Ice Crystal Sizes in High Ice Water Content Clouds. Part I: On the Computation of Median Mass Diameter from In Situ Measurements

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ABSTRACT

Engine and air data probe manufacturers, as well as aviation agencies, are interested in better characterization of high ice water content (HIWC) areas close to thunderstorms, since HIWC conditions are suspected to cause in-service engine power loss and air data events on commercial aircraft. In this context, a collaborative field campaign has been conducted by high-altitude ice crystals (HAIC) and HIWC projects in order to provide ice water content and median mass diameter (MMD) of ice crystals in the HIWC environment.

The computation of MMD from in situ measurements relies mainly on the definition of the crystal dimension $D$ and on the $m = aD^b$ relationship, which is used to convert number into mass distributions. The first part of this study shows that MMD can significantly deviate when using different mass–size relationships from the literature. Sensitivity tests demonstrate that MMD is significantly impacted by the choice of $b$. However, the larger contributor to MMD differences seems to be the choice of the size definition $D$ itself.

Since MMDs are quite sensitive to $b$, this study suggests a generic method for deducing $b$ solely from optical array probes (OAPs) image data for various size definitions. The method is based on simulations of 3D crystal objects projected onto a 2D plane, thereby relating crystal mass to 2D area (projection) and perimeter. The MMD values calculated for different size definitions are quite similar, at least much closer than MMDs derived from different $m(D)$ relationships in the literature.

1. Introduction

a. Context of the study

In the mid-1990s, a specific commuter-class aircraft experienced a series of engine power-loss events while flying near thunderstorms. Researchers concluded these engine rollback events were due to the ingestion of ice particles (Lawson et al. 1998). In 2004, as a result of an unrelated aircraft accident attributed to flight in supercooled large droplet conditions, the Engine Harmonization Working Group (EHWG) was tasked to review available in-service engine power-loss events encountered by commercial aircraft flying in mixed-phase/glaciated cloud conditions. The EHWG concluded that engine power loss in ice crystals was an industrywide problem. This newly recognized “ice crystal icing” did not require liquid water in the atmosphere, and was again attributed to flight through high mass concentrations of ice crystals, usually in mesoscale convective systems (MCSs) with relatively low radar reflectivity at the event altitude (Mason et al. 2006; Grzych and Mason 2010; Mason and Grzych 2011). However, the EHWG further concluded that the occurrence and concentrations in such high ice concentration [or high ice water content (HIWC)] areas in MCSs, as well as the ice crystal properties and formation processes in these areas, remained poorly documented. In the late 2000s, failures of air data probes on commercial aircraft were observed in increasing frequency in similar cloud conditions, and ice crystal icing was again suspected.

The EHWG created a technical plan to increase understanding of the ice crystal icing phenomenon that included the recommendation for new in situ and remote cloud characterization measurements of clouds similar to...
those that cause ice crystal icing events. Moreover, these microphysical measurements of HIWC conditions were supported with great interest by aviation safety agencies, most notably the Federal Aviation Administration (FAA) and European Aviation Safety Agency (EASA), in order to assess new regulations for aircraft engine and air data probes that have recently become law.

b. Documenting the HIWC environment

Within the overall goal to fulfill EHWG, FAA, and EASA objectives, the multinational international collaborative high-altitude ice crystals/high ice water content (HAIC/HIWC) flight measurement field program has been established (Dezitter et al. 2013; Strapp et al. 2016), and the first flight campaign has been conducted out of Darwin, Australia, in 2014. The French Falcon 20 research aircraft mainly operated in oceanic MCS, mostly at high altitudes, measuring HIWC during multiple flights.

Since the relation between ice water content (IWC) and the representative particle size seems to be of primarily importance for the study of power-loss events, specific emphasis has been placed on gathering adequate Falcon 20 measurements to determine the median mass diameter (MMD) of ice crystal populations. The MMD gives a practical single variable describing a mass-weighted size of the ice particles, analogous to the much-used median volume diameter (MVD) used by industry for conventional supercooled liquid water content (LWC). Particles smaller than the MMD account for 50% of the total water content (TWC), and particles larger than the MMD account for the other 50%. High IWCs (nominally >2 g m⁻³) with relatively low MMDs (below 300 μm) may thus denote cloud areas that are difficult to detect with onboard radar, therefore possibly representing a threat for commercial aircraft.

c. MMD computation: An open question

Unfortunately MMDs cannot be currently directly measured. However, airborne instrumentation provides measurements from which the MMD can be estimated, namely, the bulk TWC, and individual 2D ice crystal properties from optical array probes (OAPs) from which particle size distributions (PSDs) can be computed. The computation of the MMD from TWC and imaging probe PSDs is not trivial and requires the application of some hypotheses. Starting from the OAP images, image processing is used to derive the particle size D for each individual image, after which a population of crystals is analyzed and sorted by their dimension D into size bins of number concentration versus dimension (the PSD for the population of ice crystals). Then a mass–size relationship, \( m = aD^b \), is applied to convert PSDs into mass–size distributions from which the MMD can be directly calculated.

Within the HAIC/HIWC scientific group, there was no initial consensus on the best crystal size parameter or mass–size relationship to be employed for MMD calculations. Thus, there was a clear need to investigate these issues before providing results to the EHWG and aviation agencies. This paper presents the results of those investigations. We focus only the MMD sensitivity to computation hypotheses \([m(D)\text{ coefficients, size definition}].\) MMD changes as a function of cloud properties will be further investigated in Part II (Leroy et al. 2016).

The next section describes the HAIC/HIWC dataset used in this study and the associated post–project processing. In section 3, the sensitivity of the MMD to the mass–size relationship \(m(D)\) and to the size parameter \(D\) is presented. As the exponent \(\beta\) of the \(m(D)\) relation appears to be a key parameter in the process, an advanced method to retrieve \(\beta\) is proposed that allows changes in \(\beta\) as a function of time (and thus ice crystal habits and cloud conditions) and is valid for several size definitions.

2. HAIC/HIWC dataset

a. First field campaign

The first HAIC/HIWC flight campaign took place in Darwin, Australia, during the 2014 monsoon period. The French Falcon 20 aircraft performed 23 research flights, most of them in oceanic MCSs to the southwest of Darwin over the Joseph Bonaparte Gulf and off the coast of the Kimberley Plateau. The sampling strategy (Strapp et al. 2016) was based on flying long legs at constant altitude, mostly at \(-40^\circ\) and \(-30^\circ\) C levels, and to a lesser extent at \(-50^\circ\) and \(-10^\circ\) C. The results presented in this paper exclude calibration (flights 1, 5, and 21) and transit (flight 11) flights, since no cloud sampling was performed during these flights. Moreover, only data collected at constant flight levels (attempting to maintain temperature within \(\pm 2^\circ\) C of the targeted value) have been selected, whereas data sampled during ascents and descents (a minor fraction of the data) were excluded. More details regarding the dates and times of all flights, flight levels, and geographic location can be found in Leroy et al. (2016).

