Comparison of solution methods for analyzing water distribution networks under pressure-deficient conditions

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ABSTRACT

Demand-driven analysis method is used in most current existing models for hydraulic evaluation of water supply networks. This method is based on the assumption of constant use, regardless of the available pressure in nodes. Demand-driven analysis method does not have sufficient efficiency in hydraulic analysis of networks under pressure-deficient conditions. In this study, the combination of pressure-deficient network algorithm (PDNA), modified pressure-deficient network algorithm (MPDNA), and complementary reservoir solution (CRS) methods with hydraulic model have been used to analyze series networks, looped networks, and a full-scale distribution network (part of the water network in Ilam city, Iran) in critical operating conditions. The critical condition in networks is created by breakage in a pipe network and fire-fighting demands on one node. Results showed that the required flow in networks has not been quite satisfied. The supplied outflow in the series, looped networks, and zone-6 network which used the three aforementioned methods are calculated as 76.40%, 90.25%, and 98.56% of total network demand, respectively. The results also showed that the number of required iterations to achieve the solutions in the PDNA method is more than in the MPDNA and CRS methods.

Key words | artificial reservoir, pressure deficient, water distribution networks

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INTRODUCTION

The process of providing high quality water and its transportation to customers in semi-arid climates is one of the most important issues which experts are concerned with. Evaluating hydraulic networks and reducing water loss from both transmission and distribution lines may be regarded as a solution. Pressure in distribution systems should meet the minimum allowable limit in the maximum demand conditions. In other words, it should not cause bursting of pipes in the case of minimum demand on the network. Water distribution network (WDN) will be under pressure-deficient conditions if hydraulic or pressure heads in each node (consumer) are less than the required head. To resolve the pressure-deficient problem in a network, the first step is identifying areas with a low pressure.

Therefore, we need to identify these points through simulation and resolve the pressure-deficient problem in an appropriate manner. Demand at pressure-deficient nodes could not be quite satisfied. Regardless of available pressure in nodes, most existing models for hydraulic analysis of water networks assume that demand is provided and simulate the network. This type of analysis is called demand-driven simulation method. Demand-driven methods, particularly under abnormal conditions in the network, such as pipe breakage, pipe failure, valve closure, or fire-fighting demand, would not provide the required efficiency in hydraulic analysis of water distribution systems. Hence, it is necessary to consider the relationship between pressure and outflow in nodes to simulate

the actual system performance. This type of analysis is called pressure-driven simulation method. Well-known hydraulic models like Loop, WaterCad, MikeNet, Pipe Flow Expert, EPANET and WaterGems are available to analyze networks under normal operating conditions. Different relationships and methods have also been proposed to evaluate networks under critical or abnormal conditions. Typically, existing software packages have the capability of modeling pressure-dependent demands using emitter coefficients. Gupta & Bhave (1996) compared the formulation of pressure head-outflow developed by various researchers. Ozger & Mays (2003) described a semi-pressure-driven approach, demand-driven-available-demand-fraction (DD-ADF) method that uses the EPANET network hydraulic model. Ang & Jowitt (2006) calculated the partial flow in pressure-deficient nodes by introducing artificial reservoirs (ARs) and connecting them to demand nodes (DNs) with deficient pressure. This method is called pressure-deficient network algorithm (PDNA). Wu (2007) investigated the problems of adding and removing AR in large-scale networks and the inability of PDNA for handling the extended period simulation. Rossman (2007) modeled the pressure-deficient analysis in each network node which uses EPANET software by considering an emitter on the node that specifies the relationship between the outflow and the node's pressure. Wu *et al.* (2009) developed an extended global-gradient algorithm that generalized an efficient approach for pressure-dependent water distribution analysis as a unified loop-node formulation. Using complementary reservoir solution (CRS), Suribabu & Neelakantan (2011) examined a looped network under various conditions, including normal operation and at the time that each pipe network was isolated. The results showed that the proposed method can properly identify pressure-deficient nodes on the network. Babu & Mohan (2012) developed the Modified PDNA algorithm (MPDNA) for network analysis in pressure-deficient conditions and tested it for two base networks. The results showed that the algorithm can reliably predict the outflows on different nodes. Jun & Guoping (2013) developed the EPANET-MNO algorithm for network analysis in abnormal conditions, such as pipe breakage and fire-fighting demands, and compared it with different formulas which were suggested by researchers. They also demonstrated better efficiency of the algorithm developed for the extended-period analysis of an actual large-scale network. AbdySayyed & Gupta (2013)

compared the formulation of pressure head-outflow which had been discussed by various researchers and classified their methods into direct and indirect ones. Different optimization methods and techniques such as the PDNA, MPDNA, and CRS are mentioned as indirect. Muranho *et al.* (2014) developed the EPANET-WaterNetGen algorithm to analyze networks at normal pressure and pressure-deficient conditions on nodes. They modeled pressure-deficient conditions on the nodes which uses the hydraulic relationship between pressure head, outflow, and the leakage from the relevant pipe. Abdy-Sayyed *et al.* (2014) developed a simple, non-iterative method for modeling pressure deficiency in a WDN by considering the check valve, flow control valve (FCV), and a hypothetical emitter on each DN. The algorithm was integrated with the EPANET hydraulic model and the results showed better performance of the model. Sivakumar & Prasad (2014) investigated the association between the MPDNA approach and different pressure head-outflow formulas. The results indicated that the MPDNA method and the formulas are basically identical. Furthermore, they proposed modifications to the MPDNA method by which there is no need to have change in the original EPANET source code. AbdySayyed *et al.* (2015) described a modeling approach which enabled operating conditions with insufficient pressure to be simulated in a single execution of EPANET2. This is achieved by connecting a check valve, a FCV, and an emitter to the DNs. The results suggested that the procedure is robust, reliable, and fast enough for regular use. In the current study, the PDNA, MPDNA, and CRS methods are evaluated for analyzing WDNs in different operating conditions in combination with a hydraulic model.

