Transboundary water allocation in critical scarcity conditions: a stochastic bankruptcy approach

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ABSTRACT

A common problem in water resource allocation is to design a stable and feasible mechanism of water sharing in critical scarcity conditions. The task becomes very challenging when the water demand exceeds the available water resources reserves. To address this pervasive allocation problem related to transboundary rivers, the bankruptcy method is used. The bankruptcy method distributes water among riparian states when their total demand exceeds the total available water. This paper describes a new methodology for the allocation of scarce water resources in a complex system using a stochastic game theory which is an extension of bankruptcy theory. The authors have also proposed 'weighted bankruptcy' approach that can be used under a stochastic setting. The weighted bankruptcy approach favors agents with 'high agricultural productivity'. The bankruptcy rules have been applied in the water resource system in four critical scarcity scenarios. The available water is allocated under the simple and weighted bankruptcy rules. The results showed that under all four scenarios, the weighted bankruptcy rules favor the agents which have a high agricultural productivity. The stochastic bankruptcy approach under the simple and the weighted bankruptcy rules can provide important strategic information for better management and sustainable sharing of water resources.

Key words | bankruptcy theory, riparians, stochastic, sustainable sharing

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INTRODUCTION

Management and allocation of water in scarce conditions is a common problem in water resource management (Kanakoudis 2002; Kanakoudis *et al.* 2016, 2017). This problem of water allocation can be analyzed using bankruptcy game (BG) technique, which is a branch of cooperative games theory (CGT) (Young 1994). The problem of bankruptcy arises when some agents have claims on the available goods or assets, but their total claim is greater than the total available assets. The assets must be divided among the claimants or agents in such a way that each claimant or agent might receive a non-negative amount that cannot be greater than its claim. There are numerous applications of bankruptcy problems which include different real-life problems and the bankruptcy approach has proved very useful for those problems. In the literature, several bankruptcy rules and their extensions have been introduced to solve the bankruptcy problem (O'Neill 1982; Auman & Maschler 1985; Herrero & Villar 2001; Thomson 2003).

Several political disputes have been caused throughout the world due to the non-equitable distribution of water resources, the increasing consumption of resources, and the scarcity of water resources (Homer-Dixon 1994). There is a total of 148 transboundary river basins that are shared among 148 countries (De Stefano *et al.* 2012). During the past 50 years, 43 political or military acts related to shared water resources have taken place around the world (Wolf 2007). Due to factors such as climate change, growing crop production, increasing populations, and soil degradation,

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freshwater has become a source of conflict among riparian states. Hence, one of the main challenges in transboundary river management concerns how we can allocate the limited and shared available water among riparian states when it is not sufficient to satisfy the claims of all riparian states. Therefore, 'equitable' and 'reasonable' water resource reallocation faces the question of which criteria and mechanisms should be considered for this 'equitable' and 'reasonable' reallocation (Mianabadi *et al.* 2012). A variety of climatic, socioeconomic, environmental, geographical, historical, and political factors and conditions can affect water resources and, consequently, different types of water conflicts (Delli Priscoli & Wolf 2009). In one category, there are two different approaches to addressing water conflicts: first, conflicts over international water resources (Yoffe *et al.* 2003) such as aquifers, lakes, and rivers that are shared between two or more countries; second, conflicts over internal water resources shared between states, provinces, cities, or different groups within a specific country (Toset *et al.* 2000). In another category, there are two different types of water conflicts: conflicts over the quality (Perry & Vanderklein 2009) and conflicts over the availability (Ping & Ping 2010) of water resources. These conflicts can have different degrees in terms of intensity and seriousness. The complexity of water conflicts calls for accurate investigations for such conflicts to be effectively resolved. Shared water can be a source of both cooperation and conflict among riparians. The main problem arises when the total demand of riparian countries or provinces is more than the total available water. In quantitative conflict resolution, the equitable allocation of water among riparians is a complex process at both the national and international scales (Jarkeh *et al.* 2016).

To manage conflicts and allocate resources, the bankruptcy method is widely used. This method is applicable when the total claims (C) exceed the total resources or assets (E). Bankruptcy theory has been applied to problems related to the allocation of resources. Grundel *et al.* (2013) used this method for multipurpose resource allocation situations. Ansink & Marchiori (2015) used it for water resource management. In addition, Beard (2011) provided a detailed review of the connection between river sharing and the bankruptcy literature. The frequent application of the bankruptcy method reveals that it is a popular tool for resolving conflicts and for achieving agreement on water resource allocation problems. Bankruptcy theory can be used in resource allocation and dispute resolution when the total available resources are less than the total demand or claims of riparian countries or provinces (Ansink & Marchiori 2015). Auman & Maschler (1985) and O'Neill (1982) introduced bankruptcy theory, and it was later studied by various researchers (O'Neill 1982; Hendrickx *et al.* 2005; Lorenzo-Freire *et al.* 2010; Thomson 2012; Alcalde *et al.* 2014). Bankruptcy theory has been used by several researchers for water allocation among riparian countries (Madani *et al.* 2014; Mianabadi & Sheikhmohammady 2014; Mianabadi *et al.* 2015; Li *et al.* 2018). Bozorg-Haddad *et al.* (2018) used bankruptcy theory for water allocation in Iran. Degefu *et al.* (2018) applied the cooperative game theory allocation by combining the Nash bargaining theory and bankruptcy games for the water allocation among Syria, Iraq, and Turkey. Water management using system dynamics modeling in semi-arid regions was done by Reza *et al.* (2017). Sedghamiz *et al.* (2018) used a game theory approach for conjunctive use optimization model based on virtual water concept.

