Calculation of Winds Induced Bottom Wave Orbital Velocity Using the Empirical Mode Decomposition Method

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ABSTRACT
Accurate bottom wave orbital velocity (BWOV) calculation is important for understanding critical dynamic processes (e.g., turbulent mixing, sediment transport) at the bottom boundary layer of the oceans. Here we first use the empirical mode decomposition (EMD) method to calculate BWOV and evaluate its performance by comparing with two conventional methods (spectra and velocity methods) using field measurements collected from an acoustic Doppler velocimeter (ADV). The results suggest that BWOVs calculated by the EMD method were well correlated ($R^2 \approx 0.97$) to the results by the other two methods but with a few percent discrepancies. Under strong wavy conditions, BWOVs from the EMD method were 8% and 6% smaller than those from the spectra and velocity methods, respectively. Under weak wavy conditions, BWOVs from the EMD method were 14% and 11% smaller than those from the spectra and velocity methods, respectively. Statistical distributions of BWOV suggest that the EMD method calculated instantaneous BWOVs and BWOV amplitudes closely matched the Gaussian and Rayleigh distributions, respectively. Uncertainty analysis suggests that the EMD method was capable of calculating the most accurate BWOVs among the three methods. While the spectra and velocity methods can provide robust BWOV estimation, they cannot completely avoid the errors caused by wave-unrelated motions and instrumental noise.

1. Introduction
The bottom wave orbital velocity (BWOV) produced by wind waves plays an important role in many aspects of ocean engineering and oceanography, such as sediment transport, forces on seabed structures, wave–current interaction, and wave–turbulence interaction (Rosman and Gerbi 2017; Soulsby et al. 1993; Soulsby 2006). To date, studies have mainly used maximum or root-mean-square (RMS) BWOV to parameterize the influence of waves on current, turbulence, and sediment transport processes (Soulsby 1987; Soulsby and Humphery 1990;...
Williams et al. 1999). However, it has been found that different phases of waves show different influences on the aforementioned processes (Guo and Shen 2014; Qiao et al. 2016; Thais and Magnaudet 1996). Using newly developed high-resolution current meters [e.g., acoustic Doppler velocimeter (ADV), acoustic Doppler current profiler (ADCP)], we are able to distinguish the BWOV of different phases. The maximum and RMS BWOVs are insufficient to describe the characteristics of the waves at the bottom boundary layer. Therefore, more accurate BWOV calculation is required for understanding the wave related critical dynamic processes of the oceans.

Various methods have been proposed for calculating BWOV in the past few decades (Elfrink et al. 2006; Soulsby 1987; Soulsby and Humphery 1990; Wiberg and Sherwood 2008). Before the wide use of the high-frequency current meters, BWOV was mainly estimated from surface elevation measurements or its related wave parameters. In the early version of this method, providing wave height, wave period and water depth, the maximum BWOV of a monochromatic wave can be calculated based on linear wave theory (Holthuijsen 2007). However, waves in the realistic oceans are not monochromatic but have a broad spectrum of frequencies, so it is insufficient to describe near-bottom wave velocities using only a single maximum BWOV. To accurately describe the BWOV under irregular waves with different frequencies, amplitudes, and directions, the linear wave theory was extended to calculate RMS BWOV when a spectrum of waves can be presented (Soulsby 1987). The BWOV calculation method based on elevation spectra is referred as the “spectra method” in this paper. The spectra method provides a full spectral representation of surface waves, which in turn performs well on BWOV estimation and is widely used for wave buoy measurements. However, it is laborious to calculate RMS BWOV from a given surface elevation spectrum and sometimes we can only obtain certain wave parameters such as wave period and significant wave height but not a full wave spectrum. To tackle these issues, Soulsby (1987) proposed a simplified method by merely using wave height and wave period to represent a generic surface-wave spectrum from which RMS BWOV can be calculated. This simplified spectra method can be used to directly calculate RMS BWOV and shows good agreement with that from the conventional spectra method (Soulsby 1987; Wiberg and Sherwood 2008).

