Measurement of Atmospheric Aspect Sensitivity Using Coherent Radar Imaging after Mitigation of Radar Beam Weighting Effect

JENN-SHYONG CHEN
Department of Computer and Communication Engineering, Chienkuo Technology University, Changhua City, Taiwan

JUN-ICHI FURUMOTO
Research Institute for Sustainable Humanosphere, Kyoto University, Kyoto, Japan

(Manuscript received 12 January 2012, in final form 7 July 2012)

ABSTRACT

The aspect angle, a measurement of the aspect sensitivity of atmospheric refractivity irregularities, was estimated with multiple-receiver coherent radar imaging (CRI) of very high frequency (VHF) atmospheric radar. Two CRI parameters retrieved by Capon’s method were utilized to derive the aspect angle: brightness width from the vertical radar beam and the direction of arrival (DOA) of the echo center from the oblique radar beam. Differing from previous studies with CRI, the radar beam weighting effect on the CRI brightness distribution was considered, and moreover, the radar beamwidth used in this study was adaptive to the signal-to-noise ratio (SNR) of data as well as the off-beam direction angle. The study is based on statistical results. It is shown that the brightness width, a representative of the aspect angle, obtained from the modified CRI brightness distribution of the vertical radar beam was generally larger than that without correction, and it was very close to the values derived from the DOA of the 1° oblique radar beam and the power distribution of multiple beam directions. Moreover, the aspect angle derived from the DOA varied with the radar beam direction, which was similar to that obtained from the comparison of echo powers of a radar beam pair; however, the DOA approach yielded a much larger aspect angle in the low-SNR condition. This study recommended a feasibility of improving the measurements of atmospheric parameters with CRI after removing the radar beam weighting effect suitably from the CRI brightness distribution.

1. Introduction

Very high frequency (VHF) atmospheric radar is a useful instrument to study the atmosphere at all heights from near the ground up to the ionosphere. The radar echoes come mainly from the refractivity irregularities (scatterers) having length scales of the order of half the radar wavelength (e.g., the Bragg scale), and so plenty of atmospheric parameters or characteristics can be derived from the radar echoes, for example, the aspect sensitivity of refractivity irregularities, turbulence structure, vertical and horizontal winds, and so on.

Aspect sensitivity of refractivity irregularities in the clear air is a measurement of the stability of the atmosphere, which is characterized by a dependence of radar echo power on the off-zenith direction of the radar beam; that is, the radar echo power is stronger for a vertical radar beam and drops off with the off-zenith angle of the radar beam. A number of studies using VHF atmospheric radar have shown that anisotropic turbulence and specular reflectors can explain the characteristics of aspect sensitivity (e.g., Woodman and Chu 1989; Hocking et al. 1986, 1990). Therefore, the study of aspect sensitivity can yield a deeper understanding of the mechanisms of the radar echoes, for example, isotropic/anisotropic turbulence or Fresnel reflecting/scattering structures. In general, however, it is difficult to separate the respective contributions of these echoing mechanisms.

Plenty of approaches/techniques using VHF radar have been used to measure the aspect sensitivity of the scatterers, such as the comparison of echo powers between different oblique radar beams (Tsuda et al. 1986; Hocking et al. 1986; Hooper and Thomas 1995; Ghosh

Corresponding author address: Jenn-Shyong Chen, Department of Computer and Communication Engineering, Chienkuo Technology University, No. 1, Jieshou N. Rd., Changhua City 500, Taiwan.
E-mail: jschen@ctu.edu.tw

DOI: 10.1175/JTECH-D-12-00007.1

© 2013 American Meteorological Society
et al. 2004), the narrowing effect of the aspect sensitivity on the Doppler spectrum of a vertically pointed radar beam (Chu et al. 1990), spaced-antenna correlation (Briggs 1992), wind measured by oblique radar beam and spectral width (determined by the fading time of the autocorrelation function) (Hocking et al. 1990), spatial interferometric (SI) method using multiple receivers (Röttger and Vincent 1978; Vincent and Röttger 1980), echo power variations/distributions of multiple radar beams tilted at different directions (Tsuda et al. 1997a,b; Palmer et al. 1998b; Worthington et al. 1999, 2000), and so on. All the studies demonstrated a general drop-off dependence of echo power on the zenith angle. For quantitative study of the aspect sensitivity, a function of Gaussian form has been adopted to describe the drop-off dependence of echo power on the zenith angle (e.g., Hocking et al. 1986); and further, a two-dimensional Gaussian function has also been used to fit the two-dimensional echo power distribution (e.g., Palmer et al. 1998b). The standard deviation of the one- or two-dimensional Gaussian function gives the degree of aspect sensitivity, and is termed aspect angle.

