Improving Unmanned Aerial Vehicle–Based Acoustic Atmospheric Tomography by Varying the Engine Firing Rate of the Aircraft

ANTHONY FINN AND KEVIN ROGERS
Defence and Systems Institute, University of South Australia, Mawson Lakes, South Australia, Australia

(Manuscript received 25 August 2015, in final form 21 December 2015)

ABSTRACT

If the acoustic signature of an unmanned aerial vehicle (UAV) is observed as it overflies an array of ground microphones, then the projected and observed Doppler shifts in frequency of the narrowband tones generated by its engine may be compared and converted into effective sound speed values. This allows 2D and 3D spatially varying atmospheric temperature and wind velocity fields to be estimated using tomography. Errors in estimating sound speed values are inversely proportional to the rate of change in the narrowband tones received on the ground. As this rate of change typically approaches zero at least twice per microphone during the UAV’s overflight, errors in the time of flight estimates are typically too large to deliver useful precision to the tomographically derived temperature and wind fields. However, these errors may be reduced by one or two orders of magnitude by continuously varying the engine throttle rate, thereby making the tomographic technique potentially feasible. This is demonstrated through reconstruction of realistic simulated conditions for a weakly sheared daytime convective atmospheric boundary layer.

1. Introduction

Using a technique known as tomography, atmospheric temperature and wind vector profiles may be computed from sound speed observations. The technique has been widely used for many decades in oceanography, medicine, archaeology, and remote sensing of different geophysical media (Brown et al. 2014; Kak and Slaney 2001).

The first implementation of the technique to atmospheric observation was based on a series of 10-m towers that support microphones and loud speakers covering an area of 200 m × 240 m (Wilson and Thomson 1994). Other arrays have subsequently been built at the University of Leipzig (Arnold et al. 1999; Jovanović et al. 2009; Ziemann et al. 1999) and at the Boulder Atmospheric Observatory, which allows 3D tomography (Vecherin et al. 2008a; Wilson et al. 2001). A number of different techniques have also been developed (Barth and Raabe 2011; Kolouri and Azimi-Sadjadi 2012; Vecherin et al. 2008a,b, 2006, 2007), including some that passively observe atmospheric properties using impulsive noise sources, such as birds or meteors (Spiesberger and Fristrup 1990) or commercial aircraft (Ostashev et al. 2000; Wilson et al. 2001). The method has practical application in a number of research fields, including boundary layer meteorology, land–sea surface–atmosphere interactions, theories of turbulence, and wave propagation through a turbulent atmosphere. For example, the technique is applied to experimental validation of large-eddy simulation (LES) and microscale meteorological models of both convective boundary layer turbulence and the more stable nocturnal boundary layer, where it has advantages over more traditional point measurements that include its effect as a spatial filter for turbulence, the higher number of data per sensor, and minimal impact of the sensor devices on the observed atmosphere. A good survey of progress in the field is presented in Ostashev et al. (2008b).

In the case where an unmanned aerial vehicle (UAV) is used as a sound source (Finn and Franklin 2011b; Finn and Rogers 2015; Rogers and Finn 2014)—as the aircraft has no pilot and low kinetic energy—it may be safely flown at altitudes from a few meters to several kilometers, in dangerous environments and/or for long periods.
This permits examination of vertical and 3D atmospheric structure unavailable via other methods. Also, as the UAV continuously moves and emits a signature, and the number of measuring points governs the accuracy and spatial resolution of meteorological parameter estimation, one of the main issues for existing outdoor acoustic tomography—formulation of accurate reconstructions of the temperature and wind velocity fields from a spatially limited set of observations—is overcome. Multi-UAV configurations are also possible, allowing bidirectional time delay observations to be made and hence better estimates of wind velocity (Finn et al. 2014). Furthermore, the multitoneal nature of the UAV’s signature means that—regardless of local propagation conditions—at least one harmonic typically reaches each ground receiver (Finn and Franklin 2011a); and, as the harmonics are linearly related, each frequency measurement can be used to augment an estimate of a composite frequency and thus propagation delay (Rogers and Finn 2013a). As a result, in addition to the provision of information on the representativeness of point measurements and the homogeneity of other atmospheric observations, use of UAVs as a sound source offers several advantages over existing tomographic techniques and meteorological sensors (papers referenced in this paragraph give details on such benefits).

The technique has some disadvantages. For instance, as the UAV has a low top speed and conditions can change over a period of measurement, the resultant tomographic reconstruction delivers only a time-averaged version of the observed atmosphere rather than a crisp ‘‘snapshot.’’ Also, while the capital cost of the sensing equipment is relatively low, the running costs of a UAV over extended periods can be high when compared to other meteorological instruments, such as sodar, lidar, radar, anemometers, etc. However, the technique is envisaged as being useful for relatively short-term (hours–days), intensive monitoring rather than long-term (weeks–months) in situ measurements.

Ultimately, the accuracy of propagation delay measurements depends upon a number of factors: the signal processing techniques used to determine the propagation delay, assumptions inherent in the forward solution, scale size and motion of any atmospheric structures under examination, and UAV flight dynamics relative to the receiver array geometry. The last factor influences the way in which rays intersect one another and the intervening medium, and thus impact resolution and accuracy of any inverse solution. The other factors have been discussed in earlier papers (Finn and Rogers 2015; Rogers and Finn 2013a,c).

