The Impact of Beam Broadening on the Quality of Radar Polarimetric Data

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ABSTRACT

The impact of beam broadening on the quality of radar polarimetric data in the presence of nonuniform beam filling (NBF) is examined both theoretically and experimentally. Cross-beam gradients of radar reflectivity \( Z \), differential reflectivity \( Z_{\text{DR}} \), and differential phase \( \Phi_{\text{DP}} \) within the radar resolution volume may produce significant biases of \( Z_{\text{DR}} \), \( \Phi_{\text{DP}} \), and the cross-correlation coefficient \( \rho_{hv} \). These biases increase with range as a result of progressive broadening of the radar beam. They are also larger at shorter radar wavelengths and wider antenna beams.

Simple analytical formulas are suggested for estimating the NBF-induced biases from the measured vertical and horizontal gradients of \( Z \), \( Z_{\text{DR}} \), and \( \Phi_{\text{DP}} \). Analysis of polarimetric data collected by the KOUN Weather Surveillance Radar-1988 Doppler (WSR-88D) demonstrates that frequently observed perturbations of the radial \( \Phi_{\text{DP}} \) profiles and radially oriented “valleys” of \( \rho_{hv} \) depression can be qualitatively and quantitatively explained using the suggested NBF model.

1. Introduction

The Joint Polarization Experiment (JPOLE) and other validation studies demonstrate superior performance of dual-polarization radar for rainfall estimation and radar echo classification (e.g., Ryzhkov et al. 2005a). These polarimetric products, however, have been validated at relatively close distances from the radar. To our knowledge, the maximal distance at which polarimetric analysis and/or classification was ever verified using in situ measurements is 120 km (Loney et al. 2002). In most studies, the largest range to which polarimetric rainfall estimation was tested with rain gauges does not exceed 100 km (e.g., Brandes et al. 2001, 2002; May et al. 1999; Le Bouar et al. 2001; Ryzhkov et al. 2005b). On the other hand, Giangrande and Ryzhkov (2003) and Ryzhkov et al. (2005a) show that although the polarimetric method for rain measurements still outperforms the conventional one beyond 100 km from the radar, the degree of improvement decreases with distance.

Progressive beam broadening and stronger impact of nonuniform beam filling (NBF) is one of the reasons why the quality of polarimetric information deteriorates with range. Beam broadening is a common problem for both polarimetric and conventional (nonpolarimetric) radar. The issue of the vertical profile of reflectivity (VPR) correction for precipitation measurements with conventional radar is addressed in extended literature (see, e.g., the overview in Meischner 2004). Much less effort has been made to assess similar problems regarding polarimetric variables such as the differential reflectivity \( Z_{\text{DR}} \), the differential phase \( \Phi_{\text{DP}} \), the specific differential phase \( K_{\text{DP}} \), the depolarization ratio \( \text{LDR} \), and the cross-correlation coefficient \( \rho_{hv} \).

Adverse effects of NBF on polarimetric measurements are further exacerbated if the antenna patterns for horizontal and vertical polarizations are not identical. Theoretical formulas for the \( Z_{\text{DR}} \), \( \text{LDR} \), and \( \rho_{hv} \) biases caused by the antenna pattern mismatch are presented in the book by Bringi and Chandrasekar (2001). The errors in \( Z_{\text{DR}} \) due to mismatched copolar patterns together with intrinsic reflectivity gradients across the beam can be quite high at the periphery of strong storm cores (Herzegh and Carbone 1984; Pointin et al. 1988).

NBF may also cause significant perturbations of the radial profile of the differential phase (Ryzhkov and Zrnic 1998; Gosset 2004). Such perturbations of \( \Phi_{\text{DP}} \) result in spurious values of its radial derivative \( K_{\text{DP}} \) and strong biases in the \( K_{\text{DP}} \)-based estimates of the rain...
rate. These adverse effects are commonly manifested as the appearance of negative $K_{DP}$ in the regions of strongly nonuniform precipitation and become more pronounced as the physical size of the radar resolution volume increases at longer distances.

The magnitude of the cross-correlation coefficient $\rho_{hv}$ is closely related to the distribution of the differential phase within the radar resolution volume. Large cross-beam gradients of $\Phi_{DP}$ may cause noticeable decrease of $\rho_{hv}$, which is, in its turn, accompanied by higher statistical errors in the measurements of all polarimetric variables (Ryzhkov 2005).

Strong vertical gradients of radar variables are commonly observed in the presence of the bright band in startitorm rain. Beam broadening causes notable smearing of the brightband polarimetric signatures at the distances as close as 40–50 km from the radar (Giangrande et al. 2005). Such a smearing makes polarimetric classification of the melting layer more difficult, and estimation of rainfall becomes a challenge.

In this paper, we attempt to quantify the effects of beam broadening on polarimetric measurements using a simple model of NBF. We assume that the antenna patterns at the two orthogonal polarizations are perfectly matched and the biases of the measured $Z_{DR}$, $\Phi_{DP}$, and $\rho_{hv}$ are solely due to linear cross-beam gradients of different radar variables. In section 2, closed-form analytical solutions for the biases are obtained using this simplified model of gradients and the Gaussian antenna pattern. Section 3 contains analysis of the cross-beam gradients and the corresponding biases estimated from real data collected with the polarimetric prototype of the S-band Weather Surveillance Radar-1988 Doppler (WSR-88D) in Oklahoma. In section 4, we simulate the smearing effect of beam broadening on the polarimetric signatures of the melting layer for different antenna beamwidths and compare results of simulations with observational data. Finally, in section 5 we discuss practical implications of the observed effects.