A methodology for water allocation using the bankruptcy games is described in this paper for water resources under scarcity. A new methodology is also developed which considers the priorities in the allocation of water. This novelty is presented by the consideration of the user's priorities, which are settled by 'high agricultural productivity': the user with high agricultural productivity will be given preference in water allocation. First, the water distribution among the provinces of Pakistan is done under four different scenarios using the bankruptcy rules. Second, we apply an allocation procedure that includes claimants' priority by considering the agricultural productivity of the users.

BANKRUPTCY PROBLEM AND COOPERATIVE GAME THEORY

The bankruptcy methods are used in economics when the available asset (resource) is not sufficient to satisfy the claim of creditors (stakeholders). When total available resources are below the aggregate demand, the expected use of each user needs to be reduced by some amount, which can be calculated using different bankruptcy methods available in the literature (Curiel *et al.* 1987; Dagan & Volij

1993; Madani & Dinar 2011). The foundations of these bankruptcy models are set in the works of O'Neill (1982) and Auman & Maschler (1985). The links between the cooperative game theory (CGT) and bankruptcy games (BG) were consolidated by Curiel *et al.* (1987), in which they studied the class of division rules for bankruptcy as corresponding to a CGT approach to those problems.

In fact, CGT (Neumann & Morgenstern 1944; Young 1985) provides the necessary instruments to analyze allocation problems and to research a sharing mechanism which is considered efficient, just, and fair by users. A cooperative game can be seen as a kind of game where it is necessary to determine the fair allocation of 'goods' among different players (Lemaire 1984). The allocation can consider 'positive' or 'negative' goods according to a 'benefit game' or a 'cost game' solution for the users. In the first case, users collaborate with each other in order to obtain the biggest fair benefit; in the second case, users collaborate to share the smallest cost system (Young 1994).

Many allocation problems have been addressed in the literature using the CGT. Different research fields have been considered in the approaches including water resources (Deidda *et al.* 2009; Zucca 2011).

Synthetically, to define a cooperative game problem, the following definitions are needed:

$N = (1, 2, \ldots, n)$ is the set of players participating in the game
$S \in N$ is a 'coalition', and for $S = N$ we have the so-called 'grand coalition'
$v(i)$ represents the minimum cost or maximum benefit connected to the user i
$v(S)$ is the minimum cost or maximum benefit linked to the coalition S
$v(N)$ consequently, represents the cost or benefit associated with the grand coalition.

The discrete function given by the costs or benefits of every coalition is called the characteristic function and represents a key element of the cooperative game solution. An allocation is a vector $[x_1, x_2, \ldots x_n]$, where x_i is the amount of the good assigned to the *i*th player.

Regarding a benefit-sharing game, to obtain a fair solution, CGT exploits three fundamental principles (in the case of a cost game the inequality signs are the opposite).

The *efficiency principle*, which guarantees the total sharing of the grand coalition benefit among all the participants of the game:

$$\sum_{i \in N} x\,(i) = v(N) \tag{1}$$

The *rationality principle*, for which no user (or coalition) can be assigned less than its standalone benefit (i.e., opportunity benefit):

$$\sum_{i \in S} x\,(i) \geq v(S) \tag{2}$$

The *marginality principle*, which states no user should be charged more than its marginal benefit from being included in a coalition:

$$\sum_{i \in S} x\,(i) \leq v(N) - v(N - S) \tag{3}$$

The sharing of total available goods is ensured by statement (1); the incentives for the voluntary cooperation is ensured by statement (2) while statement (3) provides the consideration for equity. Conditions (2) and (3) become equivalent as condition (1) is ensured.

Two different definitions are given in CGT for the game problem solution. The first is given by the set of admissible solutions: the so-called 'core' that is the set of all allocations $x \in \mathbf{R^N}$, such that (1) and (2), or equivalently (3), hold for all S of N (Young & Okada 1982). The second type is given by a single allocation, which individuates only one solution, and this is more similar to the classic idea of the solution to a problem.

BANKRUPTCY RULES

As described above, the bankruptcy methods are used in economics when the available stock is not sufficient to satisfy the claim of the claimants or creditors. Based on this, the assumption can be made that the total water resources are not sufficient to satisfy the demand of the claimants, therefore, these rules of bankruptcy can be used for the fair allocation of water resources to satisfy all the beneficiaries (Kaveh Madani 2012). A large amount of literature on bankruptcy problems and related division rules is available. Here, we mention the papers by Gallastegui *et al.* (2002), O'Neill (1982), Ansink (2009), and Mianabadi *et al.* (2015). More studies related to the bankruptcy rules have already been discussed in the Introduction.

The set N of claimants is of the form $\{1, 2, \ldots, n\}$. Each claimant $i \in N$ advances one claim di on the estate E with $E < \Sigma_{i \in N}\, di$.

