Comparison of Two Methods to Assess Ocean Tide Models

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ABSTRACT

Two methods to assess ocean tide models, the current method and the total discrepancy method, are compared from the perspective of their relationship to the root-mean-square difference of tidal sea surface height (total discrepancy). These two methods are identically the same when there is only one spatial location involved. When there is more than one spatial location involved, the current method is the root-mean-square difference of total discrepancy at each location, and the total discrepancy method is the averaged total discrepancy. The result from the current method is always larger than or equal to that from the total discrepancy method. Monte Carlo simulation indicates that the difference between their results increases with increasing spatial variability of total discrepancy. Both of these two methods are then used to compare the two tide models of the Ocean Surface Topography Mission (OSTM)/Jason-2. The discrepancy of these two models as measured by the total discrepancy method decreases monotonically from around 11.4 to 2.2 cm with depth increasing from 50 to 1000 m. In contrast, the discrepancy measured by the current method varies from 21.6 to 2.9 cm. Though the discrepancy measured by the current method decreases with increasing depth in general, there are abrupt increases at several depth ranges. These increases are associated with large spatial variability of total discrepancy and their physical explanation is elusive. Because the total discrepancy method is consistent with the root-mean-square difference of tidal sea surface height and its interpretation is straightforward, its usage is suggested.

1. Introduction

With the development and implementation of real-time, tidal-permitting coastal ocean forecasting systems in various regions around the globe, it is important to quantify the skill of these systems. Tidal signal is ubiquitous and generally the largest signal in sea surface height of the world coastal oceans. It would be useful to develop a method to assess the accuracy of tidal prediction in these systems. The method needs to be convenient to use and its results need to be straightforward to interpret. In the past, the application of tides has been mainly for navigation purposes. The required accuracy of this type of application is on the order of decimeters.

For most satellite altimetry oceanographic applications, the tides are treated as noise and need to be removed to reveal the sea surface height variability associated with density and current changes. With the advent of satellite altimetry in the 1970s, the requirement for the accuracy of tidal models reached a new level. Using a global network of tide gauge stations, Andersen et al. (1995), Shum et al. (1997), and Fok et al. (2010) demonstrated that the accuracy of tidal models used in the satellite altimetry missions was at the 2–3-cm level in the open ocean in the 1990s and improved further in the 2000s.

Launched on 20 June 2008 (Zaouche et al. 2010), the Ocean Surface Topography Mission (OSTM)/Jason-2 is a follow-on mission to the Jason-1 and Ocean Topography Experiment (TOPEX)/Poseidon missions, which continue the satellite altimetry observation of sea surface height started by the TOPEX/Poseidon mission in 1992 (Fu et al. 1994). The application of satellite sea
surface height observation from these missions in the coastal region is still hampered for various reasons. Among them, large sea surface height variability associated with tides and degraded accuracy of tide models in coastal regions are the challenging issues. Two global tide models are used in the processing of altimetry observation from these missions (NOAA/Office of Satellite Data Processing and Distribution 2009; available from http://www.nodc.noaa.gov/SatelliteData/Jason2). One is based on a finite-element model and uses available altimetry observation to constrain the model [Finite Element Solution 2004 (FES2004), see Lyard et al. (2006)], which has a horizontal resolution of 0.125° latitude × 0.125° longitude. The other one is an empirical model using available altimetry observation with a resolution of 0.5° latitude × 0.5° longitude (GOT00; Schrama and Ray 1994; Ray 1999).

The current method used to assess tide models was proposed by Ray (1993) and Andersen et al. (1995), and the method was used in the comparison of these two tide models (Fok et al. 2010). However, the relationship between the results of the method and a straightforward measure of tidal discrepancy, such as the root-mean-square difference of hourly sea surface height, is not discussed in literature, and thus makes the results of tide model assessment not straightforward to interpret. In the context of coastal ocean forecasting systems, the research to quantify the exact accuracy of tidal models is needed in order to use satellite altimetry observation in coastal ocean forecasting systems. The accuracy or error characteristics of the altimetry observation need to be specified and used in the data assimilation process of coastal ocean forecasting systems. Tides are also highly variable in coastal regions. Better quantification and characterization of tidal discrepancy is needed when near-real-time forecasts should be made, particularly in validating the operational products.

