Power law distribution characteristics of daily streamflow in the Yellow River Basin under a changing environment
Xiu-Jie Wang, Pei-Xian Yuan, Xi-Min Yuan, Fu-Chang Tian and Xing-tong Chen

ABSTRACT
Natural streamflow series usually exhibit a power law tail with a magnitude–frequency relationship; however, it is not known how the power law distribution of daily streamflow varies under the influence of climate change and human activities. Based on the annual streamflow series of Toudaoguai, Longmen, Sanmenxia, Huayuankou, Gaocun, and Lijin stations located in the Yellow River in China, the Mann–Kendall and Mann–Whitney–Pettitt methods were used to identify change-points. Thereafter, the power law distributions of the daily streamflow series before and after abrupt changes were studied based on the two-parameter power law distribution method. The results reveal that, firstly, under the influence of human activities, abrupt changes have taken place in the streamflow series of the Yellow River, mainly in the storage year of large reservoirs. Secondly, the daily streamflow series at four out of six stations obey a power law distribution, however, with short tails. The power law characteristics of the series before the abrupt change are essentially consistent with the entire streamflow series; however, these characteristics gradually disappear after abrupt changes. Thirdly, human activities are the main factors underlying the variations in the power law distribution of daily streamflow in different periods. This study provides a way for the study of streamflow changes in the Yellow River and may also offer a scientific basis for water resources development and ecological restoration.

Key words | abrupt change, climate change, daily mean streamflow, human activities, power law distribution, Yellow River

INTRODUCTION
Numerous complex natural hazards, such as earthquakes (Corral 2005), solar flares (Lu & Hamilton 1991), landslides (Gomez et al. 2002), and forest fires (Lehsten et al. 2014), satisfy power law frequency-size statistics for the effective approximation of medium- and large-sized events. It is thus no surprise that power law distribution is also used extensively in hydrological applications (Malamud & Turcotte 2006; Peters & Christensen 2006; Segura et al. 2013). Regarding streamflow, Malamud & Turcotte (2006) demonstrated that the cumulative probability distribution of floods is in good agreement with the power law correlation. The fat-tailed power law distribution will consistently provide higher flood frequency estimates than the much thinner-tailed log Pearson type III and other thin-tailed probability distributions. Godsey et al. (2010) found that catchment travel times also typically exhibit an approximate power law distribution. Segura et al. (2013) demonstrated that the frequency distribution of daily flows is better described by power law functions than by the lognormal distribution. Therefore, the power law distribution

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may be a strong candidate for describing the distribution of daily flows (Segura & Pitlick 2010).

It is important to understand the impact of climate change and human activities on the streamflow time series and the driven mechanisms (Li et al. 2015). Distributional analysis of a river discharge time series is an important task in various areas of hydrological engineering, including the optimal design of water storage and drainage networks, management of extreme events, risk assessment for water supply, and environmental flow management, among many others (Bowers et al. 2012). From the geomorphological perspective, predictions of the frequency of intermediate to high flows are of special interest since these flows transport the majority of the sediment load and determine the morphology of the river channel (Wolman & Miller 1960; Emmett & Wolman 2010). Therefore, the power law distribution of the streamflow series is important for understanding the evolutionary trends of streamflow and making decisions regarding water management. However, few studies have been conducted in this manner on the characteristics of runoff in the Yellow River. Considering the complex geographical environment and increasing human activities in the Yellow River, this study assumes the power law distribution and examines parameter variations.

The specific aims of this study are to identify abrupt changes in daily mean streamflow of the Yellow River, to ascertain the power law variation characteristics of the daily mean streamflow in different periods, and to analyze the causes of the changes in runoff characteristics from the viewpoint of the effects of climate change and human activities.

MATERIALS AND METHODS

Study area

As the second longest river in China, the Yellow River has a total length of approximately 5,464 km and a drainage area of roughly 752,443 km². The river is divided into the upper, middle, and lower reaches based on distinctive geomorphological and climatic conditions. From the river source to Toudaoguai (its upper reaches), it is 3,472 km long with an area of 38.6 × 10⁴ km². The streamflow in the upper reaches accounts for approximately 61% of the entire basin. The upper reaches are characterized by an arid climate, with an average annual precipitation of 396 mm (Li et al. 2015). From Toudaoguai to Huayuankou (the middle reaches), it has a length of 1,206 km with an area of 362,138 km². The middle reaches are characterized by semi-arid and arid climates, with an average annual precipitation of 516 mm. This section of the Yellow River is located mainly in the Loess Plateau, where large numbers of tributaries converge into the river. The section from Toudaoguai to Longmen is the area with the most severe soil erosion in China (Peng et al. 2010); consequently, a series of water–soil conservation measures were put into practice in the late 1950s. From Huayuankou to Lijin, the Yellow River enters its lower reaches in the area known as the North China Plain. This section is characterized by a semi-humid climate, with an average annual precipitation of 648 mm. The lower reaches have an alluvial channel, which is highly unstable and liable to flood (Xu 2002). As a result, many levees were constructed on both sides, with a total length of over 1,400 km. In recent decades, several hydroelectric plants have been constructed in the Yellow River, of which Longyangxia, Liujiaxia, and Xiaolangdi have had a dominant impact on water regulation. Figure 1 illustrates the study area.