The Falcon 20 was equipped with a series of measurement devices (in situ microphysics probes, research radar, humidity sensors, etc.); however, only those instruments utilized for this study are described below. A state-of-the-art in situ microphysics package included a
cloud droplet probe (CDP-2; cloud droplet spectrometer; Baumgardner et al. 2011), 2D-stereo (2D-S; Lawson et al. 2006), and precipitation imaging probe (PIP; Baumgardner et al. 2011) OAPs. The CDP-2 sampled the smallest cloud particle sizes from nominally 3 to 50 μm. The 2D-S and PIP were used to image and size particles from 10 to 1280 μm at 10-μm resolution and 100–6400 μm at 100-μm resolution, respectively. Both the 2D-S and PIP provide size information of particles larger than the nominal maximum particle size through image reconstruction.

Finally, bulk measurements of the total water content were performed with an isokinetic evaporator probe (IKP) from Science Engineering Associates, Inc. (SEA). The prototype IKP was designed especially for HIWC measurement at high speed (Davison et al. 2008), and then a downsized version (IKP2) was designed to meet installation and certification requirements on the Falcon 20. Additional measurements of the background humidity and usual flight parameters (altitude, temperature, GPS position, etc.) were also made.

The vast majority of the data were collected in glaciated clouds. The presence of liquid water has been investigated by examining the measurements from the Rosemount Icing Detector (Baumgardner and Rodi 1989; Claffey et al. 1995; Cober et al. 2001), the CDP, and the 2D-S. For the flight-level data colder than −10°C examined for this study, the clouds were almost exclusively glaciated, and supercooled water droplets were identified for only a few seconds during flights 3, 13, and 15. This very small fraction of data, for which only small amounts (below 0.2 g m⁻³) of liquid water were present, has been excluded from the analysis and thus reduces the IKP2 data to clouds with at most traces of LWC that cannot be detected using the three instruments noted above. For all practical purposes, all of the constrained m(D) parameters described later in this study are relevant to ice-phase clouds.

b. Data processing

1) TWC FROM THE IKP

The IKP2 has been designed to measure TWC up to at least 10 g m⁻³ at airspeeds of 200 m s⁻¹, and is considered a primary bulk TWC measurement for the HAIC/HIWC flight program. The instrument is wing mounted and adjusts in real time to maintain nearly isokinetic sampling through a 7-mm-diameter inlet. The probe ingests both condensed particles (crystals, droplets) and non-condensed water vapor into the inlet without significant mass loss. An evaporator section then vaporizes all condensed water (solid, liquid) remaining after passage through the heated inlet, and subsequently measures the total water vapor with a LI-COR (model 840A) hygrometer before the air exits the probe. The total condensed water content is then calculated by subtracting the background water vapor [measured on the Falcon 20 by another LI-COR 840A and a Water Vapor Sensing System II (WVSS-II) hygrometer] from the total IKP2 vapor measurement.

The differential hygrometry method used by the IKP2 is particularly suitable for higher TWCs and lower temperatures. The minimum detectable TWC is temperature dependent due to errors related to the magnitude of the background humidity subtraction, and it varies from about 0.1 g m⁻³ at −10°C to about 0.005 g m⁻³ at −50°C. To eliminate clouds below the detection limit, low TWC clouds, and out-of-cloud periods, data points with IKP2 TWC values below 0.1 g m⁻³ have been excluded from the analysis presented here.

2) PARTICLE SIZE DEFINITION

As displayed in Fig. 1, two simple dimensions can be derived from the dimensions of a rectangular box that fully encloses the particle image (Lawson 2011): the size Dz is the box dimension along the direction of the photodiode array and Dx is the box dimension along the perpendicular direction (along the axis of the forward movement of the aircraft). The resulting mean of the box lengths Dx and Dy, Dm = (Dx + Dy)/2, is called the mean chord length and is employed in the widely used mass–size relationship used in Brown and Francis (1995, hereafter BF95).

With improving computer capabilities, the maximum dimension Dmax and the 2D area equivalent diameter (Deq) are now commonly extracted from the images. The term Deq is the diameter of the circle having the same area as the shaded pixels in the particle image: A = (π/4)Deq² (McFarquhar and Heymsfield 1996). The Deq size definition is unambiguous, whereas the Dmax value can differ from one study to another due to the way it is computed. McFarquhar and Heymsfield (1996) defined the Dmax as the largest particle dimension either along the main flow (Dx) or along the photodiode array (Dy), whichever was larger. The maximum diameter of Korolev and Isaac (2003) was calculated as the maximum Dy among all possible orientations of the recorded 2D image, whereas Heymsfield et al. (2013, p. 4124) defined the Dmax as “the smallest diameter of a circle that fully encloses the projected 2D image.” The definition used in this study also varies from the others above in that Dmax is the largest length through the center of the particle image. Recently, Wu and McFarquhar (2016) compiled various definitions for Dmax, studied their impact on PSDs and bulk parameters, and concluded that number concentrations can change by up to a factor of 6, especially
for particles smaller than 200 \( \mu \text{m} \) or larger than 2 mm. Changes in the depth of field (and thus the sample volume) are responsible for the large differences below 200 \( \mu \text{m} \). In addition, particles are rare in the largest size bins (>2 mm), and differences can be exacerbated as large particles can be placed into quite different size bins according to the chosen particle size definition.

