Quantifying the Error of Radar-Estimated Refractivity by Multiple Elevation and Dual-Polarimetric Data

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ABSTRACT

To properly use radar refractivity data quantitatively, good knowledge of its errors is required. The data quality of refractivity critically depends on the phase measurements of ground targets that are used for the refractivity estimation. In this study, the observational error structure of refractivity is first estimated based on quantifying the uncertainties of phase measurements, data processing, and the refractivity estimation method. New correlations between the time series of phase measurements at different elevation angles and between polarizations are developed to assess the bulk phase variability of individual targets. Then, the observational error of refractivity is obtained by simulating the uncertainties of phase measurements through the original refractivity estimation method. Resulting errors in refractivity are found to be smaller than 1 N-unit in areas densely populated with reliable point-like stationary ground targets but grow as the target density becomes sparse.

1. Introduction

Convection initiation and short-term quantitative precipitation forecasting are sensitive to the moisture variability at the surface (e.g., Zawadzki et al. 1981; Crook 1996; Weckwerth et al. 1999). High-spatial- and high-temporal-resolution near-surface moisture measurements are thus needed for improving the initial conditions in storm-scale numerical weather prediction (NWP) models (Emanuel et al. 1995; Dabberdt and Schlatter 1996; Fabry and Sun 2010; Hanley et al. 2011; Ha and Snyder 2014; Madaus and Hakim 2016). The refractivity (N) field estimated from weather radars (Fabry et al. 1997) provides a proxy for two-dimensional near-surface moisture distribution. The high-spatiotemporal-resolution refractivity field mimics the horizontal moisture variations, which benefits studies on convection initiation and boundary layer evolution (Weckwerth et al. 2005; Fabry 2006; Buban et al. 2007; Koch et al. 2008; Bodine et al. 2010) as well as providing valuable information for NWP models.

Since radar refractivity mimics the high-resolution moisture variability, preliminary studies have demonstrated the positive impact of ingesting the radar refractivity fields into the mesoscale NWP models to initialize the low-level moisture field (Montmerle et al. 2002; Sun 2005; Gasperoni et al. 2013; Seko et al. 2017). The results of quantitative precipitation forecasting were improved by the adjusted distribution and quantity of low-level moisture. In addition, the refractivity fields are applied to evaluate the model forecasts at different horizontal grid resolutions (Nicol et al. 2014; Besson et al. 2016). Greater differences in refractivity between radar estimates and model forecasts are particularly found in NWP models with finer horizontal resolution (Nicol et al. 2014). This suggests the critical need for assimilating radar refractivity data into advanced high-resolution NWP models. The Weather Meteorological Organization (WMO 2015) further advocates for more studies on the impact of assimilating radar-estimated refractivity data for real cases to examine the value for operational high-resolution nowcasting NWP models. In addition, while the focus of this paper is on quantifying observation errors for use in data assimilation at the convective scale, it should not be construed as implying that this is the main assimilation application of refractivity data (see the appendix for an illustration in a more regional-scale context).

The purpose of data assimilation is to find the optimal condition, known as analysis, for model integration forward in time (Kalnay 2003). The data assimilation system statistically combines observations and forecasts...
by weighting them on their error information. The updated analysis \( X_a \) can be written in matrix form as

\[
X_a = X_b + W[y - H(X_b)].
\]  

(1)

Here, \( H \) is the linearized observation operator that transfers the background field \( X_b \), the unobserved state variables in the model, into simulated observations compatible with the observations \( y \). The optimal weight matrix \( W \) equals \( B H^T (B H^T + R)^{-1} \), where \( B \) and \( R \) are the background and observational error covariances, respectively. The value of the increment \( (X_a - X_b) \) of the analysis field depends on the innovation \( [y - H(X_b)] \) and \( W \) determined by \( B \) and \( R \). Hence, \( B \) and \( R \) need to be estimated to properly assign the weights of observations in optimal estimation methods.

Quantitative knowledge on the data quality and observational errors is an essential step toward properly using data in NWP models. The characteristic of observational error of radar refractivity still remains unquantified and hinders the application of refractivity. The observational errors \( R \) used in most previous studies were assumed randomly distributed in space with a Gaussian distribution with no clear justification or basis for this choice, since \( R \) is not well known and quantified. Therefore, this study aims to investigate the quantitative characteristics of observational errors of radar refractivity. The observational error matrix in data assimilation can generally be attributed to four sources: 1) instrument and measurement error, 2) errors caused by data processing (quality control), 3) errors introduced by the observation operator (forward model), and 4) representativeness errors (i.e., different representativeness of resolved spatial scales between models and observations). Obtaining observational error is challenging, since the true value of the atmospheric state is unknown. Some methods have been proposed to estimate the magnitude and characteristics of observational errors. One is the error inventory method that analyzes all the contributions of the uncertainties to the observations. For example, Keeler and Ellis (2000) used the knowledge on data quality of radar signals to estimate the observational errors of radar reflectivity and radial wind caused by measurements and data processing. In addition, diagnostic methods are popularly applied in the model community for estimating observational errors. The observational errors are mainly obtained based on output from data assimilation systems, for example, observation minus background and observation minus analysis (Hollingsworth and Lonnberg 1986; Desroziers et al. 2005; Waller et al. 2016).