Data sources

Hydrological stations at Toudaoguai, Longmen, Sanmenxia, Huayuankou, Gaocun, and Lijin were selected for this study, which represent different climatic regions and hydrological conditions of the Yellow River. Toudaoguai station controls the wide river valley in the upper reaches and is located in the transitional zone where the channel changes from meandering to straight. Huayuankou station controls approximately 97% of the total Yellow River Basin (Tang et al. 2015). Lijin station is the final hydrological station on the Yellow River and controls the flow of the river into the sea. Longmen and Sanmenxia stations are located between Toudaoguai and Huayuankou stations, and Gaocun station is located between Huayuankou and Lijin stations (Figure 1).

The data for daily mean streamflow were derived from the Yellow River Conservancy Commission and are available for Toudaoguai, Longmen, Sanmenxia, Huayuankou,
Gaocun, and Lijin stations. The annual streamflows corresponding to the years mentioned above were calculated by the arithmetic average of daily mean streamflow, with others obtained from the Yellow River Water Resources Bulletin. All data were measured and quality controlled according to the Chinese national standard criteria. Detailed information regarding the streamflow series is listed in Table 1.

### Power law distribution analysis

#### Definitions

A continuous power law distribution is described by the probability density $p(x)$, such that:

$$p(x) \propto x^{-\alpha} \quad (1)$$

where $x$ represents the observed data and $\alpha$ is the scaling exponent. In practice, the variable $x$ exhibits power law behavior only above a certain lower bound $x_{\min}$; that is, the tail of the distribution obeys a power law relationship (Clauset et al. 2009).

In general, owing to finite sample sizes, the complementary cumulative distribution function (CCDF) tends to be more robust than the probability density function, particularly for the tail of the distribution (Clauset et al. 2009). The CCDF is expressed as follows:

$$P(x) = \Pr(X \geq x) = \int_{x}^{\infty} p(x')d(x') = \left(\frac{x}{x_{\min}}\right)^{-\alpha+1} \quad (2)$$

---

**Table 1 | Data information at six stations on the Yellow River**

<table>
<thead>
<tr>
<th>Station</th>
<th>Area (km$^2$)</th>
<th>Series length</th>
<th>Time interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huayuankou</td>
<td>730,000</td>
<td>1957–2012 Daily</td>
<td>1957–2016 Yearly</td>
</tr>
</tbody>
</table>

---

**Figure 1 | Study sites in the Yellow River Basin. Study stations and major reservoirs on the Yellow River are marked in the figure.**
Estimating parameters

The logarithm of Equation (1), namely \( \ln p(x) = -\alpha \ln x + C \), where \( C \) is the standardized constant, can be used as a basis for judging whether the random variable satisfies the power law distribution. Clauset et al. (2009) indicated that standard methods such as least-squares fitting are known to produce systematically biased estimates of parameters for power law distributions and should not be used under most circumstances.

Therefore, the maximum likelihood method and Kolmogorov–Smirnov (KS) statistic are used to estimate the parameters \( x_{\min} \) and \( \alpha \) of the power law distribution according to the steps proposed by Clauset et al. (2009).

Testing power law hypothesis

Based on the parameter estimation, the goodness-of-fit test is required to confirm whether the data are effectively fitted by the power law distribution. The hypothesis test method proposed by Clauset et al. (2009) is used in this paper. The method calculates the \( p \)-value as the fraction of KS for the synthetic dataset whose value exceeds KS for the empirical data. If \( p \geq 0.05 \), it is considered that the power law model is effectively fitted with the empirical data.

Abrupt change analysis

Owing to the spatial and temporal variability of a river hydrological series, the overall distribution of data samples before and after a change in variation trends often exhibits a significant difference. Change-point detection of a hydrological series is one statistical method for studying the response of a hydrological series to climate change and human activities. In the following analysis, the series change-points are determined by the abrupt change analysis methods, and the series is divided into series before and after abrupt changes in order to analyze the variations in the power law distribution further. We define the original series change-points as the first-level change-points, while those of the series before and after the first-level change-points are defined as second-level change-points.

Owing to the unclear physical interpretation and uncertainties involved in different methods, the application of abrupt change theory is limited. Zhang & Song (2015) argued that it is difficult to interpret certain abrupt changes as a result of unclear physical mechanisms. At times, the inappropriate use of detection methods leads to erroneous conclusions. Therefore, when detecting change-points, it is preferable to use multiple methods for comparative analysis (Gautam et al. 2010). This paper uses the non-parametric Mann–Kendall (M–K) and Mann–Whitney–Pettitt (MWP) methods, which are described as follows.