To provide guidance on the sensitivity of PSD parameters to those using the HAIC/HIWC dataset for aviation industry applications (e.g., agencies developing wind tunnel cloud simulations, aircraft and engine manufacturers, regulatory agencies), this study explores a variety of diameter definitions related to crystal mass. The size definition \( D_y \) is used to compute the appropriate effective array width in the sample volume. The term \( D_m \) is the size that was used originally in the BF95 relationship. Baker and Lawson (2006) present a more sophisticated algorithm evaluating the crystal mass from more geometric parameters (area \( A \), perimeter, maximum diameter, width) of individual crystals. Their formula for mass retrieval has been fitted to a dataset of crystals collected at the ground from precipitating winter storms, and like other empirical studies relating the mass of ice crystals physically captured at the ground to size parameters, it may not accurately describe the properties of ice crystals in deep convective clouds in high IWC conditions at high altitude. However, as Baker and Lawson (2006) also highlight the good results obtained by relating mass to the area \( A \) of the particle image, \( D_{\text{eq}} \) is chosen here as another candidate for analysis of the HAIC/HIWC dataset and for comparisons to other size definitions. As the final HAIC/HIWC MMD dataset will be distributed to the entire HAIC/HIWC community (including wind tunnel modelers, jet engine manufacturers, airframers, and other aviation agencies), changes in MMD as a function of PSD size definition need to be addressed beforehand. In this paper, we tested the impact of four size definitions \( D_y \), \( D_m \), \( D_{\text{eq}} \), and \( D_{\text{max}} \) on MMD.

### 3) OAP IMAGE PROCESSING

Computing the PSDs of hydrometeors from OAP images is not trivial because the measurements contain many complications and artifacts.

Some particles appear as partial images. In such cases, the reconstructed particle sizes are calculated according to Korolev and Sussman (2000).

In addition, a fraction of cloud particles inevitably hit the probe’s housing during sampling and may break up into multiple fragments that are recorded by the probe (Field et al. 2003; Korolev and Isaac 2005; Heymsfield 2007). During the HAIC/HIWC field campaign, the frequency of such events was reduced by using specially designed probe leading edge tips to minimize shattering. However, the remaining images related to splashing/shattering events had to be removed as effectively as possible; otherwise, PSD measurements and their derived microphysical properties would be subject to errors. Most of the images related to a shattering/splashing event were removed by a careful analysis of (i) the ratio between the particle’s area and its sizes in the \( x \) and \( y \) directions and (ii) the inter-arrival times between neighboring particles. The inter-arrival time technique is commonly used and has been described and tested by Field et al. (2006), Baker et al. (2009), Lawson (2011), and Korolev and Field (2015). In this study, the image processing rejected particles presumed to be associated with shattering if their inter-arrival times were lower than a cutoff value that was calculated once per second.
Furthermore, noisy pixels were eliminated using a technique described in Lawson (2011), although these were not a significant issue in the HAIC/HIWC dataset anyway. In addition, out-of-focus particles were identified using an algorithm that detects the area fraction of the Poisson spot in a particle image. The size of these particles was then corrected according to an algorithm described by Korolev (2007).

During the unloading of particle image data to the probe data system, OAPs are unable to continue to record images and rearm only when the unload is complete. The details of this period of probe dead time vary somewhat according to the probe and data system, but in high concentrations of particles, all probes are subject to this overload condition. The time of the overload must be taken into account in the sample volume computation (needed to estimate the number size concentrations of accepted images as a function of time). The overload time is measured directly only by the 2D-S. For the PIP, the probe continues to count the number of all particles passing through the beam even during the overload. Thus, for this probe, the image overload is estimated by comparing the number of recorded images to the total number of counted particles.

The 2D-S and PIP PSDs were merged to produce one single composite size distribution ranging from 10 \( \mu \text{m} \) up to 1 cm (Fontaine et al. 2014, hereafter F14). The 2D-S was used in the composite distribution for sizes smaller than 800 \( \mu \text{m} \). Likewise, the PIP was used for sizes larger than 1200 \( \mu \text{m} \). Between 800 and 1200 \( \mu \text{m} \) the composite distribution is a linear weighted mean of the two distinct distributions:

\[
N(D) = (1 - \omega)N_{2D-S}(D) + \omega N_{\text{PIP}}(D)
\]

for \( D \in [800; 1200] \),

whereby

\[
\omega = \frac{D - 800}{1200 - 800}.
\]

To increase the probe sample volume and improve the statistics for the largest particles, the composite PSDs were produced at a 5-s time resolution.

3. MMD sensitivity tests

a. Overview of the MMD changes when using simple mass–size relationships from literature

The simplest way to estimate the sensitivity and range of the MMDs to the image size definition and mass–size relationship is to use mass–size relationships already available in the literature.

The literature contains a variety of such relationships based on the analysis of ground and airborne data (e.g., Locatelli and Hobbs 1974; BF95; Mitchell 1996; McFarquhar et al. 2007; Heymsfield et al. 2010, hereafter H10; Cotton et al. 2013, hereafter C13; Wood et al. 2013, 2014; F14). The \( \alpha \) and \( \beta \) values in the \( m(D) \) relationships have been shown to depend on the following:

- Cloud conditions that have driven the crystal formation (dynamics, thermodynamics). H10 observed that the coefficient \( \alpha \) is about a factor of 2 higher in convective versus stratiform areas of deep convection. Heymsfield et al. (2007) highlight that \( \alpha \) is temperature dependent in synoptic clouds but rather independent of temperature in convective clouds. Heymsfield et al. (2013) also report that both \( \alpha \) and \( \beta \) increase with temperature.
- Ice crystal habit (Locatelli and Hobbs 1974, Mitchell 1996). Schmitt and Heymsfield (2010) also proposed a mass–size relationship inferring a fractal dimension that improved integrated IWC estimates when aggregates are abundant.

Mass–size relationships have also been generalized over a whole dataset yielding constant values for \( \alpha \) and \( \beta \) representative of the average properties of the dataset (H10; C13; F14). Other methods than the classical \( m(D) = aD^\beta \) have also been proposed. Baker and Lawson (2006) developed a single parameter designed to synthesize particle shape information of 2D images (combining maximum size, width, perimeter, and area), and related that parameter to the masses of ice crystals collected in physical collections, thereby enabling the estimation of individual ice crystal masses of 2D images. Recently, Erfani and Mitchell (2015) proposed second-order polynomials that can be reduced to power-law expressions, resulting in size-dependent values of \( \alpha \) and \( \beta \).

To get a preliminary overview of the MMD dependency to the mass–size relationship, in this section, we selected four of the aforementioned existing mass–size relationships, with constant values for \( \alpha \) and \( \beta \) (which have been developed based on particular datasets and are not habit dependent): BF95, H10, C13, and F14. The selected mass–size relationships are summarized in Table 1, where mass is in grams and \( D \) is in centimeters. The BF95 formulation uses the \( D_m \) size parameter, whereas the three more recent studies use the crystal maximum size \( D_{\text{max}} \). Thus, testing these four relationships will give an overview of how changes in both \( \beta \) and the definition of \( D \) impact the calculated MMD values.