MATERIALS AND METHODS

Hydraulic analysis of WDNs is influenced by both mass and energy conservation laws. These rules are a set of nonlinear equations which have been solved in various mathematical models for analyzing the networks which use methods such as Hardy Cross, linear theory, Newton–Raphson and gradient algorithms. In this study, the combination of hydraulic model with PDNA (Ang & Jowitt 2006), MPDNA (Babu & Mohan 2012), and CRS (Suribabu & Neelakantan 2011) methods have been used to analyze multiple

networks under different operating conditions. When pressure in network nodes is less than the minimum required pressure for DNs, hydraulic model should be combined with other methods proposed by researchers to ensure desirable results. PDNA, MPDNA, and CRS methods could deal with different operation modes, including normal or abnormal conditions, such as pipe breakage, pipe failure, valve closure, or fire-fighting. After creating an input file, the hydraulic simulation runs and the hydraulic head of the distribution network is retrieved through the hydraulic model. If all nodes have higher hydraulic head than the required corresponding level, the algorithm will terminate and print the results. Otherwise, the proposed model progressively introduces a set of ARs into the network to initiate nodal flows.

PDNA approach

The procedure of the network analysis for different operating conditions can be presented step by step as follows:

1. Performing the hydraulic analysis of the network with all demands set to zero.
2. Adding ARs to all nodes which have heads more than the minimum required heads with the same elevation as DNs, through a short-length, large-diameter pipe.
3. Running the hydraulic model of the updated network and removing any ARs which supply water to the WDN.
4. Repeating steps 2 and 3 until no DN has a head more than the minimum required head to satisfy nodal demands.
5. Replacing all ARs which have an inflow greater than their specified demands, with a DN of the stated demand.
6. Run the hydraulic analysis for the updated network.
7. Checking the head and outflow on every DN in ARs. If there is no DN with a head greater than the minimum head and the outflow in ARs is not more than nodal demand, the algorithm will terminate. Otherwise, we should repeat steps 2–6. The PDNA algorithm is shown in Figure 1.

MPDNA approach

In the MPDNA, the improved PDNA method will be used as a basis. The major limitation of PDNA is a necessity of multiple hydraulic simulations and dynamic addition of ARs during the successive simulations. These shortcomings have been resolved in the MPDNA through the following five steps:

1. Fixing of ARs: Adding ARs with an elevation equal to the elevation of the nodes where their heads are above the minimum required head, through a short-length, large-diameter pipe.
2. Setting of artificial flow control valve (AFCVs): AFCVs are connected with the pipes that join DN and AR, and these valves let out the water only from DN to AR as the status of the pipe is set as a control valve. Thus, reverse flow is not possible (flow from AR to DN). Max flow through AFCVn = demand at DNn.
3. Modification of nodal demands: Each AR acts as a demand-node; thus, demands at the actual DNs are set as zero in step 3.
4. Simulation of WDN: In this step, hydraulic simulation of WDN is carried out to compute the hydraulic-heads available at the DNs and discharges through the pipes.
5. Result interpretation: The discharge received by nth AR is the supply that can be provided in the nth DN. As the maximum discharge through any AFCV is the original node-demand, the flow to the AR never exceeds the demand. If the hydraulic-head available at the nth DN is less than the minimum required hydraulic-head, then the flow to nth AR will be zero. PDNA and MPDNA methods are practically the same, except for the inclusion of the AFCVs in the MPDNA.

CRS approach

In this approach, complementing reservoirs are progressively connected to the nodes which are facing pressure deficit. By addition of complementing reservoirs to the nodes, the demand at a deficit node is satisfied with minimum pressure. A critical node (CN) is a node with pressure deficiency. A complementary reservoir is an imaginary reservoir which is generally added to a CN for completing flow towards the CN until satisfying the demand at that node. The step-by-step description of the CRS approach is as follows:

1. Adding a complementary reservoir to the nodes that have maximum pressure deficiency via a large-diameter pipe
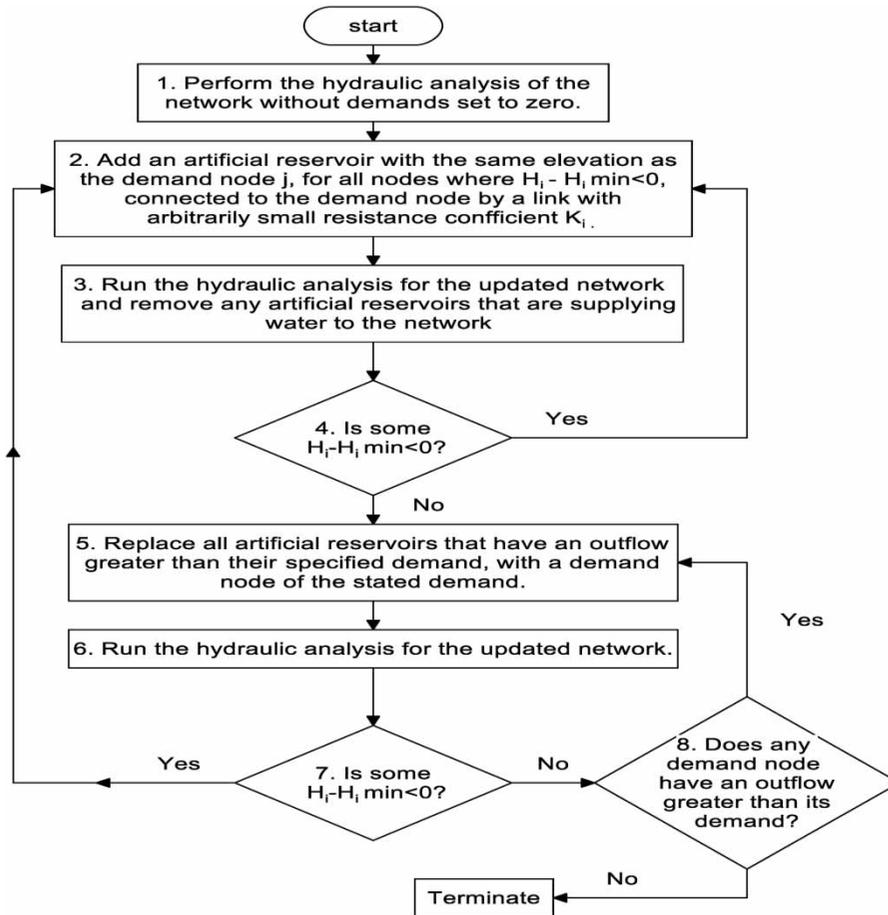
Figure 1 | PDNA algorithm.