In the last 20 years, the high-frequency current meters (e.g., ADV and ADCP) have been widely used in the field measurements. These current meters are capable of measuring water velocities with sampling frequency higher than 1 Hz, which covers the ocean wave frequency band, so the current meter measurements are capable of distinguishing the wave motions. Usually, in addition to wave motions, the measured near-bottom velocities also include low-frequency motions (e.g., tidal currents), high-frequency motions (e.g., turbulence) and instrumental noise. To accurately calculate BWOV, it is necessary to separate the wave motions from the measured bottom velocities. However, the frequencies of currents and turbulence are usually overlapped with the wave motions and it is hard to accurately separate the wave motions from the measured bottom velocities (Bricker and Monismith 2007; Trowbridge 1998). BWOV is usually calculated by removing the mean velocities (low-frequency motions) from the bottom velocity measurements or using the velocity spectrum (Madsen 1994). Neither of these two methods can accurately calculate BWOV because they treat turbulence motions and part of low-frequency water motions as wave motions, leading to an overestimation of the BWOV.

Ocean observation technology has made substantial progress over the past 20 years. To take advantage of the recent developed ocean observation instruments (e.g., the high-frequency velocity meters), it is necessary to apply new calculation methods to obtain more accurate and effective BWOV. Recently, the empirical mode decomposition (EMD) method has been used to decompose wave signals from the field observed velocities and study the mixing induced by the wave–turbulence interaction (Qiao et al. 2016). Similarly, BWOV can be extracted from the field velocity measurements using the EMD method. In this study, we review the currently popular methods of BWOV calculation and demonstrate the application of BWOV calculation by the EMD method, as presented in section 2. The field data used here are introduced in section 3. Section 4 evaluates the performance of three methods on BWOV calculation, and uncertainty analysis of BWOVs from the methods is also conducted. The conclusions are summarized in section 5.

2. Methods of BWOV Calculation

a. Spectra method

Soulsby (1987) presented the spectra method for calculating BWOV beneath waves. For small-amplitude, monochromatic waves with wave amplitude \( a = H_{\text{sig}}/2 \) (where \( H_{\text{sig}} \) is the significant wave height) and radian frequency \( \omega = 2\pi/T_p \) (where \( T_p \) is the peak wave period), linear wave theory predicts that the maximum BWOV evaluated at bottom layer (with water depth \( h \)) is...
\[ \hat{U}_{\text{max}} = \frac{H_{\text{sig}} \pi}{T_p \sinh(kh)} = \frac{\omega_s}{\sinh(kh)}. \] (1)

Wave number \( k \) is related to frequency \( \omega \) through dispersion relation \( \omega^2 = gk \tanh(kh) \), where \( g \) is gravitational acceleration. The BWVO spectrum \( S_\omega(\omega) \) can be obtained by applying Eq. (1) to each frequency band of surface elevation spectrum \( S_\eta(\omega) \):

\[ S_\omega(\omega) = \frac{\omega^2}{\sinh'(kh)} S_\eta(\omega). \] (2)

By integrating \( S_\omega(\omega) \) over each frequency band, the RMS BWVO is calculated by

\[ \hat{U}_{\text{rms}}^2 = \int_0^\infty S_\omega(\omega) d\omega. \] (3)

Using the spectra method to calculate BWVO, we need to measure time series of surface elevation \( \eta \) or its relevant parameters (e.g., water pressure from ADV, vertical acceleration from wave buoys). Moreover, to calculate \( kh \) in Eq. (1), wave period and water depth \( h \) are also needed.

