New indices for reliability assessment of water distribution networks

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ABSTRACT

This paper presents new indices to evaluate the reliability of water distribution networks (WDNs) usable in the design, planning and management of these networks. Since the pressure-driven analysis (PDA) of WDNs produces more accurate results than the demand-driven analysis, the new indices are proposed based on the PDA. In the proposed measures, nodal pressures, nodal available discharges and the energy loss per unit length of pipes are considered as the main factors influencing the reliability of WDNs. The introduced network reliability index is a combination of two indices named total nodal reliability and total pipe reliability. These indices are equal to the weighted average of all of the nodal and pipe reliabilities, respectively. A sample network is used to evaluate the new proposed index and some of the available indices and to compare their efficiencies in assessing the reliability of WDNs. The results show that the new proposed index is more efficient and outperforms the others. The introduced index is normalized and is independent of the WDN size. This index considers the qualitative aspects of WDNs besides the hydraulic aspects in evaluating reliability.

Key words | energy loss, nodal available discharge, nodal pressure, pressure-driven analysis, reliability index, water distribution networks

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INTRODUCTION

The quality of the services provided by infrastructures such as water distribution networks (WDNs) can be directly related to the reliability of these infrastructures. Reliability is the capability of a WDN to satisfy required water demands under sufficient pressure in normal and abnormal operation circumstances (Tabesh & Zia 2003). Many researchers (e.g., Tanyimboh (1993), Todini (2000), Jayaram (2006), Ghajarnia *et al.* (2009) and Ataoui & Ermini (2014), among others) have studied in this regard and have proposed indices for evaluating the reliability of WDNs.

Su *et al.* (1987) developed a reliability assessment model named the minimum cut set model. They considered the random failure of pipes and calculated the reliability as the overall probability that the WDN would satisfy the required discharges with the minimum required pressure or minimum allowable pressure (MAP) values. Jacobs & Goulter (1991) simplified the minimum cut set method. They considered the probability that a given number of pipes will simultaneously break along with the probability that the failing of a given number of pipes will lead to system failure. The reliabilities presented in the last two mentioned works are known as the WDN mechanical reliability. The other type of reliability is defined as the hydraulic reliability. To evaluate this type of reliability, only the failures resulting from hydraulic causes (such as excessive water demand) are considered (Bao & Mays 1990). Cullinane *et al.* (1992) and Gupta & Bhave (1994) merged the mechanical and hydraulic availabilities in the form of a single reliability index.

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Gheisi & Naser (2014a) applied a multi-criteria decision analysis approach for ranking some alternative distribution layouts of a WDN employing statistical flow entropy and considering various states of reliability (the probability that a WDN satisfies water demands during different states of simultaneous pipe failures). Gheisi & Naser (2014b) investigated the linear correlation between surrogate reliability indices and various states of reliability in WDNs. Resilience index (Todini 2000), network resilience (Prasad & Park 2004) and statistical flow entropy (Tanyimboh & Templeman 2000) were the surrogate reliability indices evaluated in this study. These researchers found that among the mentioned indices, the statistical flow entropy correlates more closely with the various states of reliability. This correlation grows by increasing the state of reliability. Indeed, they conclude that the WDN with higher entropy will perform better during simultaneous pipe breaks.

Ataoui & Ermini (2014) developed an approach for evaluating the overall reliability of WDNs in terms of nodal pressures and the quantity and quality of nodal available discharges using fuzzy technique. They applied demand-driven analysis (DDA), which is the disadvantage of their work. Pressure-driven analysis (PDA) that considers the pressure dependency of nodal discharges provides more realistic results than the DDA (Shirzad *et al.* 2013). Although some researchers (such as Gupta & Bhave (1994), Tabesh (1998) and Tanyimboh *et al.* (2003)) have considered the pressure dependency of nodal discharges in evaluating reliability, most of the studies have proposed a reliability index based on the DDA.

This paper introduces a reliability index based on PDA. The proposed index evaluates the WDN reliability to ensure a suitable design of these networks and establish operational and rehabilitation schedules for existing WDNs. This index is a combination of total nodal reliability index and total pipe reliability index. According to the proposed reliability index, nodal pressures, nodal available discharges and the energy loss per unit length (ELUL) of pipes are the most important factors influencing the reliability of WDNs. In this index, unlike most of the available reliability indices, in addition to pressure deficit, pressure excess due to its influence on increasing pipe breaks, leakage and water consumption, is also considered as a vital factor that decreases the WDN reliability. A sample network is used to compare the new proposed index together with some of the available indices and their efficiencies in assessing the reliability of WDNs.

CONCEPT AND THEORY

Since pipe breaks occur frequently in WDNs, the networks that are more reliable should also satisfy water demands in the case of pipe breaks. The problem that arises in the case of considering pipe breaks in calculating the reliability index (such as the index proposed by Tanyimboh (1993)) is the heavy iterative computations required for analysing the WDN in pipe break conditions and evaluating the reliability. Employing such reliability indices in the WDN optimization models would be extremely time-consuming and computationally demanding. Therefore, it is necessary to introduce a new index with a lower computational volume that is capable of evaluating the reliability and performance level of the WDN in the condition of pipe breaks.