with an elevation equal to the minimum required hydraulic head.

2. Running the hydraulic analysis for the updated network.

3. Three cases may occur as follows. (a) If there is another pressure-deficient node, the process of adding CRS to that CN would be necessary. Go to step 1 and continue. (b) If there is no other pressure-deficient node, the CRS will sometimes provide more water than what is actually required on the relevant node. This indicates that another node is experiencing pressure drop. Therefore, another CRS is required. Go to step 4 and continue. (c) If there is no pressure-deficient node and the supply from CRS is less than the demand on the relevant node, the process will terminate.

4. Finding the nodes at which the pressure is minimum and add a complementary reservoir to that node where its hydraulic gradient is equal to the previous one.

5. Running the hydraulic analysis for the updated network. If flow is taking place from node to CRS, remove that CRS; add a CRS to the nodes which have the minimum pressure and go to step 5. Otherwise, go to step 3.

The best ways to understand the correctness of the algorithm are to solve networks which have analytic solutions and their answers have reasonable accuracies. In this study, all three methods described above have been used to evaluate the different distribution networks.

RESULTS AND DISCUSSION

In this study, application of PDNA, MPDNA, and CRS methods is tested via two base networks which have been studied by various researchers and the obtained results are
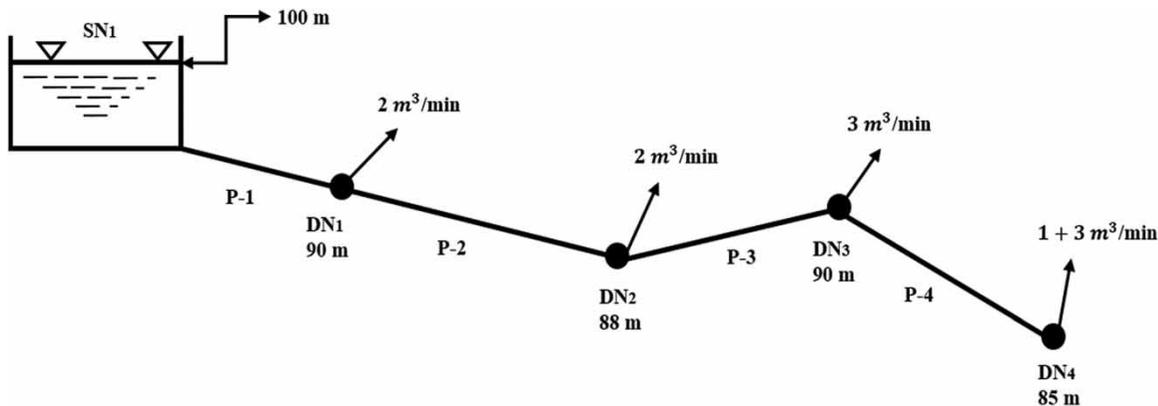
Figure 2 | Schematic of series network.

compared with results reported by previous studies. Finally, the analysis methods are addressed in a large-scale network.

First network: series network

Series network is one of the basic networks used by researchers, including Gupta & Bhave (1996), Ang & Jowitt (2006), Babu & Mohan (2012), and Sivakumar & Prasad (2014), to describe pressure-deficient networks, as shown in Figure 2. The series network is supplied by a constant head source at node 0. There are four DNs of 1, 2, 3, and 4 with demands of $2\,m^3/min$, $2\,m^3/min$, $3\,m^3/min$, and $1\,m^3/min$, respectively. The elevation at each node is shown in Figure 2 and H_i^{min} for all nodes is the elevation itself. A total of four distribution pipes 1, 2, 3, and 4 are in the series network, with diameters of 400 mm, 350 mm, 300 mm, and 300 mm, respectively. Each pipe has a length of 1,000 m and a Hazen–Williams coefficient of 130.

The above-mentioned conditions were designated as normal operation conditions which are used in the hydraulic model and the hydraulic elevations of nodes 1–4 are determined as 97.30 m, 94.27 m, 91.24 m, and 91.01 m, respectively. Hydraulic elevation on all nodes is higher than the minimum elevation, so there is no need to integrate the hydraulic model with algorithms. Assuming additional fire-fighting demands of $3\,m^3/min$ on node 4, another operating condition is applied to it. Applying the hydraulic model to fire-fighting demand indicated that the hydraulic elevation at nodes 1–4 was 95.14 m, 88.71 m, 80.16 m, and 77.13 m, respectively. The numerical values reveal that

hydraulic elevation at nodes 3 and 4 is less than the minimum elevation and the system undergoes pressure deficiency. For analyzing this situation, a combination of PDNA, and MPDNA and CRS methods with hydraulic model is used. The simulation process can be seen in Figure 3. According to the figure, it is clear that MPDNA and CRS methods are able to find a solution in a single hydraulic simulation and the PDNA method needs the following six hydraulic simulations:

Simulation 1: Demand at all DNs was set at zero to find out the static pressure-heads.