The most commonly used mass–size relationship from BF95 (more than 274 citations) is directly extracted from the relationship proposed by Locatelli and Hobbs (1974, p. 2188) for “aggregates of unrimed radiating assemblages of plates, side planes, bullets, and columns.”
BF95 has been shown to produce reasonable bulk TWC estimates that agree well with estimates from other instruments (H10). To avoid crystal masses larger than those of a corresponding solid ice sphere, the BF95 formulation is applied only for sizes larger than 97 μm. Below, ice particles are assumed to be ice spheres (β = 3) with a bulk density of 0.917 g cm^-3.

H10 used data from six field campaigns in order to gather measurements in different cloud types and for different temperatures in order to propose a universally valid mass–size relationship. A value of β close to 2.1 was found to best match the observations for both stratiform and convective cloud cases and to be applicable over a wide range of temperatures and particle sizes, but α could vary by a factor of 2 between convective and stratiform clouds. As for BF95, and in order to avoid masses larger than solid ice spheres of the same diameter, the m(D) relationship of H10 was applied only for sizes larger than 51 μm.

The C13 mass–size relationship was derived from cirrus observations and thus from low ice water content clouds (below 0.05 gm^-3) and populations of relatively small particles (800 μm as the maximum dimension). C13 suggested that for particles smaller than 70 μm, the solid sphere model should be used but with a constant density of 0.7 rather than 0.917 g cm^-3.

Finally, F14 established a mass–size relationship from measurements collected in tropical convective anvils during the Megha-Tropiques project, from two measurement campaigns performed in 2010 and 2011, yielding an averaged mass–size relationship for each campaign. Here, we will test only the relationship established for the 2011 field campaign that is focusing solely on oceanic convection (rather than continental convection in 2010), which should be more comparable to the oceanic-type MSC systems sampled during the Darwin HAIC/HIWC campaign. F14 derives β from particle imagery, and α is subsequently constrained using simulated and in situ–measured radar reflectivity. F14 is applied for particles larger than 89 μm.

These four mass–size relationships cover a wide range of different cloud conditions, temperatures, particle size distributions, and ice crystal habits that could correspond more or less to the spectrum of conditions sampled in Darwin, bearing in mind that the Darwin campaign was more specific in targeting high-altitude/high-IWC conditions in tropical oceanic MCS. For each mass–size relationship, MMDs have been calculated from the PSD with the size definition appropriate to the original study. Results are presented in Fig. 2. Each MMD dataset has been divided into 10-μm bins and counts in each bin are compared. The mean MMD values are presented in Table 2.

Figure 2 and Table 2 clearly illustrate the uncertainty in MMD resulting from differences in these four example mass–diameter formulations. As per the linear fits of Fig. 2, MMD_max values from F14 are within 10% of those of C13 at all MMDs > 100 μm, but are consistently larger than MMD_m values from BF95 for sizes larger than about 160 μm (e.g., 20% and 26% higher at 500 and 1000 μm, respectively). Looking at the counts, MMD_max values from F14 are also lower than MMD_m values from H10 with magnitude varying with size (e.g., 3%, 7%, and 12% lower at 300, 500, and 1000 μm, respectively). The differences relative to the F14 average MMD_max vary from −17% for BF95 to +3% for H10.

Differences in MMD for F14, H10, and C13 are only due to the values of the parameters α and β, whereas the choice of D_m versus D_max for BF95 and F14 contributes additional differences to MMD. Section 3b discusses in more detail the influence of α and β values on MMD, whereas section 3c will further explore the MMD sensitivity to the chosen size definition parameter.

b. Sensitivity to the mass–size relationship

1) THEORETICAL CONSIDERATIONS AND TEST METHOD

Analytically, the MMD is defined by the following equation:

$$MMD = \int_0^{\infty} \alpha D^\beta N(D) dD = \frac{1}{2} \int_0^{\infty} \alpha D^\beta N(D) dD,$$

where N(D) is the number size distribution. From this equation, Mitchell (1991) states that MMD is independent of the α coefficient. However, this relies on the hypothesis that α does not depend on the size D, which is not entirely true. Schmitt and Heymsfield (2010) deduced α from particle area analysis, with α relating to the fractal dimension of the particle. Recently, Erfani and Mitchell (2015) proposed second-order polynomials
for the mass–size relationship that can be reduced to a
power-law expression in which the final $a$ and $b$
values are dependent on $D$. All of the more classical $m = \alpha D^b$
relationships with constant values for $\alpha$ and $\beta$ produce
masses larger than corresponding solid spheres of the same
diameter at small crystal sizes, and thus each comes with
a lower threshold size $D_{\text{lim}}$. Below this size, crystals are
commonly assumed to be spheres (i.e., $\beta = 3$) with constant
ice density (e.g., 0.917 g cm$^{-3}$ for BF95 and F14; 0.7 g cm$^{-3}$
for C13). Thus, the above-mentioned equation becomes

$$\int_0^{MMD} \alpha(D) D^\beta N(D) \, dD = \frac{1}{2} \left[ \int_0^{D_{\text{lim}}} \alpha_1 D^3 N(D) \, dD \\
+ \int_{D_{\text{lim}}}^\infty \alpha_2 D^\beta N(D) \, dD \right],$$

$$\alpha(D) = \alpha_1 \quad \text{for} \quad D \leq D_{\text{lim}},$$
$$\alpha(D) = \alpha_2 \quad \text{for} \quad D > D_{\text{lim}}.$$  

At this stage, it is no longer possible to further simplify
the equation, since removal of $\alpha$ from the overall integral is
not possible.

**TABLE 2.** Average MMD value obtained for the entire HAIC/
HIWC dataset, and relative difference with respect to the F14 value
for the four mass–size relationships (Table 1).

<table>
<thead>
<tr>
<th>Mass–size relationship</th>
<th>Mean MMD value ($\mu$m)</th>
<th>Difference with respect to F14 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF95</td>
<td>452</td>
<td>-17.3</td>
</tr>
<tr>
<td>H10</td>
<td>594</td>
<td>8.8</td>
</tr>
<tr>
<td>C13</td>
<td>530</td>
<td>-3.0</td>
</tr>
<tr>
<td>F14</td>
<td>546</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 3 illustrates the evolution of the size threshold $D_{\text{lim}}$ as a function of $b$ with constant $\alpha = 0.005$ and for two different ice densities. The term $D_{\text{lim}}$ is the particle size below which the mass–size relationship $m(D) = 0.005 \times D^b$ produces a mass larger than that of an ice sphere of the same size with a constant density of 0.917 g cm$^{-3}$ (continuous line) or 0.7 g cm$^{-3}$ (dashed line).