The other important point is that pressure deficiency is not the only factor that decreases the WDN reliability, since pressure excess, because of its impact on increasing pipe breaks, leakage and water consumption, is also a vital factor in decreasing the WDN reliability. This point should also be considered in developing the reliability index.

Figure 1 shows the sample network used to demonstrate the theory and to evaluate the new reliability index proposed in this study. This network consists of 1 source node (reservoir), 8 demand nodes and 12 pipes in 4 loops (Tabesh 1998).

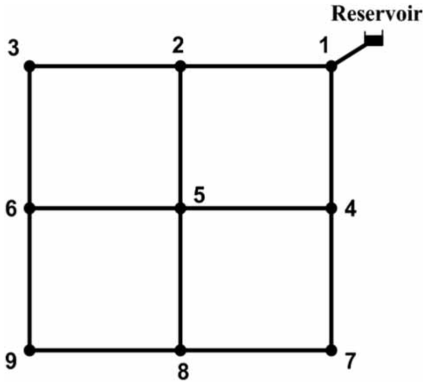

Figure 1 | The sample network (Tabesh 1998).

Table 1 | Characteristics of the pipes and nodes in the sample network (Tabesh 1998)

Pipe information				**Node information**			
Pipe	**Diameter (mm)**	**C_{HW} (−)**	**Length (m)**	**Node**	**H_j^{min} (m)**	**H_j^{req} (m)**	**Q_j^{req} (m³/s)**
1–2, 1–4	250	130	1,000	1 (reservoir)	–	100	0.2081
2–3, 4–7	175	130	1,000	2, 4	0	30	−0.0208
2–5, 4–5	145	130	1,000	3, 7	0	30	−0.0208
3–6, 7–8	115	130	1,000	5	0	30	−0.0208
5–6, 5–8	100	130	1,000	6, 8	0	30	−0.0208
6–9, 8–9	100	130	1,000	9	0	30	−0.0625

Table 1 shows the general characteristics of the sample network.

In the case of a pipe break in one of the loops of a WDN, the water demanded at consumption nodes fed by the broken pipe will inevitably travel a longer distance to reach the consumption point. For instance, in the sample network (Figure 1), the water demand at node 2 is supplied through pipe 1–2. When pipe 1–2 is broken, the water demand at node 2 will be supplied through pipes 1–4, 4–5 and 2–5. Indeed, in this condition, the travelled distance is longer and excessive energy will be lost. Therefore, in the case of pipe 1–2 breakage, the sample network would be able to satisfy the water demand at node 2, if the network could compensate for the energy loss of pipes 1–4, 4–5 and 2–5. Thus, if the nodal pressures in normal operational conditions are in the allowable range and the energy loss in the pipes is low and reasonable, the WDN will be able to compensate for the excessive energy losses and satisfy water demands in the condition of pipe breaks.

Calculations were performed to determine the appropriate and reasonable values of the energy loss in pipes. In these calculations, the velocities and ELULs of a pipe with a Hazen–Williams coefficient (CHW) of 120 and different diameters (75, 80, 100, 150, 200, 250, 300, 350, 400 and 500 mm) have been calculated for different values of discharge in the pipe. The Hazen–Williams formula is used for determining ELUL. As mentioned previously, ELUL is the energy loss per 1 km of pipe length. The lower bound of discharge in the pipe is considered equal to 2 L/s; it is gradually increased to 309 L/s. Tables 2 and 3 show the calculation results. In Table 2, bold values are ELULs in the range of 1–5 m/km. For example, when pipe discharge is 2 L/s, ELUL values for diameters of 75 (two sizes smaller), 80 (one size smaller), 100 (selected size), 150 (one size larger) and 200 mm (two sizes larger) are 4.55, 3.32, 1.12, 0.16 and 0.04 m/km, respectively. ELUL values of 1.12, 3.32 and 4.55 m/km which are in the range of 1–5 m/km are bold. According to Tables 2 and 3, the ELUL values in the range of 1–5 m/km are reasonable and appropriate for design of WDNs, because velocity value for ELULs of 1–5 m/km will be almost in the range of 0.3–2 m/s (i.e., the standard range for flow velocity), but it is not true inversely. In other words, for a given value of velocity in pipes with different diameters, ELUL has different values. On the other hand, by increasing the pipe diameter relative to the selected diameter, in addition to the increase in the network construction cost, the ELUL will decrease which can lead to water quality reduction due to the decrease in velocity. Furthermore, by decreasing the pipe diameter, the ELUL will increase too much so that pressure deficiency will occur in the WDN, even though according to Table 3 the pipe velocity is in the allowable range.