Simulation 2: AR1 to AR4 were added with DN1 to DN4, respectively.

Simulation 3: AR3 was removed as it supplied water to the system.

Simulation 4: AR1and AR2 were removed as they received more than the demand at DN1 and DN2.

Simulation 5: AR4 is removed as it received more than the fire-fighting demand at DN4.

Simulation 6: AR3 is connected again as the pressure-head at DN3 is greater than zero.

Available flow and hydraulic elevation for each node with different approaches are presented in Figures 4 and 5. Analysis of the results showed the full supply on nodes 1, 2, and 4. Node 3 provides demand equal to $0.404\,m^3/min$. Thus with a source elevation of 100 m, $8.404\,m^3/min$ of total demands ($11\,m^3/min$) is provided. The results of the three methods are completely consistent and have an insignificant difference with the results of Ang & Jowitt (2006).
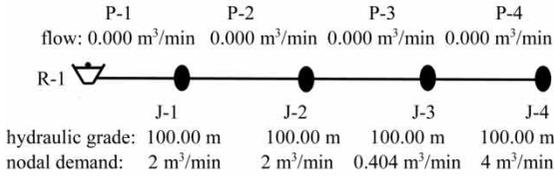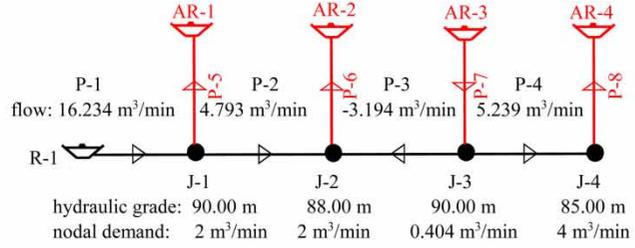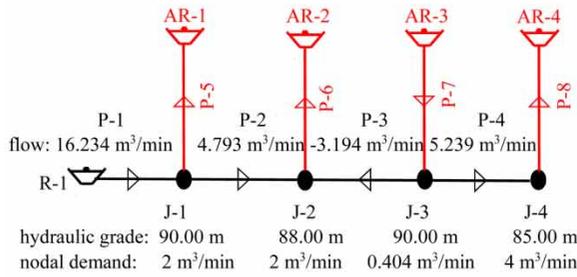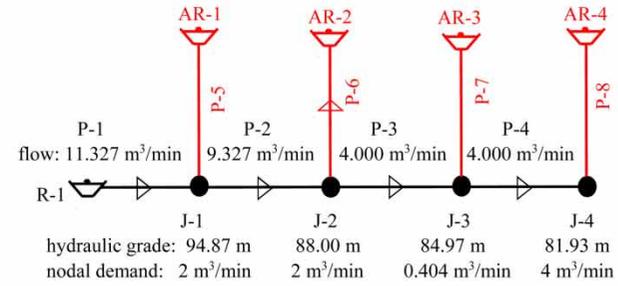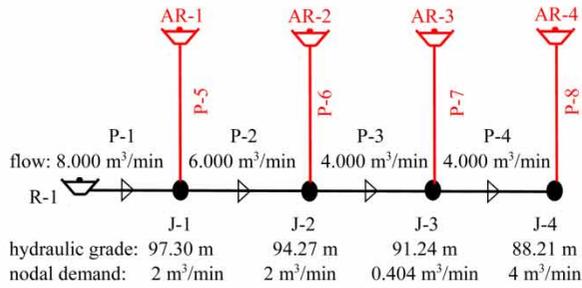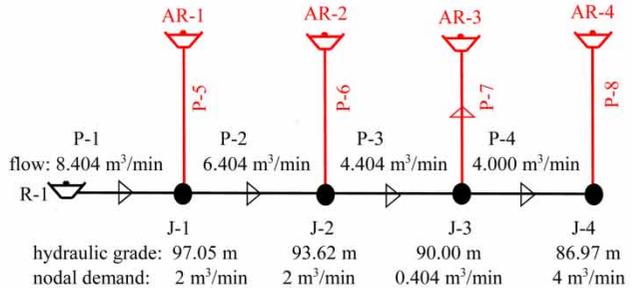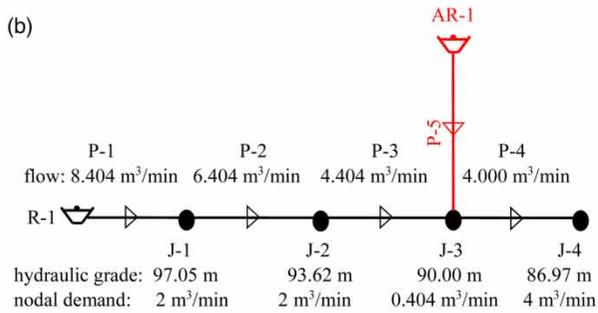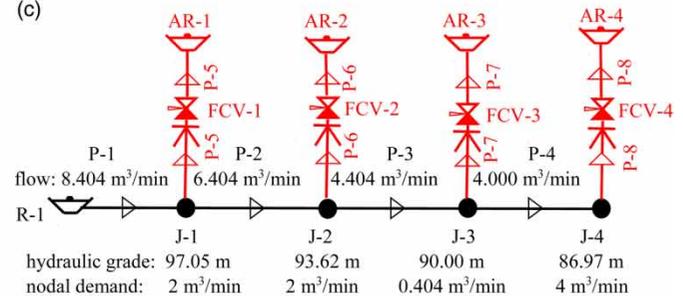
Figure 3 │ Procedure of the running model with hydraulic model for source pressure head of 100 m (fire-fighting demand on node 4). (a) PDNA, (b) CRS and (c) MPDNA.
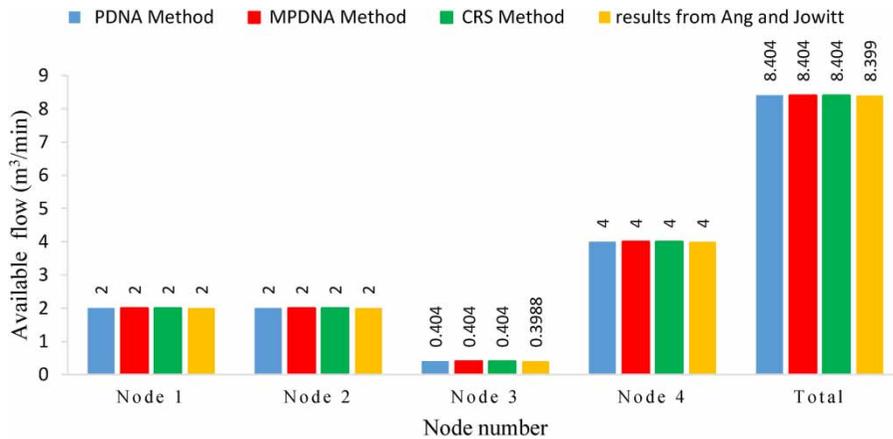
Figure 4 │ Available outflow with different approaches for source pressure head of 100 m (fire-fighting demand on node 4).