In this paper, we examine the components of error for time delay estimation for UAV-based acoustic atmospheric tomography (AAT). We show that by carefully varying the aircraft’s engine, firing sequence propagation measurement accuracy can be significantly improved, which in turn enhances the tomographic inversions. Atmospheric simulations for a canonical daytime convective planetary boundary layer (PBL) based on LES are then used to demonstrate that the accuracy of the time delay measurements are sufficient. The LES uses pseudospectral differencing in horizontal planes and solves an elliptic pressure equation. Particular attention is paid to the accuracy with which the surface layer (lowest 50–100 m of atmosphere) may be reconstructed as this region typically experiences the greatest spatiotemporal variation in temperature and wind speed; and arrangements of UAV flight path and sensor geometry do not generally permit ray paths to intersect without the UAV flying very close to the ground (<50 m).

The paper is arranged as follows. The key elements of the UAV’s acoustic signature needed for time delay estimation, together with the approaches used to solve both the forward (calculation of the propagation delay) and inverse (reconstruction of the temperature and wind velocity profiles from these observations) problems are described in section 2. The inverse solution described improves the stability, resolution, and accuracy of temperature and wind velocity profile estimates over techniques reported elsewhere. Section 3 examines the impact of varying the throttle settings of the UAV on propagation delay error estimation, and section 4 applies these predictions to tomographic reconstruction of a simulated atmosphere. Section 5 provides some concluding remarks.

2. UAV-based acoustic tomography

a. UAV signature

A time–frequency signal analysis of a propeller-driven aircraft shows its signature comprises strong narrowband tones superimposed onto a broadband random component, with almost all of the narrowband energy below 2 kHz (Ferguson and Lo 2000). These tones correspond closely to the engine firing rate $N_R$ (the number of engine rotations per second) and propeller blade rate (PBR) of the aircraft such that $PBR = N_RN_B$, where $N_B$ equals the number of blades on the propeller, with each of these sound sources (engine firing rate and propeller) generating its own set of harmonics. Thus, if such an aircraft is used as an acoustic transmitter, then there is a rich set of useful frequencies—up to about the 15th harmonic—that may be used to determine the propagation delay between the aircraft and any receiver (Finn and Rogers 2015). For a more complete description of

Spectrograms of the UAV signature show that each harmonic, \( h = 1, \ldots, N_h \) (where \( N_h \) is the number of visible harmonics), typically varies over time. However, over periods of a few seconds, variations are approximately linear with respect to time (Ferguson and Lo 2000; Finn and Franklin 2011a). The spectrograms also suggest the mean value of a normalized frequency (each tone normalized by harmonic number) has \( 1\sigma \) standard deviation of 0.13 Hz. As a result, the harmonics may be considered linearly related. The lower bound of the standard error \( \sigma_{fh} \) in the estimate \( f_h \) of each harmonic is given by the Cramér–Rao lower bound (CRLB; Rogers and Finn 2013a). Based on the linear relationship between harmonics, an estimate of a composite frequency, \( f_u = \sum_{h=1}^{N_h} f_h / h \), can be established, where the higher harmonics offer the advantage that the standard error is divided by the harmonic number, even though the signal-to-noise ratio (SNR) is typically less for these higher harmonics.

The least squares estimate of composite frequency is \( f_u = \sum_{h=1}^{N_h} \sigma_h^2 f_h / h \sigma_h^2 \), where \( \sigma_h^2 = \sum_{h=1}^{N_h} \sigma_{fh}^2 - 1 \) is the variance of this estimate. Consequently, in addition to ensuring at least one signal typically reaches the ground receivers, the linear harmonic relationship can also be used to improve estimates of the emitted and received frequencies. Moreover, the variance of the frequency estimate \( \sigma_f^2 \) may be used to weight the tomographic inversion (Rogers and Finn 2013a).

b. The forward problem

For propagation in an inhomogeneous moving medium between a moving source and a stationary receiver, the frequency received by a ground sensor is [Eq. (5.43) in Ostrashev 1997]

\[
f(t + t_p) = \frac{1 + n_u(t) u_u(t)/c_u(t)}{1 + n_u(t) [v_u(t) - u_u(t)]/c_u(t)} f_u(t),
\]

where \( n_u(t) \) is the magnitude of the unit vector normal to the wave front of the wave emitted by the UAV, \( u_u(t) \) is the velocity magnitude of the medium at the UAV at time \( t \), \( t_p \) is the propagation time, \( u_u(t) \) is the velocity magnitude of the UAV emitter at \( t \), \( f_u(t) \) is the instantaneous source frequency at \( t \), \( f_u(t + t_p) \) is the instantaneous received frequency at time \( t + t_p \), and \( c_u \) is the speed of sound at the UAV.

The UAV location and velocity at each epoch \( t \) and the position of all ground microphones may be accurately measured (using real-time kinematic carrier phase differential GPS) with the acoustic signature of the UAV (using microphones both on board and at each ground receiver), which allows for computation of \( f_u(t) \) and \( f_u(t + t_p) \). Based on meteorological observations such as wind velocity, thermodynamic temperature, mixing ratio, and specific and relative humidity made on board the UAV, the sound speed \( c_u(t) \), the unit vector magnitude normal to the wave front \( n_u(t) \) and the wind velocity magnitude \( u_u(t) \), may also be estimated.