In this research, we propose a modification to the current method used to assess the discrepancy either between tidal models or between the tidal model and observation. The proposed modification, which is consistent with the root-mean-square difference of tidal sea surface height, is called the total discrepancy method for the convenience of discussion. The relationship between the total discrepancy method and the current method is discussed, and their results are compared using Monte Carlo simulations and two tide models of the OSTM/ Jason-2 mission. When there is only one spatial location involved, the total discrepancy method is the same as the current method. The result of the total discrepancy method is directly comparable to the root-mean-square difference of tidal sea surface height when the comparison time is longer than 1 month. Where there are more than one spatial locations involved, the total discrepancy method provides a measure that is consistent with the root-mean-square difference of tidal sea surface height while the current method tends to give an overestimate. We will formally present the current method and the total discrepancy method in section 2 and demonstrate their differences in section 3. Section 4 summarizes our research.

2. Methods to measure the accuracy of tide models

a. Current method

For the convenience of discussion, we start from the harmonic representation of sea surface height associated with tides (e.g., Munk and Cartwright 1966),

$$\zeta(j, t) = \sum_{k=1}^{K} C_k(j) \cos \{\omega_k t + \theta_k(j)\},$$

where $\zeta(j, t)$ is the sea surface height for time $t$ and location $j$, $C_k(j)$ is the amplitude, and $\theta_k(j)$ is the Greenwich phase for tidal constituent $k$ with frequency $\omega_k$. The above equation is a modification of Eq. (2.9) used by Munk and Cartwright (1966) by ignoring the terms associated with nodal correction and the Dooodson number representation of tidal frequency because they are not relevant for our discussion. We would emphasize that the tidal amplitude $C_k(j)$ and phase $\theta_k(j)$ are also functions of space, and tidal accuracy assessment can involve multiple spatial locations. The explicit representation of $j$ in $C_k(j)$ and $\theta_k(j)$ is not strictly followed when the spatial dependence of $j$ is clear in the context.

The current method to assess the accuracy of tides is based on the magnitude of vector difference of in-phase [$C_k \cos(\theta_k)$] and quadrature [$C_k \sin(\theta_k)$] amplitudes (Ray 1993; Andersen et al. 1995; Fok et al. 2010). For a given constituent $k$, the magnitude of vector difference of in-phase and quadrature amplitude [noted as RMS$_k$ by Fok et al. (2010)] is

$$\text{RMS}_k = \left(\frac{1}{2N} \sum_{j=1}^{N} \left\{ [h_1^o(j, k) - h_1^m(j, k)]^2 + [h_2^o(j, k) - h_2^m(j, k)]^2 \right\}\right)^{1/2},$$

or

$$\text{RMS}_k = \left(\frac{1}{2N} \sum_{j=1}^{N} \left[ C_k^2 + C_k^m - 2C_k^o C_k^m \cos(\theta_k^o - \theta_k^m) \right]\right)^{1/2},$$

where $h_1^o(j, k) = C_k \cos \theta_k(j)$, $h_1^m(j, k) = C_k \cos \theta_k(j)$, $h_2^o(j, k) = C_k \sin \theta_k(j)$, and $h_2^m(j, k) = C_k \sin \theta_k(j)$.
in which \( h_1(j, k) = C_k(j) \cos \theta_k(j) \) and \( h_2(j, k) = C_k(j) \sin \theta_k(j) \) are the in-phase and quadrature amplitudes, and \( j = 1, \ldots, N \) is for the spatial locations involved. Here \( C_k \) and \( \theta_k \) are tidal amplitude and Greenwich phase from the observation, and the superscript (*) \( \text{m} \) stands for observation. Their counterparts from a tidal model are \( C_k^m \) and \( \theta_k^m \), in which superscript (*) \( \text{m} \) stands for model. To measure the overall difference for all \( K \) tidal constituents, the definition of variable RSS (Fok et al. 2010) is