To determine the minimum and maximum elevations on the source node several simulations are carried out which can be seen in Figure 6. The results show that if the elevation on source node reaches 85 m, the demand will not be satisfied in any node. If source elevation is equal to 109.86 m, the demand will be satisfied for all nodes. It should be noted that for the source elevation of 85 m to 96.82 m, ARs in the CRS method would provide more demand than is actually needed on the DN and the method proposed by Suribabu & Neelakantan (2011) (step 4) would not work in this case. Thus, in this study, a modified version of the CRS method is proposed to resolve shortcomings in the CRS method. In this method, a FCV is added to the pipe connecting DN and

CRS which provides more water than they actually require. Elevation and outflow from FCV are equal to the minimum required hydraulic head and demand at the relevant node, respectively. This valve is only allowed to release water from the complementary reservoir to the node. To assess the value of real demand provided by the node, the flow rate which is provided by the complementary reservoir is subtracted from the amount of the demand in the relevant node. If there is no node with negative pressure, the algorithm will terminate. Otherwise, the process continues until there is no node with pressure deficiency in the network. The results of this method are perfectly consistent with those of PDNA and MPDNA methods.
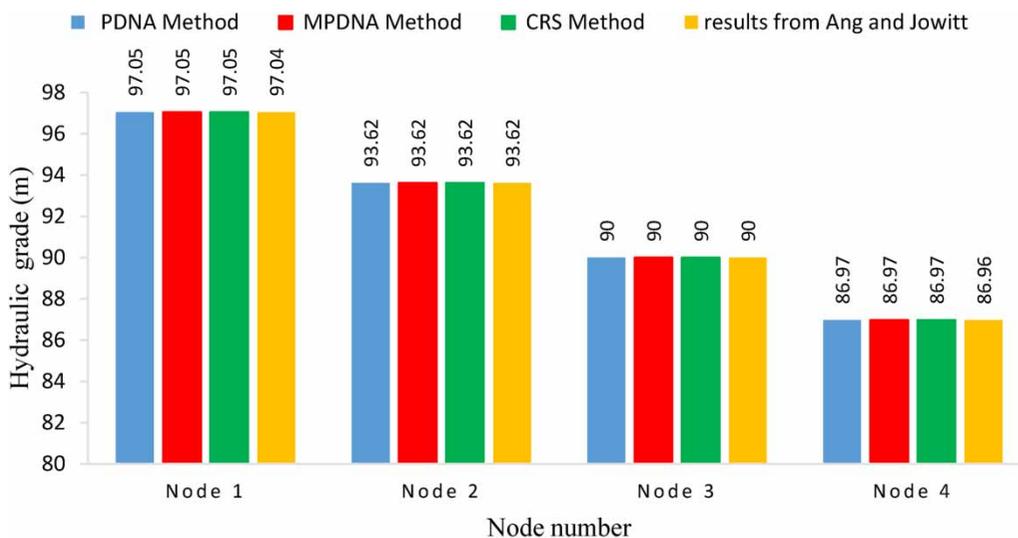

Figure 5 │ Hydraulic grade with different approaches for source pressure head of 100 m (fire-fighting demand on node 4).
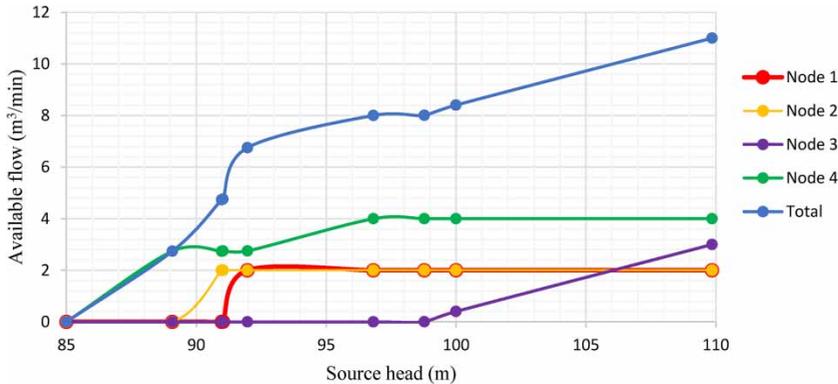
Figure 6 | Sketch of different pressure heads on the source node versus total network outflow.

