Wavelet transform analysis of reference crop evapotranspiration during the growing season in three typical regions of Inner Mongolia, China
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ABSTRACT

Management and scheduling of irrigation water requires consideration of evapotranspiration, one of the most important hydrological variables. This study investigates the variations in the daily potential evapotranspiration (ET₀), and its aerodynamic (ETₐ) and radiometric (ETᵣ) components in three areas (western, central and eastern) of the Inner Mongolia Autonomous Region (IMAR) during the growing season (April–September, 2007). In this study, a data-driven approach was followed, and the wavelet transformation analysis method was used to investigate the evapotranspiration characteristics of a relatively large geographic region. The results show that there are close correlations in the variations of ET₀ with those of ETₐ and ETᵣ. For the western area of the IMAR, the timing of the largest ETₐ is 1 month earlier and its wave period is 10 days shorter than those of ET₀ and ETᵣ. For the central area, the wave period of ETₐ is 20 days shorter, and the timing of the largest ETₐ is approximately 1 month earlier than those of ET₀ and ETᵣ. For the eastern area, there are two large fluctuations in ETₐ, and they occur 1 month earlier than those of ET₀.

Key words | China, Inner Mongolia, reference evapotranspiration, spatial variations, temporal variations, wavelet transforms

INTRODUCTION

The reference crop evapotranspiration (ET₀) is one of the most important hydrological variables, which affects the water and energy exchanges at the land surface/atmosphere interface (Liu et al. 2013; Xie et al. 2014). To estimate ET₀ requires a good understanding of the hydrological cycle. Accurate estimates of ET₀ are critical for scheduling irrigation activities and are also required for a variety of hydro-meteorological applications, e.g. certain hydrological and water-balance models and in calculating the potential/actual evapotranspiration of a region and/or a basin (Blaney & Cridge 1950; Dyck 1953; Hobbins et al. 2001a, 2001b; Xu & Li 2003; Xu & Singh 2005; Gong et al. 2006). The major factors that affect ET₀ are elevation, latitude and longitude, meteorological conditions, land use (land cover) and changing climatic conditions at different spatio-temporal scales (Jhajharia et al. 2006, 2009a, 2009b).

Many studies have analysed the methods of estimating ET₀ and their validity in different parts of the world (Jhajharia et al. 2004a, 2004b, 2006, 2009a, 2009b). Further, ET₀ has been evaluated in the context of its spatial and temporal variations. For instance, Thomas (2000) analysed the spatial and temporal characteristics of the ET₀ trends across China. Jhajharia et al. (2012) found that the reduction in ET₀ in the humid region of northeast India was mainly due to the net radiation and wind speed. In assessing a monthly reference evapotranspiration model, Butterfusco et al. (2010) found that there was spatial uncertainty in the results, which were caused by the uncertainties in the inputs, particularly...
the temperature, on a regional scale. Using the parametric t-test and the non-parametric Mann–Kendall test, Xu et al. (2006) analysed the temporal trend of ET₀ and compared it with the pan evaporation (Eₚan) trend in the Chang Jiang river basin. They found that for both the ET₀ and Eₚan between 1961 and 2000, there were significant decreasing trends. Mo et al. (2004) analysed the seasonal and annual variations of ET₀ between 1984 and 1997 for a mountainous region in the Lu Shi basin, China. Tong et al. (2007) analysed the temporal variations of ET₀ from spring wheat in the last 50 years (1955–2005) in northwest China and tested the ET₀ trend using the Mann–Kendall test and linear regression analysis. In a study in Iran, Dinpashoh et al. (2011) found that the wind speed was the most dominant factor affecting ET₀ throughout the year except for the winter months. Using the data from 34 meteorological stations between 1957 and 2007 in the Hai He river basin of China, Wang et al. (2011) analysed ET₀ spatial and temporal patterns and trends. Liang et al. (2010) analysed the ET₀ temporal variations between 1961 and 2005 in the Taoer River basin in northeast China.