In fact, the ELUL of 1–5 m/km assures the designer that the velocities in pipes are in the allowable range as well as the issues of selecting wrong and uneconomical diameters are prevented. For instance, the selected diameter for the discharge of 14 L/s is 200 mm and the ELUL is 1.41 m/km. In the case of selecting smaller diameters, the ELULs for diameters of 150 and 100 mm will be equal to 5.71 and 41.17 m/km, respectively. It means that by decreasing the pipe diameter the ELUL increases intensively. Also, in the case of selecting larger diameters, the ELULs for diameters of 250 and 300 mm will be equal to 0.47 and

Table 2 | Comparison of the ELUL in pipes of the WDN with various parameters

		ELUL (m/km)				
Pipe diameter (mm)	**Discharge range (L/s)**	**Two sizes smaller**	**One size smaller**	**Selected size**	**One size larger**	**Two sizes larger**
100	2	**4.55**	**3.32**	**1.12**	0.16	0.04
	4	16.42	11.99	**4.05**	0.56	0.14
150	5	18.13	6.12	**0.85**	0.21	0.07
	13	106.39	35.89	**4.98**	**1.23**	0.41
200	14	41.17	5.71	**1.41**	0.47	0.20
	27	138.94	19.29	**4.75**	**1.60**	0.66
250	28	20.63	5.08	**1.71**	0.71	0.33
	49	58.16	14.33	**4.83**	**1.99**	0.94
300	50	14.87	5.02	**2.06**	0.97	0.51
	80	35.52	11.98	**4.93**	**2.33**	**1.21**
350	81	12.26	5.05	**2.38**	**1.24**	0.42
	120	25.39	10.45	**4.93**	2.57	0.87
400	121	10.61	5.01	**2.61**	0.88	0.36
	172	20.35	9.61	**5.01**	**1.69**	0.70
500	173	9.71	5.07	**1.71**	0.70	0.33
	309	28.43	14.84	**5.00**	**2.06**	0.97

Table 3 | Comparison of velocity in pipes of the WDN with various diameters

		Flow velocity in pipe (m/s)				
Pipe diameter (mm)	**Discharge range (L/s)**	**Two sizes smaller**	**One size smaller**	**Selected size**	**One size larger**	**Two sizes larger**
100	2	0.45	0.40	0.25	0.11	0.06
	4	0.91	0.80	0.51	0.23	0.13
150	5	1.00	0.64	0.28	0.16	0.10
	13	2.59	1.66	0.74	0.41	0.26
200	14	1.78	0.79	0.45	0.29	0.20
	27	3.44	1.53	0.86	0.55	0.38
250	28	1.59	0.89	0.57	0.40	0.29
	49	2.77	1.56	1.00	0.69	0.51
300	50	1.59	1.02	0.71	0.52	0.40
	80	2.55	1.63	1.13	0.83	0.64
350	81	1.65	1.15	0.84	0.64	0.41
	120	2.45	1.70	1.25	0.96	0.61
400	121	1.71	1.26	0.96	0.62	0.43
	172	2.43	1.79	1.37	0.88	0.61
500	173	1.80	1.38	0.88	0.61	0.45
	309	3.21	2.46	1.57	1.09	0.80

0.2 m/km and the velocities for these diameters will be equal to 0.29 and 0.20 m/s, respectively. It means that although the increase in pipe diameter causes an increase in the WDN construction cost, the hydraulic condition of the WDN does not improve. In addition, the water quality may be reduced due to the decrease in velocity. Indeed, the selected diameters shown in Table 1 are the smallest economical diameters, and by selecting them excessive costs will be prevented and the WDN will have an appropriate quality and hydraulic condition.

Nodal pressures in normal operation conditions should be in the allowable range (30–50 m) and it is better to be near the MAP (30 m) as much as possible. According to the guidelines for the design of WDNs (IRIVPSPS 2011), the MAP in WDNs designed to serve four-floor buildings is 26 m. However, in this study, similar to most published papers, the MAP is considered to be 30 m. This value enables the WDN to also serve five-floor buildings.

In normal operation conditions, existence of some excessive reserve nodal pressure (ERNP) is necessary, because for satisfying nodal water demands in the case of pipe breakage, excessive energy losses due to the long distance passed by water should be compensated. In this study, supposing 2 km excessive passed distance and 2.5 m ELUL, 5 m ERNP is considered to compensate for the excessive energy losses in pipe break conditions. Indeed, if nodal pressures in normal conditions are equal to 35 m, in the case of pipe breaks and then 5 m decrease in nodal pressures to compensate for the excessive energy loss, 30 m nodal pressure will remain. Consequently, according to the pressure–discharge relation in the PDA, all of the required nodal discharge will be available. Then, the nodal reliability will be 1 (100%). With the decrease in nodal pressure, the nodal reliability decreases proportional to the per cent of nodal available discharge. The nodal reliability for pressures higher than 35 m will also decrease because of the increase in leakage, pipe breaks and water consumption.

The PDA is accomplished using the hydraulic analysis model proposed by Tabesh *et al.* (2014). They have developed this PDA-based model based on the gradient method. The pressure–discharge relation used in the PDA is shown in Equation (1) (Shirzad *et al.* 2013):

$$Q_j^{avl} = \begin{cases} 0 & \text{if} \quad P_j \leq 0 \\ 0.176(Q_j^{req} \times P_j^{0.51}) & \text{if} \quad 0 < P_j \leq 30\,\text{m} \\ Q_j^{req}\,(0.5 + 0.0882 P_j^{0.51}) & \text{if} \quad 30 < P_j \leq 100\,\text{m} \\ 1.424\; Q_j^{req} & \text{if} \quad P_j > 100\,\text{m} \end{cases} \tag{1}$$

in which Q_j^{avl} is the available discharge at node j, Q_j^{req} is the required discharge at node j and P_j is the pressure at node j.