The goal of this paper is to estimate the observational errors of radar refractivity through the error inventory method. In section 2, all possible sources of uncertainty in the radar phase measurements and data processing based on the original radar refractivity technique (Fabry et al. 1997) are revisited and discussed. In section 3, we propose a method to estimate the uncertainties of the radar phase measurements that have not been quantified before. The errors of radar refractivity associated with instrument measurements, data processing, and the estimation method are estimated in section 4. We then summarize our findings in section 5.

2. Sources of uncertainty in radar refractivity

In this section, we revisit the basic concept and data processing processes of radar refractivity estimation. These enable us to sort out the sources of the uncertainties in radar refractivity.

a. Basis behind radar-estimated refractivity

The time \( t_{\text{travel}} \) that electromagnetic waves travel between the radar and a stationary ground target changes with the refractivity of the air along its propagation path. Fabry et al. (1997) used the phase signal from a fixed ground target as a proxy for \( t_{\text{travel}} \) to estimate the air refractivity. The radar-measured phase \( \phi \) of a stationary point ground target depends on the time taken by a radar pulse for a two-way path:

\[
\phi(r) = 2\pi f t_{\text{travel}} = \frac{4\pi f}{10^6c} \left[ \int_0^r N(r', t) + 10^6 \right] dr',
\]

(2)

where \( f \) is a stable radar transmitter frequency, \( r \) is the one-way beam path range from the radar to the target, and \( c \) is the speed of light in a vacuum. Since \( r \) is not known with the required precision to use (2) and since the observed phase largely exceeds \( 2\pi \), the phase difference \( \Delta\phi \) of a stationary ground target between a scan at time \( t \) and at a reference time \( t_{\text{ref}} \) can be used to relax considerably the need for precision on \( r \) and to reduce the problem of the aliasing of phases:

\[
\Delta\phi = \phi_t - \phi_{t_{\text{ref}}} = \frac{4\pi f}{10^6c} \left[ \int_0^r N(r', t) dr' - \int_0^{t_{\text{ref}}} N(r', t_{\text{ref}}) dr' \right].
\]

(3)

To estimate refractivity change between two time steps (\( \Delta N = N_t - N_{t_{\text{ref}}} \)), the reference phase \( \phi_{t_{\text{ref}}} \) needs to be determined first when \( N_{t_{\text{ref}}} \) is known. Then, the temporal local small-scale variations of \( \Delta N \) can be derived from the radial gradient of the phase difference between these targets. By adding \( \Delta N \) to the known \( N_{t_{\text{ref}}} \) field, the refractivity value can be estimated.
There are several processing steps to get the refractivity field from the radar measurements as shown in Fig. 1 (Fabry and Pettet 2002). At calibration stage, $\phi_{\text{ref}}$ from a known homogeneous refractivity field $N_{\text{ref}}$ and the data quality of ground targets need to be determined. At real time stage, the $\Delta \phi$ field between the current time and the reference time is calculated. Then, the noisy $\Delta \phi$ field is processed (e.g., via smoothing weighted by the reliability of targets, dealiasing, and interpolations) to obtain a smoother $\Delta \phi$ field to easily estimate the local $\Delta N$ from the radial slope of the processed $\Delta \phi$ field. The final $N$ field is obtained as the sum of $N_{\text{ref}}$ and $\Delta N$. 

**FIG. 1.** The data processing flowchart of obtaining near-surface refractivity from radar measurements (Fabry and Pettet 2002).
The data quality of the radar refractivity is affected by both the phase measurements of ground targets and by the radar refractivity estimation method. Note that the estimation method includes the data processing procedures and the local \(N\) estimation as mentioned in the previous paragraph. Nevertheless, previous studies mainly focused on qualitative discussions of different sources of phase measurements uncertainty (see the following section), but they seldom investigated the magnitude of the uncertainties of measured phase data and the estimation method.

### b. Uncertainties of the phase measurements of ground targets

The phase change caused by the change in refractivity at a given height above the terrain is the signal of interest for radar refractivity estimation (Fabry 2004; Feng et al. 2016). However, different reasons other than this atmospheric-driven signal also lead to uncertainties in \(\phi\) or introduce errors in the refractivity estimations. Generally, contributions to the uncertainty in the phase measurement come from the phase variations associated with issues of radar hardware, radar wave propagation conditions, and characteristics of ground targets. The details of each category are described as follows.