However, studies have shown azimuthal variations in echo power, indicating tilted or corrugated reflecting surfaces that may be associated with wave activities, or the nonhomogeneous distribution of turbulence in the horizontal (e.g., Tsuda et al. 1997a). Horizontal distribution of echo power can also be skewed because of vertical wind shear, as demonstrated by Worthington et al. (1999, 2000). Moreover, the studies via comparing the echo powers of different oblique radar beams demonstrated that the estimated aspect angle varied with the tilt angles of the radar beam pair used (e.g., Hocking et al. 1990), suggesting that the scatterers responsible for different oblique radar beams have different degrees of anisotropism, namely, the ratios of horizontal scale to vertical scale of the scatterers are diverse. According to Hocking et al. (1990), the values for the aspect angle of 3°–4° imply scatterers with ratios of horizontal scale to vertical scale of about 8–10, which represents a fairly large degree of anisotropism. In view of the above studies, it is evident that different approaches/techniques for the study of aspect sensitivity can disclose various characteristics of refractivity irregularities in the clear air.

Coherent radar imaging (CRI), using multiple receivers to retrieve the angular distribution of echo power (termed brightness distribution), can also be employed to yield information about aspect sensitivity. This is achieved by fitting the brightness distribution of a vertically transmitted beam with a two-dimensional Gaussian function, leading out the parameter of (angular) brightness width that has been demonstrated to be positively correlated with aspect sensitivity (Yu 2000; Chilson et al. 2002). Such a study is analog to the formation of the angular distribution of echo powers constructed from multiple radar beams at different angular directions (Worthington et al. 1999, 2000). To improve the estimate of brightness width, however, the radar beam weighting effect on the brightness value should be removed beforehand. Usually, the radar beam weighting function is a Gaussian form shaped by a half-power radar beamwidth obtained from theoretical or simulated computation. Such a function can model the intensity distribution of the radar beam within the beamwidth appropriately, but it may not be suitable enough for off-beam directions outside the beamwidth. As a result, the compensation on the brightness value using a theoretical radar beam weighting function usually incurs spurious peaks around the edge of the brightness distribution. In view of this, a more suitable function for the intensity distribution of the radar beam is desired to improve the compensation result. Indeed, an approach to this has been introduced in our previous study by using multiple beam directions used with CRI (Chen and Furumoto 2011); an adaptable beamwidth was thereby proposed, which is of help particularly to the compensation outside the beamwidth. It should be mentioned that the proposal of adaptable beamwidth does not imply that the radar beam pattern changes from time to time. It is an artifice to compensate the beam weighting effect, which is not only associated with the radar beam pattern, but also the performance of the signal processing method and the data quality, such as the signal-to-noise ratio (SNR).

Aspect sensitivity can also be examined with the incident angle determined by CRI or SI with oblique radar beam. This is because the CRI- or SI-determined incident angle could be biased to zenith owing to the anisotropism of scatterers. By considering the beam weighting effect, the anisotropism of scatterers can be estimated by assuming a proper function, usually a Gaussian function, to the anisotropism of scatterers in advance. Such an inspection of the aspect sensitivity will be detailed in section 2.

The main goal of this paper is to demonstrate the usability of adaptable beamwidth used for CRI, which is achieved via the estimate of the aspect angle from vertical and oblique radar beams. To this end, multiple-receiver CRI and multiple beam directions were carried out with the middle and upper (MU) atmosphere radar (34.85°N, 136.11°E) in Japan. Differing from previous studies with the CRI or SI method, however, our present CRI approach uses adaptable beamwidth as well as the echo centers estimated from the oblique radar beam. The estimate of the aspect angle with CRI is addressed in section 2. Also introduced in section 2 are the two
aspect angles estimated from the comparison between echo powers of a beam pair and a power distribution constructed from different oblique radar beams, respectively; such aspect angles were examined for validating the CRI approach. The concept and equations of adaptable beamwidth is introduced briefly in section 3; readers can refer to Chen and Furumoto (2011) for the details.

In section 4, we compare the aspect angles estimated from different methods on the basis of statistical features. However, only the comparisons in the zonal and meridional directions are made because the radar beam in the present experiment is directed to only four directions: east, west, north, and south. Conclusions are stated in section 5.

2. Estimate of aspect angle

In this work, the following three methods are employed to estimate the aspect angle, in which the CRI method using oblique radar beam is introduced first and its usability can be proved by comparing with the other two existing methods.

a. By power distribution of different oblique radar beams

According to the definition of aspect sensitivity, this method can give a direct estimate of the aspect angle. To obtain a representative value of the aspect angle, however, the conditions of considerably slanted structures or greatly off-zenith discrete scattering patches in the radar volume should be excluded. These conditions usually deflect the maximum of the power distribution from the zenith remarkably. Therefore, the power distribution with the maximum at an angular location larger than 1° off zenith was not used for the computation of the mean aspect angle. The obtained mean aspect angle can be a frame of reference for validating the CRI approach given below.

b. By CRI

As mentioned in the introduction, a positive relationship between CRI brightness width and aspect sensitivity has been disclosed with a vertical radar beam (Yu 2000; Chilson et al. 2002), although the radar beam weighting effect was not removed yet. Removal of the radar beam weighting effect using a theoretical beam pattern often results in overmodified brightness values at the border of the brightness distribution and then gives unrealistic echo centers at the border. In view of this, the adaptable beamwidth proposed by Chen and Furumoto (2011), which varies with the SNR and the off-beam direction angle, can be employed to correct the brightness distribution. We have applied such an adaptable beamwidth in this study.