2. Theoretical analysis

In the case of weather scatterers, the voltage vectors of the transmitted (V) and received (V) waves are related as

$$
\begin{bmatrix}
V_h \\
V_v
\end{bmatrix} = C_1 \begin{bmatrix}
T_{hh} & 0 \\
0 & T_{vv}
\end{bmatrix} \begin{bmatrix}
S_{hh} & S_{hv} \\
S_{vh} & S_{vv}
\end{bmatrix} \begin{bmatrix}
T_{hh} & 0 \\
0 & T_{vv}
\end{bmatrix} \begin{bmatrix}
V_h' \\
V_v'
\end{bmatrix},
$$

where matrix elements $S_{hh}$, $S_{hv}$, and $S_{vh}$ represent backscattering coefficients of hydrometeors in the radar resolution volume and $T_{hh}$ and $T_{vv}$ describe phase shifts and attenuations for H and V waves along the propagation path:

$$
T_{hh} = \exp(-j\Phi_h - \Gamma_h),
$$

$$
T_{vv} = \exp(-j\Phi_v - \Gamma_v),
$$

where $\Phi_{hv}$ is the phase shift and $\Gamma_{hv}$ is the attenuation. The differential phase $\Phi_{DP}$ is defined as

$$
\Phi_{DP} = 2(\Phi_h - \Phi_v).
$$

The coefficient $C_1$ is a constant depending on radar parameters and range from the scatterers (see the appendix). If both H and V waves are transmitted simultaneously [i.e., $V' = (V', V')$], then

$$
V_h = C_i(T_{hh}^2 S_{hh} + T_{hh} T_{vv} S_{hv}) V',
$$

$$
V_v = C_i(T_{vv}^2 S_{vv} + T_{hh} T_{vv} S_{hv}) V'.
$$

In our analysis we will neglect the cross-coupling terms proportional to $S_{hv}$ in (5)–(6), which is reasonable assumption for rain and aggregated snow (Doviak et al. 2000).

Using (5) and (6), we introduce effective radar reflectivity factors $Z_{h,v}^{(e)}$ at orthogonal polarizations as

$$
Z_h^{(e)} = C_2|V_h|^2 = Z_h e^{-4\Gamma_h},
$$

and

$$
Z_v^{(e)} = C_2|V_v|^2 = Z_v e^{-4\Gamma_v},
$$

the effective differential reflectivity

$$
Z_{dr}^{(e)} = \frac{Z_h^{(e)}}{Z_v^{(e)}} = Z_{dr} e^{-4(\Gamma_h - \Gamma_v)},
$$

and the covariance

$$
R_{hv} = C_2 \overline{V_h V_v} = Z_{hv} e^{i\Phi_{DP}},
$$

where

$$
Z_{hv} = (Z_h Z_v)^{1/2} |\rho_{hv}| e^{-2(\Gamma_h + \Gamma_v)}
$$

and

$$
\Phi_{DP} = \Phi_{DP} + \arg(\rho_{hv}).
$$

In (7)–(11), intrinsic values of the radar reflectivities $Z_{h,v}$, the differential reflectivity $Z_{dr}$, and the cross-correlation coefficient $\rho_{hv}$ are defined from the second moments of the scattering matrix $S$:

$$
Z_h = C'\langle |S_{hh}|^2 \rangle, \quad Z_v = C'\langle |S_{vv}|^2 \rangle, \quad Z_{dr} = \frac{\langle |S_{hh}|^2 \rangle}{\langle |S_{hv}|^2 \rangle^{1/2}},
$$

$$
\rho_{hv} = \frac{\langle S_{hh} S_{hv} \rangle}{\langle |S_{hh}|^2 \rangle^{1/2} \langle |S_{vv}|^2 \rangle^{1/2}}.
$$
Overbars in (7), (8), and (10) mean expected values and brackets in (12) stand for ensemble averaging. The factors $C_2$ and $C'$ are constants defined in the appendix. In the absence of propagation effects and cross coupling, the effective reflectivity factors are equal to their intrinsic values.

The radar-measured reflectivities $Z_{hv}^{(m)}$ and the covariance $K_{hv}^{(m)}$ are weighted by the radar antenna pattern $I(r, r_0)$ as follows (see the appendix for details):

$$Z_{hv}^{(m)}(r_0) = \int Z_{hv}^{(e)}(r, r_0) I(r, r_0) \, dr,$$

$$K_{hv}^{(m)}(r_0) = R_{hv}(r) I(r, r_0) \, dr.$$  

In (13) and (14), it is assumed that antenna patterns are identical at the two orthogonal polarizations. The measured differential phase $\Phi_{DP}^{(m)}$ and cross-correlation coefficient $\rho_{hv}^{(m)}$ are

$$\Phi_{DP}^{(m)} = \text{arg}(R_{hv}^{(m)})$$

$$\rho_{hv}^{(m)} = \frac{R_{hv}^{(m)}}{(Z_{hv}^{(m)} \cdot Z_{hv}^{(m)})^{1/2}}.$$  