Second network: looped network

Another distribution network (used by Ang & Jowitt (2006)) including one supply source and two loops are used to evaluate these three algorithms. The network consists of a reservoir, six DNs, and eight pipes (Figure 7). All pipes are 1,000 m long with a Hazen–Williams coefficient of 130. Diameters of pipes 1–8 are 350 mm, 300 mm, 300 mm, 250 mm, 250 mm, 200 mm, 200 mm, and 200 mm, respectively. The minimum pressure which is required at each DN is equal to its elevation and the demand on each node under normal conditions is 25 L/s. By running the hydraulic model for network analysis in normal conditions, it has been observed that hydraulic heads available on nodes 2–7 are 93.57 m, 91.40 m, 90.31 m, 89.65 m, 84.43 m, and 86.42 m, respectively. Hydraulic heads on all nodes are above the
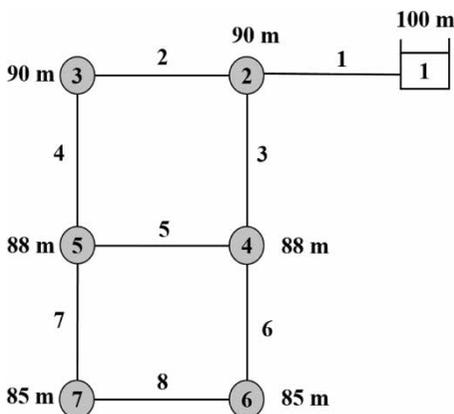
minimum required pressure. Thus, the hydraulic model is applicable under these conditions. Pressure-deficient conditions are created by applying breakage in one of the network pipes (pipe 4). In this research, pipe breakage is equivalent to closure of the relevant pipe in the hydraulic simulation. Demand-driven analysis showed that available hydraulic heads on nodes 2–7 are 93.57 m, 93.08 m, 87.15 m, 83.73 m, 81.85 m, and 81.60 m, respectively. The numerical values indicate negative pressures on nodes 4, 5, 6, and 7 and the system is experiencing pressure-deficient conditions and these methods must be integrated with hydraulic software. In each method, the node elevation is equal to elevation of ARs and they are connected to the desired point through a pipe of large diameter (350 mm) and short length (0.1 m). The procedure of running the model by various methods is shown in Figure 8. Although the results provided by the PDNA, MPDNA, and CRS are almost identical, they differ in the number of hydraulic simulations. For instance, in the PDNA method, for a pressure head of 100 m on the source node, after seven hydraulic model iterations, the demand in the 5th node is about 10.38 L/s; in other words, it is about 42% of total provided demand, and demands on other nodes have been fully satisfied. The MPDNA method is able to find the same result in a single hydraulic simulation. In the CRS method, only one AR is connected to node 5 and the demand of 10.38 L/s on this node could be satisfied and the demand on other nodes has been fully satisfied. The results are completely consistent with those of Ang & Jowitt (2006). The available outflow on each DN for a range of total source pressure
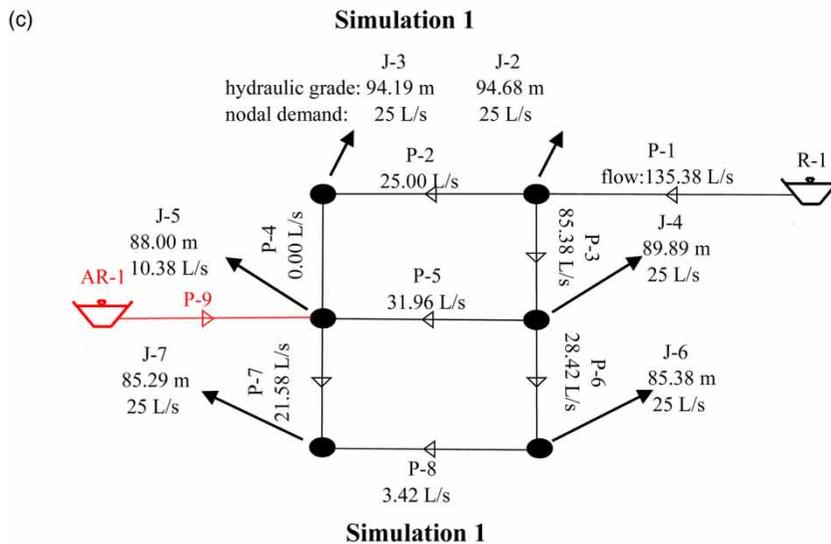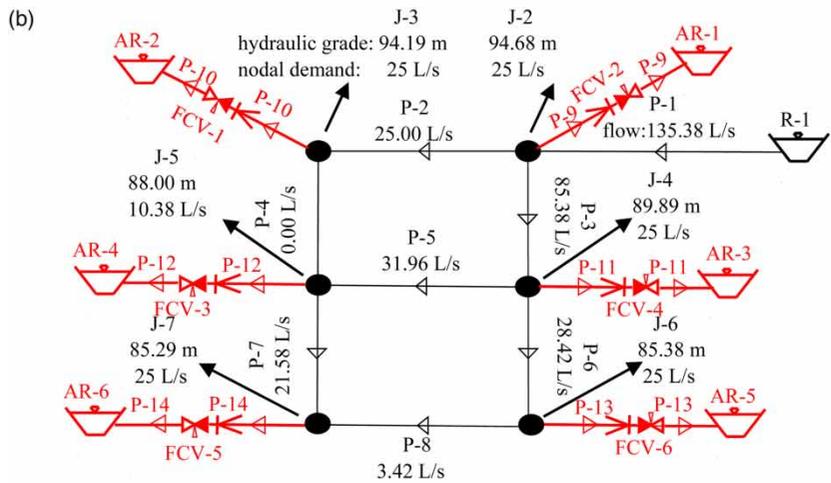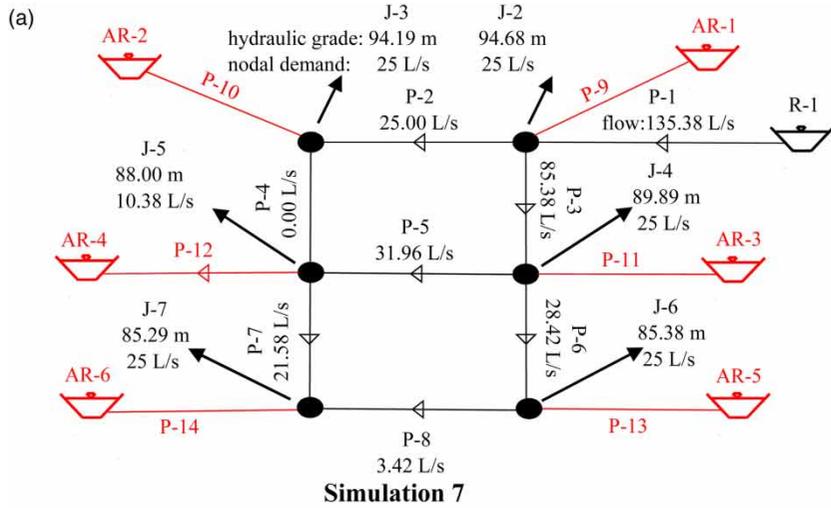