### b. Velocity method

High-frequency current meters have been widely used to measure three-dimensional water motions \( u, v, \) and \( w \) in the coastal and open oceans. Instantaneous BWVO, \( \tilde{u} \) and \( \tilde{v} \), can be calculated by removing mean velocities \( \bar{u} \) and \( \bar{v} \) from the measured velocities as

\[ (\tilde{u}, \tilde{v}) = (u, v) - (\bar{u}, \bar{v}). \] (4)

Total wave orbital velocity \( \hat{U} \) is determined by the horizontal orbital velocities \( \tilde{u} \) and \( \tilde{v} \) and their direction \( \theta \) as

\[ \hat{U} = \tilde{u} \sin \theta + \tilde{v} \cos \theta, \] (5)

where \( \theta \) can be estimated from Eq. (4) of You (2009).

In this study, we refer to this direct velocity measurement of BWVO as the “velocity method.” It should be noted that, BWVOs in the oceans calculated from Eqs. (4) and (5) also contain high-frequency turbulence and other low-frequency water motions that are unrelated to waves. These irrelevant signals are capable of inducing small errors (a few percent) to BWVO estimation (Wiberg and Sherwood 2008). The errors can be estimated by investigating the velocity spectrum \( S_\omega \) and \( S_\eta \) of the velocity measurements. Similar to the spectra method, Eq. (3) can be used to calculate RMS BWVO:

\[ \hat{U}_{\text{rms}}^2 = \int_0^\infty S_\omega(\omega) d\omega = \int_0^\infty S_\eta(\omega) d\omega + \int_0^\infty S_\xi(\omega) d\omega. \] (6)

The value of \( \hat{U}_{\text{rms}} \) calculated from Eq. (6) equals RMS \( \hat{U} \) from Eq. (5). To eliminate the influence of low- and high-frequency motions unrelated to waves, the velocity spectrum over principal wave frequencies is integrated as

\[ \hat{U}_{\text{rms,new}}^2 = \int_{\text{wave}_1} \text{wave}_h S_\omega(\omega) d\omega + \int_{\text{wave}_1} \text{wave}_h S_\xi(\omega) d\omega, \] (7)

where \( \text{wave}_1 \) and \( \text{wave}_h \) indicate the lowest and highest principal wave frequencies, respectively. Errors caused by low- and high-frequency motions unrelated to waves can be determined by RMS BWVO difference between the results calculated from Eqs. (6) and (7).

c. EMD method

EMD is first proposed by Huang et al. (1998), as a new adaptive signal time–frequency processing method that is suitable for the analysis and processing of nonlinear and nonstationary signals, such as wave motions. The EMD method decomposes the observed signal into several intrinsic mode functions (IMFs). Each IMF represents a narrowband frequency–amplitude modulation that is often related to a specific physical process. Qiao et al. (2016) used the EMD method to extract wave signals from field velocity observations. Here, we briefly illustrate an application of the EMD method on BWVO estimation based on near-bottom velocity measurements. Details on the algorithms of the EMD method are described in Huang et al. (1998).

ADV-measured near-bottom horizontal velocities \( u \) and \( v \) were taken as examples to show the extraction of wave motions from measured velocities using the EMD method. Detailed information of the field data can be found in section 3.

First, the mean velocity is removed from the measured velocities:

\[ u' = u - \bar{u}, \] (8)

\[ v' = v - \bar{v}. \] (9)

The velocity method treats the residual velocity \( u' \) and \( v' \) as wave velocity \( \tilde{u} \) and \( \tilde{v} \) [Eq. (4)]. In practice, \( u' \) and \( v' \) contain wave orbital velocities, turbulences, and low-frequency flows. The EMD method decomposes observed residual velocities into several IMFs and a residual velocity (Fig. 1):

\[ u' = u'_{\text{IMF}_1} + u'_{\text{IMF}_2} + \cdots + u'_{\text{IMF}_n} + u'_{\text{Residual}}, \] (10)

\[ v' = v'_{\text{IMF}_1} + v'_{\text{IMF}_2} + \cdots + v'_{\text{IMF}_n} + v'_{\text{Residual}}. \] (11)