- **Radar hardware:** Phase uncertainty associated with radar hardware issues includes the varying frequency of a magnetron-based radar (Parent du Chatelet et al. 2012; Nicol et al. 2013) and the phase pattern of the parabolic antenna (Feng and Fabry 2016). These systematic phase shifts can be corrected by knowing the fluctuating local oscillator frequency and the antenna phase pattern.

- **Radar wave propagation conditions:** Electromagnetic wave propagation conditions associated with the vertical refractivity gradient \(dN/dh\) and the height differences between the radar and ground targets have been considered the major problem in the original method (Fabry 2004; Park and Fabry 2010; Bodine et al. 2011). A novel method to reduce this uncertainty caused by the coupling effect of \(dN/dh\) and target heights has been proposed in Feng et al. (2016). Moreover, precipitation along the wave path also affects the phase measurements, which can be estimated from the rainfall rate (Bodine et al. 2011).

- **Characteristics of ground targets:** The uncertainties of the phases related to the characteristics of ground targets include the movements of targets, changes in how a complex target with multiple reflecting elements are illuminated by the radar (Fabry 2004), sidelobe contamination from strong neighboring targets (Besson et al. 2012; Nicol and Illingworth 2013), and the unknown target locations with respect to the center of the sampled range gate (Nicol et al. 2013). The uncertainties of target locations in range, azimuths, or heights affect the interpretations and representativeness of phase measurements, even for ideal stationary point-like targets. For example, the unknown ranges of targets in conjunction with transmitter frequency fluctuations result in random phase errors that are proportional to the magnitude of the frequency change and the pulse length (Nicol et al. 2013). In addition, if targets are not aligned on the same azimuth and refractivity changes locally, then some spatial noisiness of phase might also be introduced. Since the time between most radar volume scans is about 5–10 min, refractivity variations at small scales also introduce some fluctuations in phase that may not be properly interpreted as such, particularly when the exact location of targets is unknown.

The uncertainties in phase measurements can be classified into **biases** and **errors**. Biases are systematic given a set of conditions and can be corrected based on these known conditions, such as the measured changes in the unstable frequency of the local oscillator and estimated \(dN/dh\). Phase uncertainties caused by radar hardware, the coupling effect of changes in \(dN/dh\), and uneven target heights, as well as precipitation delay, are considered biases. These systematic biases need to be corrected during the data processing. Then, the unbiased refractivity can be properly used in quantitative applications. On the other hand, phase uncertainties that cannot be corrected are errors. The phase errors include the random phase changes introduced by the unknown target characteristics, for example, movements, complexity, and locations. Since these terms are independent, the total error variance of phase \(\sigma_{\phi,\text{total}}^2\) equals the sum of individual error variances:

\[
\sigma_{\phi,\text{total}}^2 = \sigma_{\phi,\text{(target movement)}}^2 + \sigma_{\phi,\text{(target complexity)}}^2 + \sigma_{\phi,\text{(target location uncertainty coupled with small-scale N changes)}}^2
\]  

(4)
3. Estimating phase measurement errors $\sigma_{\phi_{\text{total}}}^2$

Estimating $\sigma_{\phi_{\text{total}}}^2$ as a bulk error remains a challenge, not to mention each individual term in (4). It is an ill-posed problem to obtain the signal and noise simultaneously with limited phase measurements. Therefore, we propose a new method for quantitatively estimating $\sigma_{\phi_{\text{total}}}^2$ through more phase measurements collected at multiple antenna elevations and at two polarizations.

a. Correlation of two time series of radar signals

Let us conduct a simple thought experiment to help understand the correlation of the radar signal at two lower elevation angles. An ideal stationary point-like target is considered here and it is located well within the main lobe of the antenna, where the antenna phase pattern is constant (Feng and Fabry 2016). When there is no temporal change in refractivity, the phase measurements from an ideal target at two low antenna elevations—say, 0.3° and 0.5°—are constant in time. Meanwhile, the time series of phase measurements at horizontal and vertical polarizations at a given elevation are also constant. In other words, the phase differences between two elevations or between two polarizations stay constant in time. Nevertheless, for nonideal ground targets, the phase differences between two elevations vary temporally because of target movements and unknown target location issues associated with small-scale refractivity changes as mentioned in (4). The phase difference between polarizations also shows temporal variability caused by the complexity of targets and its changing illumination of the targets. Therefore, the information of phase difference between polarizations and between elevations provides both qualitative and quantitative insights on the phase errors.

We use the correlation coefficient $\rho$ of two time series of phase measurements $\phi_1$ and $\phi_2$ over $M$ volume scans from a given ground target to quantify the temporal variation of phase differences. Then, $\rho$ can be expressed as

$$\rho = \frac{1}{M} \left| \sum_{m=1}^{M} \exp[i(\phi_{2,m} - \phi_{1,m})] \right|. \quad (5)$$

These two time series of phase measurements can be applied for two elevations or two polarizations. When $\phi_1$ and $\phi_2$ are identical or with a constant phase difference in time, $\rho$ is equal to one. For most targets, $\rho$ decreases with increasing temporal variability of $\phi_2 - \phi_1$ as a result of target movements under various wind speeds, small-scale refractivity variations, and complicated wave interferences of nonideal point targets. We learned that $\rho$ qualitatively decreases with the temporal fluctuations of the phases, but quantifying the variance of these phase errors $\sigma_{\phi_{\text{total}}}^2$ is a problem that needs to be solved in order to estimate the error of radar-estimated refractivity.