A division rule f (E, d) associated with the bankruptcy problem gives a solution as a vector $x = (x_1, x_2, \ldots, x_n)$, such that:

$$\begin{cases} \sum_{i \in N} x(i) = E \\ 0 \leq Xi \leq di \end{cases} \tag{4}$$

In Equation (4), x_i represents the amount of the estate E assigned to the ith claimant.

The cooperative game, related to the bankruptcy problem, is defined by the characteristic function as given in Equation (5):

$$v_{E,d}\,(S) = max \left\{ \left(E - \sum_{i \in (N-S)} d_i \right), 0 \right\} S \in N \tag{5}$$

where $v_{E,D}$ (S) in Equation (5) denotes the minimal amount that the coalition $S \subset N$ will receive once the claims of the creditors outside S have been fully compensated.

In this situation, the solution to the bankruptcy problem and the related cooperative game is the same. Hereafter, we consider division rules reported by Branzei *et al.* (2008) considering a flow approach to bankruptcy problems: the proportional rule (PROP), the constrained equal award rule (CEA), the constrained equal loss rule (CEL), the Talmudic rule (TAL), and the Piniles rule (Pin) which are explained below.

Let 'n' be the number of claimants. The claimants are $n \geq 2$ and their claims are $c_i \geq 0$; $C = (C_1, \ldots, C_n)$. A bankruptcy problem in river systems is defined as F (N, E, c_i, a_i); $i = 1, 2, \ldots, n$, where N = no of agents, E = total resources, c_i = claim of the agent i, and a_i = contribution of the agent i. The aim of bankruptcy method is to determine the allocation to each agent, denoted by F (N, E, c_i, a_i) = x_i where $x_i \geq 0$; $x = (x_i, \ldots, x_n)$. For a resource allocation problem, we have:

$$E = \sum_{i=1}^{n} a_i \tag{6}$$

$$C = \sum_{i=1}^{n} C_i \tag{7}$$

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} x_i \tag{8}$$

$$0 \leq X_i \leq C_i \tag{9}$$

where Equations (6) and (7) are the contribution and claims of the agents, respectively. Equation (8) states that the assets are fully allocated. Equation (9) states that the allocation cannot exceed its claims and can never be negative.

1. Proportional rule (PRO):

The proportional rule (PRO) is given by Equation (10):

$$p_i^{pro} = \rho c_i \text{ where } \rho = \frac{E}{C} \tag{10}$$

where C is the total amount of claims and E is the total assets.

2. Constrained equal award (CEA) rule:

This rule is given by Equation (11):

$$x_i^{CEA} = \min(\lambda, c_i) \text{ where } \sum_{i \in N} \min(\lambda, c_i) = E \tag{11}$$

CEA assigns each agent an equal share λ of E, except that no creditor receives more than his or her claim.

3. Constrained equal losses (CEL) rule:

As per Equation (12), this rule is defined as:

$$x_i^{CEL} = \max(0, c_i - \lambda) \text{ where } \sum_{i \in N} \max(0, c_i - \lambda) = E \tag{12}$$

CEL allocates each claimant a share of the asset such that their losses in comparison with their claims (λ) are equal, constrained to no claimant receiving a negative allocation.

4. The Talmud rule:

The Talmud rule is derived by combining the CEL and CEA rule and is given by Equation (13):

$$x_i^{TAL} = \begin{cases} CEA\left\{\frac{1}{2}c_i, E\right\} & if\ E \leq \frac{1}{2} \\ \frac{1}{2}c_i + CEL\left\{\frac{1}{2}c_i, E - \frac{1}{2}C\right\} & otherwise \end{cases} \tag{13}$$

5. Piniles' rule:

For each c_i, Piniles' rule is calculated as given by Equation (14) (Bosmans & Lauwers 2011):

$$x_i^{Pin} = \begin{cases} x_i^{CEA}\left\{\frac{1}{2}\underline{c}, E\right\} & if\ E \leq \frac{D}{2} \\ \frac{1}{2}\underline{c} + x_i^{CEA}\left\{\frac{1}{2}\underline{c}, E - \frac{D}{2}\right\} & if\ E \geq \frac{D}{2} \end{cases} \tag{14}$$

In the above bankruptcy rules, the 'estate' amount E to be divided is the available water for users; moreover, different priorities or rights of the claimants are also considered by using the riparians' agricultural productivity.

WEIGHTED BANKRUPTCY RULE: METHODOLOGY DEVELOPMENT

A novel weighted bankruptcy mechanism has also been developed and the above-defined bankruptcy rules have also been applied that include the riparians' priority considering the agricultural productivity of each riparian: higher agricultural productivity produces higher user priority. The method will encourage the riparians to increase their agricultural productivity, which is very essential considering the scarcity of water in future. A simple method to define agricultural productivity is given by crop production per acre feet of water (US$). Therefore, the claimants 'weights' are included, and the BG allocation has been modified. These weights are evaluated considering the agricultural productivity: higher weight w_i is given to the riparian with high crop productivity; the weighted demands will be consequently defined as:

$$w_i = f(p_i,\ u_i);\ d_i^* = d_i w_i$$

All the bankruptcy rules defined above will then be applied again considering the weighted water demands. The weighted water requests are considered as the claims of the agents or riparians. If any riparian or claimant receives a greater water allocation than its original request, the assignment will be equal to its original request and available surplus will be shared among the other users using the same rule.