The CRI brightness distribution of the vertical beam can also be compared with 2D angular power distributions constructed from different oblique radar beams (Worthington et al. 1999, 2000). Unfortunately, the oblique radar beams in our experiment are only in the east, north, west, and south directions, which are not enough to construct a 2D angular power distribution, causing the comparison to be unavailable. Therefore, only the features of aspect angles in the zonal and meridional directions are discussed.

In addition to the brightness width of vertical beam CRI, the direction of arrival (DOA) of the echo center determined from CRI brightness distribution can also be applied to yield an estimate of the aspect angle, and this can be achieved with the vertical or oblique radar beam. Nevertheless, we will use only the oblique beam, considering that the error due to a slight shift of the echo center from the zenith could be substantial for the vertical beam. For homogenously distributed scatterers, the DOA of the echo center can be related to the aspect angle by the following Gaussian expression (Hocking et al. 1986):

\[
P(\theta) = \exp \left[ -\frac{(\sin \theta - \sin \theta_0)^2}{\sin^2 \theta_0} \right] \exp \left[ -\frac{\sin^2 \theta_s}{\sin^2 \theta_s} \right] \tag{1}\]

where \(P(\theta)\) is the characteristic polar diagram of backscattered power (characteristic power distribution hereafter), the first term of \(P(\theta)\) is the beam weighting effect, and the second term represents the effect of aspect sensitivity that is assumed to have a maximum weight at zenith. Also, \(\theta_T\) is the off-zenith angle of the oblique radar beam; \(\theta\) is a variable of the zenithal angle; \(\theta_s\) is termed aspect angle, which is an indicator of aspect sensitivity; and \(\theta_0\) is the half-beamwidth of the radar beam. Notice that (1) is valid for \(\theta_T\) and \(\theta_0\) less than \(\sim 10^\circ\) (Hocking et al. 1986). The brightness distribution retrieved from CRI is supposed to represent the power distribution \(P(\theta)\); hence, the DOA of the echo center can be computed from the CRI brightness distribution. Notice that there could be single or multiple echo centers in the brightness distribution. To determine the locations of multiple DOAs, we employed the contour-based approach proposed by Chen et al. (2008). Such computed DOAs are subject to aspect sensitivity and the beam weighting effect, so we term them CRI–DOA.

The process of estimating an aspect angle from the characteristic \(P(\theta)\) is as follows: use a suitable beamwidth and give various values of \(\theta_s\) to (1), and then compute the mean maximum center (DOA) of \(P(\theta)\) for each \(\theta_s\) to yield a set of DOAs; one of the computed
DOAs is expected to be the closest to the CRI-DOA, and the corresponding value of $\theta_s$ can be regarded as the optimal estimate of the aspect angle. The suitable beamwidth here is, namely, the adaptable beamwidth proposed by Chen and Furumoto (2011). Such an estimated aspect angle, however, could be a function of radar beam direction ($\theta_T$), owing to the following two reasons: (i) various oblique radar beams may observe some different shapes (or anisotropism) of scatterers, as discussed in the literature (Hocking et al. 1990; Hooper and Thomas 1995; Ghosh et al. 2004), and even the radar beams are overlapped partly; and (ii) the second term of $P(\theta)$ in (1) is a Gaussian-type function for describing the aspect sensitivity of scatterers, which may not be suitable enough for observation with a beam direction at a larger tilt angle. In fact, Hocking et al. (1990) pointed out that the falloff in echo power could be close to the exponential that flattens off at a larger zenith angle. In any case, to validate the CRI-DOA approach, the estimated aspect angle should be compared with that obtained from other comparable methods, such as the comparison of echo powers of two radar beam directions, as given in the following subsection.

The need for using adaptable beamwidth in CRI has been demonstrated further by Chen et al. (2011), with a simulation study. To validate the workability of CRI-DOA in deriving the aspect angle, more simulations were performed in this study. The results are presented in the appendix. In the main body of the text, we discuss the aspect angles obtained from practical data analysis.

c. By comparison of echo powers of different oblique radar beams

The aspect angle estimated by the method of a comparison of echo powers can refer to the following expression given by Hooper and Thomas (1995):

$$\theta_s = \sin^{-1}\left\{\frac{\sin^2\theta_{T2} - \sin^2\theta_{T1}}{\ln[P(\theta_{T1})/P(\theta_{T2})] - \sin^2\theta_o}\right\},$$

(2)

where $\theta_{T1}$ and $\theta_{T2}$ are the off-zenith angles of two radar beams, $\theta_o$ is a measurement of the radar half-beamwidth, and $P$ represents the received backscattering power. Notice that $\theta_o$ is constant in (2), which is different from the adaptable $\theta_o$ used with the CRI approach.