Note that the $\rho$ of a ground target here is calculated from the phases of a given radar-sampled range gate (resolution volume). For example, the ideal point-like ground target we mentioned is an isolated power or communication tower within the gate that dominates the returned signals of that given gate. In reality, there are usually more than one ground target within a gate. Even with high-resolution lidar measurements or high-resolution satellite images, we cannot resolve every single ground target in detail within a resolution volume for more complicated convolution computation of the radar antenna pattern and these ground targets’ reflected characteristics. The phase measurement of a given sampled range gate provides bulk characteristics of the radar wave interacting (convolving with) all the ground targets within the resolved volume. The term “ground target” provides a simple concept to understand, so we keep using “$\rho$ of ground target.” Also note that we do not calculate the correlation between the original returned signals $V$ that are generally used in the radar signal processing; $V$ is a complex number as $V = A \exp(i\phi)$, where $A$ is the amplitude of the signal and $\phi$ is the phase. It is because the returned power fluctuates in time for many reasons and then affects the magnitude and the representativeness of correlation calculated from these signals. Therefore, we use the correlation of phases to determine the temporal variation of the phases’ difference and exclude the effects of fluctuating amplitudes.

b. Types of correlation coefficient of phases

The correlation coefficient of phases can be calculated between two elevations $\rho_{\text{ele}}$, between two polarizations $\rho_{\text{pol}}$, or from a time series of successive scans at a given polarization and elevation using lagged autocorrelation $\rho_{\text{auto}}$. Here, we will discuss and compare the differences between these $\rho$ (Fig. 2) calculated from 3-h data of the McGill radar. The selected time period is 1100–1400 UTC (0700–1000 LST) 30 October 2012. The weather condition was cloudy with some showers. The average wind speed is 7 m s$^{-1}$ from surface stations and the average vertical gradient of refractivity is about $-70 \pm 15$ km$^{-1}$.

- The coefficient $\rho_{\text{ele}}$ is a useful indicator to distinguish the reliability of targets, particularly in windy conditions. This correlation is calculated from time series of phases at two low successive antenna scanning elevations, for example, 0.3° and 0.5° for the McGill radar. The time interval between these two elevations is about 10 s, and the temporal refractivity changes...
within this short time period can be assumed to be negligible. For stationary ideal point-like targets located within the radar antenna main beam, the phase difference between nearby elevations is constant in time based on the antenna phase pattern (Feng and Fabry 2016). Thus, the temporal variability of the phase difference between these elevation scans is mainly caused by the changes in target characteristics [i.e., movement and complexity in (4)]. For example, winds cause target movements and shape changes even within a few seconds, leading to a temporal phase difference between the two elevations and a reduction in the magnitude of $\rho_{\text{ele}}$. Figure 2a shows the distribution of $\rho_{\text{ele}}$ at horizontal polarization on a windy morning. Higher values of $\rho_{\text{ele}}$ shown as yellowish colors represent the reliable ground targets with smaller phase errors. On our radar, the number of targets (Fig. 2d) whose $\rho_{\text{ele}}$ is greater than 0.8 is about 15,000, which is 1% of the data within a 62.5-km radius. In addition, $\rho_{\text{ele}}$ fields at horizontal and vertical polarizations show very similar distribution, the minor differences are due to the antenna pattern and the target characteristics responding to the different polarizations (Feng 2017).

Moreover, some practical criteria are suggested for selecting the two elevation angles required to calculate $\rho_{\text{ele}}$. The angle difference between these two elevations is recommended to be less than the half of the antenna beamwidth, considering the characteristics of antenna phase pattern. A temporal interval of less than 1 min between these elevations is also recommended.

- The coefficient $\rho_{\text{pol}}$ is best used to distinguish point or point-like targets from complex ones. For point-like ground targets, the phase difference between the simultaneously transmitted and received horizontal and vertical polarizations is constant in time, since it is solely related to the discrepancy of the antenna phase patterns at each polarization. As a consequence, the value of $\rho_{\text{pol}}$ of point-like targets is expected to be close to one. For complicated targets, the phase difference between the two polarizations varies temporally and this results in a lower $\rho_{\text{pol}}$. The phase discrepancy between polarizations mostly occurs during atmospheric inversion conditions at night with strong negative $dN/dh$ values (Feng and Fabry 2013).