SOLUTION FRAMEWORK

After defining the objective, the fundamental principles of water sharing as stated in Equations (1)–(3) are defined. The disagreement points as well as the amount of water available for consumption is then determined. The allocation of water among the riparians is then done using the bankruptcy rules. The procedure for water allocation problem under water bankruptcy using the bankruptcy rules is described in Figure 1.

When the demand and the available water in the river basin changes with time, the bankruptcy rules are applied again by updating the disagreement points and the water allocation is done again using the bankruptcy rules.

CURRENT WATER DISTRIBUTION MECHANISM IN PAKISTAN AND ITS SHORTCOMING

The Islamic Republic of Pakistan is home to the sixth largest population in the world. There are four administrative units in the country, or four provinces, namely, Punjab, Sindh, Baluchistan, and Khyber Pakhtunkhwa. Pakistan also consists of small areas of Gilgit-Baltistan and federally administered tribal areas. Arable land and water are the principal natural resources of Pakistan, and agriculture significantly contributes to the country's economy. It accounts for almost 19.8% of the country's gross domestic product (GDP) (Anwar & Bhatti 2017). Of the 27% of cultivated land in Pakistan, Punjab has the highest proportion (63%), followed by Sindh (18%), and the remainder is equally divided between the provinces of Baluchistan and Khyber Pakhtunkhwa (Delli Priscoli & Wolf 2009; Ahmad 2017).

The interprovincial water sharing of surface waters in Pakistan is currently governed by the Water Apportionment

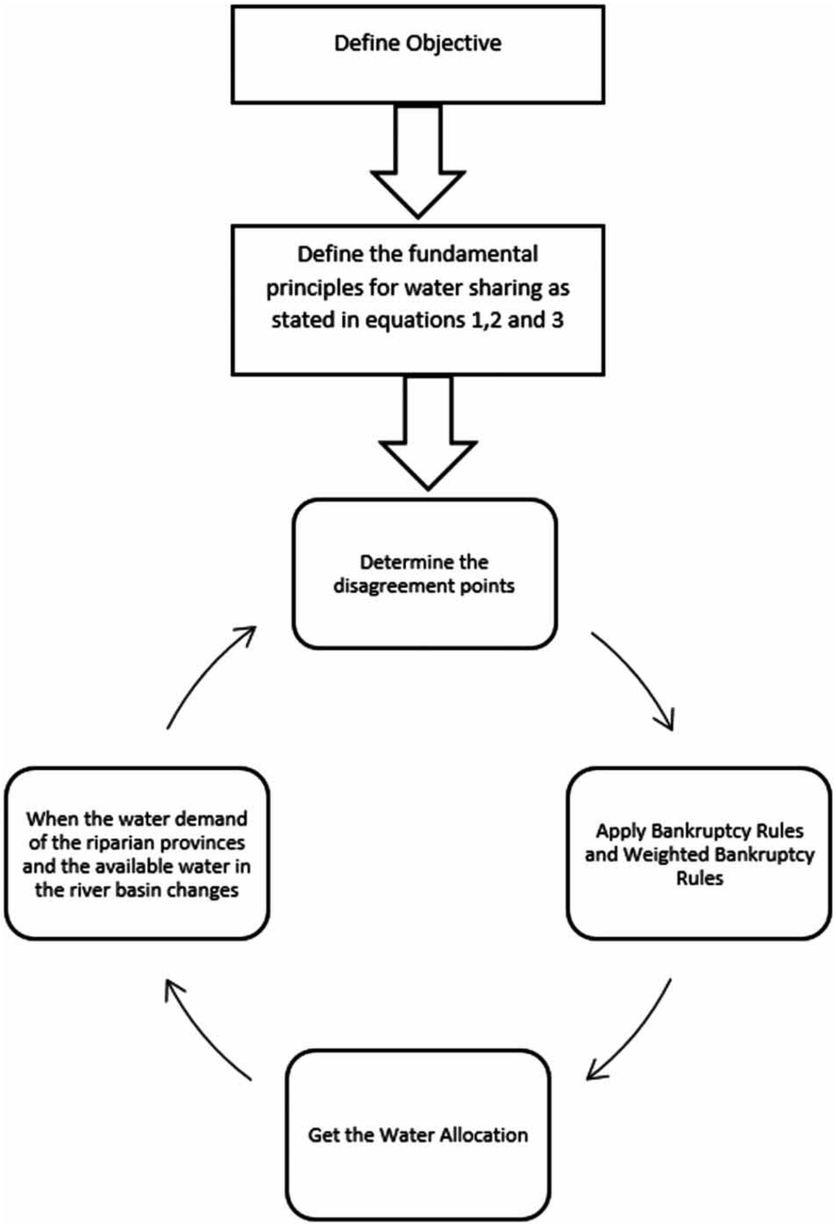

Figure 1 | Flow chart for water allocation under bankruptcy.