The values of $\Phi_{DP}^{(m)}$ and $\rho_{hv}^{(m)}$ depend on the distributions of $Z_{hv}^{(e)}$ and $R_{hv}$ within the radar resolution volume and on the shape of antenna pattern. In this study, we assume that a two-way antenna power pattern is axisymmetric and Gaussian (Doviak and Zrnic 1993):

$$I(\theta, \phi) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\theta^2 + \phi^2}{2\sigma^2}\right),$$  

where $\theta$ and $\phi$ are elevation and azimuth, respectively, and $\sigma = \Omega/(4\ln 2)^{1/2}$ ($\Omega$ is a one-way 3-dB antenna pattern width).

Next we assume that reflectivity factors $Z_{hv}^{(e)}$ and $Z_{hv}$ expressed in logarithmic scale vary linearly in both cross-beam directions, $\theta$ and $\phi$:

$$10 \log[Z_{hv}^{(e)}(\theta, \phi)] = Z_{hv}^{(e)}(0, 0) + \frac{dZ_{hv}^{(e)}}{d\theta} \theta + \frac{dZ_{hv}^{(e)}}{d\phi} \phi,$$

$$10 \log[Z_{hv}(\theta, \phi)] = Z_{hv}(0, 0) + \frac{dZ_{hv}}{d\theta} \theta + \frac{dZ_{hv}}{d\phi} \phi.$$  

Similar assumption is made for differential phase $\Phi_{DP}^{(e)}$:

$$\Phi_{DP}^{(e)}(\theta, \phi) = \Phi_{DP}^{(e)}(0, 0) + \frac{d\Phi_{DP}^{(e)}}{d\theta} \theta + \frac{d\Phi_{DP}^{(e)}}{d\phi} \phi.$$  

Note that, throughout the paper, an uppercase subscript is attributed to radar reflectivity and differential reflectivity in logarithmic scale, whereas lowercase subscript signifies the corresponding variables expressed in the linear scale. Arguments $(0,0)$ in Eqs. (17)–(19) correspond to the center of the antenna beam.

As a result,

$$Z_{hv}^{(m)} = Z_{hv}^{(e)}(0, 0) J_{hv}^{(m)} J_{hv}^{(m)},$$

$$K_{hv}^{(m)} = Z_{hv}^{(e)}(0, 0) e^{j \Phi_{DP}^{(e)}(0, 0)} J_{hv}^{(m)} J_{hv}^{(m)},$$  

where

$$J_{hv}^{(m)} = \frac{1}{\sqrt{2\pi\sigma}} \int \exp\left(\frac{0.23 dZ_{hv}^{(e)}}{d\theta} \theta - \frac{\theta^2}{2\sigma^2}\right) d\theta = \exp\left[\frac{(0.23)^2}{2} \sigma^2 \left(\frac{dZ_{hv}^{(e)}}{d\theta}\right)^2\right].$$  

$$J_{hv}^{(m)} = \frac{1}{\sqrt{2\pi\sigma}} \int \exp\left(\frac{0.23 dZ_{hv}^{(e)}}{d\phi} \phi - \frac{\phi^2}{2\sigma^2}\right) d\phi = \exp\left[\frac{(0.23)^2}{2} \sigma^2 \left(\frac{dZ_{hv}^{(e)}}{d\phi}\right)^2\right].$$  

$$J_{hv}^{(e)} = \frac{1}{\sqrt{2\pi\sigma}} \int \exp\left(i \frac{d\Phi_{DP}^{(e)}}{d\theta} \theta + 0.23 \frac{dZ_{hv}^{(e)}}{d\theta} \theta - \frac{\theta^2}{2\sigma^2}\right) d\theta = \exp\left[\frac{\sigma^2}{2} \left(i \frac{d\Phi_{DP}^{(e)}}{d\theta} + 0.23 \frac{dZ_{hv}^{(e)}}{d\theta}\right)^2\right].$$  

$$J_{hv}^{(e)} = \frac{1}{\sqrt{2\pi\sigma}} \int \exp\left(i \frac{d\Phi_{DP}^{(e)}}{d\phi} \phi + 0.23 \frac{dZ_{hv}^{(e)}}{d\phi} \phi - \frac{\phi^2}{2\sigma^2}\right) d\phi = \exp\left[\frac{\sigma^2}{2} \left(i \frac{d\Phi_{DP}^{(e)}}{d\phi} + 0.23 \frac{dZ_{hv}^{(e)}}{d\phi}\right)^2\right].$$  