RESULTS AND DISCUSSION

This paper introduces a new reliability index based on the theory and concept section. The proposed index (Equation (2)) is a combination of two indices called total nodal reliability and total pipe reliability. The total nodal reliability (Equation (3)), which is an indicator of nodal pressures, is equal to the weighted average of all of the nodal reliabilities (Equation (5)). The weight of each node is considered equal to the required discharge at that node. The total pipe reliability (Equation (4)), which is an indicator of the energy loss in pipes, is equal to the weighted average of all of the pipe reliabilities (Equation (6)). The weight of each pipe is considered equal to the length of pipe. The proposed index is written as below:

$$Re_N = \begin{cases} TRe_{Node} \times TRe_{Pipe} & \text{if} \quad NJ_{35} < \dfrac{NJ}{2} \\ \sqrt{TRe_{Node} \times TRe_{Pipe}} & \text{if} \quad NJ_{35} \geq \dfrac{NJ}{2} \end{cases} \tag{2}$$

$$TRe_{Node} = \frac{\sum_{j=1}^{NJ} (Re_{Nodej} \times Q_j^{req})}{\sum_{j=1}^{NJ} Q_j^{req}} \tag{3}$$

$$TRe_{Pipe} = \frac{\sum_{i=1}^{NL} (Re_{Pipei} \times L_i)}{\sum_{i=1}^{NL} L_i} \tag{4}$$

$$Re_{Nodej} = \begin{cases} 0 & \text{if} \quad P_j \leq 5 \\ \left(\dfrac{P_j - 5}{30}\right)^{0.51} & \text{if} \quad 5 < P_j \leq 35 \\ 1 - \dfrac{P_j - 35}{30} & \text{if} \quad 35 < P_j \leq 50 \\ 0.25 & \text{if} \quad P_j > 50 \end{cases} \tag{5}$$

$$Re_{Pipei} = \begin{cases} h_{f_i} & \text{if}\, h_{f_i} < 1 \\ 1 & \text{if}\, 1 \leq h_{f_i} \leq 5 \\ \left(\dfrac{30 - \dfrac{h_{f_i} \times L_i}{1,000}}{30}\right)^{0.51} & \text{if} \quad 5 < h_{f_i} \leq \dfrac{30,000}{L_i}, \quad h_{f_i} = \dfrac{10,680 \times Q_i^{1.852}}{D_i^{4.87} \times C_i^{1.852}} \\ 0 & \text{if}\, h_{f_i} > \dfrac{30,000}{L_i} \end{cases} \tag{6}$$

in which Re_N is the network reliability, TRe_{Node} is the total nodal reliability, TRe_{Pipe} is the total pipe reliability, Re_{Pipe_i} is the reliability of pipe i, Re_{Node_j} is the reliability of node j, NJ_{35} is the number of nodes with pressure equal to or higher than 35 m, NL and NJ are the total number of pipes and nodes, respectively, P_j is the pressure at node j, Q_j^{req} is the required discharge at node j, L_i is the length of pipe i, D_i is the diameter of pipe i, C_i is the CHW of pipe i, Q_i is the discharge of pipe i and h_{f_i} is the ELUL of pipe i.

Similar to the work of Tabesh & Zia (2003), five levels of service are considered for the reliability of WDNs (Figure 2). These levels of service are: no service (reliability = 0), the unacceptable level of service (reliability = 0.25), the acceptable level of service (reliability = 0.50), the fair level of service (reliability = 0.75) and the excellent level of service (reliability = 1).

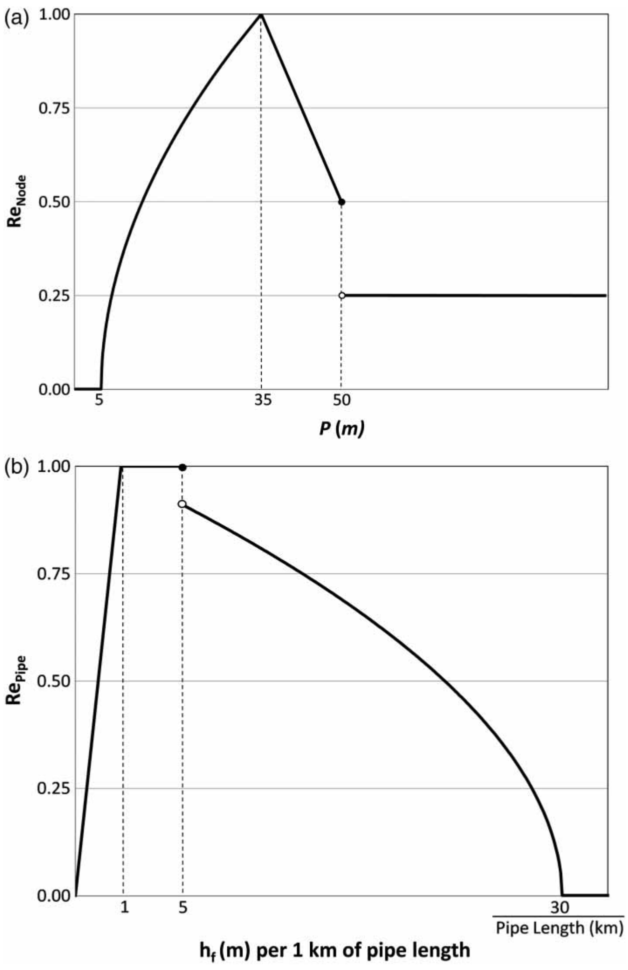

Figure 2 | Diagram of nodal and pipe reliability variations.

