Serviceability assessment and sensitivity analysis of cast iron water pipes under time-dependent deterioration using stochastic approaches

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ABSTRACT

Determination of the depth of the corrosion pit is essential to predict the serviceability of corrosion affected pipes and to instigate maintenance and repairs for the pipeline system. In this paper, a reliability-based methodology for serviceability assessment of corrosion affected pipes is presented. The depth of corrosion pit is considered as the critical parameter that causes serviceability failure when it is greater than the wall thickness of the pipe. A stochastic model for corrosion depth of cast iron pipes is developed and a time-dependent method is employed to quantify the probability of serviceability failure so that the time for the pipeline to be unserviceable and hence requiring repairs can be determined. A sensitivity analysis is also undertaken to identify the factors that affect the failure due to corrosion. The methodology presented in this study can be used as a tool for infrastructure managers in different industries such as water distribution systems, oil and gas pipelines, sewer networks, etc. The methodology can help pipeline engineers and asset managers in making decisions with regard to the serviceability of corrosion affected pipelines both for assessment of existing systems and for designing new pipelines.

Key words | cast iron pipes, corrosion pit, first passage probability, life time prediction, reliability analysis, serviceability failure

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INTRODUCTION

Corrosion in pipes is the predominant causal factor in the premature degradation of ferrous pipes, leading to ultimate structural failure. For pipes, failure does not necessarily imply structural collapse but in most cases involves the loss of serviceability, characterized by leakage. Pipes might not reach their ultimate structural failure but from a serviceability point of view they might fail when there is leakage from corrosion pits. Since the safety factors used in design for structural strength are usually larger than those for serviceability (due to the paramount importance of structural safety) the actual probability of loss of strength is smaller than that of loss of serviceability. As is well known, the costs of repairs are usually high for pipelines (in addition to the inconvenience to the public due to interruptions), it is of practical importance to accurately predict the time for repairs for deteriorated pipes to achieve a risk-cost optimized asset management of the pipeline system. This gives rise to the need for a reliability analysis on corrosion affected pipes with various failure criteria.

Determination of the depth of the corrosion pit can help to predict the serviceability of corrosion affected pipes and to instigate maintenance and repairs for the pipeline system. The reliability of pipelines subjected to corrosion has been investigated extensively in the past (e.g. Rajani *et al.* 2000; Caleyo *et al.* 2002; De Silva *et al.* 2006; Stephens & Nessim 2006; Maes *et al.* 2008; Li & Mahmoodian 2013;

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Routil *et al.* 2015). In these investigations, the pipeline system parameters have been assumed to be either deterministic quantities or random variables. Compared with deterministic approaches, stochastic methods are more realistic because in reality variables of the system are not only uncertain but also change with time. As such, it is more appropriate for them to be modeled as stochastic processes.

Assessment of pipe infrastructure requires the collection of a wide range of data for developing models for pipe deterioration, based on which the service life of pipe systems can be predicted.

In deterministic models, normally data from laboratory tests are used to produce the necessary data and establish the relationships between components. Variations and uncertainties in variables are not considered in deterministic models. Rajani *et al.* (2000) proposed a deterministic model to estimate the remaining service life of cast iron water pipes. It considers that the corrosion pits reduce the structural capacity of the pipes, but it does not consider the uncertainties involved in all factors contributing to the corrosion and resultant failures. Deb *et al.* (2002) presented a deterministic model based on analyzing the growth of corrosion pits on cast iron (CI) pipes, loss of wall thickness and the strength reduction of the pipe over time.

Probabilistic models are used when historical failure or inspection data are limited or unavailable. These models specifically analyze the effective parameters on pipe performance rather than evaluate the previous pipe failure history. Uncertainties are included by considering the variables as random variables. Usually, this method is applied to pipes where the process of deterioration and factors for failure are well understood.

Sadiq *et al.* (2004) developed a probabilistic method to predict the remaining service life of in service cast iron pipes based on Monte Carlo simulation. They defined maximum principal strain as the failure criterion and considered stresses both in the longitudinal and circumferential directions. De Silva *et al.* (2006) presented a condition assessment and probabilistic analysis to estimate failure rates in ferrous pipelines. A first-order-second moment (FOSM) analysis was combined with condition assessment data to determine the probability of failure. Davis & Marlow (2008) developed a probabilistic failure model for service life prediction of CI pipelines subject to corrosion and under internal pressure and external loading.

Mahmoodian & Li (2011) used the Monte Carlo simulation method in reliability analysis for time-dependent service life prediction of buried cast iron water pipes. The failure criterion in their study was defined as the exceedance of stress intensity factor from fracture toughness of pipe material.

Li & Mahmoodian (2013) also presented an analytical methodology for failure analysis and service life prediction of cast iron pipes using a time-dependent reliability theory. They considered ultimate failure of the pipe by using the concept of stress intensity in fracture mechanics. An empirical model was derived in their study for maximum pit growth of corrosion from the available data based on mathematical regressions.