The measured differential reflectivity is expressed as

$$Z_{hv}^{(m)} = Z_{hv}^{(e)}(0, 0) \frac{J_{hv}^{(m)} \overline{J}_{hv}^{(m)}}{I_{hv}^{(m)} \overline{I}_{hv}^{(m)}} = Z_{hv}^{(e)}(0, 0) \exp\left\{\frac{(0.23)^2}{2} \sigma^2 \left[\left(\frac{dZ_{hv}^{(e)}}{d\theta}\right)^2 - \left(\frac{dZ_{hv}^{(e)}}{d\theta}\right)^2 + \left(\frac{dZ_{hv}^{(e)}}{d\phi}\right)^2 - \left(\frac{dZ_{hv}^{(e)}}{d\phi}\right)^2\right]\right\}.$$
Since
\[ Z'^{\text{DR}} - Z'^{\text{HV}} = Z'^{\text{DR}} \quad \text{and} \quad 2\frac{dZ'^{\text{DR}}}{d\theta} \approx \frac{dZ'^{\text{DR}}}{d\phi} \quad \text{and} \quad 2\frac{dZ'^{\text{DR}}}{d\theta} \approx \frac{dZ'^{\text{DR}}}{d\phi}, \]
we can further simplify
\[ Z'^{\text{DR}} \approx Z'^{\text{DR}} \exp \left\{ (0.23)^2 \sigma^2 \left[ \frac{dZ'^{\text{DR}}}{d\theta} \right]^2 + \frac{dZ'^{\text{DR}}}{d\phi} \right\}. \]

Or, equivalently,
\[ Z'^{\text{DR}} \approx Z'^{\text{DR}} + 0.23 \sigma^2 \left[ \frac{dZ'^{\text{DR}}}{d\theta} \right]^2 + \frac{dZ'^{\text{DR}}}{d\phi} \left\{ \frac{dZ'^{\text{DR}}}{d\phi} \right\}. \]

The measured differential phase can be written as
\[ \xi_2 = \exp \left\{ \frac{(0.23)^2}{2} \sigma^2 \left[ \left( \frac{dZ'^{\text{DR}}}{d\theta} \right)^2 + \left( \frac{dZ'^{\text{DR}}}{d\phi} \right)^2 \right] - \frac{1}{2} \left[ \left( \frac{dZ'^{\text{DR}}}{d\theta} \right)^2 + \left( \frac{dZ'^{\text{DR}}}{d\phi} \right)^2 \right] \right\} \]

If \( |\rho_v| = 1 \), then
\[ \xi_2 \approx \exp \left\{ - (0.23)^2 \sigma^2 \left[ \left( \frac{dZ'^{\text{DR}}}{d\theta} \right)^2 + \left( \frac{dZ'^{\text{DR}}}{d\phi} \right)^2 \right] \right\}. \]

The coefficient \( \xi_2 \) is usually very close to 1, hence we will ignore this term in our further considerations.

Expressing the parameter \( \sigma \) via the antenna beamwidth \( \Omega \), we finally arrive at the following approximate
\[ \frac{|\rho_v^{(m)}|}{|\rho_v|} = \xi_1 = \exp \left\{ - 1.37 \times 10^{-2} \Omega^2 \left[ \left( \frac{d\Phi_D}{d\theta} \right)^2 + \left( \frac{d\Phi_D}{d\phi} \right)^2 \right] \right\}. \]

In Eqs. (36)–(38), \( \Phi_D, \Delta \Phi_D, \Omega, \theta, \) and \( \varphi \) are expressed in degrees, whereas \( Z \), \( Z' \), \( \Delta Z \), and \( Z_D \) are in decibels.

Similar formulas can be obtained for the NBF-related bias of the radar reflectivity factor at horizontal polarization:
\[ \Delta Z_H(dB) = 0.01 \Omega^2 \left[ \left( \frac{dZ'^{\text{DR}}}{d\theta} \right)^2 + \left( \frac{dZ'^{\text{DR}}}{d\phi} \right)^2 \right]. \]
3. NBF effects in the case of the mesoscale convective system

The gradients of $Z_{HV}$, $Z_{DR}$, and $\Phi_{DP}$ in Eqs. (36)–(38) can be approximately estimated from real data by comparing the corresponding variables at adjacent radials. We perform such estimation in the case of a mesoscale convective system (MCS) that was observed with the polarimetric prototype of the S-band WSR-88D radar (hereafter KOUN) in central Oklahoma on 2 June 2004. The analysis was conducted using the data from two lowest plan position indicators (PPIs) at elevations of 0.44° and 1.45°. Horizontal gradients were computed from the data collected at the lowest elevation, whereas vertical gradients were estimated using the data at both elevations.

Strictly speaking, such a procedure underestimates the magnitude of intrinsic gradients because the data are smeared with the antenna beam. Indeed, the transverse dimension of the radar resolution volume exceeds 3 km at 200 km from the radar if the antenna beamwidth is 1°. Hence, smaller-scale cross-beam nonuniformities of the precipitation field are not resolved. Nevertheless, as will be shown later, these approximate estimates of gradients prove to be very useful for evaluating the quality of polarimetric data.

A composite plot of $Z_{HV}$, $Z_{DR}$, $\Phi_{DP}$, and $\rho_{hv}$ at elevation 0.44° (Fig. 1) corresponds to the time when an extensive squall line passes over the radar and produces tremendous attenuation and differential attenuation that are clearly visible in the eastern sector. The radar reflectivity factor and differential reflectivity are deliberately not corrected for attenuation in order to estimate the gradients of $Z_{HV}$, $Z_{DR}$, and $Z_{DP}$, which are affected by attenuation according to their definition in (7)–(11). High values of $\Phi_{DP}$ in the eastern sector are accompanied by negatively biased $Z$ and $Z_{DR}$ and a pronounced drop in the cross-correlation coefficient $\rho_{hv}$. While the drop in $Z_{DR}$ well below −2 dB is caused by differential attenuation, the decrease in $\rho_{hv}$ is a result of NBF.