According to Figure 2(a), the nodal reliability for the nodal pressure of 35 m (MAP + ERNP) is 1 (the excellent level of service). The nodal reliability for nodal pressures lower than 35 m is equal to the ratio of the nodal available discharge to the nodal required discharge (Q_j^{avl}/Q_j^{req}) based on the pressure–discharge relation proposed by Shirzad *et al.* (2013). The nodal reliability for pressures lower than 5 m is 0 (no service), because nodal pressure of 5 m is served compensating for the excessive energy loss in pipe break conditions. Then, the residual nodal pressure and consequently the nodal available discharge would be 0 (no service). The nodal reliability for the maximum allowable pressure (MAAP) (50 m) is considered equal to 0.5 (the acceptable level of service). Although the pressure of 50 m is allowable, due to its impact on increasing pipe breaks, leakage and water consumption, it is an undesirable value. Pressures higher than 50 m are unacceptable, because pipe break, leakage and water consumption increase intensively at such pressures. Thus, the nodal reliability is considered as 0.25 (the unacceptable level of service).

According to Figure 2(b), the pipe reliability for ELULs in the range of 1–5 m/km is 1. It decreases linearly from 1 to 0 for ELULs lower than 1 m. Pipe reliability for the ELULs higher than 5 m decreases proportional to the decrease in Q_j^{avl}/Q_j^{req} due to the increase in pipe energy loss and the decrease in the nodal pressure. Indeed, the pipe reliability for ELULs higher than 5 m is equal to the ratio of the available discharge at the node at the end of the pipe at a pressure of 30 m minus energy loss to the available discharge at that node at a pressure of 30 m

$$\left(Re_{Pipe_i} = \frac{\frac{Q^{req}}{5.6670}(30-\text{headloss})^{0.51}}{\frac{Q^{req}}{5.6670}(30)^{0.51}} = \left(\frac{30-\text{headloss}}{30}\right)^{0.51} = \left(\frac{30-\frac{h_{f_i}\times L_i}{1,000}}{30}\right)^{0.51} \right)$$

. If the energy loss in the pipe is equal to or higher than 30 m, the residual pressure at the node at the end of the pipe will be equal to or lower than 0 m. Thus, the available discharge at that node will be 0 L/s and, consequently, the pipe reliability will be 0.

According to Equation (2), the network reliability is a function of the total nodal and total pipe reliabilities. If less than

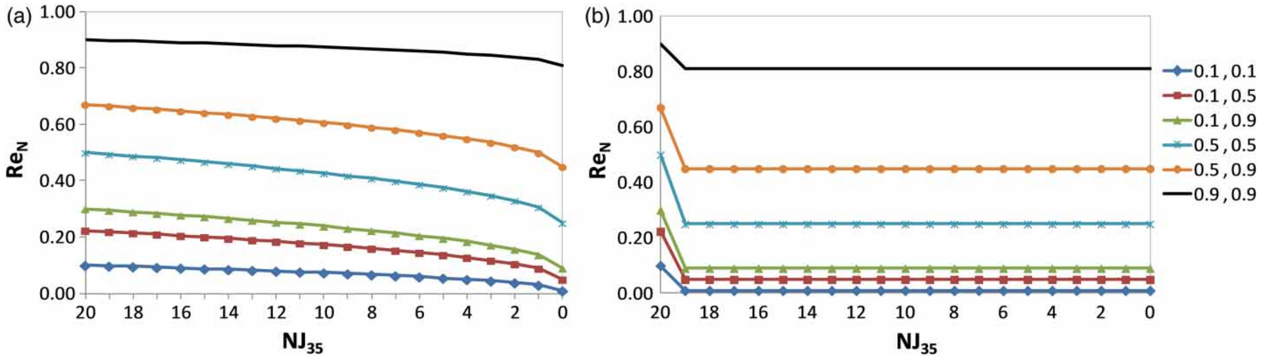

Figure 3 | Reliability variation of a WDN with 40 demand nodes.