Although structural integrity of pipelines has been the main concern in failure assessment of the infrastructure in all of the above mentioned research, serviceability assessment can be of more practical importance as the warning stage before ultimate collapse.

In serviceability assessment, it is the corrosion depth that is of the most practical significance and of real concern to pipeline engineers, operators and asset managers. Moreover, repair costs arising from corrosion induced serviceability failures, in particular pipe leakage, exceed those arising from strength failures by a substantial margin. It is therefore essential to predict accurately the serviceability of corrosion affected pipes based on the criterion of corrosion depth so as to achieve cost effectiveness in the asset management of pipeline infrastructure. Timely repairs have the potential to prolong the service life of a pipeline system.

In this paper a reliability-based methodology for serviceability assessment of corrosion affected cast iron pipes is presented. A stochastic model for corrosion depth is developed which relates to key factors that affect the depth of corrosion pit.

Serviceability of a pipeline system fails when it starts to leak, i.e. the depth of corrosion pit is equal to the pipe wall thickness. An analytical time-dependent method is employed to quantify the probability of serviceability failure of cast iron pipes due to corrosion so that the time for the pipeline to be unserviceable and hence requiring repairs, can be determined with confidence.

A sensitivity analysis is also undertaken to identify the factors that affect the failure due to corrosion. The methodology presented in this study can serve as a tool for infrastructure managers in different industries such as water distribution systems, oil and gas pipelines, sewer networks, etc.

Accurate prediction of pipe failure probability will help asset managers to better plan for water pipe networks; preventing environmental pollution, flooding and disruption of the daily life of the public. The methodology can help pipeline managers and maintenance engineers to accurately estimate the probability of pipeline failures which can be used for prioritizing pipe repairs and/or replacement based on their risk analysis. Decision making methods and risk-cost analysis are out of the scope of the current research and will not be covered in this paper.

PROBLEM FORMULATION

The practical serviceability criterion related to corrosion of pipe wall is to limit the corrosion depth within the pipe wall thickness. In the theory of structural reliability, this criterion can be expressed in the form of a limit state function as follows:

$$G(d, a, t) = d - a(t) \tag{1}$$

where $a(t)$ is the depth of corrosion pit at time t and d is the critical limit for corrosion depth, which in this case is equal to the pipe wall thickness. The depth of corrosion $a(t)$ increases with time as the corrosion process continues. With the limit state function of Equation (1), the probability of serviceability failure due to corrosion can be determined from:

$$P_f(t) = P[G(d,\ a,\ t) \leq 0] = P[a(t) \geq d] \tag{2}$$

where P_f denotes probability of an event.

Equation (2) represents a typical upcrossing problem, which can be dealt with using time-dependent reliability methods (Melchers 1999). Time-dependent reliability problems are those in which either all or some of basic random variables are modeled as stochastic processes. For serviceability problems involving the stochastic process of corrosion, as measured by corrosion depth $a(t)$, the serviceability depends on the time that is expected to elapse before the first occurrence of the stochastic process, $a(t)$, upcrossing a critical limit (the threshold), d sometime during the service life of the pipe. Equivalently, the probability of the first occurrence of such an excursion is the probability of serviceability failure, $P(t)$, during that time period. This is known as 'first passage probability' and can be determined from (Melchers 1999):

$$P(t) = 1 - [1 - P(0)]e^{-\int_0^t v d_\tau} \tag{3}$$

where $P(0)$ is the probability of failure due to corrosion of pipe wall at time $t = 0$ and v is the mean rate for the stochastic process $a(t)$ to upcross the threshold d. In many practical problems, the mean upcrossing rate is very small, so the above equation can be approximated as follows:

$$P(t) = P(0) + \int_0^t v d_\tau \tag{4}$$

The upcrossing rate in Equation (4) can be determined from the Rice formula (Melchers 1999):

$$v = v_d^+ = \int_d^\infty (\dot{a} - \dot{d}) f_{a\dot{a}}(d, \dot{a}) d_{\dot{a}} \tag{5}$$

where v_d^+ is the upcrossing rate of the stochastic process $a(t)$ relative to the threshold d; $\dot{d}$ is slope of d with respect to time t; $\dot{a}$ is the time-derivative process of $a(t)$; and $f_{a\dot{a}}()$ is the joint probability density function for a and $\dot{a}$.