M–K method

The M–K test does not require samples to follow a certain distribution and is not disturbed by several abnormal values (Kendall 1958; Mann 1945). For a continuous time series \( x_1, x_2, \ldots, x_n \), a rank series is constructed as follows (Chiyuan et al. 2010):

\[
S_k = \sum_{i=1}^{k} r_i \quad (k = 2, 3, \ldots, n) \tag{3}
\]

\[
r_i = \begin{cases} 
+1 & x_i > x_j \\
0 & \text{else} \\
\end{cases} \quad (j = 1, 2, \ldots, i) \tag{4}
\]

On the assumption that the time series is random and independent, the statistics are calculated as follows:

\[
UF_k = \frac{S_k - E(S_k)}{\sqrt{\text{Var}(S_k)}} \quad (k = 1, 2, \ldots, n) \tag{5}
\]

where \( UF_k \) is the standard normal distribution, which is the statistical series calculated for the series \( (x_1, x_2, \ldots, x_n) \); \( UF_1 = 0 \); and \( E(S_k) \) and \( \text{Var}(S_k) \) are the mean and variance of \( S_k \), respectively, calculated as follows:

\[
UF_1 = 0 \tag{6}
\]

\[
E(S_k) = \frac{n(n + 1)}{4} \tag{7}
\]

\[
\text{Var}(S_k) = \frac{n(n - 1)(2n + 5)}{72} \tag{8}
\]

By following the same procedure, \( UB_k \) is similarly obtained for the inverse series \( (x_{n-1}, x_{n-2}, \ldots, x_1) \). The
curves of $UF_k$ and $UB_k$ are, respectively, plotted. If the two curves cross one another and the intersection is in between the confidence lines ($U_o$) with a confidence level $\alpha$, the corresponding intersection time is that when the abrupt change occurred.

**MWP method**

The essence of the MWP method is to test whether two samples belong to the same population (Pettitt 1979). A time series $x_1, x_2, \ldots, x_N$ with a length $N$ is considered. Assuming that $t$ is the time of the possible change-point, the series can be divided into the two samples $x_1, \ldots, x_t$ and $x_{t+1}, \ldots, x_N$. For a continuous series, the statistic $U_t$ is based on Mann–Whitney and calculated in the following manner (Pettitt 2007):

$$U_t = U_{t-1} + \sum_{j=1}^{N} \text{sgn}(x_i - x_j) \quad (t = 2, \ldots, N) \quad (9)$$

$$\text{sgn}(x_i - x_j) = \begin{cases} 0 & x_i - x_j > 0 \\ 0 & x_i - x_j = 0 \\ -1 & x_i - x_j < 0 \end{cases} \quad (10)$$

The estimated significant probability $P_0$ for a change-point is expressed as follows:

$$P_0 = 2 \exp \left[ -\frac{6K_t^2}{N^3 + N^2} \right] \quad (11)$$

$$K_t = \max_{1 \leq C \leq N} |U_t| \quad (12)$$

If $P_0 \leq 0.5$, $t_0$ becomes the statistically significant change-point.

**RESULTS AND DISCUSSION**

**Power law distribution characteristics of long daily mean streamflow series**

In this section, we study the power law distribution characteristics of the daily mean streamflow series at Toudaoguai, Longmen, Sanmenxia, Huayuankou, Gaocun, and Lijin stations on the Yellow River. The characteristics are based on the CCDF curves in the double logarithmic coordinate and its corresponding power law fitting parameters and the hypothesis testing index. The CCDF curves of each series in the double logarithmic coordinate are obtained from the frequency statistics of daily mean streamflow data, as illustrated in Figure 2. The power law fitting parameters such as $x_{\text{min}}$ and $a$ and the hypothesis testing parameter $p$ are calculated according to the power law distribution method described above, the results of which are summarized in Table 2, where $n_{\text{tail}}$ is used to quantify the tail length that confirms to the power law distribution.

As observed in Figure 2 and Table 2, the CCDF curves and corresponding parameters signify a reasonable power law frequency–magnitude relationship for streamflows above a certain lower bound ($p \geq 0.05$), with the exception of Longmen and Sanmenxia stations. That is, the streamflow series at four out of six stations obey the power law tail distribution, but their tail lengths are all short. It should be noted that the raw river flow data may include variability resulting from both natural and anthropogenic sources (Poff et al. 1997). In recent years, climate change and human activities, in particular, have caused great disturbances to natural runoff in the Yellow River, thereby changing the runoff characteristics. Therefore, we need to explore further the power law distribution and its regularity over the series before and after abrupt changes. As for Longmen and Sanmenxia stations, the loss of the power law tail should be attributed to the ecological damage caused by the severe soil erosion in the Loess Plateau. Water–soil erosion in watersheds depends on climatic and physico-geographical factors (Abdellah et al. 2018). The combinations of wind-deposited loess soils, sparse vegetation, intensive rainfall, and a long agricultural history have resulted in a heavily dissected landscape and severe soil erosion in the Loess Plateau (Zhang et al. 2008). With poor soil–water conservation ability in the Loess Plateau, rainfall cannot be stored in soil, rushing down and causing the loss of water and soil. Consequently, the flow concentration time is shortened and peak discharge is increased, leading to sharp changes in high and low runoff and aggravating runoff instability. The severe soil erosion in the middle reaches caused the deterioration of the ecological environment and destroyed
the self-organized critical state of the ecosystem. Therefore, runoff in the middle reaches of the Yellow River lost the natural-order power law tail that characterizes its self-organized critical state. Meanwhile, we have tried to fit the daily streamflow data for Longmen and Sanmenxia stations using another two possible types of distributions, lognormal and exponential. However, the results show that neither lognormal nor exponential models fit the data satisfactorily, because $P < 0.05$. Furthermore, we can observe that the power law tail of Lijin is shorter in Figure 2. This can be attributed to the special geographical conditions in the lower reaches, which are confined by man-made levees on