Figure 7 | Looped network diagram.

Figure 8 │ Procedure of the running model with hydraulic model for source pressure head of 100 m (breakage in pipe 4). (a) PDNA, (b) MPDNA and (c) CRS.
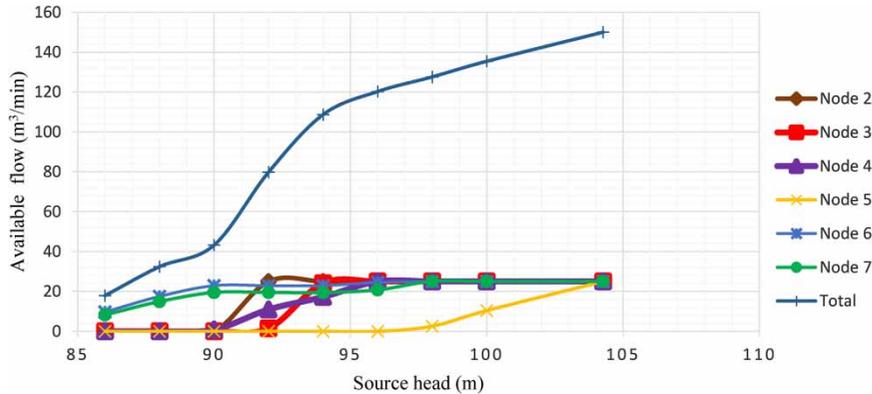
Figure 9 | Sketch of different pressure heads on the source node versus total network outflow.

head (from 86 to 104.27 m) is shown in Figure 9. The results show that if the reservoir elevation is at 104.27 m, the total demand of 150 L/s on all nodes will be provided. It should be noted that for the source elevation of 86–96 m, ARs in the CRS method provide more than the actual need on the DN and the method proposed by Suribabu & Neelakantan (2011) will not work in this case. Thus, the modified CRS method was used. The results of this method are completely consistent with those of PDNA and MPDNA methods.

Third network: large-scale WDN

It is necessary to evaluate the performance of the three methods on a large-scale network. In this study, a part of the Ilam WDN (zone-6) in Ilam province, Iran was considered. A sketch of this network is shown in Figure 10. The network consists of one reservoir, 404 DNs, and 551 pipes. Hydraulic head available in the reservoir is 1,025 m. The elevation and average daily demand of DNs varies from 923 m to
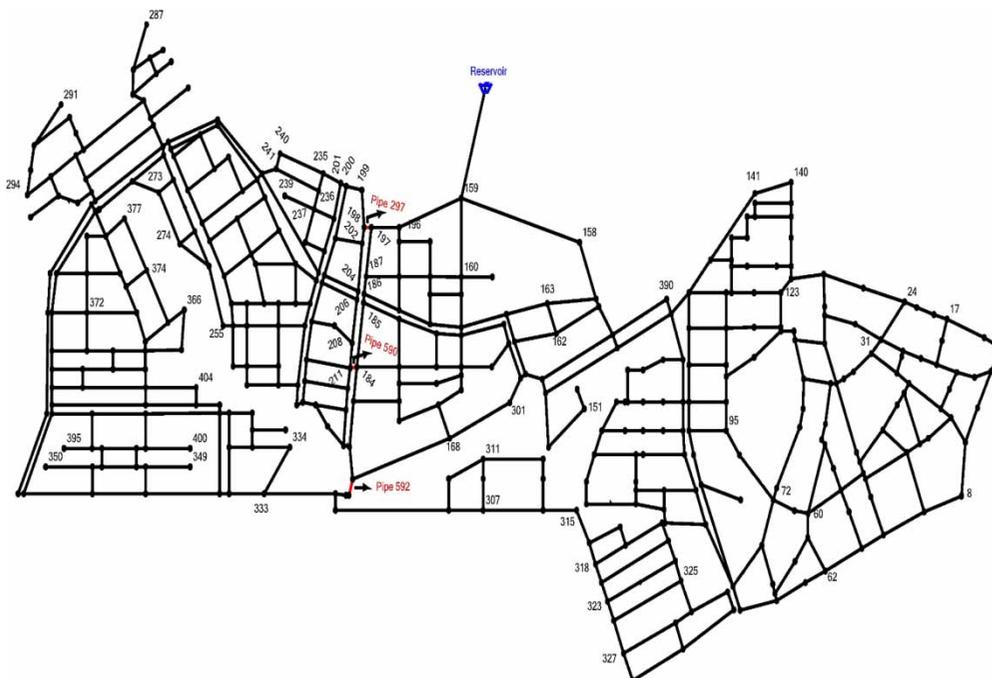