An analytical solution to the above equation has been derived by Li & Melchers (1993), as follows:

$$\upsilon_d^+ = \frac{\sigma_{\dot{a}|a}}{\sigma_a} \oslash \left(\frac{d - \mu_a}{\sigma_a}\right) \times \left\{ \oslash\left(-\frac{\dot{d} - \mu_{\dot{a}|a}}{\sigma_{\dot{a}|a}}\right) - \frac{\dot{d} - \mu_{\dot{a}|a}}{\sigma_{\dot{a}|a}} \Phi\left(-\frac{\dot{d} - \mu_{\dot{a}|a}}{\sigma_{\dot{a}|a}}\right)\right\} \tag{6}$$

where $\oslash$ and Φ are standard normal density and distribution functions, respectively; μ and σ denote the mean and standard deviation of random variables, represented by subscripts a and $\dot{a}$, and '|' denotes the condition. For a given Gaussian stochastic process with mean function $\mu_a(t)$ and auto-covariance function $C_{aa}(t_i, t_j)$, the variables in the above equation can be determined, according to the theory of stochastic processes (Papoulis 1965; Melchers 1999), as follows:

$$\mu_{\dot{a}|a} = E[\dot{a}|a = d] = \mu_{\dot{a}} + \rho \frac{\sigma_{\dot{a}}}{\sigma_a}(d - \mu_a) \tag{7a}$$

$$\sigma_{\dot{a}|a} = [\sigma_{\dot{a}}^2(1 - \rho^2)]^{1/2} \tag{7b}$$

where

$$\mu_{\dot{a}} = \frac{d\mu_a(t)}{dt} \tag{7c}$$

$$\sigma_{\dot{a}} = \left[\frac{\partial^2 C_{aa}(t_i, t_j)}{\partial t_i \partial t_j}\Big|_{\cdot i=j}\right]^{1/2} \tag{7d}$$

$$\rho = \frac{C_{a\dot{a}}(t_i, t_j)}{\left[C_{aa}(t_i, t_i).C_{\dot{a}\dot{a}}(t_j, t_j)\right]^{1/2}} \tag{7e}$$

and the cross-covariance function is

$$C_{a\dot{a}}(t_i, t_j) = \frac{\partial C_{aa}(t_i, t_j)}{\partial t_j} \tag{7f}$$

It can be reasonably assumed that the corrosion depth in the pipe does not exceed the wall thickness at the beginning of structural service, therefore, the probability of serviceability failure due to corrosion at $t = 0$ is zero, i.e., $P(0) = 0$. The solution to Equation (4) can be expressed, after substituting Equation (6) into Equation (4), and considering that d is constant ($\dot{d} = 0$), therefore:

$$P_f(t) = \int_0^t \frac{\sigma_{\dot{a}|a}(t)}{\sigma_a(t)} \oslash \left(\frac{d - \mu_a(t)}{\sigma_a(t)}\right) \left\{ \oslash \left(-\frac{\mu_{\dot{a}|a}(t)}{\sigma_{\dot{a}|a}(t)}\right) + \frac{\mu_{\dot{a}|a}(t)}{\sigma_{\dot{a}|a}(t)} \Phi \left(\frac{\mu_{\dot{a}|a}(t)}{\sigma_{\dot{a}|a}(t)}\right) \right\} d_\tau \tag{8}$$

At a time that $P(t)$ is greater than a maximum acceptable risk in terms of the probability of serviceability failure, P_a, it is the time the pipeline becomes unserviceable. This can be determined from the following:

$$P_f(T_c) \geq P_a \tag{9}$$

where T_c denotes the time the pipeline becomes unserviceable due to critical corrosion. In principle, P_a can be determined from risk-cost optimization of the pipeline during its whole service life.

For Equation (8) to be of practical use, i.e., determining the probability of serviceability failure due to corrosion over time, the key is to develop a stochastic model for the corrosion depth. This is dealt with in the next section.

MODEL OF CORROSION DEPTH

Corrosion depth

The predominant deterioration mechanism on the ferrous pipes is corrosion with the damage occuring in the form of corrosion pits. The damage to iron is often identified by the presence of graphitization, a result of iron being leached away by corrosion. The characteristics of the soil in contact with the pipe surface, such as pH, soluble salt, oxygen and moisture content, soil resistivity, temperature and presence of certain bacteria affect the corrosion rate (*Cast Iron Soil Pipe and Fittings Handbook* 2006).

As a common form of metal loss, the corrosion pit can grow with time and result in reduction of the thickness and mechanical resistance of the pipe wall. Corrosion pits have a variety of shapes with characteristic depths, diameters (or widths), and lengths. They can develop randomly along any segment of pipe and tend to grow with time at a rate that depends on environmental conditions in the immediate vicinity of the pipeline (Rajani & Makar 2000).

The metal corrosion phenomenon is complex, and the depth of corrosion pit due to both internal and external corrosions, as well as whether the rate is constant or variable, have been the subject of debate (Snoeyink & Wagner 1996; Melchers 2008). However, in all cases, the corrosion rate is generally considered to be high during early pipe ages and then stabilizes at a certain value thereafter. In the first phase a rapid growth of the corrosion pits occurs because any corrosion products formed on the pipe surface are porous and have poor protective properties. In the second phase a slow growth of the corrosion pits leading to a constant corrosion depth occurs due to auto-protection of corrosion layers. This complexity of the corrosion phenomena leads to the different corrosion models and different suggested definitions of corrosion based on the different assumptions made by researchers (Yamini & Lence 2010).