3. Radar experiment and adaptable radar beamwidth

a. Radar experiments

The MU radar consists of 25 antenna groups, as illustrated in Fig. 1, and arbitrary combinations of antenna groups for transmission and reception are available. Moreover, the radar beam can be steered to many directions between pulses, making the use of the CRI technique flexible.

Table 1 lists some major radar parameters given in the two experiments. Seven antenna groups were combined for both transmission and reception, termed Tx7/Rx7 mode here. Seven receiving channels were arranged for both experiments, and the inversion algorithm of Capon’s method was applied to CRI (Capon 1969; Palmer et al. 1998a). The radar beams in the first experiment were transmitted to zenith and 1°–4° off zenith in the east, west, north, and south directions. The second experiment, in addition to the vertical beam, used the radar beams tilting to 6°–12° off zenith, which were carried out one day after the first experiment for complementing the first experiment.

Figure 2 displays the height–time intensity of the vertical beam, with a time resolution of ~60 s. Many stable layer structures were detected in the two observations in spite of the weakening of the layer structure around the height of 4 km in the second experiment. Also plotted are the profiles of mean echo powers obtained by different oblique radar beams, from which the height intervals with larger difference in the SNR can be found at ~4 and ~8 km, suggesting that the scattering irregularities be highly anisotropic and so be very suitable for investigation of the aspect sensitivity.
Notice that the echo power of the radar beam at 12° off zenith was very low above 4 km, and so it was not employed in the study.

b. Adaptable radar beamwidth

The Tx7/Rx7 mode has been employed to find an adaptable radar beamwidth by Chen and Furumoto (2011). This was done by estimating the effective beamwidth \( u_e \), namely, the 3-dB half-beamwidth of a Gaussian-shaped beam pattern function, at various off-beam directions by means of a set of oblique radar beams. Because of the uses of the same radar, the Tx/Rx mode, and the signal processing method, it is expected that the present two experiments yield similar adaptable beam widths to the previous one, even the experiments were not carried out at the same time.

Estimations have demonstrated similar features of \( u_e \) (not shown) between the present experiment and that in Chen and Furumoto (2011). Accordingly, the equations given by Chen and Furumoto (2011) can be employed here to obtain the effective beamwidth \( u_e \), as addressed briefly below:

(i) Use the following cubic curve to find the effective beamwidth \( \theta_{oe} \) of the SNR-free condition for an off-beam direction angle \( \theta \):

\[
\theta_{oe} = c_1 \theta^3 + c_2,
\]

where \( c_1 = 0.0036 \) and \( c_2 = 2.8655 \).

(ii) Find an empirical equation to describe the relationship between effective beamwidth \( \theta_e \) and SNR. The empirical equation employed for the Tx7/Rx7 mode can be

\[
\theta_e = (\frac{\theta_{oe}}{a_0})^{2.8} \exp[-c_o \times \text{SNR}] + \theta_{oe},
\]

where \( a_0 = 3.0 \) and \( c_o = 0.03 \). The unit of SNR is decibels. Such obtained \( \theta_e \) is then assigned to the \( \theta_o \) in (1) for various SNRs.

4. Results and discussion

a. Aspect angle measured by CRI of experiment 1

The DOA of the echo center estimated from the brightness distribution of the oblique radar beam may be biased owing to the aspect sensitivity of scatterers, as indicated in (1). One case is demonstrated in Fig. 3, where the distributions of CRI–DOAs at each range gate observed by the 3° oblique radar beam are shown for the east, west, south, and north directions, respectively. It is seen clearly that many of the CRI–DOAs have zenith angles much smaller than the tilt angle of the radar beam, especially at the heights of ~4 and 6–8 km, which can be attributed to the aspect sensitivity of scatterers.

In the following, we discuss the aspect angles estimated from the CRI–DOAs of several oblique radar beams as well as the brightness width of the vertical radar beam. In the case of multiple centers, the CRI–DOA having the highest brightness level was used to estimate an aspect angle. For the oblique radar beam,
only the CRI–DOAs having zenith angles smaller than the tilt angle of the radar beam qualify for estimating the aspect angle because these echoes are supposed to be influenced by aspect sensitivity. On the other hand, for the vertical beam the brightness values within the 5° angular zone of the CRI–DOA were adopted in the computation of the brightness width.

Fig. 4 shows the distributions of aspect angles in the east and north directions, derived from the CRI–DOAs in Fig. 3 (the 3° oblique radar beam). As seen, the aspect angles vary with altitude and have smaller values around the sampling ranges of ~4 and ~7.5 km; a smaller aspect angle indicates a higher degree of aspect sensitivity. On the other hand, aspect angles above the sampling range of ~10 km are more dispersive for their low SNR (refer to Fig. 2), so the results above ~10 km are not reliable in this observation.