When the radar beam bends toward the ground, the main beam may illuminate more of the full vertical
extent of targets or surrounding targets, such as the ground and shorter buildings, that were not seen by the main beam under normal propagation conditions. Then, the complicated shape of the targets responds differently to horizontal and vertical polarizations, and a lower value of $\rho_{\text{pol}}$ is consequently expected for complicated targets. The values of $\rho_{\text{pol}}$ (Fig. 2b) are thus mostly higher than $\rho_{\text{ele}}$, particularly under the normal propagation conditions. The histogram of $\rho_{\text{pol}}$ (Fig. 2e) shows the majority of $\rho_{\text{pol}}$ centered around 0.4, which is different from the distribution of $\rho_{\text{ele}}$ (Fig. 2d), which is more skewed toward zero. Though $\rho_{\text{pol}}$ is a good indicator for the ground target properties, it cannot provide too much information on the phase error caused by target movement and small-scale refractivity changes. Additionally, a lower value of $\rho_{\text{pol}}$ might be a result of problematic unstable phases of either horizontal or vertical polarization, or of both the polarizations. The challenge of using $\rho_{\text{pol}}$ alone is that we do not know whether to discard the phases at both polarizations or to use either one of them.

- The autocorrelation $\rho_{\text{auto}}$ of phases at a given elevation and polarization but between successive volume scans is dominated by the temporal refractivity change between volume scans, whose time interval depends on radar scanning speed (e.g., 5 min for the McGill radar). For stationary point-like ground targets, the phase change caused by the refractivity variation between successive volume scans is generally larger than other causes in (4). Thus, the values of $\rho_{\text{auto}}$ (Figs. 2c and 2f) are smaller than the previous correlations, particularly at farther ranges. The variable $\rho_{\text{auto}}$ can represent the variance of phase errors. Nevertheless, $\rho_{\text{auto}}$ can be useful to determine the reliability of ground targets under only one special condition, in the presence of a homogeneous refractivity field in time and space at the calibration stage (Fabry et al. 1997).

c. Which correlation to use

The phase uncertainties $\sigma^2$ discussed in (4) caused by target movement and shape complexities are related to the near-surface wind conditions. We compared the relationship between areal-average $\rho$ within the radar coverage and the wind speed from surface stations near the McGill radar during summer days. Figure 3 shows that $\rho$ generally decreases with increasing wind speed, but the decreasing rates vary among different $\rho$. Correlation $\rho_{\text{ele}}$ is the most sensitive to the variation of wind speed. It is because target movements are strongly affected by the wind speed, and the phase differences between two elevations separated by a 10-s time interval can be substantial. For instance, a radial displacement of half a wavelength—that is, 5 cm for S-band radars—leads to a 2$\pi$ phase variation. When the wind speed is greater than 15 m s$^{-1}$, $\rho_{\text{ele}}$ lowers to a constant value around 0.3. On the other hand, a higher $\rho_{\text{ele}}$ might happen during calm environment conditions with light (or no) wind. The higher values of $\rho_{\text{ele}}$ are contributed from the nonideal targets without movements. The variation ranges of $\rho_{\text{pol}}$ and $\rho_{\text{auto}}$ show only relative minor changes with wind speed compared to the variation of $\rho_{\text{ele}}$. In summary, $\rho_{\text{ele}}$ is preferred to provide the time-dependent phase quality under various weather conditions over the other two correlations.

Based on the different characteristics of each correlation (Table 1), we suggest applying $\rho_{\text{ele}}$ for selecting reliable ground targets and for estimating the phase errors. Correlation $\rho_{\text{ele}}$ is the most representative of the phase uncertainties in (4) among other correlations. Using $\rho_{\text{ele}}$ is practical for most weather phenomena, and it does not have to commit to an artificial time like $\rho_{\text{auto}}$ in the original method. It can also be used as a weighting index of each individual ground target in the real-time data processing (step III in Fig. 1). Even though $\rho_{\text{ele}}$ still underestimates the effect of uncertainties of target locations, it is so far the best we have found. The time-dependent $\rho_{\text{ele}}$ provides an effective way for real-time ground target selection and quantification of the error of phase measurements for the radar-estimated refractivity technique. In addition, in extreme propagation conditions, $\rho_{\text{pol}}$ adds value by detecting the abnormal phase behaviors from complicated targets.
d. Link between ρ and σφ

To quantitatively describe the uncertainties of the phase error, the knowledge of the relationship between the correlation of two time series of phase and the standard deviation of the phase error, hereafter the ρ–σφ relationship, is required. The conventional equation for estimating the angular standard deviation of phase is based on a wrapped normal distribution:

\[ \sigma_{\phi} = \frac{180}{\pi} \left[ -\ln(\rho^2) \right]^{1/2}, \]

that is \( \rho = \exp[-(\sigma_{\phi}/57.3)^2] \), if \( \sigma_{\phi} \) is expressed in degrees (Mardia 1972; Weber 1997). Since the magnitude of the phase error changes with time, the length of the time series (i.e., number of volume scans) used to estimate that fluctuation must be a compromise between greater accuracy, calling for a longer time series, and adaptability to changing conditions, calling for shorter time series. Even though the ρ–σφ relation exists, we investigated the sensitivity of the sampling.