\[
\text{RSS} = \left( \frac{1}{N} \sum_{k=1}^{K} \text{RMS}_k^2 \right)^{1/2} = \left\{ \frac{1}{2N} \sum_{j=1}^{N} \sum_{k=1}^{K} \left[ C_k^{o2} + C_k^{m2} - 2C_k^{o}C_k^{m} \cos(\theta_k^{o} - \theta_k^{m}) \right] \right\}^{1/2}. (3)
\]

The variable RSS has the same unit as amplitude. To quantify the relative difference, the definition of RSSIQ (Fok et al. 2010) is

\[
\text{RSSIQ} = \left( \frac{1}{2N} \sum_{k=1}^{K} \sum_{j=1}^{N} [h_k^{o2}(j, k) + h_k^{m2}(j, k)] \right)^{1/2}, (4)
\]

which can be used as an overall measure of sea surface height variability associated with tides for all spatial locations. The relative difference is then measured by Fok et al. (2010) using

\[
\frac{\text{RSS}}{\text{RSSIQ}}. (5)
\]

If we ignore the cross terms among tidal constituents with different frequencies and \( T \) is much longer than tidal periods, then we have

\[
D = \left\{ \frac{1}{2} \sum_{k=1}^{K} \left[ C_k^{o2} + C_k^{m2} - 2C_k^{o}C_k^{m} \cos(\theta_k^{o} - \theta_k^{m}) \right] \right\}^{1/2}, (8)
\]

which is the same as Eq. (6). The variability of sea surface height associated with \( K \) tidal constituents can be measured by

\[
V = \left\{ \frac{1}{T} \int_0^T \sum_{k=1}^{K} C_k^{o} \cos(\omega_k t + \theta_k^{o}) \right\}^{2} dt \right\}^{1/2}, (9)
\]

or

\[
V = \left\{ \frac{1}{2} \sum_{k=1}^{K} C_k^{o2} \right\}^{1/2}, (10)
\]

if the same assumptions as in Eq. (8) are used. Thus, if we compare tidal sea surface height at one location, then the variable RSS used by Andersen et al. (1995) and Fok et al. (2010) and the root-mean-square difference of tidal sea surface height are consistent.

When more than one spatial location is involved, the current method used in Andersen et al. (1995) and Fok et al. (2010) can still measure the overall difference of tides in all the locations. However, its interpretation is not directly related with the root-mean-square difference of tidal sea surface height associated with \( K \) tidal constituents for all the locations because \( N \) is in the square root form in the definition of RSS. To have a formulation consistent with the root-mean-square difference of tidal sea surface height, we propose to use

\[
D = \frac{1}{N} \sum_{j=1}^{N} D(j) (11)
\]

to measure the overall discrepancy at multiple spatial locations. To measure the relative discrepancy of tides, we propose to use
The tidal analysis is conducted using T-Tide software (Pawlowicz et al. 2002). Valdez (ID 9454240; 60.5583° N, 145.7533° W) was chosen to demonstrate that the total discrepancy method uses the spatially averaged total discrepancy $\overline{D}$ to assess the tidal discrepancy. The current method uses the square root of the averaged square total discrepancy. Mathematically, RSS is always larger than or equal to $\overline{D}$. The interpretation of the current method is not consistent with the total discrepancy, which might limit its practical application. Another advantage of the total discrepancy method is that the standard deviation of $\overline{D}$ can be used to measure the spatial variability of tidal discrepancy. For the coastal region, the spatial scale of tides is smaller than that of the open ocean. Such a measure is useful. In next section, we will demonstrate the difference of the variable RSS and $\overline{D}$.