Figure 2 | CCDF curves and their power law fits for long daily streamflow series of six stations in a log-log plot. The blue curves are the overall CCDF curves; the red lines indicate the best-fitting power law tails. Please refer to the online version of this paper to see this figure in color: http://dx.doi.org/10.2166/wcc.2019.303.
both sides. The perched riverbed is generally 3–7 m above the surrounding floodplain outside the levees and more than 10 m in certain sections. Hence, the lower reaches are also referred to as the ‘secondary hanged river’. Since 1972, there has been a recurring period of no flow in the lower reaches. This dry period increased rapidly to 226 days in 1997, as recorded at Lijin (Peng et al. 2010). The small-flow frequency increased significantly, therefore, leading to a sharp decrease in the power law tail length.

**Abrupt change analysis**

The M–K and MWP methods are used to calculate the change-points of the streamline series, according to the annual streamflow data measured at Toudaoguai, Longmen, Sanmenxia, Huayuankou, Gaocun, and Lijin stations. Taking Toudaoguai, Huayuankou, and Lijin stations as an example, the M–K and MWP test results are illustrated in Figure 3, with a significance level of $\alpha = 0.05$. The intersections of the UF and UB curves, which mark the change-points at Toudaoguai and Huayuankou stations, are in the 95% confidence interval. The MWP test result satisfies the condition of $P_0 \leq 0.5$; therefore, the point corresponding to the maximum value of $U_i$ is the change-point. Both test results indicate 1986 and 1985 as the change-points of Toudaoguai and Huayuankou, respectively, which is consistent with the findings of Chinese scholars (Liu et al. 2012; Li et al. 2014; Ran et al. 2014). The M–K test at Lijin station fails to identify the change-point, because the intersection of the UF and UB curves exceeds the 95% confidence interval. The MWP test result for Lijin station is 1985. The final first-level change-points for the six stations are displayed in Table 3.

![Table 2](image)

**Table 2** | Basic parameters of power law fitting and testing for long daily mean streamflow series

<table>
<thead>
<tr>
<th>Station</th>
<th>Series length</th>
<th>$x_{\text{min}}$ (m$^3$/s)</th>
<th>$n_{\text{fail}}$</th>
<th>$\alpha$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toudaoguai</td>
<td>1954–2006</td>
<td>2,840 (1,063)</td>
<td>277</td>
<td>6.71 (1.94)</td>
<td>0.24</td>
</tr>
<tr>
<td>Longmen</td>
<td>1956–2005</td>
<td>2,840 (1,079)</td>
<td>517</td>
<td>5.43 (1.43)</td>
<td>0.00</td>
</tr>
<tr>
<td>Sanmenxia</td>
<td>1958–2005</td>
<td>3,920 (1,598)</td>
<td>452</td>
<td>6.88 (2.20)</td>
<td>0.02</td>
</tr>
<tr>
<td>Huayuankou</td>
<td>1957–2012</td>
<td>4,820 (1909)</td>
<td>358</td>
<td>6.24 (1.74)</td>
<td>0.35</td>
</tr>
<tr>
<td>Gaocun</td>
<td>1951–2008</td>
<td>5,160 (396)</td>
<td>298</td>
<td>6.33 (0.45)</td>
<td>0.74</td>
</tr>
<tr>
<td>Lijin</td>
<td>1950–2014</td>
<td>6,610 (1584)</td>
<td>103</td>
<td>11.06 (3.29)</td>
<td>0.11</td>
</tr>
</tbody>
</table>

*Note: Estimation errors for the estimated parameters are denoted in parentheses.*

After obtaining the first-level change-points, further analysis is performed to detect the second-level change-points in the periods before and after the first-level change-points. As the M–K test is more sensitive to the series length, the MWP test is used here. The specific method involves dividing the original series into two periods according to the first-level change-points; the second-level change-points are then detected for the two periods. The results are illustrated in Figure 4. Given a significance level of $\alpha = 0.05$, we can confirm whether the change-point is significant. The final results for all stations are displayed in Table 3.

From Table 3, with a significance level of 0.05, we can observe that in the series before the first-level change-points, the change-points at the six stations occurred in 1968. After the first-level change-points, the change-points of Longmen, Sanmenxia, Huayuankou, and Gaocun stations occurred in 1994. The change-points of Toudaoguai and Lijin stations appeared in 1995 and 1994, respectively, but these do not pass the significance test ($P_0 > 0.5$), indicating that no significant abrupt changes occurred in the series. Thus, it can be concluded that the second-level change-points of Toudaoguai and Lijin stations are 1968, while those of Longmen, Sanmenxia, Huayuankou, and Gaocun stations are 1968 and 1994.