The comparison between aspect angles in the experiment 1 is shown in Fig. 5, where the profiles of the mean aspect angles estimated in the east, west, north, and south directions are exhibited, respectively. Notice that the sampling range of the oblique radar beam has been transformed to height according to the off-zenith angle of the oblique radar beam. In the panels of the east and north directions, we also show the aspect angles obtained from the power distribution of the oblique radar beams (solid line with circles) for reference. The profiling curves marked with $B_w$ and $B_w'$ are the brightness widths obtained from the original and corrected brightness distributions of the vertical beam, respectively. Such estimated brightness widths can be regarded as the original and corrected aspect angles measured by the vertical radar beam, respectively. The profiling curves denoted by the numbers 1–4 are the mean values of $\theta_s$,
estimated with (1) for the radar beams tilted to the off-zenith angles \( \theta_T \) of 1°, 2°, 3°, and 4°.

For clarity, the major features indicated in Fig. 5 are listed and discussed one by one in the following:

(i) In general, the mean aspect angles in the zonal and meridional directions were similar, suggesting that the scatterers in this observation were not aligned apparently in the zonal or meridional direction. However, the orientation of the scatterers cannot be determined because of the lack of observations in other directions.

(ii) Altitudinal variations in brightness width and aspect angles, estimated from the vertical and oblique radar beams, respectively, were consistent, although differences in values can be as large as several degrees at some altitudes, for example, at ~4.5 and ~5.5 km.

(iii) For CRI using the vertical beam, removal of the radar beam weighting effect from the brightness value resulted in a brightness width \( B_{w_c} \) larger than the uncorrected one \( B_{w_o} \) by ~1°~2°. The altitudinal variation in \( B_{w_o} \) was very small, which was obviously subjected to the lack of correcting the radar beam weighting effect. In the literature,
Yu (2000) and Chilson et al. (2002, their Fig. 4) have shown that the uncorrected brightness width is generally smaller than the aspect angle derived from spectral width. Our correction of the brightness value can thus make up the underestimate caused by the beam weighting effect.

(iv) In general, $B_{wc}$ was smaller than the DOA-derived aspect angles. However, the mean profile of $B_{wc}$ was very close to that estimated from the $1^\circ$ tilted radar beam, which is logical because the two beam directions are very close and so many similar scatterers can be observed. Moreover, $B_{wc}$ was close to that of the power distribution. In view of these comparisons, the mitigation of the radar beam weighting effect on the brightness value is applicable.

(v) The aspect angle varied apparently with the off-zenith angle of the radar beam, that is, tilting the radar beam to a larger off-vertical direction resulted in a larger aspect angle. One of the interpretations for this feature is that the echoes of the vertical radar beam are mainly from the scatterers with larger anisotropism than those for the oblique radar beams, namely, there could be various shapes (anisotropism) of scatterers, as discussed in the literature (e.g., Woodman and Chu 1989; Hocking et al. 1990). Certainly, specular reflectors can also be one cause of this feature.

(vi) In spite of the dependence between the aspect angle and radar beam direction, the aspect angles obtained from different oblique radar beams were very close at the heights of $\sim 7.5$ and $\sim 4$ km. This indicates that the echoes of different oblique radar beams were mainly from the scatterers having similar aspect sensitivity, and the degree of aspect sensitivity was high.

b. Aspect angle measured by comparison of echo powers

In this part of study, Eq. (2) was employed to estimate the aspect angle $\theta_s$, in which the parameter $\theta_o$ was constant. For the Tx7/Rx7 mode, the 3-dB half-beamwidth is $\sim 3.46^\circ$ (VanZandt et al. 2002), which corresponds to the value of $\theta_o = 3.46^\circ/\sqrt{\ln 2}$ for the $\theta_o$ in (2).

Figure 6 shows the profiles of mean $\theta_s$ for all pairs of radar beams in the experiment 1, as denoted by 0/1, 0/2, $\ldots$, 3/4. The number represents the off-zenith angle of the radar beam; for example, 0/1 means the comparison between vertical and $1^\circ$ tilted radar beams. Also plotted in Fig. 6 is $B_{wc}$.

Remember that the sampling range gate of vertical and oblique radar beams covers the scatterers in different height intervals, which may become a considerable problem for the radar beam tilted to a larger zenith angle and/or at higher altitude. In experiment 1, the maximum tilt angle of the radar beam is only $4^\circ$, so the above problem can be ignored. To make the comparison of $\theta_s$ more reasonable along altitude, however, the ordinate of each curve is the mean height of the two sampling gates of the beam pair.

Compared with Fig. 5, some similar and different features can be found, as described below for different height intervals.