### 3. Comparison of methods to assess the accuracy of tide models

#### a. Monte Carlo simulation

In the derivation of the total discrepancy in Eq. (8) and the variability of tidal sea surface height in Eq. (10), we assume that the integration period is much longer than tidal periods and the cross terms can be ignored. We will quantify the errors introduced by these assumptions. With a tidal range around 4 m and an $M_2$ amplitude as 1.45 m (Table 2), the tidal gauge station in Valdez, Alaska (ID 9454240), is chosen to demonstrate that the errors introduced by such assumptions are small. Because our conclusions are presented in a nondimensional way, the selection of a specific station will have limited influence on our conclusions.

Using hourly sea surface height observation for 2004, the amplitudes and phases of major tidal constituents ($M_2$, $S_2$, $N_2$, $K_2$, $K_1$, $O_1$, $P_1$, and $Q_1$) are estimated (Table 2) by using T-Tide software (Pawlowicz et al. 2002).

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$M_2$</th>
<th>$S_2$</th>
<th>$N_2$</th>
<th>$K_2$</th>
<th>$K_1$</th>
<th>$O_1$</th>
<th>$P_1$</th>
<th>$Q_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude (cm)</td>
<td>145.19</td>
<td>49.29</td>
<td>28.89</td>
<td>13.38</td>
<td>49.61</td>
<td>30.29</td>
<td>15.36</td>
<td>5.37</td>
</tr>
<tr>
<td>Phase (°)</td>
<td>290.41</td>
<td>322.30</td>
<td>268.49</td>
<td>315.76</td>
<td>270.70</td>
<td>249.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 illustrates the difference of the current method and the total discrepancy method. When there is only one spatial location involved, these two methods give the same results. When there are multiple locations involved, their results are subtly different. The total discrepancy method uses the spatially averaged total discrepancy $\overline{D}$ to assess the tidal discrepancy. The current method uses the square root of the averaged square total discrepancy. Mathematically, RSS is always larger than or equal to $\overline{D}$. The interpretation of the current method is not consistent with the total discrepancy, which might limit its practical application. Another advantage of the total discrepancy method is that the standard deviation of $\overline{D}$ can be used to measure the spatial variability of tidal discrepancy. For the coastal region, the spatial scale of tides is smaller than that of the open ocean. Such a measure is useful. In next section, we will demonstrate the difference of the variable RSS and $\overline{D}$.

### Table 1. Comparison of the current method (Ray 1993; Andersen et al. 1995; Fok et al. 2010) and the total discrepancy method to measure the accuracy of tides. The discrepancy and relative discrepancy in tides are presented in terms of the root-mean-square difference $D$ and the sea surface height variability associated with tides $V$, which are defined in Eqs. (8) and (9), respectively. The total number of spatial locations involved is denoted by $N$.

<table>
<thead>
<tr>
<th>Method</th>
<th>One location</th>
<th>Multiple locations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Discrepancy</td>
<td>Relative discrepancy</td>
</tr>
<tr>
<td>Current method</td>
<td>$D$</td>
<td>$D/V$</td>
</tr>
<tr>
<td>Total discrepancy method</td>
<td>$D$</td>
<td>$D/V$</td>
</tr>
</tbody>
</table>

\[
\overline{D} = \frac{1}{N} \sum_{j=1}^{N} \frac{D(j)}{V(j)}.
\] (12)

The $\overline{D}$ and $\overline{D}$ are the spatial average of $D$ and $D/V$, respectively. The above modification of the current method used by Andersen et al. (1995) and Fok et al. (2010) makes the interpretation of tidal accuracy assessment straightforward because $\overline{D}$ is the averaged root-mean-square difference of tidal sea surface height for the spatial locations involved. For the convenience of discussion, this modification of the current method is called the total discrepancy method because $D$ as defined in Eq. (7) is the total discrepancy by taking into account the discrepancies in amplitudes and phases of $K$ tidal constituents.