Table 4 | The evaluated reliability indices

Index	**Researcher**	**Formula**
Nodal reliability index	Tanyimboh (1993)	$\left\{\begin{array}{l} R_j = P(0)\sum_{M=0}^{NP} r_j(M)\prod_{l=1}^{M}\frac{UA_l}{A_l} \\ \qquad = P(0)\,r_j(0) + \sum_{l=1}^{NP} P(l)r_j(l) + \sum_{l=1,m\neq l}^{NP} P(l,m)\,r_j(l,m) + \ldots \\ r_j(M) = \frac{Q_j^{avl}(M)}{Q_j^{req}}, \quad M = 1,\ \ldots,\ NP;\ \nabla j \\ P(0) = \prod_{l=1}^{NP} A_l, \quad P(M) = P(0)\prod_{l=1}^{M}\frac{UA_l}{A_l}, \quad UA_l = 1 - A_l \end{array}\right\}$
Resilience index	Todini (2000)	$\left\{ I_r = \frac{\sum_{j=1}^{NN} q_j^{req}(h_j - h_j^{des})}{[\sum_{r=1}^{NR} q_r h_r + \sum_{pu=1}^{NPU}\frac{P_{pu}}{\gamma} - \sum_{j=1}^{NN} q_j^{req} h_j^{des}]} \right\}$
Modified resilience index	Jayaram (2006)	$\left\{ MI_r = \frac{\sum_{j=1}^{NN} q_j^{req}(h_j - h_j^{des})}{\sum_{j=1}^{NN} q_j^{req} h_j^{des}} * 100\% \right\}$
Fuzzy reliability index	Ghajarnia *et al.* (2009)	$\left\{\begin{array}{l} FRI_j = MemF_j \times\ C_j^1 \times C_j^2 \\ MemF_j = \begin{cases} 0 & \text{if} \quad H_j \leq H_j^{des} \\ \frac{2}{H_j^{\max} - H_j^{des}}(H_j - H_j^{des}) & \text{if} \quad H_j^{des} < H_j \leq \frac{H_j^{des} + H_j^{\max}}{2} \\ \frac{2}{H_j^{des} - H_j^{\max}}(H_j - H_j^{\max}) & \text{if} \quad \frac{H_j^{des} + H_j^{\max}}{2} < H_j \leq H_j^{\max} \\ 0 & \text{if} \quad H_j > H_j^{\max} \end{cases} \\ C_j^1 = 1 - \frac{q_j^{req}}{\sum_{j=1}^{NJ} q_j^{req}}, \quad C_j^2 = \frac{\sum_{i=1}^{NP_j} D_{ij}}{NP_j \times D_{Max_j}} \end{array}\right\}$

50% of the demand nodes have pressures less than 35 m, the network reliability will be equal to the product of the total nodal and total pipe reliabilities. Otherwise, it will be equal to the geometric average of them. The reason for this difference is that in the case of low total nodal reliability due to nodal pressure deficiency, the WDN will be unable to completely satisfy the nodal demands in both the normal and abnormal (such as pipe breaks) operational conditions. However, if the low total nodal reliability is due to nodal pressure excess, then in the abnormal condition of pipe breaks, a proportion of the excessive pressure will be consumed to compensate for the excessive energy loss and the pressure excess will decrease. In other words, the total nodal reliability increases in the abnormal condition of pipe breaks. Indeed, the reliability index introduced in this paper, in addition to hydraulic aspects, reflects the qualitative and executive aspects of WDNs, so that a nodal reliability of 1 means the nodal pressure is 35 m and the nodal demand is completely satisfied. A pipe reliability of 1 means that the energy loss is in the range of 1–5 m/km, which indicates that the economical pipe size is selected in the design process and the velocity in the pipe is in the allowable range.

In the case of using Equation (2), the network reliability has a jump on the boundary between ($NJ_{35} = NJ/2 - 1$) and ($NJ_{35} = NJ/2$). To remove this weakness, a modification factor is added to the network reliability values ($NJ_{35} < NJ/2$). Thus, Equation (2) is modified as below:

$$Re_N = \begin{cases} (TRe_{Node} \times TRe_{Pipe}) + MF & \text{if} \quad NJ_{35} < \dfrac{NJ}{2} \\ \sqrt{TRe_{Node} \times TRe_{Pipe}} & \text{if} \quad NJ_{35} \geq \dfrac{NJ}{2} \end{cases} \tag{7}$$

$$MF = (TRe_{Node} \times TRe_{Pipe}) + \left(\frac{NJ_{35}}{\frac{NJ}{2}} \right) \times \left[\sqrt{TRe_{Node} \times TRe_{Pipe}} - (TRe_{Node} \times TRe_{Pipe}) \right] \tag{8}$$

In fact, according to Equation (7) in the case of ($NJ_{35} < NJ/2$), the network reliability will decrease proportionally to the number of nodes with pressures less than 35 m. In other words, the more nodes with pressures less than 35 m, the more the network reliability decreases. For instance, Figure 3 shows the variation of network reliability

Table 5 | PDA results of the sample network for the different values of the reservoir head

	$H_r = 30$ m		$H_r = 70$ m		$H_r = 100$ m		$H_r = 150$ m		$H_r = 200$ m	
Pipe no.	**Q_l (L/s)**	**ELUL (m)**	**Q_l (L/s)**	**ELUL (m)**	**Q_l (L/s)**	**ELUL (m)**	**Q_l (L/s)**	**ELUL (m)**	**Q_l (L/s)**	**ELUL (m)**
1–2, 1–4	58.777	5.84	86.206	11.86	99.730	15.54	114.597	20.10	121.617	22.44
2–3, 4–7	25.319	6.97	38.582	15.20	45.094	20.29	53.409	27.76	57.609	31.94
2–5, 4–5	14.832	6.47	22.657	14.18	26.611	19.09	31.570	26.20	34.391	30.70
3–6, 7–8	9.660	9.04	15.701	22.22	19.375	32.80	23.790	47.98	27.990	64.84
5–6, 5–8	6.887	9.54	11.141	23.25	13.679	34.00	16.761	49.54	19.581	66.08
6–9, 8–9	5.843	7.04	9.624	17.73	12.019	26.76	16.100	45.98	19.942	68.35
Node no.	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**
2, 4	24.16	18.627	58.14	24.967	84.46	28.024	129.90	29.618	177.56	29.618
3, 7	17.20	15.659	42.93	22.881	64.17	25.720	102.14	29.618	145.62	29.618
5	17.70	15.889	43.96	23.033	65.37	25.865	103.70	29.618	146.86	29.618
6, 8	8.16	10.704	20.71	17.218	31.36	21.035	54.16	24.451	80.78	27.630
9	1.12	11.685	2.98	19.248	4.61	24.038	8.17	32.201	12.44	39.884

