Testing reliability of the spatial Hurst exponent method for detecting a change point

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ABSTRACT

The reliability of using abrupt changes in the spatial Hurst exponent for identifying temporal points of abrupt change in climate dynamics is explored. If a spatio-temporal dynamical system undergoes an abrupt change at a particular time, the time series of spatial Hurst exponent obtained from the data of any variable of the system should also show an abrupt change at that time. As expected, spatial Hurst exponents for each of the two variables of a model spatio-temporal system – a globally coupled map lattice based on the Burgers’ chaotic map – showed abrupt change at the same time that a parameter of the system was changed. This method was applied for the identification of change points in climate dynamics using the NCEP/NCAR data on air temperature, pressure and relative humidity variables. Different abrupt change points in spatial Hurst exponents were detected for the data of these different variables. That suggests, for a dynamical system, change point detected using the two-dimensional detrended fluctuation analysis method on a single variable alone is insufficient to comment about the abrupt change in the system dynamics and should be based on multiple variables of the dynamical system.

Key words: Burgers’ chaotic map, change point detection, globally coupled map lattice, Hurst exponent, two-dimensional detrended fluctuation analysis

HIGHLIGHTS

• In a non-linear dynamical system, the spatio-temporal images of one variable also contain information about the interaction of this variable with other coupled variables of the system.
• When a parameter of the dynamical system changes abruptly, the spatial distribution of all the coupled variables should show an abrupt change simultaneously.
• It is found that the 2D-DFA method successfully detected abrupt change points in each variable of GCML based on Burgers’ chaotic map at the same time as a system parameter was changed.
• The 2D-DFA method detected different abrupt change points in the time series of the spatial Hurst for the NCEP/NCAR data of three different variables.
• This study suggests, for a dynamical system, change point detected using 2D-DFA method on a single variable alone is insufficient to comment about the abrupt change in the system dynamics and should be based on multiple variables of the dynamical system.

1. INTRODUCTION

Abrupt change in the climate refers to the change in the state of the climate that occurs in a short time and persists for a prolonged period. Abrupt climate changes have occurred in the past. These changes can be detected by means of statistical tests that can identify changes in the mean and/or some other properties of the dynamical system’s variables. According to the intergovernmental panel on climate change (IPCC) report: ‘abrupt climate change detection refers to detecting a change in some definite statistical property (e.g. mean, variance and trend) of the climate’. Abrupt climate changes have been identified in the thermohaline circulation (THC), temperature and precipitation patterns (Pitman & Stouffer 2006; Narisma et al. 2007; Cramer et al. 2014). The prolonged Sahel drought and Dust Bowl are a few recent examples of abrupt climate change (Narisma et al. 2007).

The earth’s climate is governed by a very complex non-linear spatio-temporal system. This system may be regarded as made up of several sub-systems. The output from one sub-system becomes the input for another. If one of the sub-systems evolves...
slowly compared to another, the input from the slow sub-system can be treated as a parameter for the fast sub-system. Sudden changes in this parameter can cause abrupt changes in the fast sub-system. For example, the oceans change slowly compared to the terrestrial system. Abrupt surface climate changes identified from paleo records have been attributed to the sudden weakening or collapse of THC.

Gradual changes in a parameter can also cause abrupt climate changes. An abrupt climate change occurs when the climate system is forced across some threshold causing a transition to a new state, at a rate which is faster than its cause (Alley et al. 2003). The internal thresholds within the terrestrial system also allow abrupt changes to occur when driven by external forcing such as gradual changes in solar insolation (Pitman & Stouffer 2006). Such threshold values are a fundamental property of a non-linear system, and with sufficiently large perturbations, irreversible changes can happen (Stocker 1999).

Abrupt change can also occur if a system makes a regime transition from the neighbourhood of one equilibrium state to the neighbourhood of another equilibrium state. In such cases, external triggers for transition are not required (National Research Council Reports 2002). Such changes are determined by the internal dynamics of the system (Pitman & Stouffer 2006). Palmer (1994) introduced the Lorenz model as a conceptual model for such regime changes. The attractor reconstruction method applied to time series of rainfall data was found to be consistent with this conceptual model (Yadav et al. 2005). The existence of a low dimensional attractor suggests that the rainfall data are governed by a low dimensional chaotic dynamical system (Singh et al. 2019).