This is confirmed by Fig. 2 where the fields of the parameters $\Delta Z_{DR}$, $\Delta \Phi_{DP}$, and $\xi$ computed from Eqs. (36)–(38) are displayed together with $Z_{HV}$. The $\rho_{hv}$ depression in Fig. 2d is very well correlated with the observed decrease of the measured $\rho_{hv}$ in Fig. 1. The magnitude of the negative $\rho_{hv}$ bias exceeds 0.2. Such a strong bias adversely affects the quality of the polarimetric classification of radar echoes and induces large statistical errors in the estimates of all polarimetric variables. Similar radial features or “valleys” of lower $\rho_{hv}$ are frequently observed in the KOUN polarimetric data. Their primary cause is large vertical gradient of $\Phi_{DP}$. The ray at a higher elevation overshoots precipitation at closer distances from the radar than the ray at lower tilt. Therefore, the differential phase at higher tilt stops increasing earlier (i.e., at closer slant ranges) than the one at lower tilt. While both higher and lower rays are still in rain, the differential phases at the two rays grow proportionally and the difference between them is not high. However, once the higher ray intercepts the freezing level, the corresponding $\Phi_{DP}$ stops increasing, whereas $\Phi_{DP}$ at the lower ray continues its growth. This explains a radial character of the observed artifacts and their severity, which progresses with range.

According to (38), large gradients of $\Phi_{DP}$ are responsible for the decrease in $\rho_{hv}$. In contrast, perturbations of $\Phi_{DP}$ are determined by both the gradients of the differential phase and the reflectivity factor. As a result, $\Delta \Phi_{DP}$ exhibits more complex and nonmonotonic behavior along the radial than the factor $\xi$. If the reflectivity field is relatively uniform as in the stratiform region of the MCS north-northeast of the squall line, then the gradients of $\Phi_{DP}$ dominate and apparent radial features are evident in the field of $\Delta \Phi_{DP}$.

The NBF-related bias in differential reflectivity can also be significant and may exceed several tenths of a decibel as Fig. 2b shows. Positive biases of $Z_{DR}$ are common in convective areas of the storm not far away from the radar, whereas negative biases are prevalent at longer distances in convective and stratiform parts of the MCS. The latter feature is explained by the general decrease of $Z_{DR}$ with height. The $Z_{DR}$ biases, as well as the biases in $\Phi_{DP}$ and $\rho_{hv}$, tend to increase with range as a result of beam broadening.

A similar analysis was performed on the data collected for the same storm but 2 h after the squall line passed over the radar and was viewed at a different angle (Figs. 3 and 4). At that moment, attenuation effects were much weaker and the differential phase was significantly lower. Again, the area of $\rho_{hv}$, depression is well predicted from the analysis of gradients. The perturbations of the $\Phi_{DP}$ radial profiles are also in good agreement with their estimates from the gradients in accordance with Eq. (37).

In Fig. 5, measured range dependencies of $\Phi_{DP}$ (thin curves) are compared with radial profiles of $\Delta \Phi_{DP}$ calculated from (37) (thick curves) for six successive azimuths belonging to the sector indicated in Fig. 4d. Despite many simplified assumptions made in the evaluation of $\Delta \Phi_{DP}$, the correlation between the $\Phi_{DP}$ and $\Delta \Phi_{DP}$ profiles is surprisingly high. The most pronounced excursions of the $\Phi_{DP}$ curves, such as spikes and depressions, are well reproduced in the modeled $\Delta \Phi_{DP}$. Thus, they are primarily attributed to NBF rather than pure statistical errors in $\Phi_{DP}$ estimation or
to the contribution from the backscatter differential phase.

4. Beam-broadening effects in the case of stratiform rain

The melting layer or bright band is a special case of strong vertical nonuniformity in stratiform precipitation. The bright band is associated with very well pronounced polarimetric signatures such as the sharp $Z_{DR}$ maximum and $\rho_{hv}$ minimum. These signatures have very important prognostic value because the top of the melting layer corresponds to the freezing level and its bottom represents the boundary between pure liquid and mixed-phase hydrometeors. The latter one marks the onset of the brightband contamination in radar rainfall estimates. Accurate designation of the melting layer is a key for successful discrimination between liquid and frozen hydrometeors (Giangrande et al. 2005).

Because the thickness of the bright band is only few hundreds of meters, the corresponding polarimetric signatures degrade very rapidly with range even for the radar beam as narrow as $1^\circ$. This degradation is illustrated in the range–height indicator (RHI) plot of $Z$, $Z_{DR}$, $\Phi_{DP}$, and $\rho_{hv}$.
To quantify the degree of such deterioration at longer distances from the radar one has to use a more sophisticated model of NBF than is described in section 2.

For the case illustrated in Fig. 6, we obtained average vertical profiles of all radar variables at very close distances from the radar and modeled the RHI fields of $Z_{DR}$, $\Phi_{DP}$, and $\rho_{hv}$ at the S band for different antenna beamwidths assuming the horizontal homogeneity of the intrinsic fields of these radar variables. The results of such modeling studies are presented in Figs. 7 and 8 for antenna beamwidths at $1^\circ$ and $2^\circ$. Modeled fields in Fig. 7 are very consistent with what was actually observed with the same antenna beamwidth (Fig. 6). This means that the model adequately reproduces observational data.

