Influence of cavitation phenomenon on the hydraulic behavior of leaks in water distribution systems

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ABSTRACT

This paper examines the hydraulic behavior of leakage from water distribution systems under condition of cavitation. For this purpose, an experimental model featuring small (i.e., a fraction of a millimeter) orifice openings was designed and built to simulate idealized cracks in defective pipes. During the tests, water was allowed to flow through the cracks at controlled pressures during observation of hydraulic behavior. The study showed that the cavitation phenomenon can develop in leaking orifices and that it drastically affects the pressure–flow rate relationship. The results also showed that cavitation inception is dependent on the size of the leak opening and the rate of flow but remains independent of the pressure head. The coefficient of discharge in cavitating flow conditions is influenced by the cavitation number rather than the Reynolds number. A model was used to determine the coefficient of discharge (Cd) for cavitating orifice flow, after which the calculated results were compared with those measured experimentally. Accordingly, care should be taken using the orifice flow equation to model leakage from water distribution systems.

Key words | cavitation, hydraulics, pipeline leakage, pressure management, water distribution systems

INTRODUCTION

Leakages from water distribution systems (WDSs) are a significant worldwide dilemma. The World Bank Group (2006) reported that ‘real losses’ (the recognized term for physical losses used by the International Water Association) from these systems around the world amount to more than 32 billion m³ of treated water annually. In some low-income countries, this loss represents 40–50% of water supplied, compared to an estimated global average of 20% (Growing-Blue 2016). In Saudi Arabia alone, the rate of leakage from water systems is estimated to be around 35% of water supplied (Figure 1). As losses of drinking water represent a distinguishably high cost, reductions to leakage rates are not only a prerequisite for making water distribution viable for water distribution companies but also an environmental necessity, for these leakages pose a danger to public health and safety (the leaking flow may damage the foundations of buildings and roads and be contaminated by pollutants).

As noted by Colombo & Karney (2002), leakages cause inefficient energy distribution through the network (thus wasting the energy used for pumping the water) and may also affect overall water quality by introducing infection into the water distribution networks in low-pressure conditions.

Pressure management is already frequently used as a leakage control strategy in WDSs (Farley & Trow 2005). The conventional view is that leakages from WDSs are relatively insensitive to pressure, with the leakage flow rate proportional to the square root of the pressure, i.e., having a leakage exponent of 0.5, according to the orifice flow equation:

\[ q = C_d A_o \sqrt{2gh} \]  

where \( q \) is the flow rate, \( C_d \) the discharge coefficient, \( A_o \) the orifice area (area of the leak), \( g \) the acceleration due to gravity, and \( h \) the pressure head.
However, a number of field and experimental studies have shown that leakages vary with pressure to a greater power exponent than the 0.5 of the orifice flow equation—varying between 0.36 and 2.95 (Farley & Trow 2003; Greyvenstein & Van Zyl 2007). A sound understanding of the hydraulic behavior of leakages from WDSs (along with variations in pipe pressure) is essential for assessing the detection of leaks and leakage rates, as well as for developing leakage reduction programs featuring pressure control. This paper investigates the influence of the cavitation phenomenon on the hydraulic behavior of leakage from water distribution pipes and, in particular, examines its effect on the pressure–leakage relationship.

THE PRESSURE–LEAKAGE RELATIONSHIP

A substantial amount of research has sought to improve understanding of leakage from WDSs (for a review, see Puust et al. (2010), among others). One of the most significant factors influencing leakage rates in WDSs is pressure (Farley & Trow 2003; van Zyl & Clayton 2007). As a result, active pressure control management is widely considered an essential tool for reducing losses in the short term (Walski et al. 2006).

Existing models have been proposed with which to model and assess leakage rates from WDSs. Many of these proposed models attempt to model leakages using the orifice flow equation (Wiggert 1968; Al-Khomairi 2005; Walski et al. 2006). Wiggert (1968) used the orifice flow equation to describe the relationship between pressure in a pipe and leakage rate, which suggests that leakages vary in proportion to the square root of the pressure head in the pipe, according to the orifice flow equation (Equation (1)). Al-Khomairi (2005) found through experimentation that the orifice equation gave a good estimation of unsteady leak rate history for normal leak openings. However, for long leak openings (extremely major leaks), it produced a major error in leakage rate computation.