1) Below $\sim 4$ km

(i) The aspect angle was obviously a function of the beam pair. Taking the beam pairs 0/1, 0/2, 0/3, and
0/4 as examples, the aspect angle became larger when the second beam was tilted to a farther off-vertical direction. Besides, it can be found that these estimated aspect angles were close to those obtained from the CRI–DOA (see Fig. 5), supporting the usability of CRI–DOA in estimating the aspect angle.

(ii) The aspect angles derived from the beam pairs 1/2, 1/3, 1/4, 2/3, 2/4, and 3/4 also varied with the beam pair. A similar feature has been demonstrated by Hooper and Thomas (1995).

(iii) The CRI brightness width was generally smaller than the aspect angle obtained by the comparison of echo powers. However, it was closer to that of the 0/1 and 0/2 beam pairs, stating again that the CRI brightness width of a vertical beam can present the anisotropic feature of the scatterers located around the zenith.

2) BETWEEN ~4 AND 10 KM (MIDDLE TROPOSPHERE)

(i) The aspect angles were close to each other, especially around the height of 7.5 km, as has been revealed in Fig. 5. In view of this, we can speculate that almost only one shape of scatterers, of which
the aspect sensitivity can be characterized with a Gaussian-type function, contributed to the major part of the radar echoes around the height of \( \sim 7.5 \) km.

(ii) The heights having smaller aspect angles, as observed in Fig. 5, can also be distinguished (i.e., at \( \sim 4, \sim 5, \sim 6 \) km), especially from the results of southern beams. However, the degree of recognition is not as high as in Fig. 5. This has indicated one benefit of CRI in distinguishing the difference between isotropic and anisotropic scatterers that can raise the degree of recognition of different scattering layers.

(iii) The aspect angle obtained by the comparison of echo powers was close to but generally smaller than Bw, especially around the height of 7.5 km. Also, it was smaller slightly than that derived from CRI–DOA, as compared with Fig. 5.

3) Above 10 km

In this height region, the SNR was very low so that the results were not very reliable. However, it is interesting to see the different statistical behaviors of the aspect angles at noise-governing situation. The aspect angles obtained from the comparison of echo powers varied around 5°, while those indicated by Bw were about 7°. The aspect angles derived from CRI–DOA were even larger, which varied around 10°, as shown in Fig. 5. This implies that the methods of brightness width and CRI–DOA are very sensitive to the SNR, which is actually also revealed below the height of \( \sim 10 \) km.

c. Results of experiment 2

The second experiment was carried out to supplement the first experiment, with the radar beams tilting to some larger zenith angles, as listed in Table 1. This observation also detected many echoing layers at \( \sim 4 \) km and between \( \sim 6 \) and \( \sim 10 \) km, as shown in Fig. 2. Respective mean profiles of aspect angles are exhibited in Fig. 7, but only the western and southern directions are shown for saving space in the article; the other two directions have similar features to those exhibited here.

First, Bw was larger than Bw, as shown in Figs. 7a,b, which has been demonstrated in Fig. 5. The profile of Bw discloses two height intervals with smaller values, that is, \( \sim 4 \) and \( \sim 8 \) km. The two height intervals are also indicated clearly by the profiles of 6° and 8° tilted radar beams, but they are more difficult to identify from the profile of 10° tilted radar beam, implying the 10° tilted radar beam observed mainly isotropic scatterers, which is consistent with the conclusion given in the literature (Röttger and Vincent 1978). Second, an oblique radar beam at a larger tilt angle observed a larger aspect angle and the aspect angle became larger at a lower SNR; these features are similar to the first experiment.

Regarding the results obtained from the comparison of echo powers, as shown in Figs. 7c,d, two noticeable characteristics below \( \sim 10 \) km include the following:

(i) A dependence of the aspect angle on the beam pair also existed but the differences between these profiling curves were small, as compared with Figs. 7a,b. The altitudinal variation in the aspect angle was apparent still.

(ii) Two height intervals with a smaller aspect angle were located at \( \sim 5 \) km, and between \( \sim 6 \) and \( \sim 9 \) km. The height interval located at \( \sim 5 \) km was slightly higher than that illustrated by CRI–DOA, and the second height interval did not have a remarkable minimum of the aspect angle like that indicated by CRI–DOA at \( \sim 8 \) km. Such a discrepancy could be owing to different sampling height intervals of range gates of the two oblique radar beams used for the comparison of echo powers, which cannot yield a precise comparison in height and so cause a smear of the aspect angle along height.

d. Information from standard deviation of aspect angles

The case shown in Fig. 4 gives us the impression that aspect angles are more concentrated when they are at smaller values. This implies that a smaller standard deviation of aspect angles (\( \sigma_z \) for convenience) may exist for smaller aspect angles. This is demonstrated in Fig. 8, where all mean profiles of \( \sigma_z \) of the two experiments are exhibited. For the reason of saving space, however, only the results of the eastern beam direction are exhibited; the other beam directions have the features consistent generally with the eastern direction.