The runoff changes can be related to both natural and man-made changes. Precipitation and evapotranspiration (as reflected by the temperature change) are the most obvious natural changes to the environment. The effects of human activities are more varied and may have significantly greater short-term and lasting impacts. The reasons for these changes are analyzed below, from the perspectives of both climate change and human activities.

**Impact of climate change**

*Impact of temperature.* Temperature change is a global environmental issue and controls evapotranspiration. During the past 50 years, the temperature of the Yellow River has increased significantly (Yuan et al. 2016). However, Zhang et al. (2008) found that no statistically significant change-points were identified in the potential evapotranspiration time series in the Yellow River. This implies that temperature does affect runoff, but owing to
its periodicity and perdurability, it is not sufficient to cause abrupt runoff changes in this catchment.

**Impact of precipitation.** Various studies have demonstrated that runoff is most sensitive to precipitation changes. Precipitation changes make a larger fractional contribution to streamflow changes than temperature changes, as the major driver for the spatial and temporal patterns of water resources across the Yellow River (Zhang et al. 2017). In order to study the effects of precipitation on the abrupt changes in runoff further, the MWP methods described in the section ‘MWP method’ are used to calculate the

![Figure 3](http://iwaponline.com/jwcc/article-pdf/11/4/1603/830108/jwc0111603.pdf)
change-points of the annual precipitation series, with a significance level of $\alpha = 0.05$. The final results are displayed in Table 4. It can be observed that no significant change-points exist in the entire Yellow River Basin, the upper and middle reaches. The abrupt changes in precipitation occurred mainly in the lower reaches in 1964. Therefore, the precipitation in the upper and middle reaches is not sufficient to cause abrupt runoff changes, while the effect of precipitation on the abrupt changes in the lower reaches needs to be studied further, as per the following section.

### Impact of human activities

**Impact of water diversion consumption.** Annual water consumption in the Yellow River began to be reported in 1998 (Figure 5). The water inflow conditions of the Yellow River
have been significantly modified by irrigation measures. By the 1980s and 1990s, water diversion consumption accounted for approximately 51.7–62.2% of the annual runoff of the Yellow River, resulting in a sharp decrease in the annual runoff. The total water consumed within the river basin now exceeds the streamflow that is currently observed to enter the Bo Sea (Zhao et al. 2014b). As illustrated in Figure 5, an increasing trend in annual water consumption can be qualitatively expected for the period before the abrupt changes. This large amount of water diversion will inevitably cause runoff variations. However, considering the rapid increase in population since the 1950s, as well as the rapid increase in economic activity in the Yellow River since the 1980s, it is difficult to quantify the impact on runoff, owing to the lack of available data.

**Impact of water–soil conservation measures.** Owing to the severe soil erosion in the middle reaches, a series of water–soil conservation practices in the Loess Plateau were put into practice in the late 1950s, which have been notably effective since the 1980s. These consist of building terraces and check dams and changing land cover through afforestation and grassing. Table 5 shows statistics of water–soil conservation measures in the middle reaches of the Yellow River Basin over the past six decades. The area protected by soil–water conservation measures doubled or tripled from the 1980s, reaching approximately 4.99 million hm², which is 18.1% of the area between Toudaoguai and Tongguan stations in the middle reaches, as illustrated in Figure 1. Associated with a large number of soil–water conservation measure implementations, the land use and cover exhibited significant changes, which were responsible for more than 70% of the streamflow reduction (Zhao et al. 2014b). It appears that the detected change-points in the annual streamflow of 1985 in the middle reaches were predominantly related to the soil conservation measures.

**Impact of major reservoirs.** The regulation of streamflow through dams and reservoirs is the human activity that influences streamflow regimes most directly. There are 24 particularly large reservoirs in the Yellow River, three of which are along the mainstream and the most influential: the Liujiaxia, Longyangxia, and Xiaolangdi reservoirs. The Liujiaxia Reservoir in the upper reaches was completed in 1968, with a total storage capacity of $5.7 \times 10^9$ m³. The Longyangxia Reservoir, the largest in the upper reaches, was constructed in 1986 with a total storage capacity of $27.6 \times 10^9$ m³ and operated jointly with the Liujiaxia Reservoir to regulate water discharge by storing water during the flood season and discharging it during the non-flood seasons, which has significant effects on the streamflow regime of downstream stations. The Xiaolangdi Reservoir, a major hydropower for flood control, agricultural irrigation, and sediment deposition in the middle and lower Yellow River, significantly changed the natural flow regimes in the downstream river reaches following its construction in 1994. It can be observed that the significant change-points in streamflow at mainstream stations in the years 1968, 1985, and 1994 coincided well with the operation of the three reservoirs. As illustrated in Figure 6, the annual storage variables of Longyangxia and Xiaolangdi reservoirs changed.