In recent years, wavelet transform analysis has been widely used in the field of water resources and meteorological sciences. For instance, wavelet transform analysis was used to investigate the multi-timescale features of ET₀ at the Chang Ping station in China (Ren et al. 2009). The complexity associated with ET₀ was solved based on wavelet denoising and symbolic dynamics by Chen et al. (2012). The decomposed inter-decadal and inter-annual components of the rainfall data during the rainy seasons in north China have been analysed using wavelet transformation (Lu 2002). Kişi (2010) investigated the accuracy of a wavelet regression approach for modelling ET₀. A synthetic model of wavelets was used to downscale and forecast ET₀ with support vector machines (Kaheil et al. 2008). Wavelet transform analysis in conjunction with power laws was used to perform wet-spell and dry-spell analyses of precipitation from the global climate models (Ashok et al. 2011).

In earlier studies, using the Penman–Monteith equation, as recommended by Food and Agriculture Organization of the United Nations (FAO), and meteorological data from the weather stations, ET₀ and its spatial and temporal variations were investigated (Song et al. 2013). However, ET₀ is affected by both aerodynamic and radiometric variables, as these two variables reflect the combined effects of topography, soil, vegetation and climate. The Inner Mongolia Autonomous Region (IMAR) is predominantly an agricultural region in China, where irrigation activities are highly dependent on the available water resources. Due to variations in the natural geographical and climatic conditions in the IMAR, there is no consistency in the ET₀ trend. Therefore, an analysis of ET₀ and its components (ETₐ and ET₃) for different sub-regions are needed in order to understand the ET₀ variation over one whole year. In view of the uncertainties in ET₀, the wavelet transform method has been used in this study to investigate the variations in ET₀ in the western, central and eastern regions of the IMAR during a growing season.

The main objectives of this study are: (1) to use the wavelet transform method (a data-driven approach) to examine the spatial and temporal patterns of ET₀, (2) to investigate the spatial differences of relevant hydrological variables and (3) to improve the understanding of ET₀ for agricultural water management.

**Study area**

The IMAR lies between 37°01’–53°02’ N latitude and 95°02’–125°37’ E longitude and along the northern border of China. The IMAR is the third largest province in China, with an area of 1.183 × 10⁶ km², as shown in Figure 1. The

![Figure 1](https://example.com/figure1.png)
The data include: (1) the mean air temperature, (2) the obtained from the Inner Mongolia Meteorological Bureau. Hence, the data from these three stations, there are three that are representative of the literature ranges between $-4$ C and $9.2$ C, and the minimum mean monthly air temperature ranges between $-30$ C and $-11$ C during January. The maximum mean monthly air temperature ranges between $19$ C and $26$ C during July. The annual mean precipitation ranges between $50$ mm and $550$ mm. In the IMAR, the precipitation decreases while air temperature increases from the northeast to southwest. The annual precipitation in Oroqen Autonomous County on the eastern border, and the elevation ranges from $86$ m to $3,522$ m above mean sea level. It has a typical moderate and continental monsoon climate with small and non-uniform rainfall events. There is a large difference in temperature between summer and winter. Except for Da Xing’an Mountain area which is relatively humid, most areas within the IMAR from west to east are arid, semi-arid or sub-humid. In the IMAR, the average annual sunshine is about $2,700$ hours. The annual mean air temperature varies between $-30$ C and $-11$ C during January.

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Due to the large variations in the climatic and physical conditions, the ET0-distribution in the IMAR is complex. However, from the $135$ weather stations within the IMAR (Figure 1), there are three that are representative of the ET0 variations. Hence, the data from these three stations have been used for analysis. The selected stations are in: (1) an extreme arid area in Ejina County, (2) a semi-arid area in Hohhot and (3) a relative humid area in Erguna Zuo County. They are situated within the western, central and eastern areas of the IMAR, respectively, as shown in Figure 1.

