronous stations are compared, as affirmed by WSEP86, who raise the possibility that local wave spectra may adapt due to changes in the duration of the wind. Because of the limited information provided by WSEP86 on the local wave and wind conditions present during the experiments, we cannot quantify the bias introduced by the stationarity assumption. In the swell regime, we do not expect local wave generation to play a significant role.

The main constraint to reanalyzing the WSEP86 dataset is the quality of ancillary information about the sea ice field through which waves pass while being scattered and dissipated by processes that depend strongly on the attributes of both the ice field and the ocean waves themselves. Technological limitations at the time are also a restricting factor, as GPS, cheap (i.e., expendable) and accurate inertial measurement units that upload data to satellites, and more sophisticated fast computers are now commonplace. Armed with the MSB16 scattering model and contemporary data, the authors hope to validate the model further in a fully quantitative way.

The analysis conducted in this paper has highlighted the need to conduct more in situ directional waves-in-ice experiments, with fastidious simultaneous recording of the ice conditions prevalent during the field campaigns. This is key to improving the parameterizations present in theoretical models of wave–ice interactions and therefore to providing accurate predictions of wave energy attenuation rate and the spatial evolution of direction spread in the MIZ as a function of the observable ice conditions. The first steps toward this goal have already occurred, as fieldwork conducted in the boreal fall of 2015 under ONR sponsorship is delivering new data that will undoubtedly advance our understanding of this important yet, antecedently neglected, agent of change to the sea ice in the polar regions.

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REFERENCES