The challenge is that, while we measure (or can compute) the parameters necessary to estimate \( f_u(t + t_p) \) from Eq. (1), as the value of \( t_p \) is not known accurately, the value of \( f_u(t + t_p) \) observed at the ground receivers—with which we must correspond the estimated value of \( f_u \)—cannot easily be determined. Fortunately, techniques exist to determine the value of \( t_p \) from straight ray path propagation (Wilson et al. 2001). For example, representing \( f_u(t + t_p) \) as a Taylor series (Finn and Rogers 2015) we have

\[
t_p = t_{p,0} + \left[ f_u(t + t_p) - f_u(t + t_{p,0}) \right] \frac{\partial f_u(t + t_{p,0})}{\partial t},
\]

where \( f_u(t + t_{p,0}) \) is the measured value of \( f_u \) at \( t_{p,0} = l(t)/c(t) \). \( f_u(t + t_{p,0}) \) is the value of \( f_u(t + t_p) \) computed from Eq. (1), and \( \partial f_u(t + t_{p,0})/\partial t \) is the numerical derivative of \( f_u \) at \( t_{p,0} \).

c. The inverse problem

The group velocity of a sound wave propagating in a moving medium is \( \mathbf{u}_G = c \mathbf{n} + \mathbf{V} \), where \( c = (\gamma R_e T)^{1/2} \) is the sound speed, \( \gamma \) is the specific heat ratio, \( R_e \) is the gas constant for dry air, \( T \) is the virtual acoustic temperature, \( \mathbf{V} \) is the wind vector, and \( \mathbf{n} \) is the unit vector normal to the wave front of the sound wave.

If the fields are linearized about mean values \( c_0, T_0 \) and \( \mathbf{V}_0 \), then the travel time \( (t_{p,j}) \) for sound ray \( j \) (Jovanović 2008) is

\[
t_{p,j} = \frac{l_j}{c_0} \left[ 1 - \frac{\mathbf{V}_0 \cdot \mathbf{n}}{c_0} \right] - \int\frac{\Delta T(X) + \Delta V(X) \cdot \mathbf{n}}{2T_0 c_0^2} \] dl,
\]

where \( l_j \) is the path distance, from \( \mathbf{X}_0 \) (UAV) to \( \mathbf{X}_r \) (receiver); \( c_0 \) is the mean sound speed; \( \mathbf{V}_0 \) is the mean wind speed vector over the volume; \( \mathbf{n} \) is the unit vector in the direction of sound ray \( j \); \( \Delta T(X) \) is the temperature deviation at location \( \mathbf{X} \); \( \Delta V(X) \) is the wind speed deviation at location \( \mathbf{X} \); and \( dl \) is an integration length along the ray’s path.

The quantities \( \Delta T(X) \) and \( \Delta V(X) \) may each be independently approximated by a network of weighted radial basis functions (RBFs) (Rogers and Finn 2013c;
where \( N_r \) is the number of RBF centers; \( W_{T,j} \) is the temperature weighting coefficient for RBF(j); and \( W_j = [W_{x,y}, W_{y,z}, W_{z,x}] \) is the wind weighting coefficient vector for RBF(j) in the \( x, y, \) and \( z \) directions. These equations may be expressed in matrix notation as \( F(X) = \Phi(X)W \), where \( F(X) = [\Delta T(X), \Delta V_x, \Delta V_y, \Delta V_z]^T \) is a \((4 \times 1)\) column vector of temperature and wind speed component deviations \((x, y, z)\), \( W \) is a \((4N_r \times 1)\) column vector of parameter weights, and \( \Phi(X) \) is a \((4N_r \times 4)\) matrix of RBFs.

Although less relevant for simulated temperature and wind fields, a key issue for trials data is to determine the number and locations of the RBFs and to decide whether their number and locations need be different for \( T \) and \( V \). We can also improve the estimate of the RBF coefficients (i.e., temperature and wind fields) by taking additional measurements of wind speed and temperature at the ground sensors and/or on board the UAV. We can then use the abovementioned equations to constrain the least squares adjustment by estimating the RBF coefficients, \( \Delta T = RW_T, \Delta V_x = RW_x, \Delta V_y = RW_y, \Delta V_z = RW_z \), where \( R_j = e^{-k_i|x_i-x_{i,j}|}, i = 1, \ldots, M_s \) (the number of temperature and wind speed measurements) and \( j = 1, \ldots, N_r \).

These equations can then be combined into a single matrix relationship, \( AW = b_{\text{obs}} \), where \( b_{\text{obs}} \) is a column vector containing all travel time and direct measurements of temperature and wind velocity, and \( A \) is an \((m \times n)\) matrix, which is an extension of the \( \Omega \) and \( R \) matrices, where the \((m \times N_r)\) \( \Omega \) matrix is the error function of the RBF given in Rogers and Finn (2013c).

If we assume that the error distributions are Gaussian with zero mean, then we may iteratively determine the RBF coefficients \( (W_j) \) using the posterior model and data covariance matrices, \( C_M \) and \( C_D \) (Tarantola 2005):

\[
W_{i+1} = W_i + C_M A^T (A C_M A^T + C_D)^{-1} (b_{\text{obs}} - AW),
\]

where the posterior model covariance matrix \( C_{M,i+1} = C_M - C_M A^T (A C_M A^T + C_D)^{-1} A C_M, \) the data covariance matrix \( C_D = b_{\text{obs}} - AW \), and \( p \) is an exponent chosen to suit the degree of noise found in the data.