In the context of earth sciences, change-point detection techniques have been widely used in several studies to detect abrupt changes in temperature and in precipitation (Kothiyari & Singh 1996; Gallagher et al. 2013; Bisai et al. 2014; Chen et al. 2014; Khan et al. 2014; Zarenistanak et al. 2014; Chen et al. 2016). The detection of climate shifts requires high-quality long-term observations. It also requires a deep understanding of how natural and anthropogenic factors influence the climate (Cramer et al. 2014). Any technique for detecting climate shifts should have the capability to capture non-linear interactions between different components. It has been argued (He et al. 2016) that traditional single-dimensional (1-D) abrupt change point analysis methods like those used by Mann & Whitney (1947), Yamamoto et al. (1985) and Li et al. (1996) allow only delayed change point detection. Therefore, abrupt change points detected using these methods lose their significance for real-time applications like disaster preparedness. This leads to the need for a method which can detect an abrupt change as it occurs. On the other hand, the long-term changes in the climatic variables exhibit abrupt shifts and non-linearity (Beaulieu et al. 2012). Therefore, the inherent non-linearity of natural systems requires non-linear techniques, like detrended fluctuation analysis (DFA), in order to reveal intrinsic information about such systems. The DFA method is used to obtain the Hurst exponent. Harold Edwin Hurst had studied the hydrological properties of the Nile basin. In his study, Hurst normalised the adjusted range \( R \) by the sample standard deviation \( \sigma \) to obtain what is now called the rescaled adjusted range statistic. The ratio \( R/\sigma \) increases with some power of time \( R/\sigma \propto n^H \), where \( H \) is the Hurst exponent (Hurst et al. 1965). The Hurst exponent is a useful measure for understanding the properties of a time series without making assumptions about statistical restrictions (Tatli 2015). The Hurst exponent provides a measure for long-term memory and fractality of a time series, and it has applications in fields like meteorology and finance.

The DFA method was developed from a modified root mean square analysis of a random walk, and it has proven to be suitable for analysing a time series with long-range correlation (Peng et al. 1994). DFA is widely being used to detect the long-range correlation in time series obtained from natural and artificial systems (Kantelhardt et al. 2001; Király & Jánosi 2005; Varotsos et al. 2007). In recent years, the DFA method has been generalised from DFA to multifractal DFA (MF DFA) to explore the multifractal nature hidden in the time series. A two-dimensional DFA (2D-DFA) method proposed by Gu & Zhou (2006) has been found to be an accurate method for calculating the spatial Hurst exponent \( H \) of fractal images. We use the term ‘spatial Hurst exponent’ as a shortened expression for the term ‘Hurst index of the surface’ in Gu & Zhou (2006).

The 2D-DFA method has gained a lot of interest in recent times for analysing two-dimensional images. The strength of this method is that it exploits the intrinsic behaviour of the dynamical system directly from the two-dimensional images, which preserves more information than a single time series constructed by averaging the spatio-temporal system. Spatio-temporal images contain information both about evolution in time and about spatial interactions (He et al. 2016).

The 2D-DFA has been applied to analyse changes in the roughness of fracture surfaces, landscapes, clouds and temperature fields (Gu & Zhou 2006). Whereas the traditional change point detection methods suffer from delayed detection of abrupt change in climate, the 2D-DFA method solves this issue by detecting an abrupt change point almost exactly as it occurs
(He et al. 2016; Liu et al. 2017). He et al. (2016) used the 2D-DFA method to detect abrupt changes in the dynamical system from a single variable data of an artificial system and a real-world system.

It is important to understand the difference between the abrupt change detection method of He et al. (2016) and other methods. The other methods detect an abrupt change in the series of a variable averaged over a spatial surface. The method of He et al. (2016) detects an abrupt temporal change in the spatial distribution of a variable. He et al. (2016) have argued that spatio-temporal images contain information about evolution in time as well as spatial interactions. We argue that in a non-linear dynamical system, the spatio-temporal images of one variable also contain information about the interaction of this variable with other coupled variables of the system. When a parameter of the dynamical system changes abruptly, the spatial distribution of all the coupled variables should show an abrupt change simultaneously.

In support of their method, He et al. (2016) successfully detected abrupt change points introduced by changing a parameter in a model coupled map lattice based on the Henon Chaotic Map. This shows that the abrupt change addressed by them belonged to the category of the changed dynamical system due to parameter changes and not changes in a variable of the same dynamical system. We explore the reliability of the 2D-DFA method for identifying this type of abrupt change.