To achieve this, we performed a series of simulations using various numbers of samplings \( M \). The error of the phase measurement is assumed to follow white Gaussian noise statistics with a given standard deviation \( \sigma_{\phi} \). Even though the noisy phases caused by small-scale refractivity variation and target movements might vary as a function of time, white noise statistics is the only assumption we can make given the limited observation samples and unknown wind conditions. Time series data of fluctuating phases \( \phi_{1,M} \) and \( \phi_{2,M} \) are generated independently as random white noise with a given \( \sigma_{\phi} \) and zero mean. Then, \( \rho \) of these simulated \( \phi_1,M \) and \( \phi_2,M \) is calculated for a variety of \( \sigma_{\phi} \), ranging from 0° to 120°. For example, using \( M = 36 \) volume scans of a time series, the relationship between \( \rho \) and \( \sigma_{\phi} \) is shown as blue dots in Fig. 4. The \( \rho–\sigma_{\phi} \) relationship is then fitted with a Gaussian distribution as \( \rho = \exp[-(\sigma_{\phi}/59.2)^2] \), shown by the red line.

We examined the \( \rho–\sigma_{\phi} \) relation under various samples. The \( \rho–\sigma_{\phi} \) fitting starts to converge around 30 samples where the relationship is \( \rho = \exp[-(\sigma_{\phi}/60)^2] \). The fit becomes stable when there are 60 samples and the relationship turns into \( \rho = \exp[-(\sigma_{\phi}/57.5)^2] \).

The slight difference in the fitting coefficient between 30 and 60 samples does not affect \( \sigma_{\phi} \) too much under a given \( \rho \). Therefore, sample numbers greater than 30 are acceptable to apply the fitted relationship. In this work, we used 36 samples of the time series, corresponding to 3 h of data from the McGill radar, to obtain \( \rho \) and the \( \rho–\sigma_{\phi} \) relationship. In the end, \( \sigma_{\phi} \) can be estimated based on \( \rho \) calculated from the radar observation and the newly derived \( \rho–\sigma_{\phi} \) relationship, at least as long as \( \rho \) is greater than 1/e.

4. Quantification of the error of radar-estimated refractivity \( N_{\text{error}} \)

Based on the quantitative knowledge of phase uncertainties at each gate sampled by the radar in section 3, the impact of phase quality on \( N_{\text{error}} \) can thus be quantified. In addition, even though the biases of refractivity associated with the target height and \( dN/dh \) can be corrected given reasonable assumptions for target heights, there are still concerns about the impact of the uncertainties of the target heights and estimated \( dN/dh \). The \( N_{\text{error}} \) caused by these uncertainties is also discussed and examined.

a. \( N_{\text{error}} \) caused by phase uncertainties and the estimation method

The data quality of the radar refractivity depends both on the phase measurements of ground targets and on the radar refractivity estimation method as shown in Fig. 1.
The following steps attempt to estimate $N_{\text{error}}$ that is introduced by phase errors through the data processing of phases and the $N$ estimation method: 1) For each sampled range gate, $\rho_{\text{ele}}$ is calculated from the observed phase measurements at two low antenna elevations. Consequently, $\sigma_\phi$ is obtained based on the calculated $\rho_{\text{ele}}$ and the $\rho$–$\sigma_\phi$ relationship. 2) The noisy phase at each sampled gate is simulated by random errors based on Gaussian statistics with a mean of zero and the assigned $\sigma_\phi$ of given $\rho_{\text{ele}}$. 3) We can estimate $N$ from the simulated phase field in the previous step through the method for estimating radar refractivity. Then, $N_{\text{error}}$ is obtained by comparing the estimated $N$ with the expected one.

Taking a real case for example, the refractivity error is examined and obtained based on the phase perturbation estimated from the $\rho_{\text{ele}}$ field (Fig. 5a). The errors in refractivity (Fig. 5b) are not randomly distributed as generally assumed for the observational error for data assimilation but are associated with the density of reliable targets. In areas with dense reliable ground targets of higher $\rho_{\text{ele}}$, $N_{\text{error}}$ is close to zero and is smaller than those in areas with fewer reliable ground targets. This experiment confirms that we can use the radar-estimated refractivity with higher confidence in areas having a greater number of reliable targets. Further, in order to determine the variance of $N_{\text{error}}$, this process of taking a known phase field, adding a random noise based on the expected phase fluctuation for each gate,
estimating $N_e$ and comparing it with the truth was repeated 30 times. The ensemble mean of the refractivity error field $N_{\text{error}}$ is almost zero as shown in Fig. 5c. The ensemble standard deviation of the refractivity error $\sigma_{N_{\text{error}}}$ shown in Fig. 5d is within 1 N-unit in areas with numerous reliable targets, generally in urban and suburban regions (e.g., areas with yellowish shads in Fig. 5a). Figure 6 further shows that the areal-mean $\sigma_{N_{\text{error}}}$ is inversely proportional to the areal target density fraction above a given threshold, here $\rho_{\text{ele}} > 0.6$, based on Fig. 5d. In addition, this simulation was repeated with a higher value of average $\Delta N$—here, 30 N-unit—in order to examine the impact of the phase errors on the refractivity in the presence of large phase gradients in range. The $\sigma_{N_{\text{error}}}$ field (not shown here) presents the similar pattern as it when $N$ is equal to 0 N-unit in Fig. 5d.