A twofold increase of the radar beamwidth leads to the enhanced brightband contamination of the low-altitude echoes in rain (Fig. 8). At the lowest elevations, the differential reflectivity and cross-correlation coefficients quickly acquire the values typical for melting hy-
drometeors. As in the case of the MCS, vertical non-uniformity causes wavelike perturbation of the $\Phi_{DP}$ profile in the melting layer as was explained by Ryzhkov and Zrnic (1998). Below the melting layer, the mean value of $\Phi_{DP}$ is less biased but differential phase becomes more noisy due to lowering of $\rho_v$ at the altitudes below the physical (i.e., intrinsic) bottom of the bright band.

5. Discussion

The findings in this study may have important practical implications to all users of polarimetric radar data. This is significant in view of the forthcoming polarimetric upgrade of the U.S. National Weather Service network of the WSR-88D radars. One should avoid using polarimetric variables in a quantitative manner in the areas where these variables are significantly affected by NBF. Such areas can be identified by computing horizontal and vertical gradients of the radar reflectivity, the differential reflectivity, and the differential phase as well as estimating the biases of $Z_{DR}$, $\Phi_{DP}$, and $\rho_v$ according to Eqs. (36)–(38). The procedure for gradient estimation is simple and straightforward.

If the magnitudes of $\Delta Z_{DR}$, $\Delta \Phi_{DP}$, and the difference...
1 - \( \xi \) exceed certain thresholds, then the corresponding variables (\( Z_{\text{DR}} \), \( K_{\text{DP}} \), and \( \rho_{\text{hv}} \)) should not be used for estimating polarimetric products in these areas. The choice of such thresholds is dictated by tolerable errors that depend on particular applications. For example, the \( Z_{\text{DR}} \) bias has to be less than 0.2 dB if \( Z_{\text{DR}} \) is utilized for rainfall estimation. The biases of \( \Phi_{\text{DP}} \) within \( \pm 2^\circ \) are acceptable because the statistical fluctuations of the \( \Phi_{\text{DP}} \) estimate are between 1^\circ and 2^\circ for typical dwell times used for operational weather radars. The bias of 0.02 in \( \rho_{\text{hv}} \) may also be tolerable for classification purposes.

In addition to the negative impact on the quality of polarimetric classification, the decrease of \( \rho_{\text{hv}} \) is detrimental for statistical accuracy of the estimates of \( Z_{\text{DR}} \), \( \Phi_{\text{DP}} \), and \( \rho_{\text{hv}} \) itself. Indeed, the standard deviations of the estimates for all three variables are proportional to \( (1 - \rho_{\text{hv}}^2)^{1/2} \) (Bringi and Chandrasekar 2001). This means that if \( \rho_{\text{hv}} \) drops from 0.99 to 0.90, the corresponding errors increase 3 times.

Perturbations of the \( \Phi_{\text{DP}} \) radial profile produce erroneous estimates of \( K_{\text{DP}} \) of both signs. Although negative \( K_{\text{DP}} \)s are easily identified (and sometimes taken out as unphysical), positively biased \( K_{\text{DP}} \)s usually go

Fig. 4. Composite plot of (a) \( Z \), (b) \( \Delta Z_{\text{DR}} \), (c) \( \Delta \Phi_{\text{DP}} \), and (d) \( \xi \) (multiplicative factor of \( \rho_{\text{hv}} \)) corresponding to PPI in Fig. 3. The biases of \( Z_{\text{DR}} \), \( \Phi_{\text{DP}} \), and \( \rho_{\text{hv}} \) are attributed to NBF and computed from Eqs. (36)–(38). Overlaid are contours of \( Z \). The data are displayed for SNR > 10 dB.
undetected. Since $K_{DP}$ is a slope of the $\Phi_{DP}$ radial profile, the bias in $K_{DP}$ is not necessarily zero if $\Delta \Phi_{DP} = 0$. Thus, the data with $\Delta \Phi_{DP} = 0$ in the vicinity of large $|\Delta \Phi_{DP}|$ should be also scrutinized.

The magnitudes of $\Delta Z_{DR}$, $\Delta \Phi_{DP}$, and $1 - \xi$ depend on the square of antenna beamwidth. Such a strong dependence may preclude the use of wide-beam antennas for polarimetric measurements. A twofold increase of the beamwidth from $1^\circ$ to $2^\circ$ leads to 4-times-larger biases and significant deterioration of the melting layer designation as Figs. 7 and 8 show.