Using the definition of $D$ and $V$, the variables RSS and RSS/RSSIQ used by Fok et al. (2010) can be written as

\[
\text{RSS} = \left( \frac{1}{N} \sum_{j=1}^{N} D_j^2 \right)^{1/2},
\] (13)

and

\[
\text{RSS/RSSIQ} = \left( \frac{1}{N} \sum_{j=1}^{N} \frac{D_j^2}{V_j^2} \right)^{1/2}.
\] (14)

Table 2. The amplitude (cm) and Greenwich phase (°) of eight major semi-diurnal and diurnal tidal constituents for tidal gauge station Valdez (ID 9454240: 60.5583° N, 145.7533° W). The tidal harmonic analysis is based on hourly sea surface height observation for 2004. The tidal analysis is conducted using T-Tide software (Pawlowicz et al. 2002).

<table>
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<th>Constituent</th>
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<th>$K_2$</th>
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<td>270.70</td>
<td>249.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
We can directly estimate the variability associated with tides $V$ following Eq. (9). When the length of sea surface height observation is longer than 1 month, the difference of $V$ computed from tidal amplitudes as in Eq. (10) and that computed from hourly observation as in Eq. (9) is less than 0.5 cm, which is less than a half-percent of the variable $V$ (Table 3). Thus, the errors introduced by ignoring the cross terms in the derivation of Eqs. (8) and (10) are relatively small, and the two forms of $D$ based on the root-mean-square difference of tidal sea surface height [Eq. (7)] and based on discrepancies in amplitudes and phases [Eq. (8)] are consistent when the time scale is longer than 1 month.

Random errors are introduced into the amplitudes and phases of the eight major tidal constituents for station Valdez to simulate the situation of tidal comparison at different spatial locations. The difference of RSS and $D$ is analyzed for discrepancies varying from 1% to 30% of the amplitudes and phases of respective tidal constituents (Fig. 1). To interpret the results of the Monte Carlo simulation in a more general way, the nondimensional $(\text{RSS} - D)/V$ and std$(D)/V$ is used. As expected, RSS is always larger than or equal to $D$. The difference between RSS and $D$ increases from 0.5% to 8% when the level of discrepancy in tidal amplitudes and phases increases from 1% to 30%. Equivalently, when there is large spatial variability in $D$ (red solid line in Fig. 1), the difference between RSS and $D$ is also large. In contrast to its sensitivity to the spatial variability in $D$, the difference is not sensitive to the number of locations involved, either it is 10 (black dash-dotted line) or 1000 (black solid line). Because the total discrepancy is the root-mean-square difference of tidal sea surface height and its interpretation is straightforward, the variable $D$ is a better measure than the variable RSS when more than one location is involved. The use of $D$ instead of RSS is suggested. The standard deviation of $D$ also gives a measure of how the total discrepancy varies over space.

b. Discrepancy of FES2004 and GOT00

The two tide models used in the OSTM/Jason-2 altimetry data processing are compared using the total discrepancy method and the current method to further demonstrate the advantage of the total discrepancy method. We concentrate between the latitude of 66°S and 66°N where the satellite altimetry observations from the mission are located, and on the coastal ocean (water depth is less than 1000 m) where large spatial variability of tidal model discrepancy exists. For detailed comparison of tidal models over various regions at different stages of altimetry missions, we refer to Ray (1993), Andersen et al. (1995), Shum et al. (1997), and Fok et al. (2010).