This study is based on the premise that an abrupt change in the dynamical system should lead to an abrupt change in the spatial Hurst exponent, not for just one variable of a non-linear dynamical system but for all the variables at the same time. The reliability of using abrupt changes in the spatial Hurst exponent by the 2D-DFA method for identifying temporal points of abrupt change in climate dynamics is explored. This is done first on the artificial data. We replaced the Henon map used in He et al. (2016) with the Burgers map, which has a more complex relationship between the X and the Y variables than for the Henon map. We expected this to provide a more stringent test for our premise that the 2D-DFA method applied to the data from either variable should show abrupt changes at the same time. We found that the 2D-DFA method successfully detected abrupt change points using either variable of this model system at the same time as a system parameter was changed.

The above hypothesis was tested on the real-world meteorological data by applying the 2D-DFA method for the identification of change points in climate dynamics using the NCEP/NCAR data on air temperature, pressure and relative humidity variables. It is expected that if the dynamical system changes abruptly, data on all the variables of the dynamical system should give a change point at the same time. Different abrupt change points in spatial Hurst exponents were detected for the NCEP/NCAR data of these three meteorological variables.

2. DATA AND METHODS

2.1. Data

Before applying the 2D-DFA method for detecting abrupt dynamical change on real data, it was tested on artificial data generated by a coupled map lattice as a discretised model of a spatio-temporal system. This model is based on the Burgers’ chaotic map (Whitehead & Macdonald 1984; Burgers 1995). The Burgers’ mapping is a discretisation of a pair of coupled differential equations which were used by Burgers to illustrate the relevance of the concept of bifurcation to the study of hydrodynamic flows. These equations are similar to the Lorenz model (Elabbasy et al. 2007) which has been widely used as a conceptual model to study weather and climate (Palmer 1994; Mittal et al. 2015; Singh et al. 2015). Whitehead & Macdonald (1984), in their study, stated that ‘The Burgers map is chaotic in the way that the Lorenz model is, namely the iterates orbit one unstable fixed point until a flip occurs whereupon they orbit the other such point.’

The Burgers map is defined by the following equation:

\[ \dot{X}_{n+1} = aX_n - Y_n^2 \]
\[ \dot{Y}_{n+1} = bY_n + X_nY_n \]

The Burgers map has a more complex relationship between the X and the Y variables than the Henon map, which was previously used in the study by He et al. (2016). Therefore, in the present study, a globally coupled map lattice (GCML) based on the Burgers’ chaotic map was used as a model to test our basic premise that with an abrupt change (here, by introducing sudden change in a parameter) in the spatio-temporal dynamical system, it is expected that the time series of spatial Hurst exponent obtained from the data of any variable of the system should also show an abrupt change at the same time.

The GCML was formed as described in the literature (Vasconcelos et al. 2006). A lattice point is defined by an integer pair \((p, q)\). The variables X and Y at the lattice point \((p, q)\) at time step \(n\) are denoted by \(X^{(p,q)}_n\) and \(Y^{(p,q)}_n\). If these variables evolved
without any coupling, we would have:

\[
X_{n+1}^{(p,q)} = aX_n^{(p,q)} - Y_n^{(p,q)}
\]

\[
Y_{n+1}^{(p,q)} = bY_n^{(p,q)} + X_n^{(p,q)}Y_n^{(p,q)}
\]  

(2)

However, in a coupled map lattice, the evolution at a lattice point is modified by the values of variables at neighbouring points. The lattice point \((p, q)\) has eight nearest neighbours: \((p + 1, q)\), \((p + 1, q + 1)\), \((p, q + 1)\), \((p - 1, q + 1)\), \((p - 1, q)\), \((p - 1, q - 1)\), \((p, q - 1)\) and \((p + 1, q - 1)\). In a coupled map lattice, the value of a variable at a lattice point is reduced by a factor \(\epsilon\) and gets a contribution from each of its nearest neighbours weighted by \(\epsilon/8\), so that:

\[
X_{n+1}^{(p,q)} = (1 - \epsilon)X_n^{(p,q)} + \frac{\epsilon}{8} \left\{ X_{n+1}^{(p+1,q+1)} + \hat{X}_{n+1}^{(p+1,q-1)} + \hat{X}_{n+1}^{(p-1,q+1)} + \hat{X}_{n+1}^{(p-1,q-1)} + \hat{X}_{n+1}^{(p+1,q-1)} + \hat{X}_{n+1}^{(p-1,q+1)} + \hat{X}_{n+1}^{(p+1,q+1)} + \hat{X}_{n+1}^{(p+1,q-1)} \right\}
\]

(3)

with a similar equation for the \(Y\) variable. The parameter \(\epsilon\) denotes the coupling strength.