Field and experimental studies (e.g., Farley & Trow 2003; Greyvenstein & van Zyl 2007) have shown that leakage rates could be much more sensitive to the pipe pressure head than has been suggested by the orifice flow equation (i.e., proportional to the square root of the pressure head in the pipe). It has been found that leakages differ with pressure to a power exponent value varying between 0.36 and 2.95 (Farley & Trow 2003).

Several factors have been suggested that might explain the range and typical values of leakage exponents found in the aforementioned studies, but these are not yet fully understood. In particular, van Zyl & Clayton (2007) have suggested four factors that affect the pressure–flow rate relationship in leaking water pipes: pipe material behavior (with leak area increasing with pressure), leak hydraulics, soil hydraulics,
and water demand. Farley & Trow (2003), moreover, have indicated that expansion of leakage area with pressure is likely the primary cause of certain observed leakage exponents.

Walski et al. (2006), studying the interaction of orifice and soil in pipe leakage, considered two elements of head loss – flow through an orifice and Darcy flow in soils – and consequently developed an orifice/soil (OS) number to characterize their importance. Walski et al. (2006) concluded that ‘in most real world cases, the OS number is large’, that is, the orifice head loss dominates.

For soil hydraulics, van Zyl & Clayton (2007) concluded that the relationship between head loss and flow is unlikely to be linear because of the complex interaction of soil particles with the orifice, turbulent flow in soil, the changing geometry of the unconfined flow regime, hydraulic fracturing, and piping.

Recently, van Zyl et al. (2015) studied the effect of soil fluidization outside of leaks in water distribution pipes and found that head loss from a leaking pipe can be divided into three components: through the orifice, in the fluidized zone, and through static soil. They found that most head loss occurred within the fluidized zone, with significant but lesser head loss occurring through the orifice and only a small fraction occurring in the static soil.

All this literature testifies to the monumental efforts exerted by researchers to improve understanding of leaks from WDSs and to subsequently control them and thereby solve all underlying issues. However, to the authors’ knowledge, the cavitation phenomenon and its influence on the pressure–leakage relationship has yet to be investigated. Indeed, further research studies will be needed to extend understanding of the pressure–leakage relationship in leaking water pipes and to investigate this phenomenon’s influence on the pressure–leakage relationship.

THE CAVITATION PHENOMENON

The formation and collapse of vapor bubbles in a region of high flow velocity is called cavitation (Douglas et al. 1979). A hydraulic orifice – a component of fluid systems – is often used in cavitation research, as seen in Figure 2. The cavitation phenomenon occurs when the local static pressure in the orifice drops because of an increase in flow velocity below the local vapor pressure of the liquid (Douglas et al. 1979). This triggers local vaporization of the liquid, producing vapor-filled bubbles or a cavity in the flow (Douglas et al. 1979), as seen in Figure 2(b).

Extensive research has been carried out into cavitation in the field of fluid mechanics (e.g., Douglas et al. 1979; Koivula 2002; Payri et al. 2012), for this phenomenon can affect the performance of fluid systems. Cavitation can reduce system efficiency while increasing vibration and

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Figure 2 | (a) Pressure distribution in ideal orifice flow (McCloy & Martin 1973). (b) At a particular flow stage, flow separates in the orifice forming vena-contrac.
noise levels (Douglas et al. 1979; Koivula 2002). Recent research studies focusing on cavitation have been conducted in the field of nuclear safety, which considers flow through cracks in cooling pipes (Sievers et al. 2015). The studies in this field consider the limiting of flow through cavitation.

When Payri et al. (2012) studied cavitation inception for different fuels in a nozzle (nozzle behavior in fuel injectors), they found that fluids having less viscosity tend to cavitate sooner (Payri et al. 2012). This is likely because viscosity is inversely proportional to Re; less viscous fluids, by contrast, give higher Re at exit, and cavitation appears earlier.