In this paper, a power law corrosion model is assumed since over a long period (i.e. in old pipes) the localized corrosion pits grow to the point where they are in contact with one another, making individual pits indistinguishable and causing the pipe to appear to have a uniform corrosion (Benjamin *et al.* 1996).

The power law formulation for corrosion model is as follows:

$$a(t) = kt^n \tag{10}$$

where a = depth of corrosion pit (mm); t = exposure time (yr); k, n = empirical constants largely determined from experiments and/or field data.

In practice the constants k and n are obtained by fitting the model to the experimental data, therefore having a good regression (i.e. high R-squared value) would be a proof for choosing an appropriate model.

To determine the values or range of values for k and n in Equation (10), a comprehensive data survey was undertaken and a set of data was selected judiciously for the analysis in this paper. The data were collated by Marshall (2001) on the corrosion rate of a set of cast iron pipes over a period of up to 150 years as shown in Figure 1. Based on these data a mathematical regression results in $k = 2.54$ and $n = 0.32$ for external corrosion and $k = 0.92$ and $n = 0.40$ for internal corrosion in the model of Equation (10).

With high R-squared values for regression lines in Figure 1 (internal $R^2 = 0.959$, external $R^2 = 0.857$), it can be concluded that the power law model best fits the corrosion data and it is appropriate for corrosion modelling and the calculations.

From the experimental data presented in Figure 1, it also can be concluded that for the set of cast iron pipes in this study, especially in the early ages, external corrosion has a higher rate than internal corrosion. In Figure 2, a sample of a cast iron pipe taken from London water mains in Victorian time also shows the severity of external corrosion compared with internal corrosion.

Stochastic model

As may be appreciated, the corrosion depth in a pipe wall is a very random phenomenon. It is justifiable to model the corrosion process as a stochastic process, expressed in terms of primary contributing factors, which are treated as basic random variables. It follows that the corrosion depth (i.e. Equation (10)) is a function of basic random variables

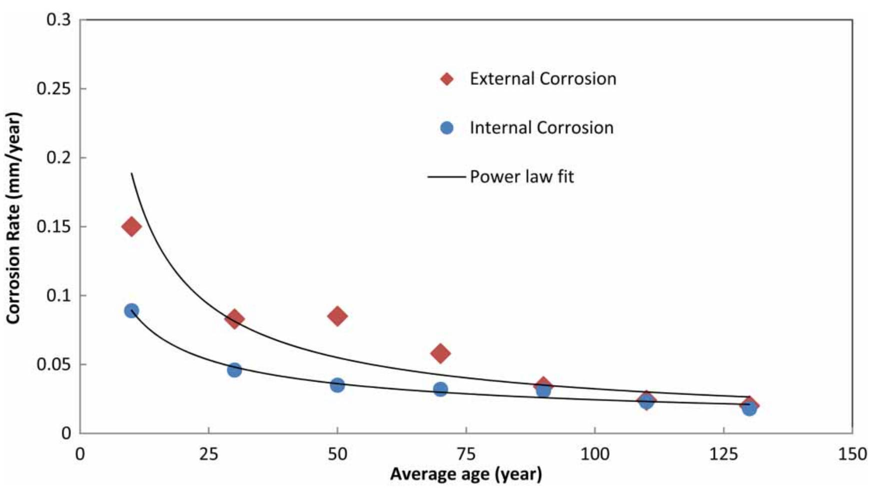

Figure 1 | Rate of internal and external corrosion for cast iron pipes (Marshall 2001).

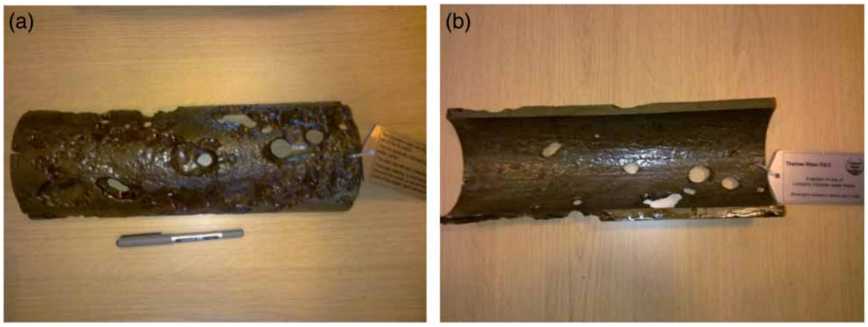

Figure 2 | A section of one of London's Victorian water mains, (a) external corrosion, (b) internal corrosion (Thames Water Ltd).

as well as time and can be expressed as:

$$a(t) = f(k, n, t) \tag{11}$$

where k and n are the basic random variables, the probabilistic information of which are (presumed) available. With this treatment, the statistics of a(t) can be obtained using the technique of Monte Carlo simulation. With the values of basic variables in Table 1, a realization of the corrosion depth is shown in Figure 3. As can be seen, the corrosion depth fluctuates with time, indicating the random nature of corrosion model parameters. It is also a vindication for stochastic approaches. Also shown in Figure 3 is the corrosion depth with a different number of simulations, i.e. sample size N. As can be seen a sample size of 1,000 can achieve a reasonable accuracy (convergence) in simulating the corrosion depth. In this paper the sample size is taken to be N = 10,000.