Each IMF represents a stationary stochastic process. If the peak frequency of an IMF is within the range of
wave frequency, the IMF is classified as a wave component. For \( u' \) and \( v' \), the peak frequencies of their decomposed IMF_5, IMF_6, and IMF_7 are within the range of wave frequencies. Therefore, IMF_5–7 are diagnosed as wave components (Fig. 2). IMF_1–4 represent high-frequency signals, such as turbulence and instrumental noise. The energy of IMF_1 shows a flat tail in a high-frequency range (larger than 10^{0.4} \text{ Hz}), representing the characteristics of white noise. IMF_8, IMF_9, and the Residual component represent low-frequency motions. Although most of the wave energy distributes in the principal wave frequency range, the wave components also contain some low- and high-frequency energy (Fig. 2). Likewise, the low- and high-frequency motions (IMF_8–9 and IMF_1–4, respectively) contain energy in wave frequency range. Thus, the spectra and velocity methods cannot avoid errors when calculating BWOV by integrating velocity spectra over the principal wave frequencies.

Figure 3 shows the waves (that are BWOVs), high-frequency motions and low-frequency motions decomposed from \( u' \) and \( v' \) using the EMD method. It is clearly illustrated that the wave motions in the velocity measurements were fully justified by the EMD method.

The high-frequency turbulent motions, instrumental noise, and low-frequency motions were effectively removed by the EMD method. RMS velocities of each component show that the high- and low-frequency components clearly influenced the accuracy of the BWOV calculation.

Another advantage of the EMD method is that it can decompose a nonstationary process (e.g., wave) into several stationary IMFs. This means, even for a nonstationary wave process, instantaneous frequencies of the IMFs can be calculated using Hilbert transform and present the energy–frequency–time distribution (Huang et al. 1998). Figure 4 shows the Hilbert spectra of the IMFs decomposed from \( u' \) and \( v' \). Wave energy variations and instantaneous frequency of the wave motions can be clearly observed, which is very useful for the studies of the interaction of multiscale dynamic processes, such as wave–turbulence interaction (Qiao et al. 2016).

3. Observations

The field data were collected off the Yellow River delta in winter (from 18 December 2016 to 11 January...
2017), about 10 km off the shoreline with a mean water depth of 10 m (Fig. 5). A bottom-mounted quadrapod equipped with a Nortek 6-MHz ADV and an RBR Tide and Wave Logger was deployed for wave measurements. The ADV was set up to continuously measure near-bed (0.67 m above the seabed) velocity and pressure with sampling frequency of 16 Hz, while the RBR Tide and Wave Logger measured wave height every 30 s.

We selected a 96-h period of time beginning on 26 December 2016 to calculate BWOV. During this period of time, two distinct regimes were observed: one with strong wave processes and the other with weak processes. The 96-h ADV data were divided into 576 equal-length segments, with each segment containing 10-min measurements of horizontal velocities $u$ and $v$, vertical velocity $w$, and bottom pressure $p$. Quality control was applied to the ADV measurements to exclude spikes and noises following procedures of Qiao et al. (2016).

Velocity measurements from the ADV show that flows at the observation site were dominated by semidiurnal...
astronomical tide $M_2$ with a maximum current speed of $0.4\,\text{m/s}$. Based on linear wave theory, wave motions at the whole water column are coherent and have a same period. Using elevation time series inferred from the ADV pressure measurements, we calculated wave peak period $T_p$ and zero-crossing period $T_z$. Here, $T_p$ is calculated as the period corresponding to the frequency in elevation spectrum $S_h$ with the most energy, and $T_z$ as the mean zero-crossing period of the highest one-third of waves. The results show that, in general, $T_p$ and $T_z$ were consistent in wavy environment (wave period > 5 s) but they were discrepant when wave processes were not significant (wave period < 5 s). During hours 60–70, even though the wave periods were short, $T_p$ and $T_z$ were also consistent due to no wave motions were observed. Following the zero-crossing method, significant wave height $H_{\text{sig}}$ was calculated by surface elevation results derived from the ADV pressure measurements. In practice, when wave motions were very weak (e.g., $T_p$ and $T_z$ were different or wave periods were shorter than 4 s), the calculated $kh$ increased dramatically, resulting in unreasonable surface elevation estimations. Therefore, we deleted the unreasonable $H_{\text{sig}}$ when there were no clear signs of wave motions (Fig. 6c). Time series of significant wave height shows a strong wave process (abbreviated name: SW; duration: 42 h) with maximum $H_{\text{sig}}$ of 3.0 m and a weak wave process (abbreviated name: WW; duration: 17 h) with maximum $H_{\text{sig}}$ around 1.0 m, and the results closely match the $H_{\text{sig}}$ obtained from the Wave Logger (Fig. 6c).