**DATASET AND METHODOLOGY**

**Data**

For the three selected stations, daily meteorological data during the growing season April–September 2007 were obtained from the Inner Mongolia Meteorological Bureau. The data include: (1) the mean air temperature, (2) the minimum and maximum air temperatures, (3) the mean relative humidity, (4) the mean wind speed and (5) the sunshine hours. Using all these data, the daily ET0, ETr and ETa were calculated using the FAO Penman–Monteith model (Allen et al. 1998):

$$ET_0 = \frac{0.408\Delta (R_n - G) + \frac{900}{273} U_2 (e_s - e_a)}{\Delta + \gamma (1 + 0.34 U_2)}$$

where $ET_0$ is the daily reference crop evapotranspiration rate (mm/d); $R_n$ is the net radiation at canopy surface (MJ/m²/day); $G$ is the soil heat flux at the soil surface (MJ/m²/day); $T$ is the mean daily air temperature (°C); $\gamma$ is the psychrometric constant (kPa/C); $U_2$ is the mean daily wind speed at 2.0 m height (m/s); $e_s$ is the mean saturation vapor-pressure (kPa); $e_a$ is the mean actual vapour-pressure (kPa); $(e_s - e_a)$ is the saturated vapour pressure deficit (kPa); and $\Delta$ is the slope of the saturated vapour pressures temperature curve (kPa/C).

In this study, as the Penman–Monteith method requires $2$ m high wind speed, the recorded $10$ m high wind speed has therefore been rescaled to $2$ m wind speed. While it is difficult to account for the many factors (e.g. land surface types and vegetation) that affect a wind profile, the logarithmic wind speed profile has been used in this study, calculated as:

$$U_2 = \frac{4.87 \cdot u_k}{\ln(67.8 h - 5.42)}$$

where $h$ is the height above the ground surface. Equation (2) is recommended for a surface with short grass (Allen et al. 1998; Gong et al. 2006; McVicar et al. 2007).

In the Penman–Monteith formula (Equation (1)), the ET0 is divided into two components, i.e. radiometric (ETr) and aerodynamic (ETa) as shown in Equation (3). Both components, ETr (Equation (4)) and ETa (Equation (5)), have been analysed in order to assess the ET0 variations.

$$ET_0 = ET_r + ET_a$$

$$ET_r = \frac{0.408\Delta (R_n - G)}{\Delta + \lambda(1 + 0.43 U_2)}$$

$$ET_a = \frac{\frac{900}{273} U_2 (e_s - e_a)}{\Delta + \lambda(1 + 0.43 U_2)}$$
Wavelet transform theory

The wavelet function \( \psi(x) \) can be defined as \( \int_{-\infty}^{\infty} \psi(x)dx = 0 \). A continuous wavelet can be obtained by compressing and expanding \( \psi(x) \):

\[
\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}} \psi \left( \frac{x - b}{a} \right), \quad a, b \in \mathbb{R}; \quad a \neq 0 \tag{6}
\]

where \( a \) is either a scale factor or a frequency factor, \( b \) is the time domain factor and \( R \) is the domain of real numbers.

If \( \psi_{a,b}(x) \) satisfies Equation (3) either for the ET0-time series \( f(x) \in L^2(\mathbb{R}) \) or for a finite energy signal, the continuous wavelet transform of the time-series \( f(x) \) is defined as:

\[
W_f(a, b) = \frac{1}{\sqrt{|a|}} \int_{\mathbb{R}} f(x) \psi \left( \frac{x - b}{a} \right) dx \tag{7}
\]

where \( W_f(a, b) \) are the wavelet transform coefficients. The characteristics of the ET0-time-series \( f(x) \) in frequency \( a \) and time domain \( b \) at the same time are reflected by \( W_f(a, b) \). When the frequency resolution of wavelet transforms is low and the time domain is high, \( b \) becomes smaller and vice versa. This is why the wavelet transform method is useful for the time-frequency localization analysis that is involved in the analysis of ET0.