Accord of 1991. The main aim of the accord was to build trust among the provinces of Pakistan. Unfortunately, this accord does not adapt to changing conditions over time; hence, it can be considered 'a glass that is half empty and half full'. The current water distributions among the provinces of Pakistan according to the Water Apportionment Accord are given in Table 1. The Indus River System Authority (IRSA) is responsible for the distribution of water among the provinces. The main shortcomings of the Water Apportionment Accord of 1991 are highlighted below.

The average canal diversions in post-Tarbela periods have only been 127 km^3, which is less than the accord's entitlements of 144.87 km^3, as shown in Tables 1 and 2. This creates problems among the provinces of Pakistan when they have to share shortages as there is no defined mechanism to share the water shortages (Condon *et al.* 2014). Currently,

Table 1 | Water allocation among provinces (*Source*: Anwar & Bhatti 2017)

Province	Water share (km^3)	Balance supply shares[a] in %
Punjab	69.03	37
Sindh[b]	60.17	37
Baluchistan	4.78	12
KPK	7.13	14
Ungauged canals[c]	3.70	
Total	144.87	100

[a]Including future storages and flood flows.
[b]Including already sanctioned urban and industrial uses for Karachi.
[c]Ungauged civil canals above rim stations in KPK.

Table 2 | Water requirement of various crops (*Sources*: Planning Commission 2010; Ahmad 2017)

Water requirements for various crops	Punjab km^3	Sindh km^3	KPK km^3	Baluchistan km^3	Total km^3
Wheat	26.72	5.17	3.05	1.20	36.12
Rice	17.66	3.97	0.41	2.10	24.16
Cotton	14.29	2.97	0.00	0.20	17.46
Sugarcane	8.07	2.72	0.88	0.01	11.68
Maize	2.04	0.01	1.69	0.02	3.76
Barley	0.10	0.04	0.10	0.04	0.28
Other crops	40.62	15.20	2.14	5.85	63.80
All crops	**109.49**	**31.07**	**8.28**	**9.42**	**157.25**
All crops after Sindh's requirement for environmental flows	**109.49**	**43.37**	**8.28**	**9.42**	**170.56**

Note: The requirements for Sindh include an additional 12.3 km^3 as environmental flows.

the two small provinces, Baluchistan and Khyber Pakhtunkhwa (KPK) are exempted by the water shortages by an act of 2003. Thus, whenever the total volume falls, the deficiencies are shared by Sindh and Punjab (Condon *et al.* 2014). Thus, there is no proper mechanism of water distribution when the total volume falls short or when the demands of the provinces exceed the total available water. Also, Khyber Pakhtunkhwa (KPK) and Baluchistan have not yet developed their irrigation system properly, therefore, they always get more water than they can use.

Another serious problem in the Water Apportionment Accord is that the water allocations are fixed which creates a quantified entitlement. Fixed water allocations' mechanisms can lead to water allocations which are unacceptable for the provinces, especially in the uncertainty, droughts, and the stochastic nature of river flow. The Water Apportionment Accord between the provinces of Pakistan was signed almost 28 years ago. Since then, the water demands of the provinces have changed due to the increase in population and the irrigated area. Therefore, the gap between the water supply and water demand has increased considerably in Pakistan. Several features and attributes of the Indus River disputes are described in the following section.

Surface water diversions

As shown in Figure 2, the Indus River, which is composed of six major tributaries, namely, the Indus, Chenab, Jhelum, Ravi, Beas, and Sutlej tributaries, is a major source of water supply for Pakistan. The water of the Indus River is shared among all four provinces of Pakistan. River flows are mainly supplied by rainfall, snowmelt, glacier melt, and runoff. According to the Indus River System Authority, the median canal diversions from 1975 to 2013 were 125.61 km^3 (Hassan 2016).

Agricultural water requirements for Pakistan

In this study, the water requirements (in feet) for various crops were taken from the Planning Commission Report whereas the cropped area was taken from the Agricultural Statistics of Pakistan.

The total agriculture water requirements or demands for Pakistan for the year 2002–2003 were estimated to be 109.7 km^3. Punjab had the highest irrigation water requirements at 78.4 km^3, followed by Sindh (20.1 km^3), KPK (6.2 km^3) and Baluchistan (5 km^3). For this particular study, the total water requirements (in feet) of the various crops of Pakistan were taken from the Planning Commission Report of the Government of Pakistan. The total area under various crops in the provinces of Pakistan was taken from the Agriculture Census of Pakistan 2010 (Government of Pakistan 2011). The total water required for various crops in cubic kilometers (km^3) was calculated as follows:

$$\text{Total water required for crop 'X'} = \text{Net crop water requirement (ft)} \times \text{Area under crop 'X'}$$

As per the Agricultural Census of Pakistan, Punjab has the largest cultivated area among the four provinces of Pakistan