In Figs. 8a,c, the profile of \( \sigma_z \) for Bw is nearly constant; by contrast, the profile of \( \sigma_z \) for Bw shows an observable variation with SNR. Such a difference in \( \sigma_z \) could be again related to the crucial effect of radar beam weighting function on the original CRI brightness. A smaller \( \sigma_z \) indicates a greater similarity in aspect sensitivity between the scatterers. In view of this, the scatterers located at \( \sim 7.5 \) km, where the values of \( \sigma_z \) of all the radar beam directions in Fig. 8a drop down apparently, were very similar, and moreover, they should be highly anisotropic because of their smaller mean values (see Fig. 5). One more noticeable feature is that the profiles of \( \sigma_z \) obtained from differently tilted radar beams were very close except for the 10° tilted radar beam in Fig. 8c, in which \( \sigma_z \) of the 10° tilted radar beam was apparently smaller than others and so implies that
the 10° tilted radar beam observed very similar scatterers; these scatterers were nearly isotropic.

As for the $\sigma_z$ obtained from the comparison of echo powers, as shown in Fig. 8b, the altitudinal variation in $\sigma_z$ was generally similar to that derived from the CRI–DOA shown in Fig. 8a. In Fig. 8d, the beam pair 8/10 is expected to indicate mainly the characteristics of nearly isotropic scatterers and so should have a smaller $\sigma_z$ like that of the 10° tilted radar beam in Fig. 8c; however, this expectation is not seen. We leave it as an open question.

5. Conclusions

In this study, the aspect angle of refractivity irregularities in the clear air was estimated by means of coherent radar imaging (CRI) of VHF atmospheric radar.
Statistical comparison between the aspect angles derived from the direction of arrival (DOA) of the echo center with oblique beam CRI, the brightness width of vertical beam CRI, the comparison of echo powers between different oblique radar beams, and the power distribution of multiple oblique radar beams was made. Differing from previous investigations of aspect sensitivity with CRI, our study of CRI is dependent on the adaptable beamwidth, in which the adaptable beamwidth is a function of the off-beam-direction angle and the signal-to-noise ratio (SNR). The following are some explicit results obtained for the altitude below ~10 km:

(i) For the vertical beam CRI, the correct brightness distribution yielded an altitudinal variation of

![Graphs showing mean profiles of standard deviation of aspect angles in the eastern direction.](attachment:image.png)
brightness width more visible than that obtained from the original brightness distribution.

(ii) The aspect angles derived from the CRI–DOA could indicate well the layers with high aspect sensitivity. However, the estimated aspect angle was positively proportional to the off-zenith angle of the radar beam, indicating various shapes or various degrees of anisotropism of scatterers in the clear air. More isotropic scatterers were observed by the radar beam at a larger off-zenith angle, which is consistent with the studies in the literature.

(iii) The corrected brightness width of the vertical radar beam was close to the aspect angle derived from 1° tilted radar beam, which is logical because the two beam directions can observe similar anisotropic scatterers or specular reflectors around the zenith. Moreover, the aspect angle indicated by the corrected brightness width was also consistent with that of the power distribution of multiple beams in the zonal and meridional directions, supporting the applicability of adaptable beamwidth.

(iv) The aspect angles derived from the CRI approach have shown altitudinal variations that are consistent with the comparison of echo powers of radar beam pairs, in spite of the slight difference in magnitude.

The above-mentioned features demonstrate the usability of CRI parameters on investigating the aspect sensitivity of the clear air in the troposphere. However, it is shown that the CRI-derived aspect angle was sensitive to the SNR, and it was remarkably larger than the values derived from the power distribution and the comparison of echo powers when the SNR was lower; this could arise from the characteristics of the Capon method used with CRI. The performance of Capon and some other inversion algorithms are SNR dependent; this is an intrinsic characteristic of these algorithms.

A further comparison of the aspect sensitivity between the CRI brightness distribution and the 2D echo power distribution formed with multiple beam directions is worth making. This, however, has not been done because the radar beam directions used in this study are only in the zonal and meridional directions, which are not enough to form the 2D echo power distribution for comparison.

Acknowledgments. This work was supported by the National Science Council of ROC (Taiwan) through Grants NSC99-2111-M-270-001 and NSC101-2111-M-270-001, and also supported by the International Collaborative Research Program of MU radar (Reference 21MU-A-43). The MU radar is operated by the Research Institute for Sustainable Humanosphere, Kyoto University, Japan.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of horizontal</td>
<td>25 m</td>
</tr>
<tr>
<td>Central heights of the radar</td>
<td>3150 m</td>
</tr>
<tr>
<td>volume and layer position: z0</td>
<td></td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>52 MHz</td>
</tr>
<tr>
<td>Receiver locations in meter (x, y): D</td>
<td>(-6, -6), (0, -6), (-6, -6), (-6, 0), (0, 0), (6, 0), (-6, 6), (0, 6), (6, 6)</td>
</tr>
<tr>
<td>Pulse length</td>
<td>2 μs</td>
</tr>
<tr>
<td>Radar half-beamwidth: θi</td>
<td>3.5°</td>
</tr>
<tr>
<td>Beam direction: θbx, θby</td>
<td>θbx = 0°, 1°, 2°, 3°, 4°, 5°, 6° in zonal, θby = 0°</td>
</tr>
<tr>
<td>Aspect angle: θi</td>
<td>4°, 5°, 6°, 7°</td>
</tr>
</tbody>
</table>