In case of a real function, the wavelet transform can be written as:

\[
W_f(a, b) = \Delta x \sum_{i=1}^{N} \psi \left( \frac{i\Delta x - b}{a} \right) f(i\Delta X) \tag{8}
\]

where \( \Delta x \) is the sampling interval and \( N \) is the number of sampling points.

Mexican hat wavelet

The Mexican hat mother wavelet (Fu 2006) is one of the most widely used wavelets because of its satisfactory performance. It is the second derivative of the Gaussian distribution function. It can be mathematically defined as:

\[
\psi(x) = \frac{1}{\sqrt{2\pi}} (1 - x^2) e^{-x^2/2} \tag{9}
\]

The wave transform coefficients \( W_f(a, b) \) can be obtained by combining Equations (5) and (6). A two-dimensional plot with coefficient \( a \) on the \( y \)-axis and coefficient \( b \) on the \( x \)-axis, is used to show the wavelet transform. This plot is able to show the characteristics of the wavelet transform of a time-series. On the same scale as \( a \), the variation of \( W_f(a, b) \) with time shows the characteristics of the time-series. A positive \( W_f(a, b) \) corresponds to ET0 with a high periodicity and vice versa. At the point where \( W_f(a, b) \) is zero, there is abrupt change. If the values of the wavelet transform coefficients are relatively large, this is an indication that there is significant variation in the corresponding time-scale.

The wavelet transform variance is the integral of the square of \( W_f(a, b) \) for \( a \) at the time-scale \( b \), as follows:

\[
Var(a) = \int_{-\infty}^{\infty} |W_f(a, b)|^2 db \tag{10}
\]

The variation of wavelet variance with the scale \( a \) can be shown in a wavelet variance plot, which shows the fluctuation of energy distribution with the scale \( a \). The wavelet variance plot can be used to determine the time-scale of main period of the ET0 time-series.

To reduce the interface effect in the wavelet analysis, the most commonly used method is to extend the time-series in both the forward and backward directions. After the wavelet transform, the part of the series being extended can then be abandoned.

Let the time-series be:

\[
X(t) = \{X(1), X(2), \ldots, X(N)\} \tag{11}
\]

where \( N \) is the length of sampling.

By extending the time-series \( N \) in the forward direction, the time-series becomes:

\[
X(t) = X(t + 1), \quad (t = 0, 1, \ldots, N - 1) \tag{12}
\]

By extending the time-series \( N \) in the backward direction, the time-series becomes:

\[
X(t + N) = X(N + 1 - t), \quad (t = 1, 2, \ldots, N) \tag{13}
\]

Using the Penman–Monteith model and the daily meteorological recorded data between April and September, 2007, ET0, ETa and ETf were calculated for the three selected stations. The ET0, ETa and ETf time-series were extended in both the forward and backward directions, according to
Equations (12) and (13). Using the Mexican hat function (MHF), the $ET_0$, $ET_a$ and $ET_r$ time-series were then analyzed at multiple time scales, according to Equations (8) and (9).

RESULTS AND DISCUSSION

The results of the wavelet analysis and their implications for the three sub-regions of the IMAR (Ejina County in the western area, Hohhot City in the central area and Erguna Zuo County in the eastern area) are discussed in this section. The differences among the three sub-regions are highlighted below.

Western area – Ejina County

As shown in Figures 2(a), 3(a) and 4(a), there are large seasonal fluctuations in the $ET_0$ in Ejina County. For the $ET_r$ and $ET_a$, the seasonal cycles are similar, in that they increase at the beginning of the growing season and decrease towards the end of the growing season. For both $ET_0$ and $ET_a$, their peaks are in June, indicating that the highest evapotranspiration is in June. On the other hand, the peak of $ET_r$ is in July. Figures 2(b), 3(b) and 4(b) respectively show the wavelet transforms of the anomaly percentages of $ET_0$, $ET_r$ and $ET_a$ for the growing season. The strength of the signals is shown by values of wavelet transform coefficient. A positive coefficient means the signal is strong, whereas a negative coefficient means the signal is weak. A coefficient equal to zero means there is abrupt change at this point. From Figures 2(b), 3(b) and 4(b), it can be seen that the isolines at the upper half of the plots are relatively sparse, which corresponds to the wave period of low frequency on a large scale. For a period of 40–50 days, the $ET_0$, $ET_r$ and $ET_a$ series show identical trends with two abrupt changes and two transitions (i.e. from low to high and then to low). For both $ET_0$ and $ET_a$, there are abrupt changes in the middle of May and
August, whereas for ET\textsubscript{a} there are abrupt changes in early May and early August.