It is worth noting that some artificial azimuthal wavy patterns with larger values are shown in both $N_{\text{error}}$ and $\sigma_{N_{\text{error}}}$ fields (Figs. 5c and 5d). These wavy patterns usually occurred at locations with fewer reliable ground targets, where the radial gradient of phases [the range derivative of (3)] is difficult to estimate from the raw observational data. We have found that these patterns also appear in the long-term climatological refractivity mean field. This highlights the need to improve the current refractivity estimation method, particularly in regions with fewer good ground targets. Other methods for refractivity estimation [e.g., least squares fit (Hao et al. 2006; Nicol and Illingworth 2013), compressive sensing (Ozturk et al. 2014), etc.] might be helpful to improve the target density problems. Evaluation of these refractivity estimation methods are worthy of further investigation.

b. $N_{\text{error}}$ from assumed target heights and $dN/dh$

A method to correct the main bias of refractivity caused by height differences between radar and targets and by changes in $dN/dh$ has been proposed in Feng et al. (2016). The bias correction is made assuming that target heights are given heights above the terrain and assuming homogeneous $dN/dh$ in space. However, there are still some uncertainties in the assumed target heights and in $dN/dh$ estimation. Simulations are thus conducted to examine the effect of these uncertainties in the assumed target heights and $dN/dh$ on $N_{\text{error}}$.

We examined the refractivity errors introduced by assumed target heights first. Taking the area of Montreal, Canada, for example, Fig. 7a shows the height differences $\Delta H$ between the McGill radar (at the origin) and the ground targets. The heights of ground targets here are assumed as the terrain height. We performed the simulation of $\Delta \phi$ with perturbed target heights with $\pm 10$ m added on the previously assumed target height and obtained the refractivity fields. The observed $\Delta \phi$ is simulated using the conditions of $\Delta (dN/dh)$ equal to $-50$ N-unit km$^{-1}$ and the randomly perturbed target heights based on the equation of phase changes [(8) in Feng et al. (2016)], and then obtained the refractivity field through the original radar refractivity technique. The simulations are executed 30 times with randomly perturbed target heights to evaluate the uncertainties of assumed target heights on $N_{\text{error}}$. Figure 7b shows that the standard deviation of $N_{\text{error}}$ from these simulations of height perturbation is sensitive to uncertainties of target heights in the area with a steep terrain height gradient, such as in the Mount Royal area ($x = 22–24$ km and $y = 10–12$ km). Elsewhere, the radar-estimated refractivity field in areas with relatively flat terrain achieve higher data quality after the bias corrections. In addition, if the uncertainties of $\Delta (dN/dh)$ are $\pm 10$ N-unit km$^{-1}$, then the result of $N_{\text{error}}$ is the same as the result of the uncertainties of a target height with $\pm 10$ m, based on the dominant term (ii) of (8) in Feng et al. (2016).

5. Summary and discussion

Radar-estimated refractivity fields provide insights into the high-spatiotemporal-resolution distribution of low-level moisture, and assimilating the refractivity field is considered critical to improve the model initial conditions. In this study, the observational errors of radar-estimated refractivity are first quantified and better understood as a prerequisite step to the ingestion of the data into NWP models. This is based on the newly developed method for quantifying the uncertainties of phase measurements and the data processing.
The observational errors of the radar-estimated refractivity associated with the uncertainties of phase measurement and the original estimation method (Fabry et al. 1997) are examined together for the first time. The magnitude of phase errors is determined based on two pieces of newly examined information: the first is the phase correlation coefficient $\rho$ from phase measurements at different elevations and the second is the derived $\rho - \sigma_{\phi}$ relationship between the correlations of phases and the standard deviation of the phase errors. The phase errors obtained here are a bulk quantity, with contributions from independent sources caused by the uncertainties of the target location, movement, and complexity within a sampled range gate. Using the phase error estimations from the observed phase measurements and the original refractivity estimation method, the time-evolving errors in refractivity are estimated.

Besides the uncertainties of the phase measurements, uncertainties caused by the assumed target height and the estimated vertical refractivity profiles for the refractivity correction are examined. In flat terrain, there are nearly negligible impacts of these uncertainties on the refractivity field. On the contrary, strong correlated uncertainties are observed in areas with steep terrain. This leads to the difficulties in correcting for refractivity errors in such regions. In summary, for better understanding of the data quality of radar-estimated $N$ obtained from different radars, the examination of $N$ error should be performed according to the environments surrounding the individual radar site, that is, terrain and reliable ground target distribution.