To assess the relative difference of these two tidal models, Fig. 2a shows the variability of sea surface height associated with eight major tidal constituents computed using Eq. (10). The computation is based on the average of the amplitudes from FES2004 and GOT00. In general,
the open ocean has variability less than 50 cm. In the North Pacific, North Atlantic, Patagonia shelf region, and northern Australia, the variability of sea surface height associated with these eight tidal constituents can reach 1 m or higher. The spatial pattern resembles the distribution of amplitude associated with semidiurnal $M_2$ constituent, which is not surprising because $M_2$ is the dominant constituent in general.
The total discrepancy around 10 cm shows up in the regions of the northeastern Pacific, northern part of the Sea of Okhotsk, east China Sea, south China Sea, north of Australia, Indonesia region, Hudson Bay, northwestern Atlantic region, and Patagonia shelf region (Fig. 2b). Some of these regions are highlighted in Fok et al. (2010). In terms of relative discrepancy $D/V$, all of the regions mentioned above show up with relative discrepancy greater than 10% (Fig. 2c). Some regions, such as the southern part of Madagascar and the Gulf of Mexico, also show up because these regions tend to have small variability associated with tides. Relatively small total discrepancy can cause large relative discrepancy in these regions, which raises challenging issues in terms of separating the sea surface height variability associated with ocean circulation and that associated with tides. The difference of the current method and the total discrepancy method shows up if there are multiple locations involved. For regions mentioned above and marked in Fig. 2b that have a discrepancy greater than 10 cm, the difference between RSS and $D$ varies from 1.4 to 11 cm; the difference is large for regions that have large spatial variability (Table 4). For example, the Hudson Bay region has the largest standard deviation of $D$ (20.87 cm) and also the largest difference (11 cm).

For global coastal ocean, all of the grid points within every 50-m depth range (0–50, 50–100 m, . . . , etc.) are grouped together to compute RSS and $D$ (Fig. 3). The discrepancy between FES2004 and GOT00, as measured by the total discrepancy method, decreases from 11.4 cm right along the coast where the water depth is less than 50 m to 2.2 cm where the water depth is between 950 and 1000 m. As a reference, the averaged total discrepancy in the open ocean where water depth is deeper than 1000 m is 1.0 cm. On the other hand, the RSS proposed by Fok et al. (2010) tends to give a larger value of the discrepancy between the two tidal models than

Table 4. The difference of the total discrepancy method and the current method for selected regions that have large tidal discrepancy. The averaged total discrepancy ($\overline{D}$) and its standard deviation (in parentheses) are shown in the second column. The variable RSS (Fok et al. 2010) is shown in the third column. The difference of RSS and $\overline{D}$ ($\text{RSS} - \overline{D}$) is shown in the fourth column. The fifth column is the domain of computation.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\overline{D}$ (std dev; cm)</th>
<th>RSS (cm)</th>
<th>($\text{RSS} - \overline{D}$)</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northeast Pacific</td>
<td>3.65 (9.45)</td>
<td>10.13</td>
<td>6.48</td>
<td>50°–66°N, 180°E–120°W</td>
</tr>
<tr>
<td>Southeast China</td>
<td>6.33 (10.69)</td>
<td>12.42</td>
<td>6.09</td>
<td>5°–45°N, 100°–130°E</td>
</tr>
<tr>
<td>Indonesia region</td>
<td>4.69 (4.53)</td>
<td>6.52</td>
<td>1.83</td>
<td>10°S–5°N, 100°–140°E</td>
</tr>
<tr>
<td>Hudson Bay</td>
<td>14.29 (20.87)</td>
<td>25.29</td>
<td>11.0</td>
<td>50°–66°N, 95°–60°W</td>
</tr>
<tr>
<td>Gulf of Mexico</td>
<td>2.17 (2.80)</td>
<td>3.54</td>
<td>1.37</td>
<td>18°–31°N, 98°–70°W</td>
</tr>
<tr>
<td>Northeast Brazil</td>
<td>2.56 (6.76)</td>
<td>7.23</td>
<td>4.67</td>
<td>5°S–10°N, 60°–30°W</td>
</tr>
<tr>
<td>Patagonia region</td>
<td>3.88 (4.76)</td>
<td>6.14</td>
<td>2.26</td>
<td>56°–30°S, 70°–50°W</td>
</tr>
</tbody>
</table>