As shown in Figures 2(c), 3(c) and 4(c) respectively, the largest variance is on Day 45 (i.e. end of June) for ET\textsubscript{0}, on Day 40 (i.e. early June) for ET\textsubscript{a} and on Day 50 (i.e. mid-July or end of June for ET\textsubscript{r}). However, at the smaller scale, the fluctuations are more complicated and less certain. Figures 2(c), 3(c) and 4(c) also show that during the growing season, the wave period is approximately 45 days for ET\textsubscript{0}, 40 days for ET\textsubscript{a} and 50 days for ET\textsubscript{r}. These results indicate that the changes are dependent on several parameters, such as the roughness of surface and the leaf area index. Hence, to assess these parameters accurately they need to be rescaled in the timeline. Further, the time lag can be attributed to the windy climatic condition in the western area of the IMAR, which has affected ET\textsubscript{a} after April. Hence, the ET\textsubscript{a} fluctuations are 1 month earlier than those of ET\textsubscript{0} and ET\textsubscript{r}. The ET\textsubscript{a} results obtained in this study are closely related to ET\textsubscript{0}, which is consistent with the findings by Huo et al. (2004). They found that ET\textsubscript{a} is dominant in the more arid area of the IMAR (Huo et al. 2004).

Central area – Hohhot city

Figures 5–7 show the evapotranspiration results for Hohhot city. Figures 5(a), 6(a) and 7(a) show the anomaly percentages of the ET\textsubscript{0}, ET\textsubscript{a} and ET\textsubscript{r}. Figures 5(b), 6(b) and 7(b) show the wavelet transform, and Figures 5(c), 6(c) and 7(c) show the MHF wavelet variance. The seasonal cycles are similar for the three variables. For both ET\textsubscript{0} and ET\textsubscript{a}, the peaks are in the middle of May, whereas the ET\textsubscript{r} peak is in the middle of July. Further, both ET\textsubscript{a} and ET\textsubscript{r} decrease gradually with time. For a period of 40–50 days, the ET\textsubscript{0} and ET\textsubscript{r} time-series follow identical trends with two abrupt changes and two transitions. For both ET\textsubscript{0} and ET\textsubscript{r}, the abrupt changes are at the end of May and end of August, respectively. On the other hand, the abrupt changes in ET\textsubscript{a} are at the end of April and during the middle of June. Further, as shown in Figures 5(b), 6(b) and 7(b), the wave
periods for ET$_0$ and ET$_a$ are both 10 days, while the wave period for ET$_a$ is 30 days.

The periods and dates corresponding to the largest variations in ET$_0$ and ET$_r$ are similar to those in Ejina County. Figures 5(c), 6(c) and 7(c) show that during the growing season, the wave periods are approximately 10 days and 45 days for ET$_0$, 10 days, 30 days for ET$_a$, and 50 days for ET$_r$. These wave periods are an indication that solar radiation has an effect on the ET$_0$ in the Hohhot area, which is consistent with the findings by Huo et al. (2004), showing that ET$_r$ is beginning to affect ET$_0$ in the semi-arid area of the IMAR (Huo et al. 2004).

Eastern area – Erguna Zuo County

Figures 8–10 show the evapotranspiration results of Erguna Zuo County. For both ET$_0$ and ET$_r$, there are significant seasonal fluctuations during the growing season, with peaks in mid-August. On the other hand, for ET$_a$, there are two peaks in early May and late August. For ET$_0$ and ET$_r$, and for a period of 40–50 days, there are two abrupt changes, occurring in early June and late August and two transitions. As shown in Figures 8(b), 9(b) and 10(b), for ET$_a$, and for a period of 20–25 days, there are three abrupt changes and three transitions (i.e. from low to high and then to low and to high again).