The biases of $\Phi_{DP}$ and $\rho_{hv}$ are wavelength dependent because the differential phase and its gradients are inversely proportional to the radar wavelength $\lambda$. The impact on $\Delta \Phi_{DP}$ is proportional to $\lambda^{-1}$, whereas the $\rho_{hv}$ bias is approximately proportional to $\lambda^{-2}$. Enhanced attenuation and differential attenuation at shorter wavelengths may either increase or decrease the gradients of $Z$ and $Z_{DR}$. In some situations, these changes in the $Z$ and $Z_{DR}$ gradients may offset the increase in the gradient of $\Phi_{DP}$ and its greater impact on the NBF-related biases in $\Phi_{DP}$ and $\rho_{hv}$. However, cursory analy-
ysis of the C- and X-band-simulated and observed polarimetric data reveals stronger NBF effects compared to the S band (Ryzhkov and Zrnic 2005). Although range coverage of the shorter-wavelength radars is usually smaller than the one for S-band weather radars and the antenna beam is not as broad at closer distances, all mentioned problems should be taken seriously. In convective situations, both attenuation and beamwidth effects may restrict the use of polarimetric methods on short-wavelength radars (particularly with antenna beams wider than 1°).

We emphasize that Eqs. (36)–(38) cannot be used for correction of $Z_{DR}$, $\Phi_{DP}$, and $\rho_v$ because the bias estimates are very approximate due to many simplifying assumptions made in derivation of these equations. Instead, we recommend using $\Delta Z_{DR}$, $\Delta \Phi_{DP}$, and $\xi$ as quality indexes for the corresponding radar variables. Such an approach is used in the algorithms for hydrometeor classification and rainfall estimation developed at the National Severe Storms Laboratory (NSSL) for operational utilization with the polarimetric prototype of the WSR-88D radar. According to this approach, each ra-
dar variable is supplemented with its confidence factor that may depend on \( \Delta Z_{\text{DR}}, \Delta \Phi_{\text{DP}}, \) and \( \xi \) along with a signal-to-noise ratio, the total differential phase (which characterizes potential impact of attenuation/differential attenuation), the magnitude of \( \rho_{hv} \) (which characterizes the noisiness of polarimetric data), etc.

6. Conclusions

In this study, we evaluate the impact of nonuniform beam filling (NBF) on the quality of polarimetric measurements. It is shown that such an impact can be quite significant, especially at longer distances from the radar due to progressive broadening of the antenna beam.

Relatively simple analytical formulas have been obtained for the NBF-induced biases of the differential reflectivity \( Z_{\text{DR}} \), the differential phase \( \Phi_{\text{DP}} \), and the cross-correlation coefficient \( \rho_{hv} \) assuming linear gradients of radar reflectivity \( Z_H \), \( Z_{\text{DR}} \), and \( \Phi_{\text{DP}} \) in the cross-beam directions within the radar resolution volume. It is found that the biases are proportional to the square...
of the antenna beamwidth. The bias of $Z_{DR}$ does not depend on the radar wavelength, whereas the biases of $\Phi_{DP}$ and $\rho_{hv}$ increase at shorter wavelength (proportionally to $\lambda^{-1}$ in the case of $\Phi_{DP}$ and to $\lambda^{-2}$ in the case of $\rho_{hv}$). Thus, the NBF effects are stronger at C and X bands than at the S band.

Horizontal and vertical gradients of $Z_H$, $Z_{DR}$, and $\Phi_{DP}$ were estimated from polarimetric data collected by the S-band KOUN WSR-88D radar in a mesoscale convective system. Joint analysis of the measured fields of polarimetric variables and their NBF-induced biases computed from the cross-beam gradients proves that nonuniform beam filling combined with beam broadening is responsible for such commonly observed artifacts as radial “valleys” of $\rho_{hv}$ depression and oscillatory behavior of the $\Phi_{DP}$ profiles. The latter usually manifests itself as the appearance of negative $K_{DP}$. It is also shown that polarimetric signatures of the melting layer rapidly degrade with distance as the antenna beam widens.

Although correcting $Z_{DR}$, $\Phi_{DP}$, and $\rho_{hv}$ for such biases is not practical because the biases cannot be esti-

![Fig. 8. Same as in Fig. 7, but for the beamwidth of 2°.](image-url)
mated with sufficient accuracy, their approximate estimates are important as “quality indexes” of the corresponding polarimetric variables. One should abstain from any quantitative use of the variable if the respective NBF-caused bias exceeds the threshold of acceptability.

These considerations should be taken into account in using polarimetric data at different wavelengths and various angular resolutions and in developing robust algorithms for polarimetric hydrometeor classification and rainfall estimation.

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APPENDIX

Equations for the Covariance $R_{hv}$

The voltage vectors of the transmitted ($V^t$) and received ($V^r$) waves in the case of individual scatterer are related as (Bringi and Chandrasekar 2001)

$$
\begin{pmatrix}
V^t_h \\
V^t_v
\end{pmatrix} = C_2 \begin{pmatrix}
T_{hh} & 0 \\
0 & T_{vv}
\end{pmatrix} \begin{pmatrix}
V^r_h \\
V^r_v
\end{pmatrix},
$$

(A1)

where matrix elements $s_{hh}$, $s_{vv}$, and $s_{hv}$ represent back-scattering coefficients of the scatterer, and $T_{hh}$ and $T_{vv}$ describe phase shifts and attenuations for H and V waves along propagation path:

$$
T_{hh,vv} = \exp(-\beta h_{hv} - \Gamma_{hv}),
$$

(A2)

where $\Phi_{hv}$ is the phase shift, $\Gamma_{hv}$ is the attenuation, and $\Phi_{DP} = 2(\Phi_{hv} - \Phi_{v})$ is a differential phase. The coefficient $C_1$ is defined as

$$
C_1 = \frac{G \lambda f^2}{4\pi R^2},
$$

(A3)

In (A3), $G$ is the antenna gain, $\lambda$ is the radar wavelength, $R$ is the distance between the radar and scatterer, and $f^2$ is the normalized one-way antenna power pattern. It is assumed that the antenna patterns for orthogonal polarizations are the same.