Power spectrum analysis was used to characterize the wave processes in frequency domain. The mean power spectra of velocity fluctuations in the SW and WW processes were calculated by applying the Welch method (Press et al. 1992) to velocity fluctuations. The most significant feature of the power spectra in the SW process was horizontal wave motions induced a high-energy band with a frequency range from 0.06 to 0.35 Hz.
and a peak frequency of 0.115 Hz (Fig. 7a), which means the horizontal wave orbital velocities dominated the water motions. In contrast, the vertical velocity fluctuations contain less energy and the energy at wave frequencies was not prominent compared with the energy at other frequencies. This is because waves in shallow water move in ellipses and the ellipses grow flatter toward the bottom, indicating that the vertical orbital velocities degenerate faster than the horizontal one. Compared with the SW process, the WW process had one order of magnitude smaller energy, a narrower frequency band of 0.1–0.35 Hz and a higher peak frequency of 0.18 Hz (Fig. 7b).

### 4. Results and discussion

#### a. BWOV calculated by the various methods

We calculated RMS BWOVs using the spectra, velocity and EMD methods. Using the high-frequency (16 Hz) pressure measurements from the ADV, we obtained the elevation spectrum $S_h$ and calculated RMS BWOV based on the spectra method [Eqs. (2) and (3)]. As mentioned in section 3, the surface elevations estimated from the bottom layer pressure measurements were unreasonable when there was no clear sign of waves. As a result, only BWOVs in the SW and WW processes were calculated by the spectra method. The velocity and EMD method used the high-frequency (16 Hz) velocity measurements from ADV to calculate instantaneous BWOV based on Eqs. (4), (10), and (11), respectively.

In general, BWOVs calculated by the various methods were highly consistent (Fig. 8). The maximum value of BWOVs for the SW process was 0.75 m s$^{-1}$ and a lower value for WW was around 0.22 m s$^{-1}$. There were differences among the various methods. Under slight wavy conditions, BWOVs based on the velocity method showed clearer tidal frequency variations compared with those on the EMD method, suggesting that a part of low-frequency tidal velocities was misdiagnosed as BWOV by the velocity method. Especially during 10–20 h, BWOVs based on the velocity method were significantly higher than those on the spectra and EMD methods (Fig. 8). The analysis of the ADV velocity measurements shows that as the wind-wave energy...
increasing, the ADV produced a large amount of noise, which cannot be eliminated by the quality control procedures introduced in section 3, leading to an overestimation of BWOVs based on the velocity method.

Linear regression analysis was conducted to quantitatively compare the BWOVs calculated by the spectra, velocity and EMD methods in the SW and WW processes, respectively (Fig. 9). In the SW process, the spectra method calculated the highest BWOVs, about 8% higher than those calculated by the EMD method. BWOVs calculated by the velocity method were 6% higher than those calculated by the EMD method. In the WW process, BWOV differences among the three methods increased: BWOVs calculated by the spectra method, velocity and EMD methods in the SW and WW processes, respectively (Fig. 9). In the SW process, the spectra method calculated the highest BWOVs, about 8% higher than those calculated by the EMD method. BWOVs calculated by the velocity method were 6% higher than those calculated by the EMD method. In the WW process, BWOV differences among the three methods increased: BWOVs calculated by the spectra method, velocity and EMD methods were 14% and 11% higher than that calculated by EMD method, respectively. Correlations between the calculated BWOVs and the $H_{\text{sig}}$ were also estimated in the SW and WW processes, respectively (Figs. 9c,d). In the SW process, BWOVs calculated by the three methods were closely related to $H_{\text{sig}}$, with coefficient of determination $R^2 \geq 0.94$, while in the WW process, correlations between the calculated BWOVs and $H_{\text{sig}}$ were decreased significantly to $R^2 \leq 0.80$. Correlations between BWOVs and the peak wave period $T_p$ were not high (Figs. 9e,f), with coefficients of determination $R^2 \leq 0.65$ and $R^2 \leq 0.39$ in the SW and WW processes, respectively.