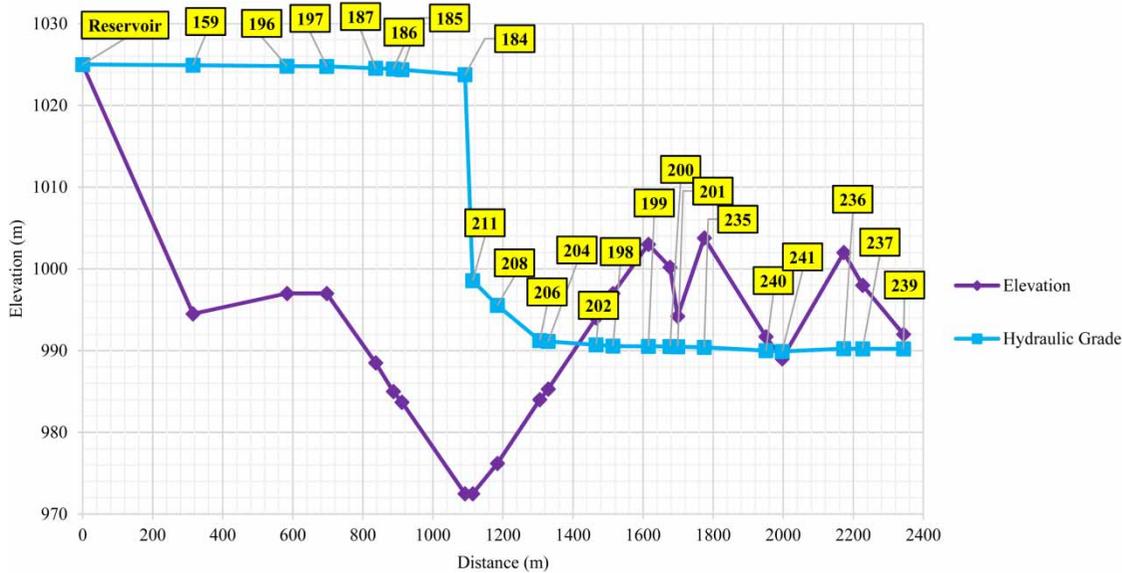**Figure 10** | Sketch of the Ilam WDN (zone-6).

Figure 11 | Longitudinal profile of the reservoir to node 239.

1,003.8 m and 0.035 L/s to 3.373 L/s, respectively. The total demand of the system is 161.431 L/s. The length and diameter of the pipes range from 0.6 m to 468.6 m and 90 mm to 700 mm, respectively. Hazen–Williams coefficient for all pipes ranges from 101 to 140. By running the hydraulic model for network analysis in normal conditions, it has been observed that the pressure on all nodes is above the minimum required pressure. Thus, the hydraulic model is applicable under these conditions. A pressure-deficient condition is modeled by applying breakage in pipes 297, 590, and 592. Demand-driven analysis indicated negative pressures on nodes 198, 199, 200, 201, 202, 203, 235, 236, 237, 239, 240, and 241, and the system experiences pressure-deficient conditions. Analysis of the results showed that the PDNA and MPDNA methods are able to find a solution in 401 and 1 hydraulic simulation iterations, respectively. The CRS method was unable to solve the problem, while the modified CRS method was able to solve this network in 20 hydraulic simulation iterations. Longitudinal profile of the reservoir to node 239 is presented in Figure 11. This figure shows that when breakage occurs in pipes 297, 590, and 592, the hydraulic gradient on nodes 198, 199, 200, 201, 202, 235, 236, 237, 239, and 240 are less than the node elevation, therefore the network will not be able to supply the required demand. The supplied and required outflows in other nodes in the network are equal (Table 1). The demand supplied from the main reservoir to the network for all

Table 1 | The results obtained from the hydraulic modeling of the Ilam WDN during breakage in pipes 297, 590, and 592 by three methods

Node number	Demand (L/s)	Outflow (L/s) (Breakage in pipes 297, 590, and 592)	Hydraulic grade (m)
Reservoir	161.431	159.106	1,025
159	1.701	1.701	1,024.91
196	0.618	0.618	1,023.74
197	0.404	0.404	1,024.37
187	0.183	0.183	1,024.47
186	0.394	0.394	1,024.55
185	0.324	0.324	1,024.80
184	0.247	0.247	1,024.77
211	0.156	0	990.55
208	0.135	0	990.52
206	0.177	0	990.51
204	0.131	0	990.48
202	0.352	0	990.71
198	0.306	0.306	991.13
199	0.308	0.308	991.23
200	0.361	0.361	995.53
201	0.321	0.321	998.56
235	0.174	0	990.40
240	0.308	0	990.23
241	0.526	0	990.21
236	0.220	0	990.21
237	0.144	0	990.01
239	0.333	0.333	989.90

nodes is equal to 159.106 L/s, representing 98.56% of the network demand that is supplied.

CONCLUSIONS

In this study, the combination of hydraulic model with PDNA, MPDNA, and CRS methods was used in normal conditions in order to analyze a WDN under different operating conditions. The results showed that the hydraulic model is able to solve the system operating problems, but in abnormal conditions, such as pipe breakage or providing the required fire-fighting demands on nodes, it must be combined with other formulas and methods. The efficiency of combining these methods with a hydraulic model was evaluated in two base networks and one large-scale network. Critical conditions in networks were applied through breakage in pipes and fire-fighting demands on one node. The results showed that the number of iterations required for achieving the results in the PDNA method are greater than those which are needed for MPDNA and CRS. Furthermore, for some reservoirs' total pressure heads, complementary reservoirs provide more demand than is actually needed on DN and the CRS method will not work in this case. Thus, MPDNA and modified CRS methods are superior to PDNA and CRS methods and could be used for critical operating conditions.

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