Figure 3 illustrates the evolution of the size threshold $D_{\text{lim}}$ as a function of $b$ with $\alpha$ fixed to a constant value of 0.005 (roughly the value used in F14, and lying between 0.0025 for C13 and 0.007 for H10) and assuming an ice density $\rho$ of 0.917 or 0.7 g cm$^{-3}$. For $b$, the literature yields values from 1.4 to 3, with lower values usually associated with aggregates of dendrites and higher values with graupel and hail. Figure 3 reveals that $D_{\text{lim}}$ is lower than the size of our first PSD size bin (10 $\mu$m) for $b$ larger than 2.4 for both ice density of 0.917 and 0.7. However, $D_{\text{lim}}$ starts increasing with decreasing values of $b$. For $b = 2$ and 1.7, $D_{\text{lim}}$ increases to approximately 100 and 300 $\mu$m (for $\rho = 0.917$), respectively, below which the algorithms would treat the particles as solid ice spheres. The increase of $D_{\text{lim}}$ with decreasing $b$ is even larger when $\rho = 0.7$. Thus, when decreasing $b$ below 2, the crystal masses of the smallest bins are decreased due to the solid sphere assumption and the MMD computation is biased (see next section).

2) RESULTS

All PSD results presented in this section were obtained using the $D_{\text{max}}$ size parameter. Figure 4 displays the impact of $b$ on MMD$_{\text{max}}$. The red curve shows the mean MMD$_{\text{max}}$ (using the constant-temperature-level PSD data from Darwin) calculated for fixed values of $b$ between 1.4 and 3 and for a fixed $\alpha$ value of 0.005. The previously described effect of increasing $D_{\text{lim}}$ with decreasing $b$ then produces increasing MMDs for $b$ lower than 1.8 because more and more the smaller crystal sizes are associated with a smaller mass.

To overcome the $D_{\text{lim}}$ issue, we tried constraining $\alpha$ values so that the integrated IWC is equal to that from simultaneous IKP2 measurements:

![Figure 4](http://journals.ametsoc.org/jtech/article-pdf/33/11/2461/3384902/jtech-d-15-0151_1.pdf)
\[ \int_{D_{\text{lim}}(a)}^{D_{\text{lim}}(a)} N(D)\alpha_i D^3 \, dD + \int_{D_{\text{lim}}(a)}^{\infty} N(D)\alpha D^\beta \, dD = \text{TWC}. \]

This way, \( \alpha \) evolves with time but stays coherent with the measured total mass no matter the value of \( \beta \) that is tested. And, on average, \( \alpha \) values now strongly decrease for low \( \beta \), ensuring small \( D_{\text{lim}} \) values (150 compared to 600 \( \mu m \) in Fig. 3) even for \( \beta \) below 1.8. The black line in Fig. 4 represents the mean MMD obtained with this method and confirms that there is no more increase of the MMD for \( \beta \) until 1.4.

Moreover, for \( \beta \) larger than 1.9, Fig. 4 demonstrates that the trends in the change of MMD\(_{\text{max}} \) as a function of \( \beta \) for a constant \( \alpha = 0.005 \) are quite similar to those for \( \alpha \) constrained by the IKP2 TWC measurement (red vs black curve, respectively). This demonstrates that, as long as \( D_{\text{lim}} \) stays rather small, the influence of \( \alpha \) on MMD\(_{\text{max}} \) is negligible and that the impact on MMD\(_{\text{max}} \) almost entirely stems from changes in \( \beta \).

To further quantify the MMD dependency on \( \beta \), the shaded area on Fig. 4 spans the 25th (P25) and 75th (P75) percentiles (time-varying \( \alpha \) case) and finally the blue line represents the relative change in the mean MMD\(_{\text{max}} \) relative to that for \( \beta = 2 \). Increasing the value of \( \beta \) is found to have several major effects. First, the mean MMD\(_{\text{max}} \) increases from roughly 400 to 900 \( \mu m \) as \( \beta \) increases from 1.4 to 3 (black curve). Compared to the calculated mean MMD\(_{\text{max}} \) of roughly 550 \( \mu m \) for \( \beta = 2 \), the mean MMD for \( \beta = 1.4 \) and \( \beta = 3 \) are lower by 25% and higher by 75%, respectively. In the range of \( \beta = 1.8-2.2 \), which is expected to be the realistic range for natural ice crystal populations in our study, the mean MMD\(_{\text{max}} \) reaches ranges between 490 and 608 \( \mu m \), representing a relative change of 24%.

Moreover, Fig. 4 illustrates that the MMD\(_{\text{max}} \) distribution widens with increasing \( \beta \). For \( \beta = 1.8 \), half of the MMD\(_{\text{max}} \) values are found between 380 and 550 \( \mu m \) (P25–P75), whereas for \( \beta = 2.2 \) the corresponding range is 440–720 \( \mu m \). As \( \beta \) increases, mass is shifted to larger crystal sizes, generating increased MMD\(_{\text{max}} \) values.

Finally, mean MMD values obtained with the four mass–size relationships used in the previous section are also shown in Fig. 4 (black crosses for mean MMD, and P25 and P75 whiskers). Mean values for H10, C13, and F14—all using the PSD size parameter \( D_{\text{max}} \)—fall along the red curve of the mean MMD\(_{\text{max}} \) curve for a constant \( \alpha = 0.005 \). The C13 use of an ice density of 0.7 g cm\(^{-3} \) for particles smaller than \( D_{\text{lim}} \) (as compared to 0.917 g cm\(^{-3} \) for H10 and F14) has almost no impact on the MMD\(_{\text{max}} \). Indeed, McFarquhar et al. (2007) already concluded that particles smaller than 0.128 mm did not significantly contribute to the total PSD mass in their MCS dataset, noting that the third moment of the PSD is much more sensitive to the larger particles than to the small ones. So, even if both the density and the concentrations in the first bins of the PSD are quite uncertain (Jensen et al. 2013), the MMD is not very sensitive to the smallest crystal sizes of the PDS for these MCS datasets. For example, if the PSD is deliberately truncated below 55 \( \mu m \) rather than starting at nominal 15 \( \mu m \), then the average increase in the final MMD values in our dataset would be only about 2 \( \mu m \).

The BF95 mean MMD\(_{m} \) is clearly below the red curve. The change from \( \beta = 1.9 \) (BF95) to \( \beta = 2.05 \) (F14) could explain only a 5% change in the mean MMD, if both relationships would have used the \( D_{\text{max}} \) size parameter. However, the difference between BF95 and F14 in mean MMD is 17% (cf. Table 2), indicating that the majority of this difference results from the different size parameters used in the \( m(D) \) relationships, that is, \( D_{m} \) in BF95 and \( D_{\text{max}} \) for F14.