b. Statistical distribution of BWOVs

Both laboratory experiments and field observations demonstrate that the distribution of instantaneous BWOVs follows the Gaussian distribution, while the distribution of BWOV amplitudes obeys the Rayleigh distribution (Sultan and Hughes 1992; You 2009). Here, we compared the instantaneous BWOV and BWOV amplitude distributions derived from the velocity and EMD methods to evaluate the performance of the methods. The statistical distribution of BWOVs by the spectra method was not conducted, because the spectra method only calculates RMS BWOV and is incapable of producing instantaneous BWOVs and BWOV amplitudes. The BWOV amplitude $U$ of a wave was calculated from the instantaneous BWOVs using the zero-up crossing method $U = 0.5 \times (\bar{U}_{\text{max}} - \bar{U}_{\text{min}})$, where $\bar{U}_{\text{max}}$ and $\bar{U}_{\text{min}}$ are the maximum and minimum instantaneous BWOVs of a wave, respectively.

Following the algorithms used in You (2009), we calculated the statistical distribution of instantaneous BWOVs and BWOV amplitudes using BWOV results derived from the velocity and EMD methods (Fig. 10). Figures 10a and 10b show probability distributions $f(x)$ of normalized instantaneous BWOVs ($x = U/U_{\text{rms}}$), which are used to specify the probability of a random instantaneous BWOV falling within a particular range of instantaneous BWOVs. The distribution of instantaneous BWOVs from the velocity method perfectly followed the Gaussian distribution in both SW and WW processes. The distribution of instantaneous BWOVs from the EMD method also followed the Gaussian distribution very well. Figures 10c and 10d show probability distributions $f(x)$ of normalized BWOV amplitudes ($x = U/U_{\text{rms}}$), which are used to specify the probability of a random BWOV amplitude falling within a particular range of BWOV amplitudes. The distribution of BWOV amplitudes from the EMD method closely followed the Rayleigh distribution in the SW and WW processes. Consistent with the distribution of instantaneous BWOVs, BWOV amplitudes calculated by the EMD method also show a higher proportion of small BWOV amplitudes than the Rayleigh distribution. In the SW process, the BWOV amplitudes from the EMD method show slightly low proportion in the range of $0.9 < U/U_{\text{rms}} < 1.6$. In the WW process, BWOV amplitudes from the EMD method showed a slightly lower proportion in the range of $0.2 < U/U_{\text{rms}} < 0.9$. However, BWOV amplitudes from the velocity method showed totally different distribution patterns compared with Rayleigh distribution: the proportion of low BWOV amplitudes from the velocity method was much higher than the Rayleigh distribution. We checked the BWOV amplitudes from the velocity method and found that the velocity method calculated instantaneous BWOV included lots of high-frequency (>1 Hz) velocity fluctuations that produced large amounts of low BWOV amplitude when using the zero-crossing method to define BWOV amplitude. These high-frequency velocity fluctuations were probably turbulence motions and instrumental noise. The velocity method misdiagnosed these high-frequency signals as BWOVs, leading to BWOV amplitudes distribution discrepancy compared with the Rayleigh distribution. The instantaneous BWOVs from the velocity method were also contaminated by the high-frequency motions, but they perfectly
Fig. 9. (top) The RMS BWOV from the spectra (blue) and velocity (orange) methods vs the RMS BWOV from the EMD method in the (a) SW and (b) WW processes. (middle) The RMS BWOV from the spectra (blue), velocity (orange), and EMD (yellow) methods vs the significant wave heights $H_{\text{sig}}$ in the (c) SW and (d) WW processes. (bottom) The RMS BWOV from the spectra (blue), velocity (orange), and EMD (yellow) methods vs the peak wave heights $T_p$ in the (e) SW and (f) WW processes. Numbers of points used in SW and WW processes are 252 and 102, respectively.
followed the Gaussian distribution (Fig. 10). This is because the high-frequency motions were uniformly distributed at each instantaneous BWOVs and their impacts cannot be detected by the probability density distribution.