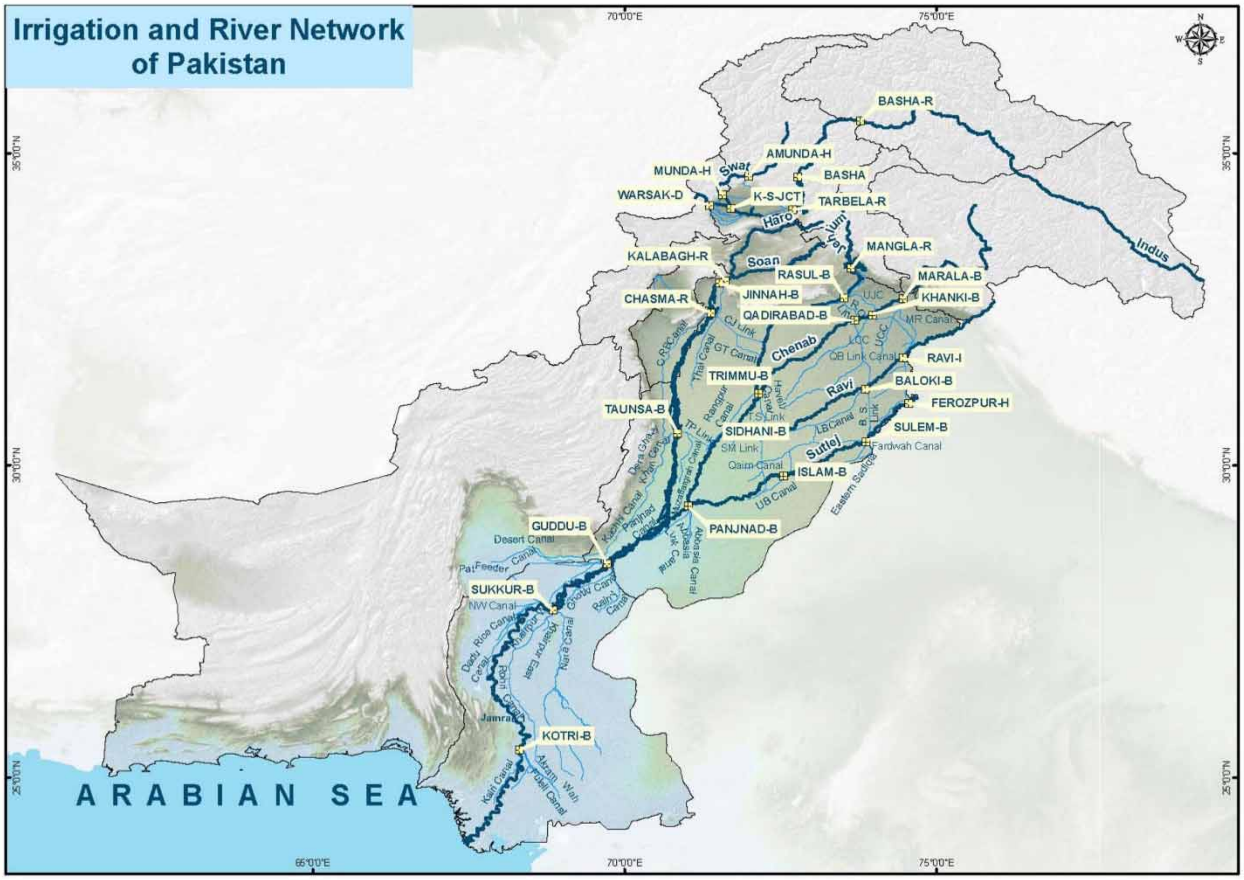

Figure 2 | Irrigation and river network of Pakistan (This map was extracted from Indus River System Authority, IRSA). Available online: http://pakirsa.gov.pk/images/IrrigationNetwork_Pakistan.jpg.

which accounts for about 56.6%, followed by Sindh, KPK, and Baluchistan. According to this study, the agriculture water requirements for Pakistan are 157.25 km^3 (excluding the environmental flows). The water required downstream of Kotri as environmental flows for the province of Sindh is 12.3 km^3. The 1991 Water Accord between the provinces also recognized the need for a residual flow to the delta of 12.3 km^3, but no formal policy or institutional mechanism has yet been adopted to make environmental water allocations. There is a clear need for such environmental allocations to be made, not only for a minimum residual flow, but also to provide a measure of flow variability that mimics the natural river regime (Archer *et al.* 2010). The total water requirement including the environmental flows, therefore, is calculated as 170.56 km^3, which also included 12.3 km^3 as environmental flows for the province of Sindh. Punjab had the highest water requirements (109.48 km^3), followed by Sindh (43.07 km^3), Baluchistan (9.41 km^3), and KPK (8.27 km^3), as shown in Table 2. In order to check the reliability of our estimated water demands for agriculture, we compared our calculated water demands with other studies. According to the report published by the Ministry of Food, Agriculture and Livestock Islamabad, in 2004, the total agricultural water requirements in Pakistan would be around 150 km^3 by 2020 (Hanif *et al.* 2004). According to Ahmad (2012), 'the demand of water to meet net crop needs would be 154.5 km^3 by 2020'. The estimates in these two reports suggest that our calculation of the agricultural water requirements of 157.25 km^3 is reliable.

