

Stochastic simulation of groundwater dynamics based on grey theory and seasonal decomposition model in a coastal aquifer of South China

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ABSTRACT

Accurate and reliable prediction of groundwater level is a critical component in coastal water management. It is important to find a suitable model with acceptable accuracy. This paper presents a comparative study with seasonal decomposition method of time series analysis and GM (1,1) method, which are applied to test the model accuracy by simulating groundwater levels of two representative wells located at a coastal aquifer in Fujian Province, South China. The average monthly groundwater level was monitored during 2006–2011, the data set from 2006–2010 was used for model establishment, and that of 2011 used for predicting the dynamic change of groundwater level. The results indicate that the multiplicative model fits better than the additive model for seasonal decomposition method. Finally, the effectiveness of the model and prediction accuracy was evaluated based on root mean square error (RMSE) and regression coefficient (R^2). From the obtained results, it is concluded that the GM (1,1) method can be a promising tool to simulate and forecast groundwater level and serve as an alternative physically based model.

Key words | coastal aquifer, grey theory, groundwater level, seasonal decomposition model, simulation

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INTRODUCTION

Groundwater is one of the major sources of supply for domestic, industrial and agricultural purposes. It serves as a dependable supply source, especially in some countries where there is a lack of surface water. However, the irrational utilization of groundwater not only exacerbates the contradiction between supply and demand, but also causes many ecological, environmental and geological problems. These problems, caused by overexploitation of groundwater and falling water tables, have brought an extremely disadvantageous influence to bear on human life and social economic development, such as desertification, land salinization, ground subsidence, salt-water intrusion, land crack, mining area geological disasters and so on (Dong 2009). Therefore, in order to effectively manage groundwater, it is important to model and predict fluctuations in groundwater levels. Predicting the potential change in

groundwater levels is critical for sustainable management of water resources (Emamgholizadeh *et al.* 2014) and, in particular, is of the utmost importance for both present and future generations (Mohanty *et al.* 2010).

Groundwater level, as a dynamic response of external stresses (precipitation, extraction, evaporation, climate and so on), can be considered as an output of the groundwater system, and characterized by trend, periodicity, dependability and randomness. The traditional deterministic methods usually are not capable of solving such problems; subsequently, stochastic time series theory has been explored to solve hydrological problems (Bras & Rodriguez-Iturbe 1985; Lin & Lee 1992; Brockwell & Davis 2010; Doglioni *et al.* 2010). There has been reported various stochastic mathematical models in groundwater dynamic forecasting, for instance, time series analysis,

fuzzy mathematics method, radial basis function network model, artificial neural network, regression analysis, grey Markov chain model, grey system method and so on (Salas 1993; Yeh *et al.* 1995; Sudheer & Jain 2003; Yeh & Chen 2004; Nayak *et al.* 2006; Sreenivasulu & Deka 2011; Zhu & Hao 2013; Shirmohammadi *et al.* 2013; He *et al.* 2014; Emamgholizadeh *et al.* 2014). A time series model is an empirical model for stochastically simulating and forecasting the behaviour of uncertain hydrologic systems (Kim *et al.* 2005; Adhikary *et al.* 2012; Li *et al.* 2014). Various stochastic time series models, such as the Markov, Box–Jenkins (BJ) seasonal auto-regressive integrated moving average (ARIMA), depersonalized auto-regressive moving average (ARMA), periodic auto-regressive (PAR), transfer function noise (TFN) and periodic transfer function noise (PTFN), have been applied for these purposes (Mirzavand & Ghazavi 2015). Time series modelling is a useful tool for detecting trends, developing hydrologic or climatic models, forecasting of hydrologic time series and prediction of future climate scenarios (Coulibaly *et al.* 2001). The application of time series analysis in forecasting groundwater level assumes groundwater system as black or grey box, extracting inherent information by analysing long-term observation series, without the necessity of obtaining other hydrologic parameters, and thus provides the convenience of groundwater dynamic forecasting on a large scale. Most of the research articles (Li & Zhang 2007; Zhou *et al.* 2007; Yang *et al.* 2009; Liang 2011; Lu *et al.* 2014) decompose the groundwater level time series into trend, periodicity and random components to study their characteristics, then combine the three together as an additive or multiplicative model to forecast groundwater levels. Among the numerical techniques, the grey numerical method has become very popular and is recognized as a powerful numerical tool. The theory of the grey system was established during the 1980s as a method for making quantitative predictions. As far as information is concerned, the systems which lack information, such as structure message, operation mechanism and behaviour document, are referred to as grey systems, where ‘grey’ means poor, incomplete, uncertain, etc. It has received increasing application in the field of hydrology (Xu *et al.* 2008). There are several models for grey theory,

and among them, the GM (1,1) method is relatively simple, but can provide a high precision of prediction.