In the case of many scatterers filling the radar resolution volume, the basic Eq. (A1) can be rewritten as

$$
V^t_h = \frac{P_t^1 G \lambda}{4\pi} \sum_i h_i e^{-2\pi f^2 |\mathbf{K}_i|},
$$

(A4)

$$
V^t_v = \frac{P_t^1 G \lambda}{4\pi} \sum_i v_i e^{-2\pi f^2 |\mathbf{K}_i|},
$$

(A5)

where $P_t = |V^t|^2$, $|V^i| = (V^i, V^v)$ and

$$
h_i = s_{hh} e^{-2\pi f^2 R_i^2},
$$

$$
v_i = s_{vv} e^{-2\pi f^2 R_i^2}.
$$

(A5)

Index $i$ in (A4) and (A5) stands for a number of scatterer. In our derivation we neglect the cross-coupling terms proportional to $s_{hv}$.

The measured covariance $R_{hv}^{(m)}$ is defined as

$$
R_{hv}^{(m)} = C_2 \sum_i |V^t_h V^r_v|,
$$

(A6)

where

$$
C_2 = \frac{2^{10}}{\pi^2 P_t G^2 \sigma^2 |\mathbf{K}_n^2|}.
$$

(A7)

The overbar in (A6) means averaging in time. In (A7), $R_0$ is the distance to the center of the radar resolution volume; $c$ is the speed of light; $\tau$ is the radar pulse duration; $\Omega$ is the one-way 3-dB antenna pattern width, $K_w = (\varepsilon_{w} - 1) / (\varepsilon_{w} + 2)$, where $\varepsilon_{w}$ is the dielectric constant of water. Substituting (A4) into (A6), we obtain

$$
R_{hv}^{(m)} = C_3 \sum_i h_i^* V_i e^{2\pi f^2 |\mathbf{K}_i|},
$$

(A8)

The summation and time averaging in Eq. (A8) can be replaced by integration over the radar resolution volume:

$$
C_3 = C_2 \frac{P_t G^2 \lambda^2}{4\pi^3} = \frac{2^{10}}{\pi^2 \tau^2 \Omega^2 |\mathbf{K}_n^2|}.
$$

(A9)
\[
R_{hv}^{(m)} = C_\lambda \int n(\langle s_{hv}^h s_{vv}^v \rangle) \frac{e^{-2(\Gamma_h + \Gamma_v)}}{R^4} e^{i\Phi_{DP}} \rho h^2 \sin \theta \, d\theta \, d\phi,
\]

where brackets stand for ensemble averaging and \( n \) is the concentration of scatterers.

According to the definition of the cross-correlation coefficient \( \rho_{hv} \),
\[
n(\langle s_{hv}^h s_{vv}^v \rangle) = n \rho_{hv} (\langle s_{hv}^h \rangle^2)^{1/2}(\langle s_{vv}^v \rangle^2)^{1/2}
\]

(11)

and
\[
n(\langle s_{hv, vv}^2 \rangle) = \frac{\pi^4}{4\lambda^4} |K_{inv} Z_{hv}|^2 \text{ (Doviak and Zrnic 1993)}.
\]

Hence,
\[
R_{hv}^{(m)} = \frac{16 \ln 2 R_0^2}{\pi c \Omega^2} \int Z_h^2 Z_v^2 \rho_{hv} e^{-2(\Gamma_h + \Gamma_v)} e^{i\Phi_{DP}},
\]

(14)

where \( \Phi_{DP} = \Phi_{DP} + \arg(\rho_{hv}) \).

If variables \( Z_{hv}, \Gamma_{hv}, \rho_{hv}, \) and \( \Phi_{DP} \) are constant within the radar resolution volume, then the measured covariance \( R_{hv}^{(m)} \) is equal to its intrinsic value
\[
R_{hv} = Z_h^2 Z_v^2 \rho_{hv} e^{-2(\Gamma_h + \Gamma_v)} e^{i\Phi_{DP}},
\]

(15)

Because
\[
\int \frac{f_j^2(\theta, \phi)}{R^2} \sin \theta \, d\theta \, d\phi = \frac{\pi c \Omega^2}{16 \ln 2 R_0^2}
\]

in the case of the Gaussian axisymmetric antenna pattern (Doviak and Zrnic 1993).

If the covariance \( R_{hv} \) varies within the radar resolution volume but its variation along the radial direction is neglected due to much smaller radial dimension of the radar volume compared to its transverse dimensions at longer ranges from the radar, then the general expression (13) can be simplified as follows:
\[
R_{hv}^{(m)} = \int R_{hv}(\theta, \phi) I(\theta, \phi) \, d\theta \, d\phi,
\]

(16)

where
\[
I(\theta, \phi) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\theta^2 + \phi^2}{2\sigma^2}\right)
\]

(17)

and \( \sigma = \Omega/4(\ln 2)^{1/2} \).

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