Table 1 | Values of basic variables, worked example

Symbol	Parameter	Corrosion type	Mean	Coefficient of variation
k	Multiplying constant	Internal	0.92	0.196
		External	2.54	0.197
n	Exponential constant	Internal	0.4	0.200
		External	0.32	0.188

To account for the randomness of the corrosion depth, a random variable, ξ_a, is introduced. ξ_a is defined in such a way that its mean is unity, i.e. $E(\xi_a) = 1$ and its coefficient of variation, λ_a, is a constant. The simulation results suggest that λ_a has an average of 0.24. Thus, Equation (11) can be expressed as a stochastic process:

$$a(t) = a_c(t).\xi_a \tag{12}$$

where $a_c(t)$ is treated as a pure time function determined by corrosion depth equation (e.g. Equation (10)). The mean and

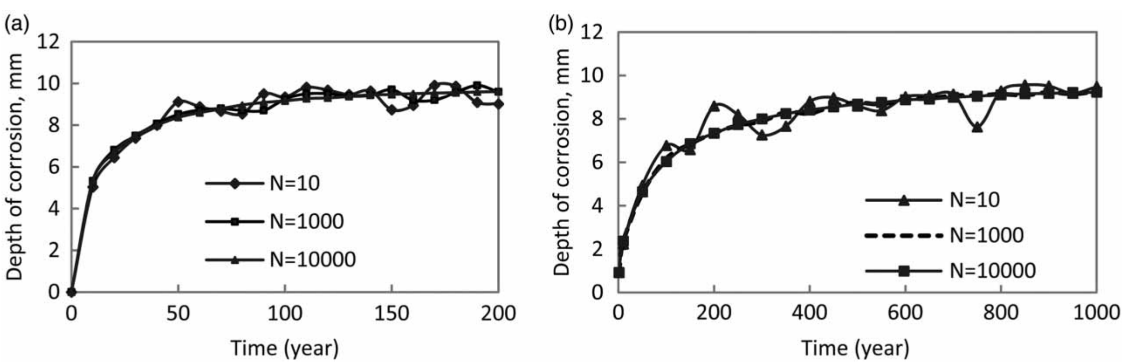

Figure 3 | Simulation of corrosion depth, (a) external corrosion, (b) internal corrosion (N = sample size).

auto-covariance functions of $a(t)$ are (see e.g. Li & Melchers 1993):

$$\mu_a(t) = E[a(t)] = a_c(t).E[\xi_a] = a_c(t) \quad (13)$$

$$C_{aa}(t_i, t_j) = \lambda_a^2 \rho a_c(t_i) a_c(t_j) \quad (14)$$

where ρ is auto-correlation coefficient for $a(t)$ between two points in time t_i and t_j. With $\mu_a(t)$ and $C_{aa}(t_i, t_j)$, Equations (7a)–(7f) can be used to determine other statistical parameters of $a(t)$.

PROBABILITY OF FAILURE

To examine the presented method for a practical problem, an example of a cast iron pipe in the UK water distribution system is taken. The pipe wall thickness, d, is 10 mm. Statistical data of external and internal corrosion parameters for use in this study, presented in Table 1, were extracted from UK Water Industry Research (Marshall 2001). With the values of variables given in Table 1, the probability of serviceability failure due to corrosion can be computed using Equation (8) and the results are shown in Figure 4 for different coefficients of correlation. Figure 4 indicates that the effect of the auto-correlation of the corrosion process between two points in time (i.e. ρ) on failure can be negligible. This may be of practical significance since ρ_a is not readily available and therefore assumption of no correlation may not lead to a significant difference. On the other hand, the theory of stochastic processes (Papoulis 1965) and the research experience (Li & Melchers 1993) suggest that the assumption of no auto-correlation between different time points generally leads to greater estimates of the probability of the occurrence of events, which is conservative for the assessment of structural deterioration.

Finally, as a complete picture of serviceability assessment of corrosion affected pipes, the time for the structure to be unserviceable, i.e. T_c, due to corrosion, can be determined for a given acceptable probability P_a. This is straightforward. For example, from Figure 4(a), it can be obtained that $T_c = 53$ years for $P_a = 0.1$ (and auto-correlation of $\rho = 0.5$). If there is no intervention during the service period of (0, 53) years for the pipe, such as maintenance and repairs, T_c represents the time for interventions or the end of service for the pipe, based on the performance criterion of corrosion depth. The information of T_c (i.e. time for interventions) is of significant practical importance to pipeline engineers and asset managers of pipeline systems in decision-making with regard to its repairs and/or rehabilitation which are usually dependent on the budget situation of the day. Therefore, when to intervene is the first question for decision-makers.