The intensity of cavitation in fluid systems can be estimated by a nondimensional number, the cavitation number \((K)\), according to Nurick (1976):

\[
K = \frac{p_i - p_v}{p_i - p_b}
\]

where \(p_i\) is the pressure upstream of the orifice, \(p_v\) the saturated vapor pressure, and \(p_b\) the pressure downstream of the orifice.

The cavitation number \((K)\) can also be expressed as a form of the Euler number that is affected by two factors: pressure and velocity. These are combined with density in the cavitation number:

\[
K = \frac{2(p_d - p_v)}{\rho V^2}
\]

where \(p_d\) is the pressure downstream from the orifice, \(p_v\) the saturated vapor pressure, \(\rho\) the water density, and \(V\) the average velocity of the liquid at the orifice, and can be calculated as \(Q/A\), where \(Q\) is the rate of flow and \(A\) the cross-sectional area of the orifice.

Typically, inception of cavitation occurs with cavitation numbers on the order of 1 (Testud et al. 2007). However, in some cases, cavitation has been observed to occur at greater cavitation numbers (Kelly et al. 2011). The intensity of cavitation is strongly dependent on orifice geometry at the inlet point and on injection velocity (Koivula 2002). The sharper the edges of the orifice at the entrance, the more likely cavitation is as a result of fluid flow separation at the orifice flow entrance.

Since water is considered a less viscous fluid, and because water mains normally have supply heads on the order of 30 m (i.e., are considered high-pressure systems), through comparison with the results of Payri et al. (2009, 2012) it may be anticipated that the cavitation phenomenon will develop in an orifice of a leaking water pipe and thus affect the hydraulic behavior of the orifice. To the authors’ knowledge, cavitation in leaking water pipes and its effect on the pressure–leakage relationship has yet to be investigated.

**EXPERIMENT DESCRIPTION**

The experimental setup, whose schematics are shown in Figure 3, consists of an engineered leak orifice (test section), a water tank, a pump, valves with which to control pressure and leakage rate, electromagnetic flow meters, and pressure sensors for measuring pressure upstream and downstream of a leak. The experimental setup also includes a data acquisition system with which to read and record data.

Specially constructed boxes measuring 100×100×100 mm and having narrow (typically fractions of a millimeter) slot orifices running the full width of the boxes...
model fractured water distribution pipes. The engineered leak orifices were manufactured using laser cutting machines. They were made of two square-edged stainless steel plates 10 mm thick and butted against each other atop the machined boxes, then welded to the desired aperture. The slot sizes were adjusted using flat feeler gags measuring between 0.23 mm and 0.65 mm. The form of the slots remained unchanged, with square-edged orifices 10 mm thick and 88 mm long (Figures 4 and 5).

Water supply and control systems

The water supply and control systems consist of a water tank, pump, control valves, and piping systems. The water tank measured 600 mm long, 350 mm high, and 380 mm wide and was filled with clean water that was then pumped into the piping system. The pump used in this study was a multistage submersible pump manufactured by Pedrollo, model NKm 4/3. The maximum pressure head that could be supplied by the pump was 50 m of water, and the maximum flow rate was 3,200 L/h. A 25.4 mm diameter flexible hose was used to connect the test section and the water tank. Pressure and flow rate in the system were controlled using two valves (A and B; see Figure 3). One valve was used to divert the flow from the pump to the water tank when tests were conducted at low pressure (and low flow rate) so that the pump could be relieved during this stage. The other valve was used to control flow rate or pressure in the system through the crack. The water was then returned to the water tank through circulated pipes. Circulating the water into the storage tank avoided the problem of depressurizing the system, which might otherwise have caused air bubbles in the system, thus affecting the measured heads.

Measurement of pressure and flow rate

Pressure upstream and downstream of the engineered leak orifice was measured using a differential pressure transducer manufactured by Setra: multisense model 231, with LCD unit and operating with a full-scale 345 kPa.

A magnetic flow meter, model Signet 2551, with LCD display unit, was used to measure the inlet flow in the system. The meter has a function for averaging the rate of flow (i.e., setting a time over which the meter averages the flow signals). During the experimental run, this function was adjusted to 25 seconds to help smooth the display on the LCD. According to the specifications, the meter achieves ±2% accuracy in readings and an operating range of up to 10 m/s.