It is known that there is no an universal model that can be applied to any given case in making a simulation of groundwater dynamics. Any mathematical model has its advantages and disadvantages, with different simulation accuracy for a certain observed time series, so it is essential to discover the suitability of a model in a given case. Also, there has been no reporting on the comparative study of decomposition models with the grey method. In order to obtain the suitability of different simulation methods for a given case, in this paper the GM (1,1) method and seasonal decomposition method, multiplicative and additive methods, have been applied to simulate groundwater water tables in a coastal aquifer at Fujian Province, South China, and the simulated results are compared by evaluating root mean square error (RMSE) and regression coefficient (R^2).

METHODOLOGY

Seasonal decomposition method

Time series analysis has been widely used in groundwater resource evaluation, forecast and management due to its simplicity. Multiplicative and additive models are two common models of the seasonal decomposition method, used to analyse the groundwater level data. The model equations can be expressed as follows:

Multiplicative model: $Y = T \times S \times I$

Additive model: $Y = T + S + I$

where Y represents the observed groundwater level series, T stands for smoothed trend-cycle (abbreviated as *stc*), affected by long-term trend factors, S indicates season factors component (abbreviated as *saf*). I is irregular component (abbreviated as *err*). The irregular change is a contingency, randomness, burst factor. Specific steps are as follows:

1. Enter the groundwater level data of an observed time series and define the time variable.
2. Draw scatter plot based on the above data, observe the sequence diagram and determine whether it is a stationary sequence.

3. Analyse and process the data. Click the analyse menu bar, select the prediction, and then select the seasonal decomposition of the prediction menu bar, select the multiplicative model or additive model.
4. Explain and illustrate the output results. The results obtained four new additional variables, which are the irregular component (*err_1*), the sequence after adjustment of the seasonal (*SAS_1*), seasonal factor (*saf_1*), and trend cycle component without seasonal and irregular change (*stc_1*).
5. Draw scatter plot of the four variables mentioned above. According to the chart *stc* of long-term trends, we can build a linear regression model with polynomial to obtain the relationship between *stc* and *t*. Make use of this expression to predict the forecasting value of future trend cycle components *stc*.
6. According to the 12 values of seasonal factor *saf* of the multiplicative model, we can get the forecasting value of groundwater level via:

$$Z = stc * saf \tag{1}$$

However, the additive model is affected by *stc* and *err*, and the predicted groundwater level can be obtained with:

$$Z = stc + err \tag{2}$$

where *Z* is predictive value of groundwater level, *stc* is the projections of trend cycle component, *saf* represents seasonal factors and *err* denotes the irregular component.

Then the accuracy for *stc* prediction can be examined by regression coefficient (R^2) due to the value in the forecasting process that will produce a certain deviation. Therefore, during the fitting process where the R^2 value is bigger, the fitting is better. R^2 value closer to 1 indicates a perfect fitting between the observed and simulated values.

GM (1,1) method

Grey system theory is a multidisciplinary theory dealing with those systems' lack of information, which uses a black-grey-white colour spectrum to describe a complex system whose characteristics are only partially known or

known with uncertainty (Deng 1982, 1989). The dynamic of groundwater level is regarded as a typical grey system problem, where the grey GM (1,1) model can better reflect the changing features of groundwater level (Yang et al. 2011). It especially has the unique function of analysing and modelling for short time series, less statistical data and incomplete information of the system, and has been widely applied. GM (1,1) represents an order, a variable, and is a special case of the N variables of differential equation GM (1, N). The essence of GM (1,1) is to accumulate the original data in order to obtain regular data. By setting up the differential equation model, we obtain the fitted curve in order to predict the system. The modelling process is as follows: First, observed data are converted into new data series by a preliminary transformation called AGO (accumulated generating operation). Then, a GM model based on the generated sequence is established. Subsequently, the prediction values are obtained by returning the AGO's level to the original level using IAGO (inverse accumulated generating operation) (Xu et al. 2008).