SENSITIVITY ANALYSIS

The effect of variables on the reliability of the pipeline can be evaluated by sensitivity analysis. The effect of changes in pipe initial wall thickness on the probability of failure has been shown in Figure 5. The graphs show that the thicker the pipe wall, the longer the service life of the pipe. Although this correlation can be obvious without any calculations, the amount of increase in service life is of practical interest for design engineers in the design of new

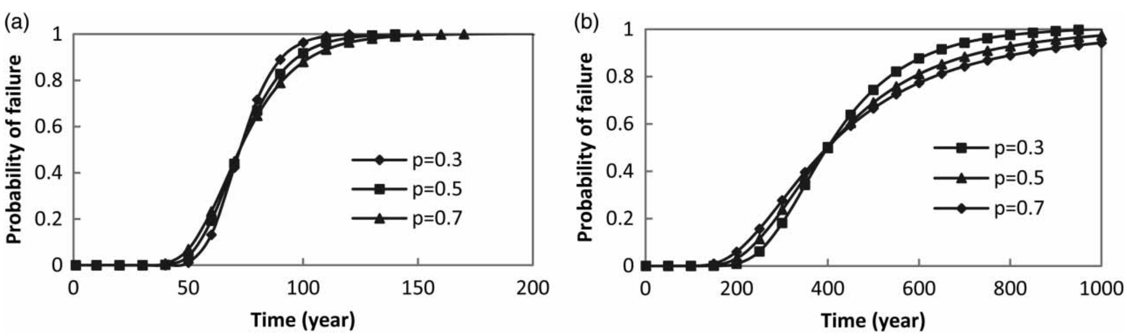

Figure 4 | Probability of serviceability failure due to corrosion of pipe wall for different coefficients of correlation (ρ_w), (a) external corrosion, (b) internal corrosion.

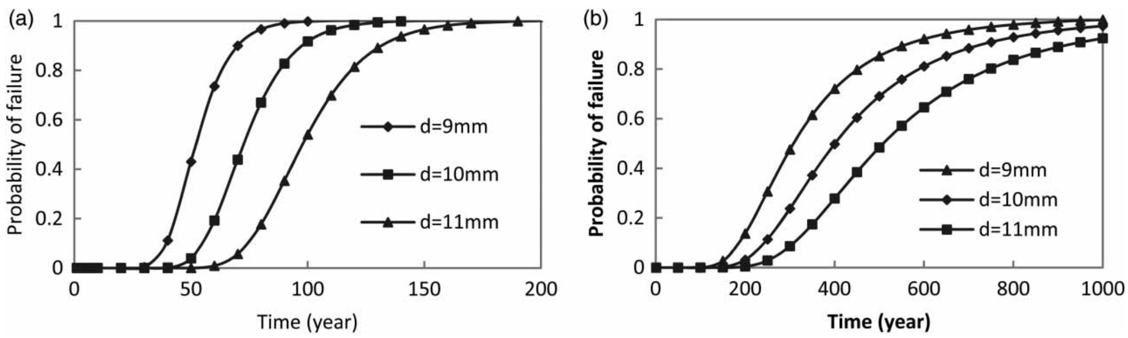

Figure 5 | Probability of serviceability failure due to corrosion for different wall thickness values, (a) external corrosion, (b) internal corrosion.

pipeline. The designer can analyze how using a thicker pipe can prolong the service life of the pipe. For instance, assuming $P_a = 0.1$ as acceptable failure probability, the result presented in Figure 5(a) shows that by increasing the wall thickness from 9 to 10 mm, the service life of the pipe increases from 43 to 53 years. Changing the wall thickness from 9 to 11 mm will dramatically increase the service life of the pipe from 43 to 78 years. This considerable difference in service life may encourage the asset managers for more capital investment to have thicker pipes with significantly longer service life.

In view of variables that affect the corrosion process, it is of interest to identify the degree of contribution of variables so that more research can focus on the most effective variable. The contribution of these variables in the failure function is calculated by using the relative contribution concept.

The relative contribution (α_x^2) of a random variable (x) to the variance of the failure function (G) can be evaluated from (Ahammed & Melchers 1994):

$$\alpha_x^2 = \frac{\left(\dfrac{\partial G}{\partial x}\sigma_x\right)^2}{\sigma_G^2} \tag{15}$$

where σ_x is standard deviation of the random variable and σ_G^2 is variance of the limit state function. α_x^2 represents the relative contribution of random variables (k and n) in the violation of the limit state function. Figure 6 shows the degree of contribution of each variable during the time of service. The results show that in the early ages, the multiplying constant (k) has a higher contribution in failure function and as time passes, the contribution of exponential constant (n) becomes higher.