APPENDIX

Simulation Study of Adaptable Beamwidth for Aspect Sensitivity

The output voltage of a receiver for the echoes backscattered from the refractivity irregularities in the radar volume is simply described as (Chen et al. 2011)

\[
S \propto \int_v L(a, R)W(a, R) \exp(-j2kR) dv, \tag{A1}
\]

where \( W(a, R) \) gives the weighting function in range and angular directions, \( k \) is the wavenumber of the carrier frequency, \( R \) is the range, and vector \( a \) indicates the angular direction; \( \int_v \) denotes the radar volume of integration. The aspect sensitivity function can be included in \( L(a, R) \), which also expresses the distribution of scatterers. For CRI, the cross-correlation function between signals from the \( i \)th and the \( j \)th receivers can be written as

\[
R(D_j, D_j, k) = S_iS_j = \int_v L^2(a, R)W^2(a, R) \exp(j\phi) dv, \tag{A2}
\]

\[
\phi = ka \cdot D_j - ka \cdot D_i, \tag{A3}
\]

where \( D \) is the location of the receiver. The present simulation assumes a horizontal scattering layer located at the center of the radar volume, and \( L^2(a, R) \) and \( W^2(a, R) \) are expressed by Gaussian functions in both angular and range/height directions as shown:

\[
L^2(a, R) = L^2(a, z) = \exp\left(-\frac{(z - z_0)^2}{2\sigma_i^2}\right)\exp\left(-\frac{\theta_x^2}{2\sigma_x^2}\right)\exp\left(-\frac{\theta_y^2}{2\sigma_y^2}\right), \tag{A4}
\]
where \( z \) is the variable of height, \( z_o \) and \( s_l \) are the mean altitude and standard deviation (thickness) of the scattering layer, respectively. The second and third terms of \( L^2(a, R) \) represents aspect sensitivity with an aspect angle of \( \theta_s \). Also, \( \theta_x \) and \( \theta_y \) represent the variable of the angle in the zonal and meridional directions, respectively; \( \theta_{bx} \) and \( \theta_{by} \) indicate beam direction; \( \theta_t \) is beam-width; and \( \sigma_r \) is given as \( 0.35ct/2 \), where \( c \) is the speed of the light and \( t \) is the pulse length.

Table A1 lists the parameters for simulation, in which the half-beamwidth was 3.5°, the aspect angle was given from 4° to 7°, and the radar beam was tilted from vertical to 6° in the zonal direction. The Capon method was used to yield the brightness distribution that contains the effects of radar beam weighting, aspect sensitivity, Capon beam weighting, and, of course, some inherent computation error. The following adaptable beamwidth \( \theta_e \) was chosen for correcting the beam weighting effect:

\[
\theta_e = b \times \theta^3 + \theta_{oe},
\]

where \( b = 0.0025 \), \( \theta_{oe} = 3.5° \), and \( \theta \) is the off-beam angle (in degrees). Note that \( \theta_e \) can be regarded as the parameter related to the radar beam and the Capon beam weighting effects. With the same computing processing of the aspect angle from the brightness distribution proposed in this study, the result is shown in Figure A1. The left panel shows the adaptable beamwidth (A6), and the right panel displays aspect angle varying with the beam direction of CRI (the solid curves with symbol). For the vertical beam CRI, however, the width of the brightness distribution was estimated to represent the aspect angle. Widths of brightness distribution without correction of the beam weighting effect were also estimated for comparison. To improve the reliability of the estimated aspect angle, each case was performed 50 times with small perturbation, randomly produced between \( -5 \times 10^{-6} \) and \( 5 \times 10^{-6} \), in the calculated cross-correlation function of each receiver pair. The arithmetic mean of the 50 estimates is that shown in the right panel. Also shown is the standard deviation of the 50 estimates, presented by the vertical stick. Notice that the four sticks of each group give the results for the simulation cases with aspect angles of 4°, 5°, 6°, and 7°, respectively.

As seen, for each case of aspect angle given in the simulation, the aspect angles retrieved from the oblique beams are generally consistent with the model in spite of some biases. The standard deviations of the aspect angles are mostly smaller than 0.5°, except for the beam directions of 1° and 2°. The vertical beam CRI also yields a reasonable aspect angle after correction of the beam weighting effect. Without correction of the beam weighting effect, however, the aspect angles of the vertical beam CRI are about 3°, which underestimates the value evidently. In view of this, the workability of the adaptable beamwidth is demonstrated to some degree.

REFERENCES