The specific steps of the GM (1,1) model can be summarized as follows:

1. Suppose the observed original data as

$$x^{(0)}(t) = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\} \tag{3}$$

2. Accumulate the observed data above once to generate a new sequence, that is

$$x^{(1)}(t) = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\} \tag{4}$$

where, $x^{(1)}(m) = \sum_{i=1}^m x^{(0)}(i)$, $m = 1, 2, \dots, n$;

3. According to the newly generated sequence, the differential equation is established as follows:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = u \quad t = [0, \infty] \tag{5}$$

4. Suppose $A = (a, u)^T$, *a* and *u* can be obtained by least squares estimation

$$A = (a, u)^T = (B^T B)^{-1} B^T Y_N \tag{6}$$

in which,

$$B = \begin{pmatrix} -\frac{1}{2}(x^{(1)}(1)) + x^{(1)}(2) & 1 \\ -\frac{1}{2}(x^{(1)}(2)) + x^{(1)}(3) & 1 \\ \dots & \\ -\frac{1}{2}(x^{(1)}(n-1)) + x^{(1)}(n) & 1 \end{pmatrix}, Y_n = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(n) \end{pmatrix} \quad (7)$$

5. Let $x^{(0)}(1) = x^{(1)}(1)$, and we get the GM (1,1) forecasting model:

$$x^{(1)}(t+1) = \left[x^{(0)}(1) - \frac{u}{a} \right] e^{-at} + \frac{u}{a} \quad (8)$$

6. To obtain the prediction values by returning an AGO's level to the original level using IAGO

$$\begin{cases} \hat{x}^0(1) = x^{(0)}(1) \\ \hat{x}^0(t+1) = \hat{x}^1(t+1) - \hat{x}^0(t) = (1 - e^a)(x^{(0)}(1) - \frac{u}{a})e^{-at} \end{cases} \quad (9)$$

In this paper, before forecasting the groundwater level, the after-test residue method (Chen et al. 1994) should be used to test the accuracy of the two methods and to find whether these models are suitable for predicting. The two methods mentioned above can be used to calculate the predicted value when the parameters P and C are within the acceptable range, otherwise it is necessary to re-adjust the parameters. The evaluation criteria for the prediction precision are shown in Table 1.

Table 1 | Forecast accuracy evaluation criteria

Predicted grade	P	C
Good	>0.95	<0.35
Qualified	>0.80	<0.50
Just qualified	>0.70	<0.65
Disqualification	≤0.70	≥0.65

CASE STUDY

Study area

Groundwater is mainly affected by atmospheric precipitation. Some of the precipitation forms the surface runoff, with some infiltrating underground to recharge the downstream aquifer, while some directly recharges the groundwater. Dongshan County is a coastal island located at the most southern part of the 'golden delta' of Fujian Province. It has a subtropical marine monsoon climate, with four mild seasons, almost no winter, and is located between 117°17' E–117°35' E longitude, 23°33' N–23°47' N latitude, consisting of Dongshan island and 44 small islands and covers an area of about 248.34 km² (Figure 1). The annual average temperature is 20.83 °C, with an average rainfall of about 1,172.9 mm. The rainfall occurs mainly from May to September, accounting for more than 67% of total annual precipitation. The main geological coverage of the study area is coastal plain, plateau and hills. The alluvial plain comprising sand, and gravel with clay accounts for more than 80% of the total area. Owing to the topographic features of Dongshan County, the characteristic movement of groundwater is flow from the high ground to the low, and finally into the sea. Groundwater level data in this study area were obtained by monitoring the groundwater level every 5 days for a period of 6 years from January 2006 to December 2011. GM (1,1) method and seasonal decomposition method are tested with the data recorded at Dongshan hydrological station.

Modelling

To analyse and forecast the groundwater table in Dongshan County with the two methods mentioned above, taking Kangmei observation well (3506260008) and Chencheng observation well (3506262024) as examples, in which monthly average groundwater level was monitored during 2006–2011, the data set from 2006–2010 is used for model establishment and that of 2011 is used for predicting the dynamic change of groundwater level.