c. Uncertainty analysis

Comparisons of BWOVs from the spectra, velocity and EMD methods showed that all the methods gave reasonable calculations of BWOV, but there were also discrepancies among the methods (Figs. 8–10). These methods are based on different physical assumptions and use different mathematical algorithms, which often give discrepant calculations of BWOV. To accurately calculate the BWOV, the performance of the various methods for BWOV calculation needs to be evaluated.

First, the sources of possible error of the spectra method were analyzed. Based on the linear wave theory, the spectra method builds a relationship between surface elevations and BWOVs. In coastal waters, the steepness of wind waves may increase significantly and the wave amplitudes are comparable to wavelength, so that the waves cannot be considered linear, and BWOV calculation based on the linear wave theory will become inaccurate (Wiberg and Sherwood 2008). It is also well known that longer-period waves penetrate more readily to bed than those of shorter-period waves, which leads to the period of bottom waves longer than that of surface waves (Soulsby 1987). The inconsistency of the wave periods may generate errors to BWOV calculation when calculating $kh$ in Eq. (1). Another important source of error is that elevation spectrum used by the spectra method contains instrumental noise, high-frequency turbulence and low-frequency motions that are not associated with waves (Figs. 2 and 3). These wave-unrelated motions are misdiagnosed as wave motions by the spectra method, which certainly overestimated the BWOV calculation. The influence of wave-unrelated motions on the BWOV calculation was estimated (Table 1) and the results suggested that the wave-unrelated motions led to RMS BWOV overestimation of 2.0% and 3.6% in the SW and WW processes, respectively. It should be noted that the wave-unrelated motions are very likely to introduce larger errors than our estimations because we only estimated the errors caused by velocities outside of the principal wave frequencies. And the errors caused by wave-unrelated velocities inside of the principal wave frequencies were ignored here (Fig. 2).

The velocity method treated measured velocity fluctuations as wave motions to calculate BWOV, which, as the spectra method did, led to BWOV errors caused by motions not associated with waves (Figs. 2 and 3).
In Table 1, the errors of the RMS BWOV caused by the low- and high-frequency signals unrelated to waves in the SW and WW processes. The ranges of wave frequency (wave_l and wave_h) correspond to those indicated in Figs. 2 and 7.