![Fig. 3. The discrepancy between tidal model FES2004 and GOT00 in the coastal region as measured by the total discrepancy method (solid line) and the current method (dashed line). The standard deviation of total discrepancy is also shown (vertical lines).]
that of $\bar{D}$. The difference tends to be large for regions with large standard deviations of $\bar{D}$, such as regions with depths around 150, 350, and 550 m, which are consistent with the results from Table 4 and Fig. 1. The discrepancy measured by the total discrepancy method shows a monotonically decreasing tendency with increasing water depth. Though the general tendency of decreasing discrepancy with increasing water depth is similar, the discrepancy measured by the current method has some abrupt increases at the depth from 100 to 150, from 300 to 350 m, etc. As mentioned above, these increases are also associated with large spatial variability of $\bar{D}$. Because the analysis domain of Fig. 1 is global, these increases are hard to explain physically and might be caused by the overestimation of RSS compared with $\bar{D}$.

The relative discrepancy as measured by both methods ($\mathcal{R}D$ and RSS/RSSIQ) is very close (Fig. 4). Both methods indicate that the relative discrepancy is around 32.3% when the water depth is shallower than 50 m and decreases to about 9.1% when the water depth is between 950 and 1000 m. In the open ocean, the relative discrepancy is 4.64% (Fig. 4). The reason for the consistency of relative discrepancy from both methods is that though the variable RSS provides an overestimate comparing with $\bar{D}$, the variable RSSIQ can also provide an overestimation of tidal variability as measured by variable $V$. The two relative discrepancy measures for the total discrepancy method and the current method, that is, the variable $\mathcal{R}D$ and the variable RSS/RSSIQ, respectively, are comparable eventually. Large difference between $\mathcal{R}D$ and RSS/RSSIQ occurs where there is large difference in $\bar{D}$ and RSS, such as the coastal regions where the water depth is 150, 350, and 550 m. The fact that the sea surface height variability either measured by $V$ or the variable RSSIQ decreases with increasing water depth might be responsible for the large difference between $\mathcal{R}D$ and RSS/RSSIQ in regions where the water depth is deeper than 600 m.

4. Conclusions

When there is only one spatial location involved, the current method to assess the accuracy of tide models (Ray 1993; Andersen et al. 1995; Fok et al. 2010) is consistent with the total discrepancy, that is, the root-mean-square difference of tidal sea surface height over a period much longer than the semidiurnal and diurnal time scales. When there are more than one spatial locations involved, however, the current method tends to give an overestimate of the total discrepancy. As an alternative approach to the current method, the total discrepancy method is proposed such that its result is consistent with the averaged total discrepancy. The difference of the current method and the total discrepancy method can reach 8% of the sea surface height variability associated with tides based on Monte Carlo simulations. The proposed total discrepancy method and the current method are then used for the comparison of two global tidal models used in the altimetry observation, FES2004 and GOT00. For the eight major semidiurnal and diurnal constituents, the discrepancy of the two tidal models as measured by the total discrepancy method monotonically decreases from 11.4 cm where the water depth is less than 50 m to 2.2 cm where the water depth is 1000 m. The discrepancy as measured by the current method varies from 21.6 to 2.9 cm. Though the discrepancy from the current method decreases with increasing water depth, there are abrupt increases at depth from 100 to 150, from 300...
to 350 m, etc. These increases are associated with large spatial variability in total discrepancy and might be caused by the overestimation of the current method. In summary, the proposed total discrepancy method can provide an estimate of tidal discrepancy that is consistent with the root-mean-square difference of tidal sea surface height and straightforward to interpret, and its usage is suggested. The large discrepancy in coastal region is still a challenging issue in the application of altimetry observation.

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