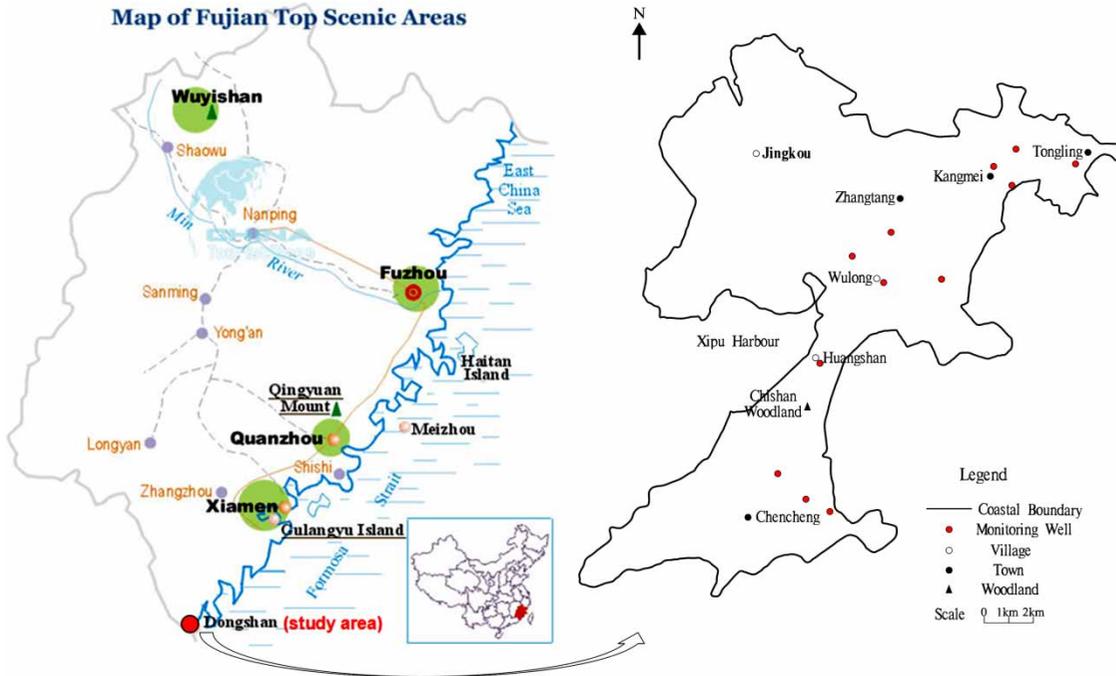


Figure 1 | Outlined location map of the study area with sampling points.

Seasonal decomposition method

From the scattered plot of groundwater level data set from 2006 to 2010 (Figure 2), it can be seen that the water level series is not steady. Therefore, it is suitable for seasonal decomposition model.

The observed data are analysed by SPSS software, and four types of data, the irregular component (*err_1*), the adjustment sequence (*SAS_1*), seasonal factor (*saf_1*) and trend cycle (*stc_1*) were obtained. These are illustrated in Figures 3(a)–3(d).

In order to improve the fitness, a trend line was added to get the fitting expressions in Figure 3(d) using Excel

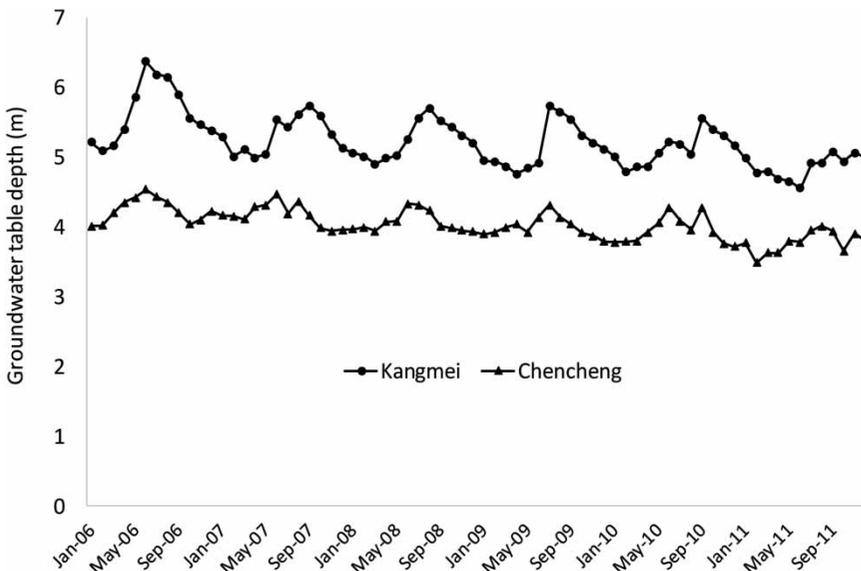


Figure 2 | The observed groundwater levels.

software. It can be seen that: (1) as R^2 value is closer to 1, the polynomial fit is better; (2) parameters C were calculated for Kangmei and Chencheng, with values of the multiplicative model (0.514, 0.615) being smaller than that of the additive model (0.851, 0.871). The smaller C is, the better it fits. Therefore, the fit of the multiplicative model is better than that of the additive model. The goodness of fit between the observed values and the calculated of the groundwater level from 2006 to 2010 is shown in Figure 4.