The contribution from more distant points is weighted as a negative power of the distance, so that:

\[
X_{n+1}^{(p,q)} = (1 - \epsilon)X_n^{(p,q)} + \frac{\epsilon}{\phi(A)} \sum_{r=1}^{N} \left\{ \frac{\gamma^{(p+r,q)}X_{n+1}^{(p+r,q)}}{\phi(A)} + \frac{\gamma^{(p-r,q)}X_{n+1}^{(p-r,q)}}{\phi(A)} + \frac{\gamma^{(p+r,q)}}{\phi(A)} + \frac{\gamma^{(p-r,q)}}{\phi(A)} \right\}
\]

(4)

where \(\phi(A) = 1/8 \sum_{r=1}^{N} r^{-A}\) and \(N = \min(n_x, n_y)/2\), \(n_x = 150\) and \(n_y = 100\) are the number of grid points in \(x\)- and \(y\)-directions. The parameter \(A\) governs the range of the coupling. The larger the value of \(A\), the faster the decay in the contribution from distant points.

The parameters for the coupled map lattice were chosen from the literature, i.e., \(A = 8\), \(\epsilon = 0.8\) (He et al. 2016) and the values \(a = 0.75\) and \(b = 1.75\) (Sprott 2003). Values on the boundary were held constant; \(X_{n+1}^{(p,q)} = X_n^{(p,q)}\) and \(Y_{n+1}^{(p,q)} = Y_n^{(p,q)}\) if \((p, q)\) belongs to the boundary (Vasconcelos et al. 2006; He et al. 2016). At any time step \(n\), the size of the matrices \(X\) and \(Y\) is \(150 \times 100\). The initial values of these matrices were chosen randomly in the range from \(-0.1\) to \(0.1\). After the first 200 transient points, changes in the Hurst exponent \(H\) became stable. Therefore, the first 200 points were discarded.

The 2D-DFA method described below was employed for computing, as a function of time, the spatial Hurst exponents for the 2D-DFA method was introduced originally to investigate the long-range dependence in a DNA sequence (Peng et al. 1994). The DFA was generalised by Gu & Zhou (2006) for exploring long-range correlation properties of a two-dimensional surface. The 2D-DFA method is summarised below:

### 2.2. Methodology

#### 2.2.1. 2D-DFA

DFA is widely employed for estimating long-range correlations of a time series (e.g., meteorological time series, time series in economics and heart rate time series). The DFA method was introduced originally to investigate the long-range dependence in a DNA sequence (Peng et al. 1994). The DFA was generalised by Gu & Zhou (2006) for exploring long-range correlation properties of a two-dimensional surface. The 2D-DFA method is summarised below:

### Step 1. Consider a two-dimensional matrix \(X(i, j)\), representing the value of the variable \(X\) at the lattice point \((i, j)\), where \(i = 1, 2, \ldots, M\) and \(j = 1, 2, \ldots, N\). The matrix is partitioned into \(M \times N\) disjoint square segments of size \(s \times s\), where \(M = [M/s]\) and \(N = [N/s]\). The segments can be denoted by \(X_{i,s}^{(i,j)}\) such that:

\[
X_{i,s}^{(i,j)} = X(l_1 + i, l_2 + j), \text{ for } 1 \leq i \leq s, 1 \leq j \leq s, \text{ where } l_1 = (w - 1)s, l_2 = (w - 1)s
\]

(5)
Step 2. For each segment, the cumulative sum $u_{v,w}(i, j)$ is defined as follows:

$$u_{v,w}(i, j) = \sum_{k_1}^{i} \sum_{k_2}^{j} X_{v,w}(k_1, k_2) \quad 1 \leq i \leq s, 1 \leq j \leq s$$