An inverse problem is described as well posed if one or more of the following conditions are met: there exists some solution, there is only one solution, and the solution depends continuously on the observations (Snyder and Trampert 1999). A problem is said to be sparse if the solution derives from a very small number of nonzero entities in the inversion. Time delay tomography in which we seek to estimate both the scalar temperature and vector wind parameters is typically both an ill posed and sparse problem. Thus, we must “damp” the least squares adjustment such that instead of solving \( AW = b_{\text{obs}} \), we actually solve \((A^T A + \alpha^2 L)W = A^T b_{\text{obs}}, \) a process known as Tikhonov regularization (Aster and Thurber 2013).

The matrix \( L \) is typically the identity matrix or (for higher-order Tikhonov regularization) a finite difference approximation proportional to the first or second derivative, which imposes a penalty on the solution to \( W \) such that flatter solutions are favored. The Tikhonov parameter \( \alpha \) and the “roughening” matrix \( L \) must generally be selected empirically. We found a first-order roughening matrix, that is,

\[
L = \begin{bmatrix}
-11 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & -11
\end{bmatrix},
\]

most effective. Furthermore, the error distributions are Gaussian with known standard deviations, \( \alpha = \sigma_t^2 \), where \( \sigma_t^2 = (b_{\text{obs}} - AW)^T (b_{\text{obs}} - AW)/N \) and \( \sigma_y^2 = W^T W/M \) (Aster and Thurber 2013). Thus, having first selected an appropriate value for \( \alpha \) (we use 0.8), it may then be adjusted during each iteration of the least squares.

3. The effect of throttle variation

If a function, \( y = g(x, t, z) \), the error in \( y \) is given by

\[
\delta_y = \sqrt{\left( \frac{\partial g(x,t,z)}{\partial x} \right)^2 + \left( \frac{\partial g(x,t,z)}{\partial t} \right)^2 + \left( \frac{\partial g(x,t,z)}{\partial z} \right)^2},
\]

where \( \delta_x, \delta_t, \) and \( \delta_z \) are the errors associated with each parameter \( x, t, \) and \( z \). The error in the time delay estimate derived using Eq. (2) is therefore

\[
\sigma_{\delta t} = \sqrt{\left(\frac{\sigma_{\delta x}^2 + \sigma_{\delta t}^2 + \sigma_{\delta z}^2}{\alpha^2} \right) + \left( \frac{(f_c - f_m)^2}{\alpha^2} \right)},
\]

where \( f_c \) and \( f_m \) are the central and mean frequencies, and \( \sigma_{\delta x}, \sigma_{\delta t}, \) and \( \sigma_{\delta z} \) are the standard deviations of the errors in the parameters \( x, t, \) and \( z \).
where \( \sigma_{\text{prop}} \) is the error associated with the nominal propagation; \( \sigma_{fu} \) and \( \sigma_{fr} \) are the errors associated with estimating the computed and measured frequencies \( f^c \) and \( f^M \), respectively; and \( \sigma_s \) is the error associated with obtaining the derivative of \( \partial f_w(t + \tau_p)/(\partial t) \).

In other words, errors in time delay estimation for UAV-based AAT comprise three components: errors in the nominal delay [term 1 in Eq. (7)], errors in measuring the frequencies \( f_u \) and \( f_r \) (term 2), and errors due to unmodeled or unknown states of the problem (term 3).

The first of these components comprises errors in determining the nominal ray path; that is, term 1 comprises errors in the straight ray and average sound speed but does not include errors in ignoring the effects of atmospheric refraction: the latter fall into the third term. Errors in term 1 should not be confused with errors resulting from the use of a straight ray path approximation. Errors in the nominal delay are usually very small and in simulation they can be reduced to zero. In field trials, they comprise only errors associated with determining the start and end coordinates of the rays as, by definition, there is no error in the \( c_0 \).

The second term comprises errors in measuring \( f_u \) and \( f_r \), and represents the effects of noise due to any signal processing techniques employed (Rogers and Finn 2013a). As indicated in the previous section, the multisonal nature of the UAV’s signature offers two advantages. The first is that, irrespective of meteorological conditions, atmospheric moisture, refraction, absorption, scattering, turbulence, etc., at least one harmonic typically reaches each ground receiver—even at ranges of 2–3 km (Finn and Franklin 2011a). The second advantage is that linearity in the harmonic relationship can be used to estimate improvements of the composite frequency by discarding observations of harmonic frequencies with values falling outside \( \sigma_r \) recomputing \( f_u \) using only “valid” harmonics: a process that may be iterated until convergence is achieved. Indeed, empirical evidence shows that—once outliers are removed—standard deviations \( \sigma_{fu} \) and \( \sigma_{fr} \) of the composite frequency estimates \( f_u \) and \( f_r \), respectively, are typically around 0.01 Hz.

The final term—due to unknown or unmodeled factors—depends on the product of \( \sigma_r \) and the difference between \( f^M(t + \tau_p,0) \), which is measured directly, and \( f^c(t + \tau_p) \), which is derived from Eq. (1). The error in the derivative \( \partial f_w/(\partial t) \) may be estimated from noise in consecutive frequency measurements, that is, \( \sigma_r \approx 2^{1/2} \sigma_{fr} \).