<table>
<thead>
<tr>
<th>Water–soil conservation measures</th>
<th>Area controlled by various measures during different periods (103 km²)</th>
<th>1959</th>
<th>1969</th>
<th>1979</th>
<th>1989</th>
<th>1996</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terrace</td>
<td></td>
<td>0.53</td>
<td>2.41</td>
<td>5.62</td>
<td>10.06</td>
<td>14.41</td>
<td>28.54</td>
</tr>
<tr>
<td>Afforestation</td>
<td></td>
<td>1.97</td>
<td>5.21</td>
<td>13.56</td>
<td>33.31</td>
<td>44.40</td>
<td>58.61</td>
</tr>
<tr>
<td>Grassing</td>
<td></td>
<td>0.42</td>
<td>0.66</td>
<td>1.57</td>
<td>5.51</td>
<td>6.37</td>
<td>14.07</td>
</tr>
<tr>
<td>Check dam</td>
<td></td>
<td>0.036</td>
<td>0.185</td>
<td>0.467</td>
<td>1.009</td>
<td>1.224</td>
<td>1.31</td>
</tr>
<tr>
<td>In Total</td>
<td></td>
<td>2.96</td>
<td>8.47</td>
<td>21.22</td>
<td>49.89</td>
<td>66.40</td>
<td>102.53</td>
</tr>
</tbody>
</table>

**Figure 6.** Annual storage variable of the three major reservoirs in the Yellow River Basin from the start of water storage.
significantly. At Toudaoguai station, the runoff during the flood seasons accounted for 63% of the annual runoff before 1968, decreased to 54% during the single reservoir operation at Liujiaxia, and decreased to 38% when the Liujiaxia and Longyangxia reservoirs were jointly operated (Ran et al. 2010). We can conclude that the abrupt change at Toudaoguai station in 1986 was predominantly related to the operation of Longyangxia Reservoir, while the abrupt change in 1994 at other stations was predominantly related to the operation of the Xiaolangdi Reservoir. However, the abrupt changes in 1985 in the middle and lower reaches may be owing to the combined effects of the above factors. Considering the decreased changes in the water storage of the Liujiaxia Reservoir, its contributions to the abrupt change in 1968 should be studied further.

**Power law distribution characteristics of short daily mean streamflow series before and after abrupt changes**

According to the abrupt change analysis results, there are obvious first-level and second-level change-points in the streamflow trends of the Yellow River. Human activities have a significant influence on runoff regime and change the natural runoff characteristics. With an in-depth understanding of the mechanisms of runoff generation and confluence, it is now generally recognized that different runoff distributions represent essentially different physical processes. In order to ascertain the power law variation characteristics of runoff, the daily mean streamflow series at each station were divided into series before and after the abrupt change, according to theoretical change-points. The CCDF curves (Figure 7) and corresponding parameters (Table 6) for each series were then obtained to analyze the variations and causes further. From Figure 7 and Table 6, it can be observed that the series before the first-level change-points at all stations, except Longmen and Sanmenxia, satisfy the power law relationship for the streamflow above a certain lower bound. Moreover, the lower bound \( x_{min} \), scale index \( \alpha \), and power law tail length \( n_{tail} \) are basically consistent with the original series. After the first-level change-point, the lower bound value and scale index of each station are significantly reduced, and the tail length is greatly increased. For Toudaoguai and Longmen stations, the power law distribution parameters of the series before 1968 are basically consistent with those of the series before the first-level change-points; furthermore, the lower bound value and scale index of the series after 1968 are significantly reduced. However, for Sanmenxia, Huayuankou, Gaocun, and Lijin stations, the series before and after 1968 do not follow such regularity and exhibit a similar lower bound value and a scale index, indicating that the changes in 1968 at stations below Sanmenxia are not significant. That is, the impact range of the Liujiaxia Reservoir is limited. Considering the insignificance of the changes in 1968 below Sanmenxia, it can be inferred that the detected abrupt precipitation change in the lower reaches in 1964 is not sufficient to cause abrupt change. This implies that precipitation does contribute to the decrease in runoff, but it is not sufficient to cause abrupt changes in the whole Yellow River. With regard to Longmen, the streamflow series before and after 1994 also exhibit a similar lower bound value and scale index. Thus, the change-point of 1994 at Longmen station is not significant, which implies that the Xiaolangdi Reservoir has little impact on the reaches above Sanmenxia. The specific impact ranges of these reservoirs require further study according to the operation process. Furthermore, for Longmen station, it can be observed that the series after 1985 and 1994 are in good agreement with the power law tails. It should be attributed to the ecological effects of soil–water conservation measures in the middle reaches. With the implementation of the conversion of cropland to forest project and the large-scale ecological restoration of the Loess Plateau, the soil erosion control of the Loess Plateau has achieved remarkable results and the ecology has gradually recovered. As illustrated in Table 5, the area of soil–water conservation measures multiplied from the 1980s. Thereby, the series after 1985 restores the natural-order power law tail, mainly reflected after the 1990s. As for Sanmenxia, it does not restore the natural-order power law tail, because the soil–water conservation measures are concentrated in Toudaoguai to the Longmen interval, and the ecological restoration has a long effect time.