The canal/irrigation water supplies for Indus River

Various researchers have different views regarding the flows of the Indus River and its tributaries. According to

Water and Power Development Authority (WAPDA), the average annual river flow is approximately 138 MAF or 170 km^3, of which 128 km^3 is diverted for irrigation (Bakhsh *et al.* 2011). Qureshi (2011) stated that the average flow of the Indus River and its tributaries is 175 km^3 of water. Of this, 165 km^3 is from the western rivers (Jhelum Chenab and Indus) whereas 10 km^3 is from the eastern rivers (Beas, Ravi, and Sutlej). Most of this, 128 km^3, is diverted for irrigation; according to Hussain *et al.* (2011), the total water supply for the agriculture sector is 130 km^3. According to another report by the Ministry of Food, Agriculture and Livestock, Islamabad, Pakistan, the total surface water diversions for Pakistan are 130 km^3 (Hanif *et al.* 2004).

Indus river supplies and, consequently, the canal water diversions for agriculture in Pakistan are highly variable, as shown in Figure 3. The canal diversions vary from 137 km^3 (111.1 MAF) to 94 km^3 (76.2 MAF). Thus, the variability between the highest and lowest canal diversion is 43 km^3 (34.9 MAF) due to the stochastic nature of river flows. The canal diversions are also affected due to less storage capacity. From 1975 to 2013, the lowest canal diversion was 94 km^3 (76.2 MAF), which severely affected the agricultural sector in Pakistan. 141 km^3 (114.32 MAF) of the canal water supplies was apportioned to the provinces according to the Water Accord (Table 1) on the condition that more dams would be constructed and additional storage created. Since the signing of the Water Accord in 1991, the agricultural water demands have increased but no new storage reservoir has been constructed. The median canal diversion from 1975 to 2013 was 125.61 km^3 (101.84 MAF), which is less than 141 km^3 (114.32 MAF), as decided by the Water Accord.

BANKRUPTCY GAMES APPROACH APPLIED IN WATER RESOURCE ALLOCATION IN THE PROVINCES OF PAKISTAN

This paper aims to show the distribution of limited amount of water available to satisfy the riparians or claimants using the BG procedures. The case of Indus River system is considered. Four different scenarios are developed for the water allocation, as shown in Table 3. First the distribution of water for all four scenarios is done using the five bankruptcy rules given earlier, then, a novel allocation procedure is applied which includes claimants' agricultural productivity. More agricultural productivity would result in higher priority in water allocation.

The core solutions of the cooperative game are defined by the principles of efficiency, rationality, and marginality, as defined in Equations (1)–(3). The lower bound for each user is given by the rationality principle and the upper bound is given by the marginality principle. Table 4 summarizes the results, $x(i)$ represents the water allocation for the

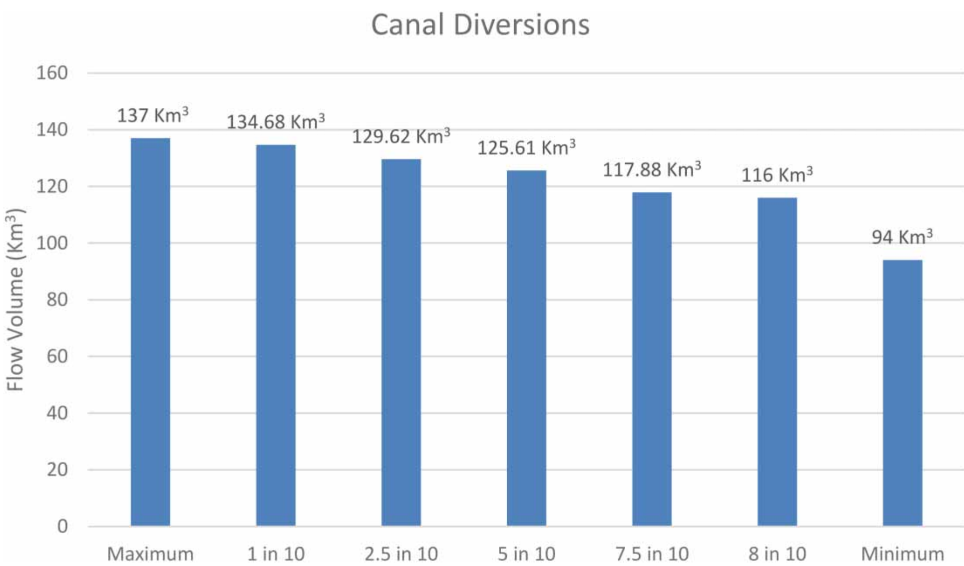

Figure 3 | Canal diversions (km^3) during 1975–2013.

*i*th user expressed in Mm^3/year; the parenthesis value represents the assigned percentage in the four deficit scenarios with respect to the available resource. In Table 4, the upper and lower bounds of water allocation inside the core can be considered as limits of 'feasible values' that could be accepted by each user.

Figure 4 summarizes the results of five bankruptcy rules under different scenarios. These results are expressed as percentage values. These values are therefore not compared to the total assets but to the user's claims. For example, in Scenario 1, according to proportionate rule, all four provinces, namely, Punjab, Sindh, Baluchistan, and KPK are awarded 73% of their claims. Similarly, in Scenario 1, according to Piniles' rule, Punjab is awarded 58% of its claims whereas Sindh, Baluchistan, and KPK are awarded 100% of their claims. It is evident from Figure 4 that a fixed proportional allocation that shares the deficit among users is given by the PROP rule. CEA favors the agents having small claims whereas CEL favors the agents having large claims. Talmud and Piniles' rules have somewhat similar results and their results fall in between CEL and PROP rules.