Monitoring and data equalization system

A PLC (programmable logic control) was used to monitor the output of differential pressure transducers and flow meter for the system due to converting the 4–20 mA calibrated sensors to read out data shown on the HMI screen. As shown in Figure 6, each instrument was connected to the PLC analog input, and the PLC then converted the input signals (4–20 mA) to digital form inside the PLC's memory and via the communication cable, where the data transfer could be read out on the HMI.

Detection of cavitation

Due to the complexity of obtaining information about the hydraulic behavior of fluid flow inside an orifice in a real system, nonintrusive techniques have been developed to determine the inception of cavitation. One of the most widely used criterion to determine the inception and development of cavitation is that proposed by Nurick (1976), whereby rate of flow obtained in stationary conditions increases in a linear function of the square root of the pressure drop until a point is reached at which it stabilizes as a result of the
The flow at this point is called ‘choked flow’, and the pressure conditions needed to achieve this condition are called critical cavitation conditions (Figure 7). The dotted line in Figure 7 represents the classical flow rate and implies that there is a sudden transition from nonchoked flow to fully choked flow.

In reality, pressure drops approaching but still below the calculated value of critical pressure drop $\Delta P_{\text{Crit}}$, usually feature some formation of vapor bubbles and some degree of cavitation (Monsen 2015). Figure 7 shows what really happens as flow transitions from nonchoked to fully choked flow.

The condition of choked flow directly affects the coefficient of discharge $C_d$ in the orifice flow equation, which is defined as the ratio of actual to theoretical flow rate (given by Bernoulli’s equation):

$$C_d = \frac{Q_{\text{act}}}{Q_{\text{th}}} = \frac{Q_{\text{act}}}{A_o \sqrt{2gh}}$$

where $Q_{\text{act}}$ is the actual flow rate in a stationary condition, $A_o$ the cross-sectional area of the orifice (area of the leak), $g$ the acceleration due to gravity, and $h$ the pressure head.

In the case of noncavitating flow, the coefficient of discharge is a function of the Reynolds number (Payri et al. 2009). The Reynolds number (Re) for a general leak opening or orifice can be calculated as:

$$Re = \frac{\rho V d}{\mu}$$

where $\mu$ is the dynamics viscosity, $V$ the mean velocity at the orifice, $d$ the diameter of the orifice, and $\rho$ the density of water.
However, when the flow begins to cavitate, the coefficient of discharge depends mainly on the cavitation number \( K \), as analyzed by Nurick (1976):

\[
C_d = c_c \cdot K^{1/2}
\]

(6)

\( K \) is the cavitation number and \( c_c \) the coefficient of contraction, which is represented by the ratio of the actual area \( A_c \) at the contraction to the orifice cross-sectional area \( A \). In this way, by monitoring the choking flow rate and reduction in the coefficient of discharge, one can predict the point at which cavitation appears in the orifice (Payri et al. 2012).

Test program and procedure

Tests were conducted at controlled pressure, upstream of the orifice, that ranged between 1 and 250 kPa, which represents a range of pressure such as might occur in WDSs. All experiments were performed at a water temperature of 20°C. During these tests, water was allowed to flow through the cracks as its hydraulic behavior was observed. Hydraulic characterization of the orifice was performed by monitoring steady-state flow behavior (i.e., measuring the rate of flow of the orifice while injecting water at stationary conditions).

The intensity of cavitation was estimated by the nondimensional number \( K \): cavitation number), which relates the static pressure (which resists cavitation) to the dynamic pressure (which promotes cavitation) using Equation (5). The higher the cavitation number, the less likely cavitation is. Lamb (1987) reports a critical cavitation number between 0.2 and 1.5, below which cavitation is likely to appear in orifices depending on their size and shape.

Test procedure

Prior to commencement of the test using the described apparatus, some arrangements needed consideration. First, the engineered leak orifice (i.e., the test section) was measured precisely using a set of flat feeler gauges. Due to limitations in the design, the sides of the crack opening were not perfectly parallel. Accordingly, a number of measurements were taken at different points along the crack and an average value calculated. Very small (i.e., fractions of millimeter) cracks, or slots, were used for the tests, ranging between 0.23 mm and 0.65 mm. After the size of the engendered leak crack was taken, it was then connected into the system, which was then primed as a water supply hose was used to load the tank with water. The pump was then turned on, allowing the water to flow from the tank throughout the system and out of the crack. After the system was primed (i.e., purged of all air, including bubbles), the apparatus was ready for testing.