According to the above analysis, it can be observed that variations in the streamflow trends change the power law characteristics of the natural streamflow. The streamflow series of the six stations before 1968 can be regarded as...
baseline periods. The power law parameters of the original and baseline series are basically consistent, indicating that the streamflow characteristics were altered after abrupt changes, and the probability of extreme flows no longer follows the law of natural-order power law distribution. Following the abrupt change, the lower bound value of the series is significantly reduced, but the series does not pass the hypothesis test, indicating that only the natural series exhibits the power law distribution characteristics, and the streamflow power law characteristics gradually disappear after the abrupt change. The disappearance of the power law tail suggests changes in the runoff characteristics.

Figure 7 | CCDF curves of daily streamflow series before and after abrupt changes in the log-log plot. The black lines represent the best-fitting power law tails.
which usually leads to a loss of system integrity and hydrographic diversity, such as changes in the power law distribution of vegetation (Sandhu et al. 2016), uniformity of wetland areas (Van & Basu 2016), and loss of the wetland system of self-organization patterns (Foti et al. 2012). All of these changes will feed back into the runoff cycle through the runoff generation and the confluence process in the basin.

Under the influence of human activities, such as water-soil conservation and water diversion, particularly the operations of Liujiaxia and Longyangxia reservoirs in the upper reaches and the Xiaolangdi Reservoir in the middle reaches, the high pulse duration is reduced, the flow magnitude of the Yellow River is substantially smaller, and the high flows are cut as well as postponed temporally. The storage of flood flows in the reservoirs is clearly linked to the reduction in high flows, but the reduction in the median flow also reflects water extraction from the reservoirs. Bowers et al. (2012) found that, for a river flow signal, a noise signal exceeding approximately 2% of the measured process is likely to cause the KS statistic to reject the power law hypothesis. A reservoir operation is equivalent to injecting a Gaussian flow into the downstream; therefore, it will inevitably lead to changes in the runoff power law distribution characteristics and even reject the power law distribution assumption.

The generation mechanism of power law distribution can be explained by self-organized critical phenomena.

### Table 6 | Basic parameters of power law fitting and testing for short daily mean streamflow series before and after abrupt changes

<table>
<thead>
<tr>
<th>Station</th>
<th>Change-point</th>
<th>Series length</th>
<th>$x_{\text{min}}$ ($m^3/s$)</th>
<th>$n_{\text{tail}}$</th>
<th>$\alpha$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toudaoguai</td>
<td>1986</td>
<td>1954–1986</td>
<td>2,840(399)</td>
<td>271</td>
<td>6.61(0.80)</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>1987–2006</td>
<td></td>
<td>758(44)</td>
<td>1,170</td>
<td>4.14(0.11)</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1954–1968</td>
<td>2,310(134)</td>
<td>303</td>
<td>6.01(0.39)</td>
<td>0.39</td>
</tr>
<tr>
<td>Longmen</td>
<td>1956–1985</td>
<td></td>
<td>2,840(661)</td>
<td>493</td>
<td>5.41(0.83)</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>1986–2005</td>
<td></td>
<td>846(29)</td>
<td>1,495</td>
<td>3.99(0.08)</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1956–1968</td>
<td>2,480(194)</td>
<td>431</td>
<td>4.80(0.32)</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>1969–1985</td>
<td></td>
<td>640(482)</td>
<td>3,905</td>
<td>2.79(0.83)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1994</td>
<td>1986–1994</td>
<td>830(45)</td>
<td>806</td>
<td>3.75(0.10)</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>1995–2005</td>
<td></td>
<td>823(155)</td>
<td>702</td>
<td>4.33(0.41)</td>
<td>0.14</td>
</tr>
<tr>
<td>Sanmenxia</td>
<td>1958–1985</td>
<td></td>
<td>3,920(405)</td>
<td>433</td>
<td>6.77(0.69)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1986–2005</td>
<td></td>
<td>853(53)</td>
<td>2,040</td>
<td>3.45(0.07)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1958–1968</td>
<td>4,350(799)</td>
<td>164</td>
<td>7.47(1.51)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1969–1985</td>
<td></td>
<td>3,880(1,364)</td>
<td>188</td>
<td>7.21(2.11)</td>
<td>0.00</td>
</tr>
<tr>
<td>Huayuankou</td>
<td>1957–1985</td>
<td></td>
<td>4,710(211)</td>
<td>387</td>
<td>6.19(0.35)</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>1986–2012</td>
<td></td>
<td>761(13)</td>
<td>4,102</td>
<td>3.17(0.04)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1957–1968</td>
<td>4,720(207)</td>
<td>244</td>
<td>6.16(0.53)</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>1969–1985</td>
<td></td>
<td>4,330(1,521)</td>
<td>215</td>
<td>6.13(1.63)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1994</td>
<td>1986–1994</td>
<td>1,010(174)</td>
<td>1106</td>
<td>3.41(0.21)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1995–2012</td>
<td></td>
<td>664(19)</td>
<td>2,911</td>
<td>3.15(0.05)</td>
<td>0.00</td>
</tr>
<tr>
<td>Gaocun</td>
<td>1951–1985</td>
<td></td>
<td>5,000(307)</td>
<td>334</td>
<td>6.20(0.39)</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>1986–2008</td>
<td></td>
<td>794(74)</td>
<td>2,310</td>
<td>3.10(0.25)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1951–1968</td>
<td>4,910(261)</td>
<td>278</td>
<td>6.09(0.41)</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>1969–1985</td>
<td></td>
<td>4,180(1,015)</td>
<td>234</td>
<td>6.18(1.23)</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>1994</td>
<td>1986–1994</td>
<td>928(43)</td>
<td>1,100</td>
<td>3.46(0.09)</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>1995–2008</td>
<td></td>
<td>583(58)</td>
<td>2,108</td>
<td>3.07(0.11)</td>
<td>0.00</td>
</tr>
<tr>
<td>Lijin</td>
<td>1950–1985</td>
<td></td>
<td>6,610(1365)</td>
<td>103</td>
<td>11.06(3.20)</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>1986–2014</td>
<td></td>
<td>2,640(906)</td>
<td>244</td>
<td>6.76(2.32)</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>1950–1968</td>
<td></td>
<td>6,610(1656)</td>
<td>93</td>
<td>10.84(3.44)</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>1969–1985</td>
<td></td>
<td>5,220(876)</td>
<td>64</td>
<td>9.32(1.97)</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Note: Estimation errors for the estimated parameters are denoted in parentheses.
In the late 1980s, Bak et al. (1987) proposed the concept of self-organized criticality, which refers to a complex system with a continuous energy supply, consisting of many basic units and with a nonlinear interaction between the basic system units. The system can spontaneously evolve into a critical state of local dynamic instability and dynamic stability in macro statistics, without the drive of external input of substances and energy. Power law distribution is considered as the expression of the self-organized critical phenomenon (Bak & Weissman 1997). When the self-organized critical mechanism of the system is destroyed by external disturbances, the system elements will no longer exhibit the power law characteristics. Power law distribution is a typical heavy-tailed distribution, and the probability of extreme events is high. It can be concluded from the analysis that the small-flow frequency in the Yellow River is gradually increasing, while the large-flow frequency is gradually decreasing, which changes the runoff distribution characteristics. This can also explain why the power law parameters of the original series are consistent with the baseline series. Power law distribution implies that interactions take place between events occurring at different times in the system. The change in the runoff distribution characteristics mentioned above means that the system has moved from ‘the edge of order and chaos’ with self-organization, maximum complexity, evolution, and innovation into a relatively ‘dead’ stable state, which has a significant influence on the entire ecosystem (Lian et al. 2016). Pandey et al. (1998) attributed the power law behavior of streamflow to the power law behavior of rainfall. The cumulative probability distributions of daily rainfall totals exhibit similar power law behavior to the daily mean streamflow in a natural state. However, the inherent link between the power law distribution characteristics of runoff and rainfall requires further verification. The power law distribution of rainfall and its impact on runoff should, therefore, be studied further.