The above-mentioned results demonstrate that MMD is very sensitive to the \( \beta \) exponent of the mass–size relationship. In addition, the size definition itself used in PSD is of primary importance for the derived MMD, as demonstrated by the fact that the MMD difference between BF95 and F14 cannot be explained solely by different \( \beta \) values. The impact of the size definition on MMD is discussed in more detail in the next section.

3) Sensitivity to the size parameter

Figure 5 illustrates how the mean MMD changes with \( \beta \) when using the different size definitions \( D_{y}, D_{eq}, D_{m}, \) and \( D_{\text{max}} \), and when all \( \alpha \) values have been constrained using the IKP2 TWC measurements. As expected, the resulting mean MMDs are all smaller than those obtained with the \( D_{\text{max}} \) definition but the trends are quite similar. The terms MMD\(_{y} \) (purple curve), MMD\(_{y} \) (red curve), and MMD\(_{eq} \) (blue curve) are on average about 10%, 15%, and more than 20% lower, respectively, than the MMD\(_{\text{max}} \) (black curve) mean values.

The mean MMD values obtained with mass–size relationships from the literature are also included as in Fig. 5, and the BF95 point now appears close to the MMD\(_{m} \) curve.

In summary, the above-mentioned sensitivity studies demonstrate that the choice of both the exponent \( \beta \) in the \( m(D) \) relationship and the definition of the size of a particle derived from OAP 2D imagery are of primary importance when computing MMDs.

Concerning the size parameter, different MMDs (i.e., MMD\(_{\text{max}} \), MMD\(_{y} \), MMD\(_{eq} \), MMD\(_{m} \), MMD\(_{eq} \)) may be produced that are consistent with the end users’ needs or size standards. However, knowing that MMD changes with \( \beta \), we
still need to choose the most suitable value for $b$ to represent the dataset. In the past, $m(D)$ parameterizations have used constant values of $a$ and $b$ and a specific defined size parameter. In the following section we present a general methodology to retrieve $a$ and $b$ coefficients for the four presented size definitions. Moreover, this method produces time-dependent (5-s time steps) $a$ and $b$ values based on spatiotemporal changes in measured cloud particle shapes.

4. Mass–size relationships constrained by reference TWC (from IKP2) and particle images’ (from OAP) measurements

a. Method

F14 proposed a method to determine the coefficients $a$ and $b$ of the mass–size relationship $m = aD^b$ using both OAP hydrometeor imagery and the Radar System Airborne (RASTA; Protat et al. 2009; Delanoë et al. 2013) reflectivity. Projections of simulated 3D ice crystals on a 2D plane were used to produce 2D binary images—in a manner similar to that of OAPs—and then these simulated images were used to study the link between their 3D volume and 2D parameters (area, perimeter, size, etc.). From these simulations, F14 established a relation linking the exponent $b$ in the $m(D)$ relationship to the exponent $\sigma$ of the area–size relationship $A = \gamma D^\sigma$ (where $A$ is the 2D area of the binary particle image) that can be extracted from simulated (as well as “real”) 2D binary images. In a similar manner, Schmitt and Heymsfield (2010) used 3D simulations of aggregates, proposing a relationship between the 2D and 3D fractal dimensions $D_{2D}$ and $D_{3D}$, respectively, which were then used as the values $\sigma$ and $b$, respectively. This method allows for the dynamic calculation of $b$ (or $D_{3D}$) from $\sigma$ (or $D_{2D}$) derived from OAP 2D images, as shown in Heymsfield et al. (2013).

Subsequently, F14 constrained the $a$ parameter by matching the PSD-derived reflectivity (from theoretical simulations) to the measured reflectivity from RASTA radar. As a consequence $a$ and $b$ are no longer unique and constant values to be applied for an entire dataset but vary with time (e.g., along the flight trajectory, from one flight to another) according to variations in the OAP ice crystal shapes.

F14 used the $D_{\text{max}}$ computed from binary OAP images. In this study, we modify and extend their $m(D)$ retrieval method, making it more general and applicable to other size definitions. To do so, we strengthen the dependency of $b$ on particle shape by studying both the surface-size $A(D)$ and perimeter-size $P(D)$ relationships (Duroure et al. 1994). This enables the extension of the method to $D_{\text{eq}}$, where by definition $\sigma = 2$ for all particle images, while $b$ is...
varying. Adding the perimeter-size relationship $P = \delta D^{\tau}$ allows for a shape dependency of $\beta$ when using $D_{\text{eq}}$.

As in F14, in this study the parameterization of $\beta$ as a function of $\sigma$ and $\tau$ has been established using simulations of 3D particles and their arbitrary projections to 2D images. In total, 45 different 3D crystal shapes (stellar, columns, plates, capped columns, rosettes, and their aggregates) were simulated, and 2D projections were created for random 3D orientations. The sizes (corresponding to the four primary dimension definitions used in this study), perimeter, and surface area were then computed from the 2D binary projections (image), while the true total volume is known for each 3D particle. Scatterplots of the mass (surface area and perimeter) as a function of size for each size definition and for a random series of particle orientations yielded for each value of $\beta$ the corresponding $\sigma$ and $\tau$ exponents for each crystal shape. The ensemble of $(\beta, \sigma, \tau)$ values obtained for equal proportions of the 45 different ice crystal shapes is presented in Fig. 6. Different symbols are assigned to the different size definitions: green circles for $D_m$ (mean of $D_x$ and $D_y$), red diamonds for $D_{\text{eq}}$, black plus signs for $D_{\text{max}}$, and blue squares for $D_y$.

![Fig. 6. Relationship between the exponents $\beta$, $\sigma$, and $\tau$ of the $m(D)$, $A(D)$, and $P(D)$ power-law relationships deduced from 3D crystals simulations. Each point corresponds to a different crystal shape (cf. F14). The different symbols correspond to the different size definitions: green circles for $D_m$ (mean of $D_x$ and $D_y$), red diamonds for $D_{\text{eq}}$, black plus signs for $D_{\text{max}}$, and blue squares for $D_y$.](image)

The accuracy of the fits has been estimated by comparing $\beta = f(\sigma, \tau)$ calculated from the linear fit to the initial model $\beta$ values and the results are presented in Table 3. Table 3 shows that the best scores are obtained for $D = D_{\text{eq}}$, where the coefficient of determination ($r^2$) is the highest (0.89) and the mean and relative errors are the lowest ($1.4 \times 10^{-5}$ and 1.4%, respectively). For the other three size definitions, the mean error is of the order of $10^{-3}$ for $D_m$ and $5 \times 10^{-3}$ for $D_{\text{max}}$ and $D_y$. The relative errors are around 2% for $D_m$ and $D_{\text{max}}$ and slightly above 3% for $D_y$. Whereas the $r^2$ value exceeds 0.8 for $D_m$ and $D_{\text{max}}$, it reaches only 0.67 for the $D_y$ definition.