The tests were conducted at controlled pressure (which ranged between 1 and 250 kPa) upstream of the leaking orifice. During the test, water was pumped through the orifice. Initially low pressures (i.e., 1 kPa) – and thus small flow rates – were applied, that were then incremented until the required pressure or capacity of the apparatus was reached. With each increment of pressure, flow rates were recorded and were allowed to stabilize for some time (5 minutes was found to be sufficient). Pressure in the pipe (i.e., upstream of the leak) and flow rate were read out from the HMI, as mentioned. The same procedures were repeated with each increase in pressure.

RESULTS AND DISCUSSION

Pressure head–flow rate relationship

The pressure head–flow rate relationship for different leak slots – 0.23 mm, 0.42 mm, 0.65 mm – was obtained experimentally as well as theoretically (using the standard orifice flow equation) and is plotted in Figures 8–10, respectively. Inspection of the pressure head–flow rate relationship shows some remarkable behavior (Figures 8–10). The data show that the rate of flow initially varies with the pressure head according to the orifice flow theory (i.e., flow rate varies with the square root of the applied pressure head). However, at a particular flow rate – i.e., 23 L/min for the 0.23 mm orifice opening – a deviation from the orifice flow equation was observed. At this point, higher pressures were required to deliver more flow. For example, in the case of the 0.23 mm orifice opening, increasing the pressure head from 18 m to 20 m resulted in a tiny increase in the rate...
of flow from 22.8 L/min to 23.6 L/min only. Similar behavior was observed for the other tested orifice openings – 0.42 mm, 0.65 mm – and occurred at flow rates of 34 L/min and 60 L/min, respectively.

This deviation from the orifice flow equation is attributed to the effect of inception of the cavitation phenomenon (i.e., flow separation) in the orifice, in what is also known as ‘choking flow’. This phenomenon occurs at an exceedingly high...
velocity in the orifice when the pressure of the liquid drops below its vapor pressure. Through this mechanism, a vapor region is thought to form inside the orifice, reducing the effective area of the flow until a point is reached at which there is no further increase in the rate flow as the pressure increases (Figures 9 and 10). This observation is consistent with the findings of Koivula (2002), who also observed a similar phenomenon in a small orifice.

Fluid mechanics theory also supports the view that the cavitation phenomenon can occur in these conducted experiments. Looking to the nondimensional parameter cavitation number $K$ for estimating the resistance of flow to cavitation, using Equation (3), the higher the cavitation number ($K$), the less likely cavitation is. A critical cavitation number of between 0.2 and 1.5 is reported, below which the cavitation is likely to appear (Lamb 1987).

When the downstream pressure at the orifice is taken as atmospheric $P_{atm} = 101.3$ kPa, corresponding to a velocity through the orifice of 15.6 m/s, and the density and vapor pressure of water at 20 °C is 998.2 kg/m³ and 2.335 kPa, respectively, we obtained a cavitation number ($K$) of 0.8, indicating that the cavitation phenomenon may occur at this point.

The condition of cavitation ‘choked’ flow directly affects the coefficient of discharge $C_d$ through the orifice. This parameter can be estimated for the tested conditions using the orifice flow equation, with known orifice geometry and pressure head:

$$C_d = \frac{Q_{act}}{A_o \sqrt{2gh}}$$

(7)

where $Q_{act}$ is the actual flow rate at stationary condition, $A_o$ is the cross-sectional area of the orifice (area of the leak), $g$ is the acceleration due to gravity, and $h$ is the pressure head.

Figures 11 and 12 show the evolution of the coefficient of discharge in the function of the Reynolds number for orifices of 0.42 and 0.65 mm, respectively. The coefficient of discharge is fairly constant in the noncavitating conditions, in which an average value of $C_d$ could be obtained. It is interesting to note that in noncavitating conditions, a value of 0.88 was obtained for $C_d$ for the tested 0.42 and 0.65 mm orifice openings (Figures 11 and 12).
Figure 11 | Coefficient of discharge ($C_d$) as a function of Reynolds number (Re) for 0.42 mm orifice opening.