The power law tail of runoff is of great significance for the integrity and diversity of a river basin circulation system, which may provide decision support for water resources development, ecological restoration, and comprehensive management. It remains debatable whether a power law estimate of the streamflow is preferable to other statistical distributions; however, the power law is certainly a conservative approach. In this study, we could have considered other statistical distributions (such as lognormal (Botter et al. 2007)); however, because we obtained effective results with the two-parameter power law distribution and achieved our research objective, we opted not to use these other distributions, as these methods introduce additional parameters. For the statistical distribution of streamflow, every method offers applicability and limitation, and whether this constitutes a problem for the researcher and the selection is largely dependent on his or her scientific goals.

CONCLUSIONS

The study of power law distribution may provide a new means for understanding the changes in hydrological properties. Based on the investigation of the streamflow series of six hydrological stations in the Yellow River, the following conclusions can be drawn:

(1) Under the influence of human activities, abrupt changes have taken place in the Yellow River streamflow series. The change-points at Toudaoguai station occurred in 1968 and 1986, at Longmen, Sanmenxia, Huayuankou, and Gaocun stations in 1968, 1985, and 1994, and at Lijin station in 1968 and 1985. However, from the above analysis, the changes at Longmen station in 1994 and the changes at stations below Sanmenxia station in 1968 are not significant.

(2) The daily streamflow series at four out of six stations follow a power law tail; however, runoff in the middle reaches lost the natural-order power law tail due to the ecological damage caused by the severe soil erosion in the Loess Plateau. The power law distribution of daily streamflow changes in different periods. The power law characteristics of the series before abrupt changes are basically consistent with the original series. After abrupt changes, the power law parameters change significantly. The stations in the lower reaches do not follow a power law tail, while Longmen station restores the power law tail with the ecological effects of soil-water conservation measures in Toudaoguai to Longmen interval. As Toudaoguai station does follow a power law distribution, this indicates that variations in the upstream
are relatively small, while those in the downstream are relatively large.

(3) Human activities are the main factors underlying the changes in the power law distribution of daily streamflow at different periods. Human activities cause abrupt changes and change the power law characteristics of the natural streamflow. Thereby, the daily streamflow lost the natural-order power law tail after the abrupt change. Considering that the watershed is a combined product of natural and anthropogenic causes, the question of diagnosing and quantifying their relative impacts on streamflow variation offers more practical benefits and requires further investigation.

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