GM (1,1) method

Following the modelling steps described above, we take the data of the groundwater level of Kangmei observation well (3506260008) as an example to perform a test. Taking the data of January 2006–2010 as initial data, that is:

$$x^{(0)}(t) = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\} = (5.21167, 5.28833, 5.06333, 4.94833, 5.005)$$

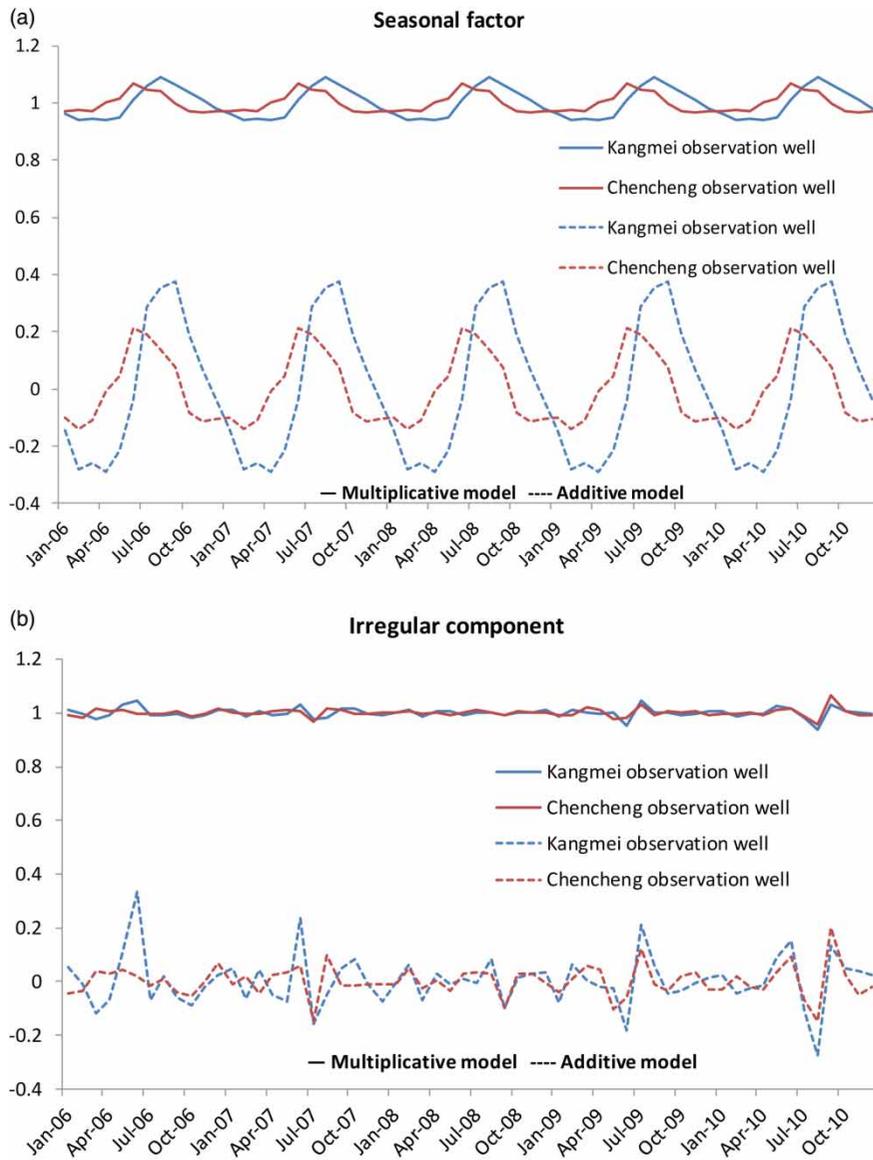


Figure 3 | (a) Seasonal factor (*saf_1*), (b) irregular component (*err_1*), (c) adjustment sequence (*SAS_1*), (d) trend cycle (*stc_1*). (Continued.)