To calculate \( f^c(t + \tau_p) \), Eq. (1) shows the UAV velocity, and the wind velocity and sound speed on board the UAV must be known. The direction of the wave front \( \mathbf{n}_u \) must also be determined. This may be achieved by determining the wind vectors from the UAV’s autopilot [note: in the absence of onboard observations of 3D wind velocity, errors are introduced into \( \mathbf{n}_u \), which results in second-order error terms that can be ignored (Finn and Rogers 2015)]. The group velocity of a ray in a moving medium is \( \mathbf{v}_{\text{ray}} = c_0 \mathbf{n}_u + \mathbf{v}_u \) [Eq. (3.31) in Ostashev 1997] such that the direction of the sound ray from the UAV to the ground microphone, characterized by the unit vector \( \mathbf{v}_{\text{ray}} \), may be estimated from the straight ray path approximation and its magnitude calculated from

\[
|\mathbf{v}_{\text{ray}}| = \sqrt{c_0^2 + (\mathbf{v}_u \cdot \mathbf{v}_{\text{ray}})^2 - \mathbf{v}_u \cdot \mathbf{v}_u + \mathbf{v}_u \cdot \mathbf{v}_{\text{ray}}}. \tag{8}
\]

The direction normal to the wave front can then be calculated approximately from

\[
\mathbf{n}_u = \frac{\mathbf{v}_{\text{ray}} - \mathbf{v}_u}{c_0}, \tag{9}
\]

based on values of \( \mathbf{v}_u \) and \( c_0 \) determined from meteorological observations of wind velocity and temperature taken on board the UAV. Experimental evidence (Rogers and Finn 2013b) shows \( f^c - f^M \approx 0.1 \) Hz.

The second and third terms in Eq. (7) are divided by \( \partial f_w/(\partial t) \) and \( (\partial f_w/(\partial t))^2 \), respectively. Moreover, as \( \partial f_w/(\partial t) \) must approach zero at some point in the overflight, the measurement of \( \tau_p \) becomes highly inaccurate at this point; and flight profiles must be designed correctly. In fact, unless the engine firing sequence of the UAV is carefully managed, a substantial proportion of the flight path may be unusable (Fig. 1).

Figure 1 (top) shows ground frequency profiles representative of a UAV flying at a nominal velocity of 28 m s\(^{-1}\). The UAV starts at an altitude of 300 m and a horizontal range of 1.4 km, and has a nominal composite frequency, \( f_u(t) = 50 \) Hz (a nominal frequency equivalent to a two-cylinder, four-stroke engine rotating at 3600 rpm). The black line represents a flight with constant horizontal and vertical velocity, that is, \( f_u(t) \) derives from an engine firing sequence that is constant with respect to time. The blue line represents a flight for which the UAV engine firing sequence increases linearly over the length of the flight path; that is, \( \partial f_u/(\partial t) \) is constant (changing linearly). The red and green lines represent flights for which \( f_u(t) \) is cycled as a sinusoid: once (red) and twice (green) over the duration of the flight.

Figure 1b (bottom) shows the resulting quantity of a flight profile for which \( \partial f_w/(\partial t) < 0.1 \) Hz s\(^{-1}\); about 81%, 29%, 11%, and 7% for each of the flight profiles. For ranges and altitudes greater than 1.4 km and 300 m, the percentage of usable data diminishes.

Figure 1 also accounts for a change in the UAV’s horizontal and vertical velocity imparted by a change in...
its throttle setting. This is accounted for by assuming that the forces on the aircraft are in quasi equilibrium; that is, for a steady-state flight drag \( D = k_1 u_x^2 = T \) and weight \( W = k_2 u_y^2 = L \), where \( k_1 \) and \( k_2 \) are constants; \( u_x \) and \( u_y \) are the horizontal and vertical velocities of the UAV, respectively; \( T \) is thrust; and \( W \) is weight. The thrust (and hence lift) is assumed linearly related to the rotation rate of the engine. The engine must rotate at 4500 rpm to generate the minimum airspeed (80 km h\(^{-1}\)) and produces a maximum of 6400 rpm for a top (horizontal) speed of 115 km h\(^{-1}\). The maximum vertical velocity for the UAV induced by throttle variation alone is 11 km h\(^{-1}\). All of these flight parameters are based on the Aerosonde UAV, details for which are available online (at www.aerosonde.com; Rogers and Finn 2013b).

If signals are propagated through a known atmosphere from–to known coordinates (i.e., \( \sigma_{p0} = 0 \)) and we use an estimate of the signal processing and unknown–unmodeled errors obtained from trials data (Rogers and Finn 2013a, b) (i.e., \( f_c^c - f_c^M = 0.1 \) Hz, \( \sigma_{f_c^c} = \sigma_{f_c^M} = 0.01 \) Hz, and \( \sigma_{\sigma_f} \approx 2^{1/2} \sigma_{f_c^M} \)), then the impact of \( \partial f_c^c / \partial t \) on the error in \( t_p \) may be determined using Eq. (7).