for different values of the total nodal and total pipe reliabilities and different numbers of demand nodes with pressures less than 35 m in a hypothetical WDN with 40 demand nodes. The curves shown in this figure are relevant to the different combinations of the total nodal and total pipe reliabilities ((0.1, 0.1), (0.1, 0.5), (0.1, 0.9), (0.5, 0.5), (0.5, 0.9) and (0.9, 0.9)). Figure 3(a) and 3(b) are based on Equations (7) and (2), respectively. According to Figure 3(a), the network reliability calculated by Equation (7) varies gradually and softly without a jump on the boundary between ($NJ_{35} = NJ/2 - 1$) and ($NJ_{35} = NJ/2$).

Table 4 shows the available reliability indices evaluated in this study using the sample network (Figure 1). Implementing the PDA, the sample network is analysed for the different values of the reservoir head. The results are shown in Table 5. Table 6 also shows the reliability values of the sample network based on various indices.

According to Table 6, the increase in the reservoir head and consequently the increase in nodal pressures up to values higher than the MAAP leads to the increase in the reliability index proposed by Tanyimboh (1993). This is in contradiction to the fact that the higher the nodal pressures, the more pipe breaks and leakage. Indeed, in this index, the bad influences of excessive pressure increases on the WDN have not been taken into consideration. The resilience index proposed by Todini (2000) and the reliability index proposed by Jayaram (2006) have negative values for some cases. These indices increase by the increase in nodal pressures up to values higher than the MAAP. These instances indicate the deficiency of these indices. The index proposed by Ghajarnia (2009) is the only one among the available reliability indices assessed in this paper that considers the excessive pressure increases as a factor decreasing the reliability. This index is superior to the others from this point of view. Furthermore, none of these indices are normalized and this is one of their deficiencies. Indeed, it is better to use normalized reliability indices (i.e., the indices have values between 0 and 1) for comparing the reliability of WDNs in different sizes (i.e., the WDNs have different total numbers of demand nodes). The other point about these indices is that the index proposed by Tanyimboh (1993) and Tayimboh *et al.* (2001) is the only one that considers pipe breaks when evaluating WDN reliability.

According to Tables 5 and 6, the total nodal reliability for the reservoir head of 30 m is low and equal to 0.4132, because the nodal pressures are lower than the MAP. However, the total pipe reliability is high and equal to 0.8635 due to the relatively appropriate values of the energy loss in pipes. In this case, the network reliability is equal to the product of the total nodal and total pipe reliabilities ($0.4132 \times 0.8635 = 0.3568$), which is proportional to the unacceptable/acceptable levels of service. This means that the WDN is unable to satisfy the nodal demands completely during the normal operation condition and thus it is also unable to perform perfectly during abnormal conditions (such as pipe breakage). Therefore, WDNs like this would have low reliability.

The total nodal reliability for the reservoir head of 70 m is also low (0.4108), because the pressures at nodes 2, 3, 4, 5 and 7 increased and are near the MAAP, even though pressures at nodes 6, 8 and 9 are reaching near to the MAP. In this case, the total pipe reliability decreases to 0.6327 due to the increase in the energy loss in pipes and getting distant from the allowable range (1–5 m/km). Since during the abnormal condition of pipe breaks, the excessive pressures

Table 6 | Reliability of the sample network for the different values of the reservoir head

						The new proposed index		
Case no.	**H_r (m)**	**Tanyimboh (1993)**	**Todini (2000)**	**Jayaram (2006)**	**Ghajarnia *et al.* (2009)**	**TRe_{Node}**	**TRe_{Pipe}**	**Re_N**
1	30	0.5646	1.3787	−0.5999	0.0587	0.4132	0.8635	0.3568
2	70	0.8281	−0.0131	−0.0122	2.2014	0.4108	0.6327	0.5098
3	100	0.9580	0.2111	0.4633	0.2065	0.3121	0.3617	0.3360
4	150	1.1008	0.2961	1.3344	0.6888	0.2704	0.1971	0.2309
5	200	1.1682	0.3394	2.3056	0.0507	0.3224	0.0825	0.1631

Table 7 | The PDA results of the sample network for the different values of pipe diameters