<table>
<thead>
<tr>
<th>Segment-averaged $U_{\text{rms}}$ (m s$^{-1}$)</th>
<th>SW process (252 segments)</th>
<th>WW process (102 segments)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectra method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_{\text{rms}} = \frac{\int_{0}^{\infty} S_U(\omega) d\omega}{\int_{0}^{\infty} S_{U}^{2}(\omega) d\omega}$</td>
<td>0.2481</td>
<td>0.1157</td>
</tr>
<tr>
<td>$U_{\text{rms}} = \frac{\int_{\text{wave}<em>l}^{\text{wave}<em>h} S</em>{U}(\omega) d\omega}{\int</em>{\text{wave}_l}^{\text{wave}<em>h} S</em>{U}^{2}(\omega) d\omega}$</td>
<td>0.2444</td>
<td>0.1123</td>
</tr>
<tr>
<td>RMS error</td>
<td>0.005</td>
<td>0.004</td>
</tr>
<tr>
<td>(2.0% of $U_{\text{rms}}$)</td>
<td>(3.6% of $U_{\text{rms}}$)</td>
<td></td>
</tr>
<tr>
<td>Velocity method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_{\text{rms}} = \frac{\int_{0}^{\infty} S_U(\omega) d\omega}{\int_{0}^{\infty} S_{U}^{2}(\omega) d\omega}$</td>
<td>0.2436</td>
<td>0.1113</td>
</tr>
<tr>
<td>$U_{\text{rms}} = \frac{\int_{\text{wave}<em>l}^{\text{wave}<em>h} S</em>{U}(\omega) d\omega}{\int</em>{\text{wave}_l}^{\text{wave}<em>h} S</em>{U}^{2}(\omega) d\omega}$</td>
<td>0.2289</td>
<td>0.1015</td>
</tr>
<tr>
<td>RMS error</td>
<td>0.029</td>
<td>0.011</td>
</tr>
<tr>
<td>(12.7% of $U_{\text{rms}}$)</td>
<td>(10.8% of $U_{\text{rms}}$)</td>
<td></td>
</tr>
</tbody>
</table>

In Fig. 8, the influence of low-frequency motions (tidal currents) on the velocity method calculated BWOVs was clearly observed. And the statistical distributions of BWOV amplitudes demonstrated that the BWOVs calculated by the velocity method contained high-frequency turbulence and noise signals that led to unusual BWOV amplitude distributions (Fig. 10). What is clear is that the velocity fluctuations that were not associated with waves caused obvious errors in BWOV calculation from the velocity method. Following Eqs. (6) and (7), we estimated the errors and the results suggested that the wave-unrelated motions led to an overestimation of RMS BWOV about 12.7% and 10.8% in the SW and WW processes, respectively. Similar with the spectra method, the wave-unrelated signals inside the principal wave frequencies cannot be removed from the BWOVs that produced inevitable errors (Fig. 2). The wave-unrelated motions had much more impact on the BWOV calculation from the velocity method than that from the spectra method. It is presumably because the velocity sensor of the ADV used by the velocity method produced more noise than the pressure sensor used by the spectra method under wavy conditions.

The EMD method only extracts wave-related IMFs to calculate BWOV, so it avoids the main errors in the spectra and the velocity methods, which are caused by wave-unrelated motions. There is a possible source of errors from the mode mixing problem of the EMD method. For intermittent oscillations signal (like wave motions), the EMD-decomposed IMFs may consist of signals of widely disparate scales, or a signal with similar scale residing in different IMFs (Wu and Huang 2004). Therefore, the wave signals may be mixed into the wave-unrelated IMFs or the wave-unrelated motions are misdiagnosed as waves, which leads to possible errors in BWOV calculation. In this study, an ensemble EMD method (Wu and Huang 2009) developed to fix the mode-mixing problem is used to calculate BWOV. Our results show that the mode-mixing problem did not introduce obvious errors to the BWOV calculation.

5. Conclusions

We applied the EMD method on BWOV calculation and evaluated its performance by comparing with the spectra and velocity methods. The results suggest that the spectra and velocity methods overestimated the BWOVs about a few percent (6%–14%) because they misdiagnosed wave-unrelated motions and instrument noise as wave velocities. Even though pretreating using a bandpass filter on the measured velocity and pressure measurements can remove the wave-unrelated signals outside of the wave frequencies, the wave-unrelated signals inside of the wave frequencies cannot be removed by the spectra and velocity methods, consequently resulting in inevitable errors to BWOV calculation. Moreover, the spectra method is not capable of providing instantaneous BWOV, which is far more valuable than RMS BWOV for engineering studies. The EMD method, on the other hand, was not affected by the wave-unrelated motions, because it only extracts wave-related IMFs for BWOV calculation. Besides, combining with Hilbert transform, the EMD method is capable of providing detailed energy and frequency variations of the wave process, which is useful for further dynamic studies. To sum up, the EMD method performs well on BWOV calculation and is capable of providing more accurate BWOV than the spectra and velocity methods.

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