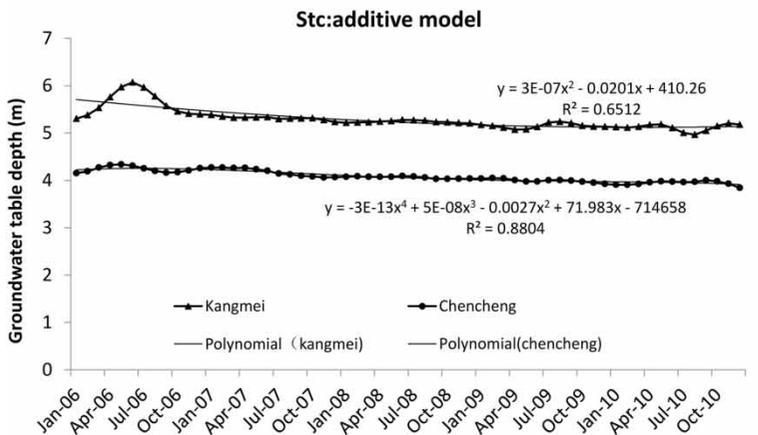
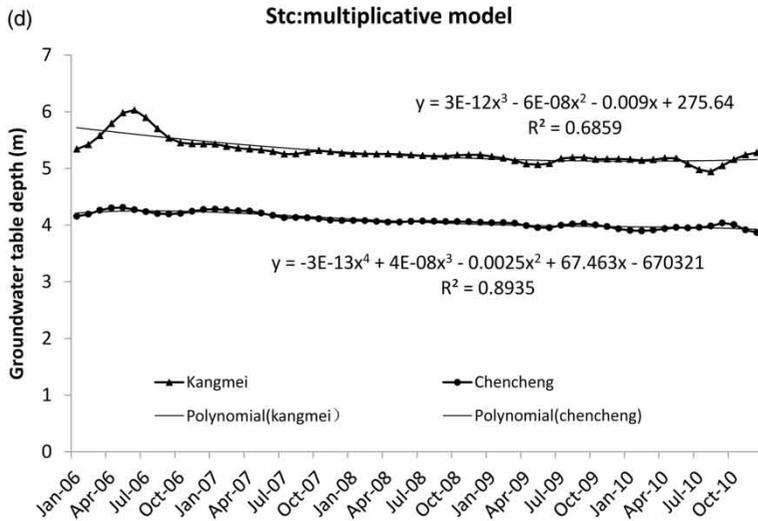
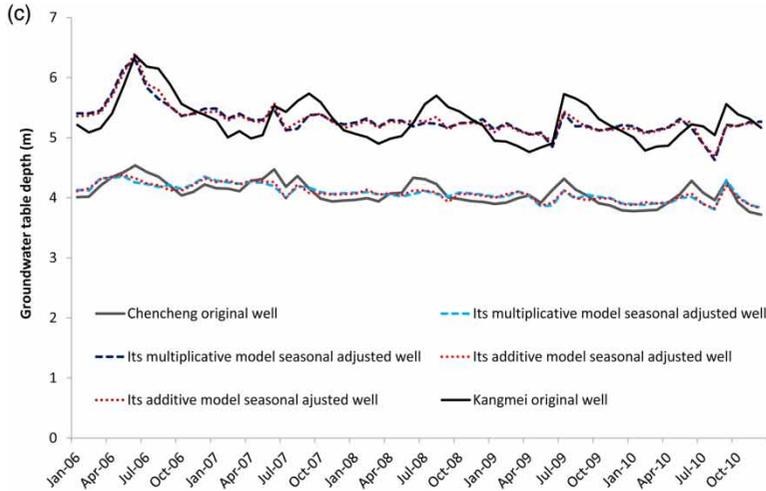


Figure 3 | Continued.