The results are shown in Fig. 2. The continuous black line represents the nominal propagation delay. The two other sets of lines represent the distributions of propagation delay errors due to the signal processing term (continuous green) and both signal processing and unknown–unmodeled terms (dotted red). The upper two images show estimated propagation delays for fixed and linearly increasing throttle settings (Fig. 2, left and right, respectively). The lower images show the same parameters for cycled throttle settings (the left/right images are for one/two cycles, respectively). Although the overflight with two throttle cycles has more singularities, the rate at which \( \partial f_c^c / \partial t \) approaches zero near each singularity means that overall there are more good data. Also, with the exception of the profile for which there are no throttle cycles, it can be seen that signal processing errors dominate the time delay estimation errors.

Estimates of time delay error are plotted for constant and twice-cycled engine/throttle profiles (Fig. 3); considerable improvement in measurement accuracy for a time-varying \( f_a(t) \) is evident (note the change of scale on the \( y \) axis for the top–bottom images). The profile generated using a constant throttle setting has only 5.8% observations with a standard error less than 10 ms, whereas the varying throttle setting delivers 88.5% observations better than this figure.

Increasing the number of frequency variations or the maximum throttle variation further reduces root-mean-square (rms) error by up to an order of magnitude (Fig. 4): 97.4% of the errors are \(<10\) ms and 79.4% \(<1\) ms. The number and extent of frequency variation are limited by the performance envelope of the UAV and the signal processing requirement that \( \partial f_c^c / \partial t \) is constant over periods of about 1–2 s; that is, \( 2^{15} - 2^{16} \) point fast Fourier transforms (FFTs) are used to determine the “instantaneous” frequencies, \( f_a \) and \( f_c \), from the 44.1-kHz digitally sampled sequences of acoustic data; and any jitter or lack of stationarity in the sampled signal will increase \( \sigma_{f_a} \) and \( \sigma_{f_c} \), respectively. A reasonable compromise is a throttle variation cycle of 10 s, which suggests 0.1–0.2-ms errors in time delay estimation. This
compares to 1–2-ms errors for two-cycle excursions or 10–20-ms errors (or significantly more) for no variation.

A secondary impact of the increased time delay measurement noise is to reduce the geometric distribution of high-quality time delay observations available to the inverse solution. The result is a sparse, narrow arc of good data directly above the sensing microphone (Fig. 5) and much poorer observations away from the vertical. This has several effects. It reduces the number of intersecting ray paths, which—as velocity field errors have a tendency to grow in the direction perpendicular to individual ray paths—degrades velocity field estimates. It also reduces the number of ray paths that usefully intersect the lower elements of any RBF lattice used to represent the atmosphere. This effectively reduces the contribution of these lower RBFs to the inverse solution (i.e., the problem becomes sparser and ill conditioned). This, in turn, has the effect of impeding discrimination between the scalar temperature and directional wind speed effects on sound speed and propagation delay, which are coupled in accordance with Eq. (3). Thus, to accommodate the increased measurement noise, the number of RBFs must be reduced.

Additionally, the choice of parameter $p$, which is used to model $C_D$ is affected. If the data are expected to be accurate, then a large error in the solution has a high importance and typically needs to be strongly deweighted—one may use higher values of $p$. On the other hand, if we expect a large scatter in the measurements, large errors have a similar impact to small ones, so a lower value of $p$ provides better estimates.

4. Acoustic propagation in simulated atmosphere

The principal difficulty in simulating realistic atmospheric profiles lies in the dominance of the nonlinear flow effects and the wide spectrum of scale sizes involved. Sullivan and Patton (2011) generated a suite of atmospheric simulations for a canonical daytime convective PBL using LES. The volume of atmosphere represented is $5120 \text{ m} \times 5120 \text{ m}$ (horizontal) $\times 2048 \text{ m}$ (vertical) for a uniform grid mesh of
and the simulations are carried forward in time for 25 large-eddy turnover times (about 38 min). The LES dataset provides temperature and 3D wind velocity vector at each point in the space–time lattice. The equations and input parameters used to simulate this weakly sheared daytime convective PBL are contained in Sullivan and Patton (2011).

Figure 6a (top left) shows a vertical cross section of temperature through the LES dataset typical of that used in this analysis. The top-right figure shows an expanded section of the image on the left. The bottom figure shows a similar image (also expanded from that shown in the top left) with wind velocity overlaid. Arrows point in the direction of wind flow, with magnitude representing wind speed (the maximum wind speed is 2.7 m s\(^{-1}\)). The greatest variation in temperature occurs in the lowest regions of the atmosphere, where for typical real-world UAV trajectories the ray paths are unable to intersect one another or the lower RBF centers, if RBFs are evenly distributed over the area under observation. There is also subtle variation in the background structure of the wind and temperature profiles at higher altitudes that we would likely wish to observe, should we employ UAV-based tomographic techniques.

There are, of course, many real-world sources of error for the techniques described in this paper, including Finn and Rogers (2015), who use ray path approximation; the effects of refraction on the propagating ray; multipath (e.g., from tall grasses, trees, shrubs, leaves, buildings, ground surfaces, interfaces, other obstacles, and partial transmission through atmospheric structure); the lack of wave coherence due to atmospheric turbulence, signal, and measurement jitter in the UAV tones; statistical errors in front-end time delay estimation and signal processing (e.g., the assumption of a stationary signal over the sampling period of a fast Fourier transform), wind noise and interference on ground microphones; UAV position and velocity errors; microphone location errors; variations in the sampling rate of the analog-to-digital converter (ADC); and uncertainty regarding moisture content in the atmosphere. As we make use of known atmospheric moisture content, we briefly discuss this error but refer readers to Finn and Rogers (2015) for a more detailed understanding of the potential effects of these errors on the resulting tomographic inversions.