Omission sensitivity is another measure of the sensitivity of random variables to the probability of failure which can be estimated based on first order reliability or FOR method (Madsen *et al.* 1986; Ditlevsen & Madsen 1996). In this method the probability of failure is expressed

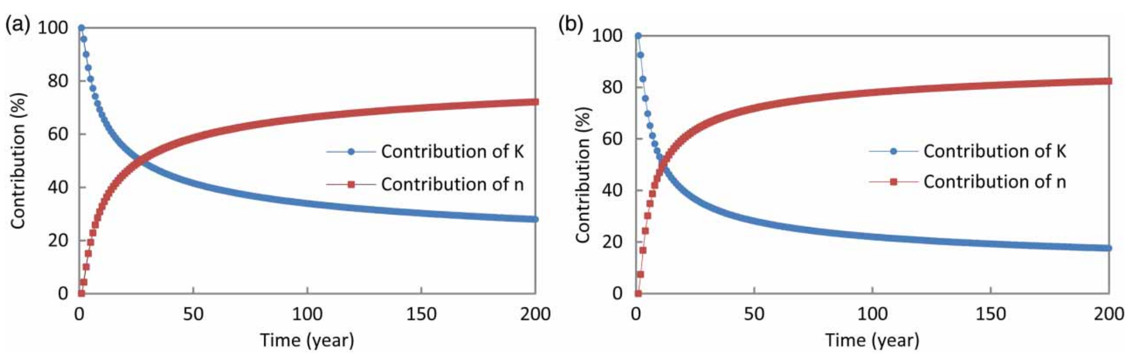

Figure 6 | Relative contribution to the variance of failure function for corrosion multiplying constant (K), and corrosion exponential constant (n), (a) external corrosion, (b) internal corrosion.

in terms of a reliability index, β, using the well known relationship (Melchers 1999):

$$P_f(t) = \Phi(-\beta(t)) \tag{16}$$

In computing the reliability index, the basic random variables X are transformed into standardized normal space U and the limit state function, $G(X, t)$, transformed to $g(U, t)$ (Melchers 1999).

The effect of randomness of variables on the probability of failure can be measured by an emission sensitivity factor. According to Ditlevsen & Madsen (1996), the omission sensitivity factor with respect to random variable u_i can be determined by:

$$\zeta_{u_i}(t) = \frac{\beta(t)|U_{i(t)=u_i}}{\beta(t)} = \frac{1-\alpha_i u_i/\beta(t)}{\sqrt{1-\alpha_i^2}} \tag{17}$$

where α is the normal unit vector to the limit state surface $g(U, t)$ at checking point u^* and time t (Melchers 1999). As can be seen the omission sensitivity factor measures the relative error in the value of reliability index β if an input random variable is replaced by a fixed value (i.e. treated as a deterministic variable). Thus when the relative error of random variables (i.e. omission sensitivity factor) is around 1, ($\zeta_{u_i} \approx 1$), it may be appropriate to treat them as deterministic variables if full statistical information is not available.

Using statistical values of basic variables given in Table 1, the omission sensitivity factor can be computed and is shown in Figure 7. A similar trend as in Figure 6 can be seen from this figure for both variables subjected to internal and/or external corrosion. This result again confirms that the reliability of the pipe at the early ages is more sensitive to the exact values of k. As time passes the sensitivity to k decreases and the effect of n on the reliability of pipe becomes more considerable.

Further sensitivity studies were carried out to investigate the effect on probability of failure of the level of variability (i.e. coefficient of variation) of each of corrosion model parameters (i.e. k and n). The coefficient of variation for each of these parameters was varied from 0 to 0.5 in steps of 0.1. The coefficient of variation of the other variable was kept constant at the values given in Table 1. Figures 8 and 9 illustrate the results for three different pipeline elapsed lives (t). Generally the probability of failure is more sensitive to the variation of the coefficient of variation of exponential constant (n).

It is also observed that the variability of the parameters (k and n) for low values of t, has more significant effect on the probability of failure. In other words, the sensitivity of corrosion parameters is more dependent on the actual value of coefficient of variation in the early ages. In such cases, more concern should be taken in order to determine relevant parameter values.

CONCLUSIONS

A reliability based methodology for serviceability assessment of cast iron water pipes has been formulated and applied to the