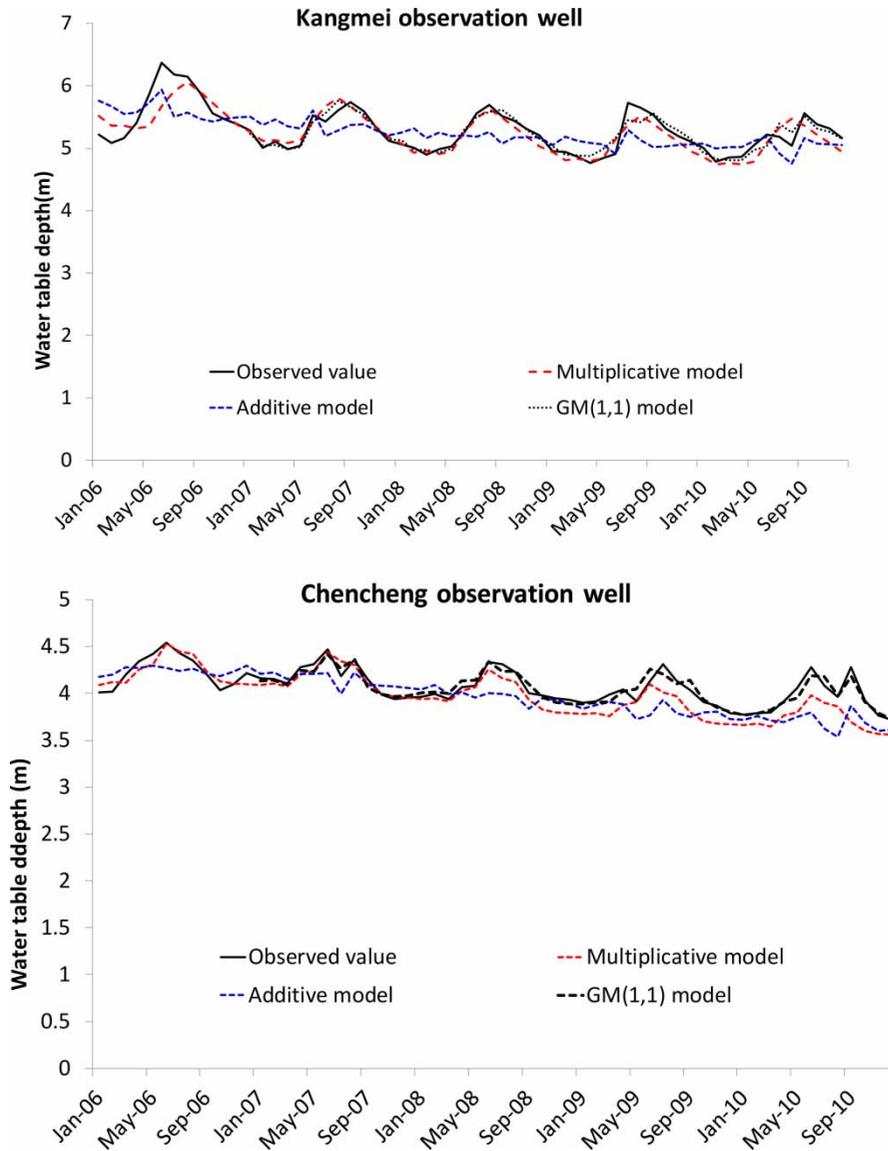


Figure 4 | Model fitting results.

1. By the formula (e), (f) and (g), we obtained $a = 0.019$, $u = 5.374$, then the differential equation is:

$$\frac{dx^{(1)}(t)}{dt} + 0.019x^{(1)}(t) = 5.374 \tag{10}$$

2. Suppose $x^{(0)}(1) = x^{(1)}(1) = 5.21167$, we can get

$$x^{(1)}(t + 1) = \left[5.21167 - \frac{5.374}{0.019} \right] e^{-0.019t} + \frac{5.374}{0.019} \tag{11}$$

$m = 1, 2, 3, \dots$

Therefore, the groundwater level prediction model of January is

$$\hat{x}^{(1)}(t + 1) = -274.3698e^{-0.019t} + 279.5814 \tag{12}$$

Therefore, we can get the predicted groundwater level of January of 2011, 4.8368 m, and the groundwater level for each month in 2011 can be obtained with Equation (12). From the accuracy degree of the GM (1,1) method, the Kangmei observation well (0.2939, 0.9999) and Chencheng observation well (0.3052, 0.9998), we can see the parameters c and p both achieve a ‘good’ degree, conforming to the values