At 20°C at sea level the effect of a 1°C increase in temperature reduces the speed of sound in air (343.2 m s\(^{-1}\)) by about 0.57 m s\(^{-1}\). The effect of moisture
content at the same temperature and pressure causes a variation of only 0.9 m s$^{-1}$ between dry and fully saturated air. Thus, a 10% error in estimation of humidity along the ray path corresponds to about 0.02% error in pathlength. The effect of refraction due to vertical humidity gradients is also important (Finn and Rogers 2015), but this effect is secondary compared to the wind and temperature gradient.

Here we simply apply levels of error in line with the estimates noted and/or derived in that earlier paper. These are superimposed onto the “true” value of each ray, applied as a bias and random noise represented by additive Gaussian white noise. Bias was calculated as a percentage of propagation delay and the random Gaussian errors are then applied around this value. They represented the following:

- A signal sampling regime that represents a 102-dB dynamic range, a 44.1-kHz analog-to-digital converter on both UAV and ground microphones, combined with a 2$^{15}$-point FFT with 4 times the oversampling and 50% overlap between sample blocks (ray paths from the UAV at 2 Hz);
- A signal processing regime that represents observed SNR applied to data sampled synchronously at 44.1 kHz at the ground microphones (1–10 ms);
- Positional errors in the location of the microphones and the UAV at each epoch as if obtained using real-time kinematic carrier phase differential GPS (0.1 m; Parkinson and Spilker 1996); and
- A refractive error in propagation delay equivalent to 0.1% of the nominal ray pathlength.

Variations in the sound speed along the length of the ray cause it to refract (Ostashev et al. 2008a). Relative to the nominal straight-line approximation for a ray, these variations in sound speed cause curvature in the ray (which is equivalent to an extension in the ray pathlength) and a deviation from the nominal travel time along it due to the difference in sound speed from its nominal value. A detailed analysis of the impact of these combined effects on the reconstructed temperature and wind fields depends upon ray path geometry and the atmospheric profiles through which the ray propagates, and this is beyond the scope of this paper.

We therefore employ an approximate approach to estimate the potential errors in pathlength caused by refraction. We ignore the effect of sound speed variations along the ray path in favor of the effects of ray path extension, noting that to a first order the effects combine to deliver a time delay estimate proportional to the integration along the ray. Using Eq. (3.57) in Ostashev (1997) and propagating the ray through the known vertical profile of wind and sound speed, the propagation delays were determined by numerically integrating each ray segment along a straight-line path between the start and end of the ray. Equation (3) was used to determine the sound speed for each individual ray segment. The integration steps were aligned with the cell size of the LES data.

Thus, while use of the ray length on its own is an imperfect guide by which to calculate refraction errors in an inhomogeneous atmosphere, the computationally much simpler approach used in this paper does permit error analysis of the type used below. However, as the quoted values (as a percentage of ray pathlength) represent the combined impact of ray curvature and sound speed variation along the (curved) ray, the ray path errors are likely to represent the lower end of estimates for the true errors; that is, a ray path with a quoted error of 0.1% may, in fact, have an error greater than this.
The scenarios made use of a simulated linear array of ground sensors located over baselines ranging from 100 to 500 m. A range of sensor separations were used in the simulations. These ranged from 10 to 100 m, but such that at least 10 ground sensors covered any baseline. The effects of sensor separation is discussed in Rogers and Finn (2013c), Kozick and Sadler (2000), and Ash and Moses (2005).

A UAV was then “flown” through the target atmosphere at a speed of 28 m s\(^{-1}\). The iterative inversion procedures described in section 2 were used to estimate temperature and wind fields within the 2D cross section. Although LES allows for simulation of time-varying atmospheres, wind and temperature profiles were assumed to be frozen over the observation period.

Figure 7 shows comparisons between target and tomographic estimates of temperature and wind profiles derived using the error regimes shown in Fig. 4 (errors of 0.1 ms for signal processing and \(\leq 0.01\%\) for ray path errors are superimposed onto each ray path). Ground sensors were spaced at 12.5-m intervals. Temperature is coded according to the color scale at the right of each image and wind velocity is shown using arrows, scaled according to maximum wind speed for the target atmosphere (2.9 m s\(^{-1}\)). Accurate reconstruction of target atmospheres is possible. Simulations for the same target atmosphere, but for which the number of throttle variations is reduced, are shown in Fig. 8 (errors of 1 ms and 0.1\%). These produce significantly poorer estimates.

The error plots for both Figs. 7 and 8 are shown in Fig. 9. There are two component errors: direct and indirect. The direct error is the degree of mismatch between the target LES profiles and their replication using RBFs: the more RBFs used, the more faithfully the target data are replicated, and the lower the direct error. Ideally, massively dense RBF sets are used. However, the ill-posed and sparse nature of the inverse problem forces a reduction in the number of RBF involved in the inversion, and hence its resolution.