Finally, for the HAIC/HIWC campaign as proposed in this study, the TWC measurements from IKP2 are used instead of the radar reflectivity to constrain the $\alpha$ factor [cf. section 3b(2)]. Table 4 summarizes the former estimates.

### Table 3. Estimation of the quality of the $\beta = f(\sigma, \tau)$ relationship deduced from 3D ice crystal simulations.

<table>
<thead>
<tr>
<th>Size parameter</th>
<th>$r^2$</th>
<th>Mean error for $\beta$</th>
<th>Mean relative error for $\beta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\text{max}}$</td>
<td>0.815</td>
<td>$5.2 \times 10^{-3}$</td>
<td>2.29</td>
</tr>
<tr>
<td>$D_{\text{eq}}$</td>
<td>0.888</td>
<td>$1.4 \times 10^{-5}$</td>
<td>1.39</td>
</tr>
<tr>
<td>$D_y$</td>
<td>0.670</td>
<td>$5.6 \times 10^{-3}$</td>
<td>3.28</td>
</tr>
<tr>
<td>$D_m$</td>
<td>0.855</td>
<td>$1.1 \times 10^{-3}$</td>
<td>1.88</td>
</tr>
</tbody>
</table>
findings for $\beta = f(\sigma)$ and related calculations of $\alpha$ from T-matrix simulations of the radar reflectivity (F14; valid only for $D_{\text{max}}$), with an additional column summarizing the new results of the current extended study with $\beta = f(\sigma, \tau)$, valid for all four diameter definitions, and with $\alpha$ constrained from IKP2 measurements of TWC.

b. Results

Figure 7 presents the overall range of retrieved $\beta$ and $\alpha$ values computed using the method of this study, as described in the previous paragraph (section 4a). McFarquhar et al. (2007) examined microphysical data collected upstream and downstream of convective lines, and using a PSD size definition of $D_{\text{max}}$, they observed noticeable discrepancies in the values of $\alpha$ and $\beta$ from flight to flight; with $\beta$ ranging from 1.3 to 2.2, for example. For the current study, computing $\beta$ from $\sigma$ and $\tau$ results in a lower variability in the HAIC/HIWC dataset. Although Fig. 7 shows that 5-s $\beta$ values ranged from 1 to 3, the 25th and 75th percentiles only extended from roughly 1.95 to 2.15 for $D_{\text{max}}$. From flight to flight (not shown), the mean $\beta$ for $D_{\text{max}}$ also only varied between 1.9 and 2.1 thus showing less variability than McFarquhar et al. (2007).

Concerning the $\alpha$ parameter, Heymsfield et al. (2013) highlighted its considerable scatter when used as a scale factor to reproduce independent bulk TWC measurements from PSDs, and suggested a value of 0.006 cgs (centimeter–gram–second) which is close to the median value for $D_\gamma$ and $D_m$ of the study presented here. With respect to $D_{\text{max}}$ our study leads to a slightly lower value of $\alpha$ (0.004). Still one has to keep in mind that the magnitude of $\alpha$ changes with $\beta$ ($\alpha$ is in g cm$^{-3}$), which makes comparisons more complex.

Finally, one has to be cautious when comparing mean values of $\alpha$ and $\beta$, since some studies have shown that mass–size coefficients might be a function of temperature (Schmitt and Heymsfield 2010; Heymsfield et al. 2013, F14). Then the mean mass–size coefficient for a flight depends on sampled flight altitudes and thus the whole campaign sampling strategy (legs at constant altitude vs ascending/descending spirals into clouds, etc.).

For all noncircular images, $D_{\text{eq}}$ is smaller than $D_{\text{max}}$. Consequently, to get the same final mass for the particle, $\beta$ must be larger for $D_{\text{eq}}$ than for $D_{\text{max}}$. Similarly, since the $D_m$ size definition using the mean chord length should produce intermediate values between $D_{\text{eq}}$ and $D_{\text{max}}$, the $\beta$ values corresponding to $D_m$ should be in between those for $D_{\text{eq}}$ and $D_{\text{max}}$. Accordingly, the $\beta$ values using our method are largest and smallest for the $D_{\text{eq}}$ and $D_{\text{max}}$ size definitions, respectively, with $D_m$ and $D_\gamma$ in between.

Finally, with respect to $\alpha$, Fig. 7 demonstrates that $\beta$ and $\alpha$ seem to be related (as previously shown by H10 and F14) and follow quite similar trends varying the size

![Image](https://example.com/image.png)

**FIG. 7.** Range of (left) $\beta$ exponent and (right) $\alpha$ factor when $\beta$ is deduced from the OAP images (linear relationship with $\sigma$ and $\tau$) and $\alpha$ is subsequently constrained by the IKP2, as a function of the particle size definition $D_{\text{max}}$, $D_{\text{eq}}$, $D_\gamma$, and $D_m$. Box plot limits represent 25th–75th percentiles, whereas whiskers give data minimum and maximum values. In the right panel, a logarithmic scale is used for the y axis.
definition: the highest values are obtained for $D_{eq}$, progressively decreasing for $D_m$, $D_y$, and finally $D_{max}$.

A comparison of the final MMDs for the HAIC/HIWC dataset, computed with the $\beta$ and $\alpha$ values of Fig. 7 for the four different size definitions, is presented in Fig. 8. Somewhat surprisingly, the scatterplot shows a rather good agreement between the different MMDs. $MMD_{max}$ and $MMD_m$ are well aligned along the (1:1) line with a correlation coefficient of 0.972. $MMD_y$ (in blue) and $MMD_{eq}$ (in red) show more dispersion (with correlation coefficients of 0.93 and 0.92, respectively). The largest difference in the mean MMD values for the whole dataset of 5% is found between $MMD_{max}$ and $MMD_m$. Using the current method with a dynamic $\beta = f(\sigma, \tau)$ estimated from OAP 2D imagery specific for the size definition, we greatly reduce the differences in calculations of MMD introduced by the size definition.