The timing of the largest variations in ET$_0$ and ET$_r$ in the eastern area is 1 month earlier than those in the western and central areas. From Figures 8(b), 9(b) and 10(b), it can be seen that there are two large fluctuations in ET$_a$ (i.e. a 50-day period in early June and a 10-day period in mid-August). Figures 8(c) and 9(c) show that for both ET$_0$ and ET$_r$, there is a fluctuation period of approximately 50 days, and Figure 10(c) shows that for ET$_a$, there is a fluctuation period of 25 days during the growing season.

As shown in Figures 8–10, it can be seen that ET$_r$ is largest in June and July. These results are consistent with the global trend of radiation. This can be attributed to the large area of the IMAR, with widely varying topography and climatic
conditions. It should be noted that the results of ET₀, ET𝛼 and ETᵣ as presented in this study are for a typical dry year with normal precipitation, in which the relationships among the hydro-meteorological parameters are generally close.

**CONCLUSIONS**

A comprehensive evaluation of the daily reference crop evapotranspiration (ET₀), aerodynamic (ET𝛼) and radiometric (ETᵣ) in the IMAR has been carried out in this study. The variability of the ET₀, ET𝛼 and ETᵣ was investigated in three regions of the IMAR during a typical growing season (April–September, 2007). The wavelet transform method (a data-driven approach) was used to analyse the meteorological data from the three selected stations. Such analyses are important to understand the regional differences in ET₀, ET𝛼 and ETᵣ information which is useful for irrigation water management.

The results of this study show significant periodicities in the estimated daily ET₀, ET𝛼 and ETᵣ over the entire growing season. Further, the results show that there are close relationships of ET₀ with ET𝛼 and ETᵣ. The timing of the largest ET𝛼 is 1 month earlier than those of the ET₀ and ETᵣ. In the eastern area, the wave period is 40 days, which is 10 days shorter than that in the western area. The characteristics of the ETᵣ wave are closer to those of the ET₀ wave than those of the ET𝛼 wave. This is an indication that there is a strong influence of ETᵣ on ET₀. This may be attributed to the relatively intense solar radiation in the western area during early April. However, the ETᵣ wave is strongly affected by wind. In the central area, the wave period of ET𝛼 is 20 days shorter than those of ET₀ and ETᵣ. Further, for both ET₀ and ET𝛼 within a 10-day period, there are some sporadic small-scale fluctuations, which can be attributed to the short duration rainfall events. For both the western and the central areas, there are two abrupt changes, two transitions and the wave periods for ET₀, ET𝛼 and ETᵣ are all approximately 50 days. However,
for the ET₀ and ETₐ in the eastern area, there are three abrupt changes and three transitions. For the ET₀ in the eastern area, the wave period is approximately 25 days.

The findings from this study can be used to improve the ET₀ estimation at both regional and local scales. They can also be used as a technical support for irrigation water management decisions and scheduling. This study has shown that using the wavelet transformation method to analyse ET₀ enables a more rational understanding of the regional differences in ET₀, ETₐ and ET₉.

While this study has used the wavelet transform method to carry out a comprehensive evaluation of ET₀, ETₐ and ET₉, so as to rationally understand the evapotranspiration characteristics and its variation, there are limitations. First, the modelled results are only based on one meteorological station in each sub-region. Second, only data of a typical year (2007) have been used. Moreover, the results may be limited due to the fundamental assumptions and approaches used in the model. These shortcomings can be overcome by comparing the results of this study with those from other meteorological stations in other regions with similar soil, vegetation, topography, meteorology and climate conditions. Therefore, while further exploration of this approach is needed, this study has provided useful experience and knowledge for the investigation of crop evapotranspiration.

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REFERENCES


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