The indirect error is the accuracy with which the inversion is able to faithfully represent the “best” RBF fit to the target data. In other words, it is possible to have
substantial direct error but negligible indirect error if the original LES profiles are heavily spatially averaged through use of a small number of RBF. Thus, a key measure of success for the overall technique is given by comparison of the direct (asterisks) and indirect errors (triangles) as the link shows correspondence between the tomographic inversion and the LES data at the same atmospheric scale size and RBF resolution (Fig. 9).

To a first order the indirect error is governed by the number of RBFs (and hence separation distance between them). While the Eikonal assumptions for propagation through such a medium would be questionable, for RBF separation distances <1 m, temperature differences between the estimated and target atmospheres are negligible (<0.01°C). If the RBF separation distance is increased to 10 m (as per Fig. 7), then correspondence falls to about 0.2°C (1σ). Pressure to increase separation distance between RBF (and hence reduce resolution and accuracy of the solution) is predominantly driven by the need to manage the number of degrees of freedom (DOFs) in the inversion. Too many DOFs lead to numerical instabilities, as errors in one (poorly observed) RBF coefficient typically propagate to others. This is because the intercellular relationship between coefficients is (at present) unconstrained, for example, by a correlation distance such as used in stochastic inversion (Ziemann et al. 2002).

5. Conclusions

Previous work, including some by the authors, has shown that AAT using a propeller-driven UAV as a sound source is technically feasible. The faithfulness with which temperature and wind velocity profiles of the

FIG. 7. (top) Target temperature and wind profiles and (bottom) those estimated using tomography. Errors due to signal processing and ray path propagation are 0.1 ms and 0.01%, respectively.

FIG. 8. Another example as in Fig. 7. Here errors due to signal processing and ray path propagation are 1 ms and 0.1%, respectively.
atmosphere may be replicated is, in part, limited by the accuracy with which the signal time of flight between the UAV and ground sensors may be measured. If the narrowband tones generated by the UAV’s engine and propeller are correctly combined, then we avoid the need for additional payloads on board the UAV, which significantly improves its endurance capabilities. Unfortunately, the measurement accuracy of the required techniques is related to the inverse of the rate of change of received frequency, and for a UAV flying straight and level at constant velocity, this derivative approaches zero at both extremes of any flight path. The distribution of errors associated with the propagation latencies therefore become very large. Such high levels of time delay measurement error impose further restrictions on the tomographic solution through the ill-posed nature of the inverse problem, which results from the poor ray–path–medium intersection geometry.

Simple throttle control may be used to vary the engine firing rate of a propeller-driven UAV. In this paper we show that this throttle control may be used to simultaneously vary both the narrowband tones of any emitted signature and the approach velocity of a UAV toward any ground microphones. Propagation latencies may then be measured an order of magnitude more accurately than previous research has suggested. This finding has implications for flight performance characteristics of UAVs used in AAT, their overflight trajectories, and the rate at which meteorological and position data are sampled on board. These implications have yet to be fully explored, but in combination they overcome a number of problems linked to poor ray geometry and the ill-posed nature of the tomographic inversion. The requirement for time variation in the UAV signature, however, suggests a need for signal processing techniques that do not assume time stationarity in the emitted–received tones. This is the subject of ongoing work by the authors.

A novel inversion approach for UAV-based AAT is also suggested. This fresh approach allows significantly more DOF to be included in the tomographic inversion than earlier RBF techniques have permitted. As a result, higher-resolution atmospheric structure can now be derived and represented using UAV-based AAT and with greater accuracy. This is demonstrated through application of the inversion techniques to LES temperature and wind profiles representative of realistic conditions for a weakly sheared daytime convective atmospheric boundary layer.

Previously, UAV-based AAT analyses have been carried out on synthetic temperature and wind fields generated using RBF. As the RBFs used in the earlier studies imposed scale size variations of the order of 100–200 m, these analyses were unable to examine the effects of small-scale (<25 m) size variations of wind and temperature on tomographic reconstruction techniques. The analysis in this paper relies on high-resolution (2 m × 2 m × 5 m) LES data, and it shows that the effects of such small-scale perturbations manifest themselves by increasing the measurement accuracy requirements. In other words, it suggests that small-scale variations in sound speed increase the “noise” on time delay estimates such that time delay observations with errors <0.1 ms and refraction <0.01% of true path-length may be required to deliver accurate reconstructions of target atmospheres. This is an order of
magnitude more stringent than previous research has suggested. Other inversion strategies that accommodate more sophisticated refraction models may yield more robust results. Unfortunately, variation in the atmosphere over the period of UAV overflight may result in tighter measurement accuracy requirements.

Acknowledgments. The authors are grateful to both the Australian Research Council and the Sir Ross and Sir Keith Smith Fund for their support to this project. We are also grateful to our colleagues Maurice Gonella of Aerosonde, Peter May and John Nairn of the Bureau of Meteorology (BoM), and Greg Holland and James Done of the National Center for Atmospheric Research (NCAR). Their ongoing assistance and support in this project have been most valuable.

REFERENCES


Acknowledgments. The authors are grateful to both the Australian Research Council and the Sir Ross and Sir Keith Smith Fund for their support to this project. We are also grateful to our colleagues Maurice Gonella of Aerosonde, Peter May and John Nairn of the Bureau of Meteorology (BoM), and Greg Holland and James Done of the National Center for Atmospheric Research (NCAR). Their ongoing assistance and support in this project have been most valuable.