	D = 100 mm		D = 145 mm		D = 175 mm		D = 250 mm		D = 300 mm	
Pipe no.	**Q_i (L/s)**	**ELUL (m)**	**Q_i (L/s)**	**ELUL (m)**	**Q_i (L/s)**	**ELUL (m)**	**Q_i (L/s)**	**ELUL (m)**	**Q_i (L/s)**	**ELUL (m)**
1–2, 1–4	23.248	90.80	56.978	78.21	85.575	66.48	131.271	25.85	140.620	12.08
2–3, 4–7	6.163	7.77	20.636	11.92	33.711	11.84	54.809	5.13	58.930	2.41
2–5, 4–5	5.704	6.73	18.672	9.91	30.463	9.82	49.569	4.26	53.301	2.00
3–6, 7–8	1.753	0.76	8.838	2.48	16.085	3.01	28.509	1.53	30.795	0.73
5–6, 5–8	2.793	1.79	12.187	4.50	21.240	5.03	36.368	2.40	39.212	1.13
6–9, 8–9	1.539	0.59	10.846	3.62	20.992	4.92	38.757	2.70	41.949	1.29
Node no.	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**	**H_j (m)**	**Q_j^{avl} (L/s)**
2, 4	9.20	11.382	21.79	17.669	33.52	21.401	74.15	26.892	87.92	28.389
3, 7	1.43	4.410	9.87	11.798	21.68	17.626	69.02	26.300	85.50	28.135
5	2.47	5.822	11.88	12.971	23.71	18.447	69.89	26.402	85.91	28.178
6, 8	0.68	3.007	7.39	10.179	18.68	16.333	67.50	26.120	84.78	28.058
9	0.08	3.078	3.77	21.692	13.75	41.984	64.80	77.514	83.49	83.898

Table 8 | The reliability of the sample network with a reservoir head of 100 m and different pipe diameters

Case no.	Diameter (mm)	TRe_{Node}	TRe_{Pipe}	Re_N
6	100	0.0733	0.6813	0.0500
7	145	0.3300	0.7646	0.2523
8	175	0.7157	0.7503	0.5370
9	250	0.2500	0.8789	0.4688
10	300	0.2500	0.9157	0.4785

at nodes 2, 3, 4, 5 and 7 decrease slightly and the pressures at these nodes reach near 35 m due to the excessive energy loss in pipes, the WDN will perform better during the abnormal condition of pipe breaks. In other words, the network reliability for the reservoir head of 70 m is increased relative to that for the reservoir head of 30 m and is 0.5098 (the acceptable/fair level of service).

By increasing the reservoir head to 100, 150 and 200 m, the nodal pressures increase and exceed 35 m more and more. Energy losses in pipes increase too. Thus, the total nodal reliability, total pipe reliability and, consequently, the network reliability decrease. Since $NJ_{35} \geq NJ/2$ for the reservoir heads mentioned above, the network reliabilities calculated based on Equations (2) and (7) are the same.

For more evaluation of the reliability index proposed in this paper, the sample network is analysed with a reservoir head of 100 m and different values of pipe diameters (100, 145, 175, 250 and 300 mm). Tables 7 and 8 show the PDA results and reliability values, respectively.

According to Tables 7 and 8, by increasing the pipe diameters from 100 mm to 175 mm, the nodal pressures increase due to the decrease in the pipe energy losses. Thus, the total nodal reliability, total pipe reliability and, consequently, the network reliability increase. By increasing the pipe diameters to 250 and 300 mm, the nodal pressures exceed the MAAP and the total nodal reliability decreases. In addition, pipe energy losses improve and the total pipe reliability increases. Since the decrease in the total nodal reliability is more than the increase in the total pipe reliability, the network reliability is also decreased. Since $NJ_{35} \geq NJ/2$, the network reliabilities calculated based on Equations (2) and (7) are the same.

According to the obtained results, the new index proposed in this paper can be used for evaluating the reliability, performance and efficiency of WDNs appropriately.

CONCLUSION

In this paper, a new index is introduced for evaluating the reliability of WDNs based on the PDA. This index is a combination of two indices called the total nodal reliability and the total pipe reliability. According to the proposed index, nodal pressures cannot be the sole indicator of WDN reliability, since energy loss and other characteristics of pipes also influence the performance and efficiency of WDNs, especially during abnormal operation conditions (such as pipe breaks). Some of the advantages of the reliability index proposed in this paper are as follows:

- Since the PDA produces more realistic results than the DDA, the new proposed index based on the PDA is more appropriate for evaluating the reliability of WDNs.
- This reliability index is normalized ($0 \leq \mathrm{Re}_N \leq 1$) and is independent of the WDN size (the number of demand nodes). Thus, this index can be used to compare the reliability of different WDNs correctly.
- The proposed index can also be an indicator of the WDN performance during the critical condition of pipe breaks. Indeed, this index obviates the need of employing other reliability indices (such as the one proposed by Tanyimboh (1993)). Therefore, to evaluate the WDN reliability, there is no need to implement iterative calculations and time-consuming hydraulic analyses for different states of pipe breaks. In other words, after a single run of the PDA-based model for any simple or real large WDN and determining the hydraulic parameters such as nodal pressures and pipe flows, the network reliability can be easily calculated.
- This index considers both the qualitative and hydraulic aspects of WDNs in evaluating reliability.

In this study, steady-state hydraulic analysis was implemented. In future studies, an extended period analysis of WDNs can be considered to improve the reliability index.

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