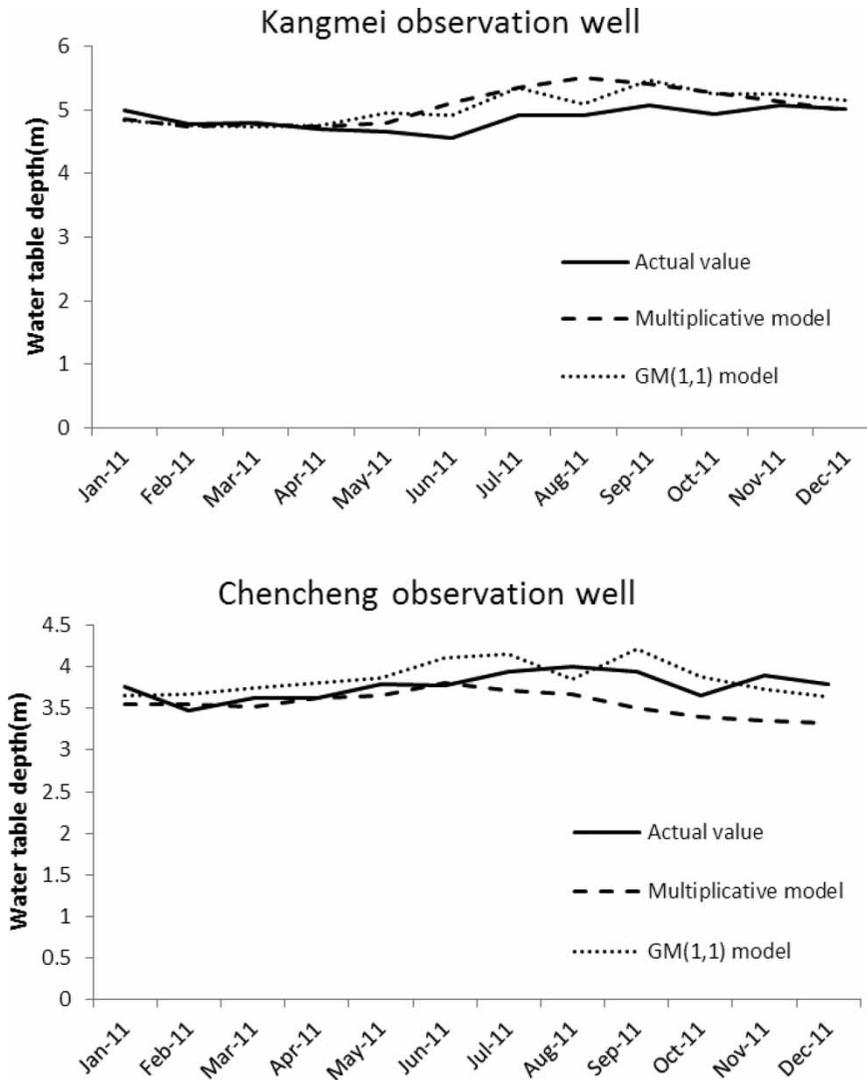


Figure 5 | The predicted groundwater level for 2011.

of accuracy test parameters of the forecast models. It can be seen in Figure 4 that a higher fitting accuracy is achieved.

RESULTS AND DISCUSSION

Owing to the better fit of the multiplicative model for seasonal decomposition method, we select the multiplicative model and GM (1,1) to make a comparison for data of 2011. Figure 5 and Table 2 summarize the prediction results for each method. Two common standard statistical measures, RMSE and regression coefficient (R^2), are employed to evaluate the performances of the forecasting

Table 2 | Model prediction accuracy results

Well Model	Kangmei observation well		Chencheng observation well	
	GM (1,1)	Seasonal decomposition	GM (1,1)	Seasonal decomposition
RMSE	0.2536	0.3053	0.1961	0.2936
R^2	0.9970	0.9957	0.9971	0.9935

models. The mathematical expression is expressed as (Sreekkanth et al. 2009):

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{N}} \quad (13)$$

$$R^2 = 1 - \frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N y_i^2 - \frac{\sum_{i=1}^N \hat{y}_i^2}{N}} \quad (14)$$

where N is the total number of value, y_i is the observed value and \hat{y}_i is the predicted value.

R^2 measures the degree of correlation among the observed and predicted values. R^2 values range from 0 to 1. The coefficient of determination describes the proportion of the total variance in the observed data that can be explained by the model. RMSE evaluates the residual between observed and forecasting value. The nearer RMSE value is to 0, the more accurate the forecasting is. The best fit between observed and calculated values will be obtained with R^2 as 1 and RMSE as 0. It can be seen in Figure 5 that the GM (1,1) model has good prediction effect.

From the accuracy degree of the forecast models of Table 2, it can be seen that a higher R^2 value and a lower RMSE of GM (1,1) was obtained for the two selected wells. Therefore, it is concluded that the GM (1,1) model fits the hydrogeological conditions better than the seasonal decomposition model in this case study. On the other hand, the comparison between the two wells indicates that Chencheng observation well had a lower RMSE and higher R^2 , which can be explained by its small fluctuations of groundwater table depth (Figure 2). The method is simple, so the model can be practical. However, if the original data sequence has large fluctuations, the model should be optimized using three point smoothing of the residual model of predicted value and the actual value to obtain a better prediction. In the GM (1,1) prediction model, parameters a and b are fixed once determined, and regardless of the numbers of values, parameters will not change with time. The feature limiting GM (1,1) is that it is only suitable for short-term forecasts because many factors will enter into the system with the development of the system with time. The accuracy of the prediction model will become increasingly weak with the time away from the origin, and thus the predictable significance will diminish.

CONCLUSION

Groundwater is an important part of water resources, and is also the main source of agricultural irrigation, industrial

and domestic water. It is very important to monitor groundwater levels and follow up its dynamic changes. Hence it is essential to obtain a proper method to forecast the groundwater dynamics. Owing to the variety and uncertainty of affecting factors on groundwater levels, stochastic models have been widely applied to groundwater level simulation. However, usually a simple model cannot provide enough accuracy while conducting long-term forecasting, therefore it is necessary to compare different methods in simulating and forecasting an observed time series for a given case. This paper compares the seasonal decomposition method of time series analysis and the GM (1,1) method to simulate and predict groundwater levels in Fujian Province, South China. The effectiveness and the final prediction accuracy of the methods are evaluated with root mean square error (RMSE) and regression coefficient (R^2). It is observed that the GM (1,1) model has a higher accuracy in this case. In the case of relatively small fluctuations in the raw data series, the prediction accuracy is high using a grey forecasting model. Application of grey theory to prediction of groundwater level is a novel research area. The model is proposed by virtue of the dynamic characteristics of groundwater level, which increased the forecast precision. Therefore, the method is reliable and effective. However, it is recommended that many methods should be employed to test the reliability and objectivity of the selected model when considering the uncertainties that affect groundwater level dynamics.

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