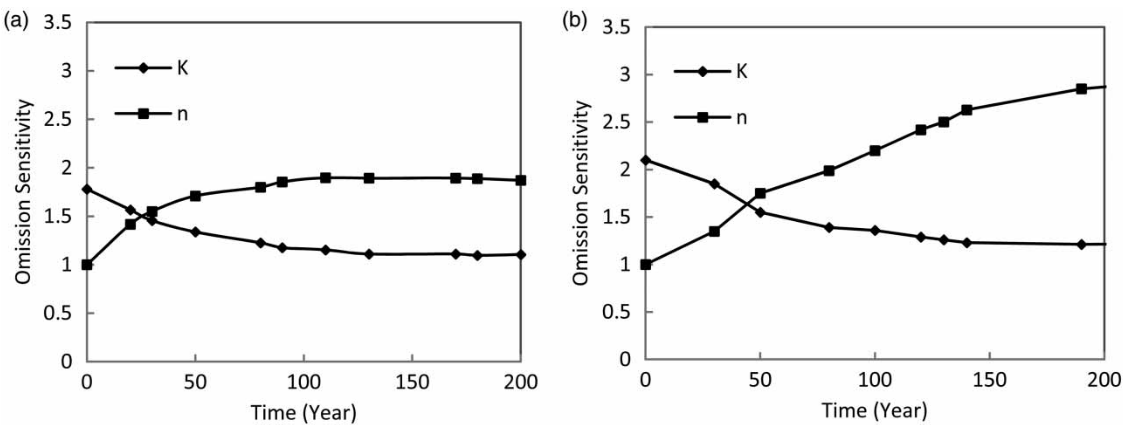

Figure 7 | Omission sensitivity for corrosion multiplying constant (*K*), and corrosion exponential constant (*n*), (a) external corrosion, (b) internal corrosion.

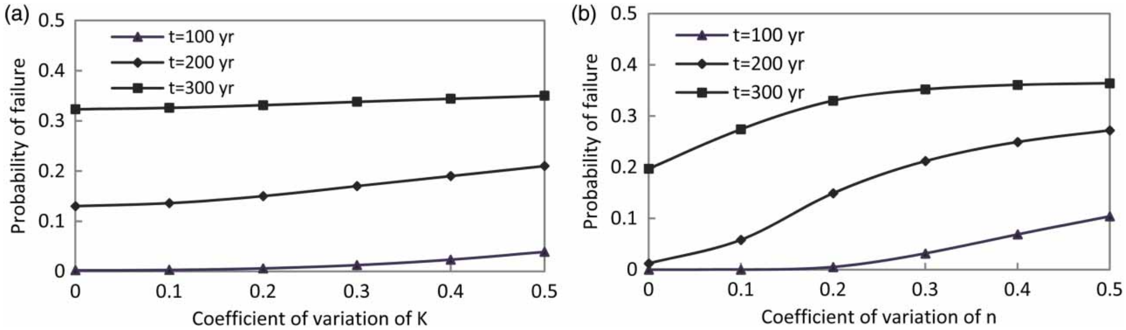

Figure 8 | Probability of failure due to internal corrosion vs coefficient of variation for various values of pipeline elapsed life, (a) corrosion multiplying constant, *K*, (b) corrosion exponential constant, *n*.

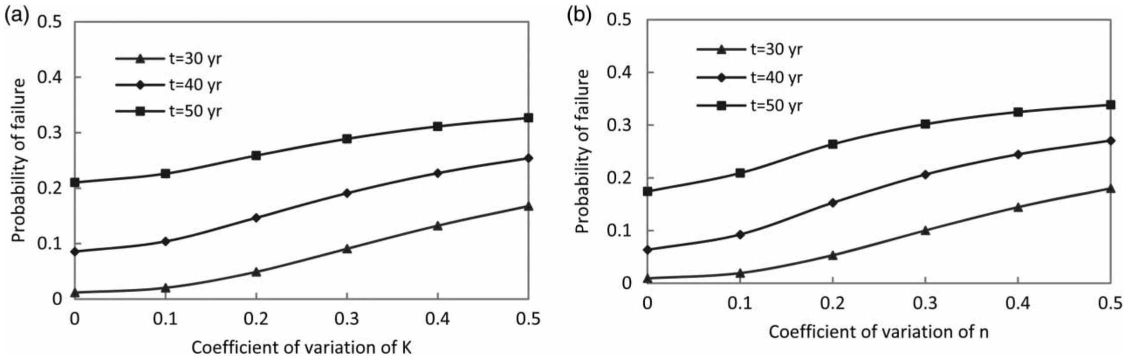

Figure 9 | Probability of failure due to external corrosion vs coefficient of variation for various values of pipeline elapsed life, (a) corrosion multiplying constant, *K*, (b) corrosion exponential constant, *n*.

limit state function which defines serviceability failure as the condition when corrosion depth becomes greater than pipe wall thickness. The stochastic nature of the problem was considered by introducing uncertain variables as random variables.

The concept of corrosion depth increment has been employed to establish the limit state function for determining the probability of serviceability failure. A widely used model for a corrosion pit has been adopted based on data mining and mathematical regressions. From the results of a sensitivity analysis it has been found that the likelihood of pipe collapse increases with the decrease of the wall thickness of the pipes for both external and internal corrosion and that for a given diameter, the likelihood of pipe collapse for pipes with external corrosion is much higher than that for pipes with internal corrosion.

Sensitivity analysis showed that among corrosion constants, the multiplying constant (k) has a higher contribution in failure function at the early ages and as time passes, the contribution of exponential constant (n) become higher. The result also showed that the sensitivity of corrosion parameters is more dependent on the actual value of coefficient of variation in early ages.

It can be concluded that a time-dependent reliability method is a rational tool for serviceability assessment of corrosion affected pipes with a view to determine the time of repairs and/or rehabilitation for the pipeline.

The method also can be used for optimizing the design of new water mains by knowing the sensitivity of the system to the pipeline variables.

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