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**APPENDIX**

**Ingesting Continental-Scale Radar Refractivity into Regional Models**

The radar-estimated refractivity might be implemented in operational radar networks in the near future (Bluestein et al. 2014). Previous studies have experimentally integrated some radars for expanding the coverage of the radar-estimated refractivity field.
(Hao et al. 2006; Roberts et al. 2008; Fritz and Chandrasekar 2009; Ozturk et al. 2014). Even if the main advantage of radar-estimated refractivity is currently thought to provide the rapidly evolving low-level moisture distribution at mesoscale, there are many other potential applications for NWP models. Here, we further investigate whether the radar refractivity from an operational radar network could constrain the regional NWP models. The uncertainty of the areal-mean radar-derived refractivity is based on the observational error quantification. With the increasing sample size of refractivity data in a given area, the accuracy of the areal-mean observation data would be higher and its uncertainty lower. Regional models have larger grid spacing than convective-scale models, and the average of few pixels of radar-derived refractivity field would better represent the average condition of the whole model grid box than a point measurement such as surface stations.

The information added by observations to model variables through data assimilation are determined by both the background and observation error covariances. The background error statistic provides an estimation of forecast uncertainty and quantifies the linear multivariate relationship within the model state. The correlation length of the background error covariance is conventionally used to indicate the impact distance that the information added by observation, or increment, is spread in space. Thus, this correlation length of background error of a regional model will be preliminary examined to demonstrate the impact regions of assimilating radar refractivity. There are three common ways to diagnose background error statistics: the innovation method using the difference between observations and background (Hollingsworth and Lonnberg 1986), the National Meteorological Center (NMC) method using the difference fields between two forecasting times (Parrish and Derber 1992) and the ensemble method using the ensemble distribution as an estimate of error statistics (Fisher 2003).

We examined the background error covariance based on ensemble model outputs from the Regional Ensemble Prediction System (REPS), version 2.2.0, of Environment and Climate Change Canada (ECCC). This model covers North America and adjacent oceans. The horizontal grid is 600 × 635 latitude–longitude with a 0.1375° resolution (about 15 km), and there are 48 vertical levels. A 20-member forecast is produced. We used 12-h forecasts for all calculations to avoid the correlations coming from the global system and correlated noises used for model initial perturbations. The prognostic variables are winds, temperature, specific humidity, pressure, liquid water content, and turbulent kinetic energy. A detailed model description can be found on the ECCC website (http://collaboration.cmc.ec.gc.ca/cmc/CMOI/product_guide/docs/tech_specifications/tech_specifications_REPS_e.pdf).

The background error correlations of refractivity from the McGill radar and state variables over the whole spatial grid are examined. Figure A1a is a monthly average (July 2016) of the autocorrelation of the refractivity errors between the point closest to the McGill radar and other model grid points. The correlation pattern shows anisotropy with elliptical contours whose major axis tilts to the east–west direction. The autocorrelation length of refractivity error is about 300 km in radius, which implies that the information of the point observation is spread over a substantial area. The correlation pattern of refractivity error and specific humidity error (Fig. A1b) shows a pattern similar to the autocorrelation of the refractivity error field. In contrast, the correlation has a much smaller magnitude for the temperature error fields (Fig. A1c). This can be
explained from the fact that refractivity in the surface layer in summer depends much more on the specific humidity variations than on temperature variations.

Would the impact of assimilating refractivity from the operational radar network be the same throughout the large domain? We performed a simple examination of the autocorrelations corresponding to various geographic conditions and for two different forecasting times at 0000 and 1200 UTC. Figure A2 shows the correlation of the background error in refractivity with that at the center of each subdomain. The correlation length is generally shorter and anisotropic in complex topography, but it is longer over the Midwest plains and ocean areas. In areas with longer correlation lengths, the average radar-estimated $N$ could provide useful information on the initialization of the regional NWP models. In areas with shorter background error correlation lengths, a higher density of observations is needed to reduce the forecasting uncertainties. Furthermore, the error correlation length also shows diurnal variations being generally longer during early morning (Fig. A2a; 1200 UTC) and shorter in the late afternoon (Fig. A2b; 0000 UTC).

This preliminary result suggests that assimilating near-surface refractivity in larger-scale regional models could have a significant value for some areas with longer correlation lengths of the background errors in specific humidity fields. Nevertheless, model background error statistics depend on the numerical model, the evolving atmospheric states, and stages of the storms. A more detailed analysis of the model background error is thus required. For example, more ensembles are needed to ensure the reliability and accuracy of the correlations (Stuart and Ord 1986). In addition, evaluating the impact of assimilating refractivity data from the radar network on the analysis of thermodynamic fields and on forecasting is worthy of further investigation.

**REFERENCES**