Step 3. For each segment, we adopt the following polynomial function to approximate the trend of the constructed surface $u_{v,w}$:

$$u'_{v,w}(i, j) = ai + bj + c$$

where $1 \leq i \leq s, 1 \leq j \leq s$ and $a, b$ and $c$ are free parameters to be determined through the least square method. We can then obtain residual matrix, given as follows:

$$E_{v,w}(i, j) = u_{v,w}(i, j) - u'_{v,w}(i, j)$$

The detrended fluctuation function $F(v, w, s)$ of each segment is defined via the sample variance of the residual matrix, given as follows:

$$F^2(v, w, s) = \frac{1}{s} \sum_{i=1}^{s} \sum_{j=1}^{s} [E_{v,w}(i, j)]^2$$

Step 4. The overall detrended fluctuation is calculated by averaging over the entire segment, given as follows:

$$F^2(s) = \frac{1}{M_t N_t} \sum_{v=1}^{M_t} \sum_{w=1}^{N_t} F^2(v, w, s)$$

Step 5. By varying the value of $s$ in the range from 6 to $\min(M, N)/4$, we can determine the scaling relation between detrended fluctuation $F(s)$ and size scale $s$, which is $F(s) \sim s^{2H}$, where $H$ is the Hurst exponent of the surface.

The Hurst exponent $H = 0.5$ indicates that the values of the variable at different lattice points are uncorrelated. $0.5 < H < 1$ indicates that the values are correlated, whereas $0 < H < 0.5$ indicates anti-correlated values.

If there is an abrupt dynamic change in a spatio-temporal system, the time series of spatial Hurst exponents will exhibit a non-stationary change. However, if there is no abrupt dynamic change in the dynamic system, changes in the Hurst exponents will be stationary.

2.2.2. Change point detection in time series of the Hurst exponents $H$

In this study, in order to detect an abrupt change in the time series of 2D spatial Hurst exponents, the moving $t$-test (MTT; Li & Shi 1993; Li et al. 1996) is used. This test detects abrupt change by determining whether there is a significant difference between the average values of two subseries at a significance level of $\alpha = 0.001$, i.e., 99.9%. For a given time series $x_n$, let $x_1$ and $x_2$ be two subseries before and after the datum point. Then the $t$ value is defined by the following equation:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $\bar{x}_1$ and $\bar{x}_2$ are the means, $s_1$ and $s_2$ the standard deviations, and $n_1$ and $n_2$ are the sizes of the two subseries.

Two subseries of the same length ($n_1 = n_2$) before and after the datum point were considered. Thus, the two series lie in a window to the left and right of the datum point $i$, which is at the centre of the window. The $t$ values were calculated using Equation (11), as the window slides forward and the datum point is moved continuously. If $|t_i| > t_\alpha$, where $\alpha$ indicates the significance level and the series had an abrupt change at the datum point $i$. A flow diagram of the methodology used in this study is given in Figure 1.
3. RESULTS AND DISCUSSION

The 2D-DFA method was applied on both $X$ and $Y$ variables of the GCML based on the Burgers’ chaotic map to obtain the spatial Hurst exponent at different time steps $n$. The power-law scaling between the detrended function $F(s)$ obtained from Equation (10) and scale $s$ for the spatial images of GCML at time step $n = 201$ is shown in Figure 2 and the $H$ is equal to 0.98 for both images, which indicates the presence of strong interaction in the coupling model. The mutual coupling interaction between the lattice points is the main cause of such strong correlation. The computed Hurst exponents $H$ as a function of $n$ are shown in Figure 3. It is evident that the Hurst exponents $H$ obtained from the $X$ and the $Y$ variables exhibit similar behaviour, i.e., the mean value of $H$ is independent of $n$, when there is no abrupt change.

We applied two abrupt change scenarios for the GCML. In scenario 1, the coupling strength $\epsilon$ was changed from 0.8 to 0.4 at the time step $n = 2,001$, and from 0.4 to 0.85 at the time step $n = 4,001$. The time series of the spatial Hurst exponents for the two variables are shown in Figure 4. Both the time series exhibit an abrupt change precisely at the time steps $n = 2,001$ and $n = 4,001$.

In scenario 2, the parameter $b$ was changed from 1.75 to 1.80 at the time step $n = 2,001$ and at time step $n = 4,001$, the value of $b$ was restored to its previous value of 1.75, whereas the parameter $a$ was changed from 0.75 to 0.80. Figure 5 shows that for
this scenario also the spatial Hurst exponent time series for either of the two variables exhibit abrupt changes at precisely the same time steps.