Figure 12 | Coefficient of discharge ($C_d$) as a function of Reynolds number (Re) for 0.65 mm orifice opening.
The analytical solutions using an average value of $C_d$ obtained from these experiments match well with the data in noncavitating flow conditions (Figures 8–10). Nonetheless, when the orifice is cavitating, the coefficient of discharge experiences a notable decrease (Figures 11 and 12). This decrease is directly related to the collapse of the rate of discharge as a result of cavitation, as observed in Figures 8–10.

Using the model proposed by Schmidt & Corradini (1997), the flow coefficient ($C_d$) for cavitating orifice flow can be calculated. In this model, a constant value of flow coefficient $C_d$ is used for noncavitating flow, and for cavitating flow, $C_d$ is calculated as a function of cavitation number using Equations (3) and (6).

When cavitation occurs in the orifice, the vapor region occupies a fraction of the orifice cross-sectional area ($A$) and passes through the vena-contracta $A_c$ (Figure 13).

Using values for the coefficient of discharge $C_d$ of 0.88 and the coefficient of contraction $c_c$ of 0.68, as obtained from these experiments, the flow behavior for both cavitating and noncavitating flow can be predicted. Figure 14 shows the calculated pressure-flow rate relationship compared with that measured experimentally. Comparison of the data shows a good match between calculated and measured flow values.

The predicted flow rate was calculated using the orifice flow theory (Equation (1)). In noncavitating flow conditions, a constant value for the coefficient of discharge ($C_d$) was used, whereas in cavitating flow conditions, $C_d$ values varying as a function of the cavitation number were used. The varying $C_d$ values in cavitating flow conditions were calculated using Equation (6).

Based on hydraulic theory, the value of the coefficient of contraction $c_c$ for an orifice ranges between 0.63 and 0.69 depending on the size and geometry of the orifice. For this experimental setup, the value of $c_c$ was determined by assuming a value for $c_c$, then calculating the rate of flow, to get the

![Figure 13](Image)

**Figure 13** Contraction at the orifice inlet due to cavitation phenomenon.

![Figure 14](Image)

**Figure 14** Measured and predicted pressure-flow rate relationship for noncavitating and cavitating flow conditions, orifice opening of 0.65 mm.
CONCLUSIONS

This paper investigated the hydraulic behavior of leakages from WDSs, in particular by examining the effect of the cavitation phenomenon on the pressure–leakage relationship. For this purpose, a special experimental model was designed and built to simulate idealized cracks in defective water pipes. Using this model, water was allowed to flow through the cracks at controlled pressures during observation of hydraulic behavior.

Based on this study, the following may be concluded:

- The phenomenon of cavitation can develop in leaking orifices of defective water pipes and thus cannot be ignored when modeling leakage rates from WDSs. Development of cavitation in leaking orifices drastically affects the pressure–flow rate relationship. At the inception of cavitation, the rate of flow starts to collapse (i.e., no further increase in the flow rate is seen as pressure increases).
- Cavitation occurs at a high velocity in the leaking orifice when the pressure of the liquid drops below its vapor pressure. By this mechanism, a vapor region forms inside the orifice, reducing the effective area of the flow. At this point, the flow is ‘choked’.
- Cavitation inception is dependent on opening size and rate of flow through the leak but is independent of the pressure head.
- The coefficient of discharge in cavitation flow condition is influenced by the cavitation number rather than the Reynolds number. In conditions of noncavitating flow, however, a constant value for the coefficient of discharge can be assumed, which is a function of the Reynolds number.

A theoretical model has been presented to determine the coefficient of discharge ($C_d$) for cavitating orifice flow. In this model, a constant value for the coefficient of discharge is used for noncavitating flow, and for cavitating flow, $C_d$ is obtained as a function of cavitation number. Comparison of the data shows a good match between model-calculated and measured values.

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REFERENCES

Lamb, W. S. 1987 Cavitation and aeration in hydraulic systems. BHR Group, Bedfordshire, UK, 114 pp. 

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