For $MMD_{max}$ larger than 0.1 cm, some $MMD_{eq}$ values appear to be larger than those for $MMD_{max}$. These points are mostly associated with lower TWC values (less than 1 g m$^{-2}$) with relatively large ice crystals in low concentrations. A possible explanation for this behavior might be related to the fact that OAP images of large ice crystals are often truncated and that the reconstruction method of Korolev and Sussman (2000) used in this study relies on a spherical/circular hypothesis, leading to $D_{eq} = D_{max}$ for such images and thus a possible underestimation of the real $D_{max}$. Then, since $\beta$ is larger for $D_{eq}$ than for $D_{max}$, the resulting $MMD_{eq}$ can be larger than $MMD_{max}$. This feature is indeed a weakness of the actual MMD computation method for $MMD_{max}$, at least when using the reconstruction method. However, it must be pointed out that only a few data points are concerned for the current dataset. The histogram in Fig. 8 shows that MMDs larger than 0.1 cm account for only 5% of the data points. Moreover, only a small fraction of that 5% will lead to $MMD_{eq}$ larger than $MMD_{max}$. Finally, Fig. 9 presents box plots of the MMDs calculated for the four size definitions of this study, thereby illustrating that P25, median, and P75 values are slightly lower for $MMD_{eq}$ than for $MMD_{max}$. In addition to Fig. 9, Table 5 presents a statistical analysis of the differences between the MMDs. For example, for the $MMD_{max}$ row, mean differences (values in bold) are always negative, thus demonstrating that on the
average, they are larger than those of the three other MMD size definitions.

Figure 9 confirms that the MMDs for the four chosen size definitions are nevertheless quite similar when using the method of this study, namely, calculating $\beta$ from OAP images and subsequently constraining $\alpha$ from IKP2 measurements of TWC. The mean absolute differences between the MMDs computed from different size definitions vary between 30 and 50 $\mu$m (cf. values in italics in Table 5), which are rather small compared to the initial differences obtained when using mass–size relationships from the literature (the results from BF95 and F14 are included in Fig. 9 for comparison).

### 5. Conclusions

In this study, we investigated the impact of the mass–size relationship $m = \alpha D^\beta$ and the definition of the parameter used to define the size of a 2D particle image on the median mass diameter (MMD) values obtained from optical array probe PSD mass distributions using the HAIC/HIWC dataset. Four mass–size relationships (Brown and Francis 1995; H10; C13; F14) from the literature have been tested and compared, yielding major discrepancies in the final MMDs. As those relationships differ in both the parameters $\alpha$ and $\beta$ of the mass–size relationships and the size definition for the particles, the sensitivity of the MMDs with respect to these two factors has been explored separately.

Tests using the same particle size definition with various values for $\beta$ ranging between 1.4 and 3 demonstrate that relatively small differences in $\beta$ value produce significant differences in the MMD. For example, increasing $\beta$ from 2 (the value used by C13) to 2.2 (used by H10) results on average in an 11% increase in MMD$_{max}$. Then, modifying the definition of the size parameter results in another major change in the MMD: for example, with a constant $\beta = 2$, 10% and 22% decreases in MMD are obtained when using $D_m$ and $D_{eq}$, respectively, relative to the value obtained using $D_{max}$. Combining the impact of $\beta$ and the definition of the size parameter then explains the resulting differences in MMD calculated for the four mass–size relationships from the literature noted above. Thus, it is very important when quoting MMD values from different datasets to clearly identify the size parameter and $\beta$ value used for MMD computation.

Since our sensitivity studies have demonstrated that the MMD is quite sensitive to the $\beta$ parameter, this parameter must be chosen carefully. F14 introduced a method using the $D_{max}$ particle size definition that dynamically deduces $\beta$ from OAP images. They established a link between $\beta$ and $\sigma$, the exponent from the area–size relationship $A = \gamma D^\sigma$, that can be directly computed from OAP binary images as a function of time, thus introducing a moving adjustment to $\beta$ to reflect changes in the ice crystal habits along the flight trajectory. This study extends the approach of F14 to other image size definitions, including $D_y$, $D_m$, and $D_{eq}$. In our method, $\beta$ is linked to both $\sigma$ and $\tau$, the latter being the exponent of the perimeter-size relationship $P = \delta D^\tau$, which can also be deduced from OAP images. Consequently, MMD can be calculated for different particle size definitions according to the different users’ size standards and needs, thereby utilizing a common concept in MMD calculation. The new method also constrains $\alpha$ by matching the integrated PSD TWC to that of the bulk measurement of the IKP2 instrument.

With this new method the statistics of the final MMD$_{max}$, MMD$_{eq}$, MMD$_y$, and MMD$_{eq}$ datasets are surprisingly similar. Although the method shows some weakness for a fraction of the large particles for which the optical array probe produces truncated images, and for which MMD$_{max}$ is sometimes smaller than MMD$_{eq}$, on average MMD$_{max}$ values are mostly larger than those of the three other MMDs, consistent with what is expected based on simple logic. However, on average the four different MMDs differ only by 30–50 $\mu$m (typically <5%). Thus, for many practical applications, it is now possible to provide MMD estimates for a specific size definition ($D_m, D_y, D_{eq}$, or $D_{max}$) most appropriate to an application, and one that will not vary greatly according to that size definition as long as the $\alpha$ and $\beta$ parameters of the mass–size relationship are carefully derived and adjusted dynamically to the current

<table>
<thead>
<tr>
<th>$\beta = f(\sigma, \tau)$</th>
<th>MMD$_{max}$</th>
<th>MMD$_{eq}$</th>
<th>MMD$_y$</th>
<th>MMD$_{eq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = f$(TWC from IKP2)</td>
<td>$\beta$</td>
<td>$\alpha$</td>
<td>MMD$_{eq}$</td>
<td>MMD$_y$</td>
</tr>
<tr>
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<td>$\beta$</td>
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<td>MMD$_y$</td>
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<td>$\beta$</td>
<td>$\alpha$</td>
<td>MMD$_{eq}$</td>
<td>MMD$_y$</td>
</tr>
</tbody>
</table>

**Table 5.** Mean difference (bold; $\mu$m) and mean absolute difference (italic; $\mu$m) between different MMDs calculated for the different size definitions $D_{max}$, $D_{eq}$, $D_m$, and $D_{eq}$, after constraining $\alpha$ with TWC from IKP2.
characteristics of the OAP particle imagery and sufficiently accurate independent bulk TWC measurements.

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