Figure 6 shows that for both the scenarios using either of the variables \( X \) or \( Y \), the MTT method correctly detects the time steps at which the system dynamics changes (Table 1). The contour plots of spatial images for \( X \) and \( Y \) variables of GCML.
formed using Burger’s chaotic map at time steps $n = 201, 500, 2,001$ and $4,001$ for scenario 1 are shown in Figures 7 and 8. The figure shows that the interaction between lattice points is found to be strengthened after the transient state. Similar contour images are also found for both variables of GCML in the case of scenario 2 (figure not shown).
We have also tested the abrupt change point detection in the GCML model for different values of coupling range parameter $A$ (figure not shown). The GCML model (artificial data) explicitly introduces impact from distant points. As the parameter 'A' of the model is reduced, the range of lattice points that influence the evolution at any lattice point increases. Thus, feedback could have some impact on the model. We have checked and found that the abrupt change in a parameter is detected identically by both variables even for very small values of $A$.

We next applied the 2D-DFA method to real-world data. The 2D-DFA method was used for the detection of abrupt change points from NCEP air temperature, pressure and relative humidity data. Figure 9 shows daily average air temperature anomaly data in the region (60°S–60°N, 180°W–180°E) for January 1, 1950. The log $F(s)$ vs. log $s$ plot obtained from the daily average air temperature anomaly for January 1, 1950 is shown in Figure 10. The value of $H$ for daily anomaly of air temperature is 0.97 (Figure 10), while the values of $H$ for the same period of daily average anomaly of pressure and relative humidity are 1.01 and 0.85, respectively (figure not shown). The time series of Hurst exponents $H$ calculated using 2D-DFA for the three NCEP variables data are shown in Figure 11. The Hurst exponents of $H > 1.0$ (Figure 11) indicate that

<table>
<thead>
<tr>
<th>Variable</th>
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Figure 7 | Contour plot for the X variable of GCML for scenario 1, when the artificial changes in the coupling strength $\epsilon$ were introduced at time steps $n = 2,001$ and $n = 4,001$. (a) $n = 201$, (b) $n = 500$, (c) $n = 2,001$ and (d) $n = 4,001$. 

Table 1 | Detected abrupt change in the mean value of Hurst exponent ($H$) in NCEP/NCAR variables by using the MTT method
a relatively strong interaction exists for the daily average temperature and pressure in different regions. In other words, the variation of the daily average temperature and pressure in the spatial domain is not random (He et al. 2016). The $t$-statistics for detecting an abrupt change in the dynamics using MTT on spatial Hurst exponents obtained from the data on the three NCEP variables are shown in Figure 12. The detected change points are listed in Table 1. It is found that the change points detected using MTT on mean of $H$ for three different NCEP/NCAR variables are not identical (Table 1).

4. CONCLUSIONS

In this study, we have tested our hypothesis, i.e., if a spatio-temporal dynamical system undergoes an abrupt change, it is expected that the time series of spatial Hurst exponents obtained for each of the variables of the dynamical system will
also undergo an abrupt change at the same time. It is found that the Hurst exponents time series calculated using 2D-DFA method successfully detected abrupt changes at the same time in each \( X \) & \( Y \) variable of GCML based on the Burgers' chaotic. The result from the artificial data supported or hypothesis for both scenarios.

In contrast, the 2D-DFA method when applied to NCEP spatio-temporal data of three different variables – air temperature, pressure and humidity – detected different change points. The climate variables considered for the identification of abrupt change in the dynamical system are closely related and measured in the same region. These variables are governed by coupled partial differential equations. When a parameter of the system changes, it is difficult to understand why different variables exhibited abrupt change at different times. The interaction between the meteorological elements and the feedback relationship are non-simultaneous, so it may be the reason that abrupt changes detected from the different elements do not coincide.

By detecting an abrupt change in a single variable, one cannot infer that the dynamical system changed abruptly, since a single variable can show abrupt changes without the dynamical system changing abruptly (Yadav et al. 2005). Therefore, the

**Figure 10** | \( \log(F(s)) \) vs. \( \log(s) \) plot for air temperature anomaly data on January 1, 1950.

**Figure 11** | Hurst exponent \( H \) using the 2D-DFA method for NCEP (anomaly) data on air temperature, pressure and relative humidity variables from January 1950 to December 2017.
conclusion that the whole dynamical system changed at identified change point, by using MTT on time series of spatial Hurst exponents calculated via the 2D-DFA method, should be based on more than one meteorological variable of the dynamical system. On the other hand, the conclusion based on identified change point from a single variable that abrupt change occurred in the whole dynamical system cannot be considered a reliable indicator of abrupt dynamical system change.

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DATA AVAILABILITY STATEMENT

All relevant data are available from https://www.esrl.noaa.gov/psd/data/gridded/data.ncep.reanalysis.surface.html.

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