The crop production per acre feet of water, weights, and the weighted water demands are shown in Table 5. The bankruptcy rules are then applied considering the weighted water demands and the results are shown in Figure 5. A comparison between Figures 4 and 5 highlights how weighted water requests modify BG assignments including priorities. From Figure 5, it is evident that due to their higher weights, KPK and Baluchistan are given more preference and hence their satisfaction level is more than Punjab and Sindh in almost all the bankruptcy rules, even when considering the minimum canal water diversions (Scenario 3). Moreover, the allocation rules used here give solutions that are still inside the core boundaries (Table 4) and they belong to the set of

Table 3 | Water availability, claims, and deficit under different scenarios

Reference scenarios	Water availability (km^3)	Claims (km^3)	Deficit (total) (km^3)
Scenario 1 (5 in 10)	125.61 km^3	Punjab (P) = 109.49 km^3; Sindh (S) = 43.37 km^3; Baluchistan (B) = 9.42 km^3; KPK (K) = 8.28 km^3; **Total = 170.56 km^3**	44.95
Scenario 2 (8 in 10)	116.00 km^3	Punjab (P) = 109.49 km^3; Sindh (S) = 43.37 km^3; Baluchistan (B) = 9.42 km^3; KPK (K) = 8.28 km^3; **Total = 170.56 km^3**	54.56
Scenario 3 (minimum)	94.00 k	Punjab (P) = 109.49 km^3; Sindh (S) = 43.37 km^3; Baluchistan (B) = 9.42 km^3; KPK (K) = 8.28 km^3; **Total = 170.56 km^3**	76.56
Scenario 4 (5 in 10) (15% increase in water demands under future scenario)	125.61 km^3	Punjab (P) = 126 km^3; Sindh (S) = 49.87 km^3; Baluchistan (B) = 10.83 km^3; KPK (K) = 9.52 km^3; **Total = 196.12 km^3**	70.51

Table 4 | Indus River Basin core solutions (km^3/year and percentage with respect to available resources)

Scenario-1 (Canal diversion: 5 in 10)	**Scenario-2 (Canal diversion: 8 in 10)**
x(P) + x(S) + x(B) + x(K) = 125.61	x(P) + x(S) + x(B) + x(K) = 116
64.54(51%) ≤ x(P) ≤ 109.49 (87%)	54.93(44%) ≤ x(P) ≤ 109.49 (87%)
0(0%) ≤ x(S) ≤ 43.37 (34.5%)	0(0%) ≤ x(S) ≤ 43.37 (34.5%)
0(0%) ≤ x(B) ≤ 9.42 (7.5%)	0(0%) ≤ x(B) ≤ 9.42 (7.5%)
0(0%) ≤ x(P) ≤ 8.28 (6.6%)	0(0%) ≤ x(P) ≤ 8.28 (6.6%)
Scenario-3 (Canal diversion: minimum)	**Scenario-4 (Canal diversion: 5 in 10) (15% increase in water demands)**
x(P) + x(S) + x(B) + x(K) = 94	x(P) + x(S) + x(B) + x(K) = 125.61
33(35%) ≤ x(P) ≤ 94 (100%)	55.28(44%) ≤ x(P) ≤ 125.61 (100%)
0(0%) ≤ x(S) ≤ 43.37 (34.5%)	0(0%) ≤ x(S) ≤ 43.37 (34.5%)
0(0%) ≤ x(B) ≤ 9.42 (7.5%)	0(0%) ≤ x(B) ≤ 9.42 (7.5%)
0(0%) ≤ x(P) ≤ 8.28 (6.6%)	0(0%) ≤ x(P) ≤ 8.28 (6.6%)

Figure 4 | Division rule solutions (percentage with respect to individual water demand).

Table 5 | Weighted water requests using priorities

	Punjab (P)	Sindh (S)	Baluchistan (B)	KPK (K)	Total
Crop production per acre feet of water (US$)	189	80	277	307	–
Weight	1.89	0.80	2.77	3.07	–
Actual water demand (km^3/year) **Scenario 1, 2, and 3**	109.49	43.37	9.42	8.28	**170.56**
Weighted water demand (km^3/year) **Scenario 1, 2, and 3**	207.0	34.7	26.0	25.4	**293.1**
Actual water demand (km^3/year) **Scenario 4**	126	49.87	10.83	9.52	**196.12**
Weighted water demand (km^3/year) **Scenario 4**	238.1	40	30	29.2	**337.3**

admissible solutions satisfying efficiency, rationality, and marginality principles. The main advantage of the weighted bankruptcy rules as compared with conventional ones is that it allows consideration of the agricultural productivity of states as an important factor which may facilitate negotiations among riparian countries or provinces.

In the Indus River Basin, climate change can have undesirable effects. Increased temperature and evapotranspiration and reduced precipitation may result in considerable changes in river runoff. It is predicted that runoff will decrease in the range of 5–40% for a majority of the basin for the period 2090–2099 relative to 1980–1999 (Granit & Joyce 2012). Therefore, for further studies, it is proposed to consider the impact of climate change on the total flow and contribution rate of states to allocate the shared water among states according to different schemes. It can potentially decrease future conflict over

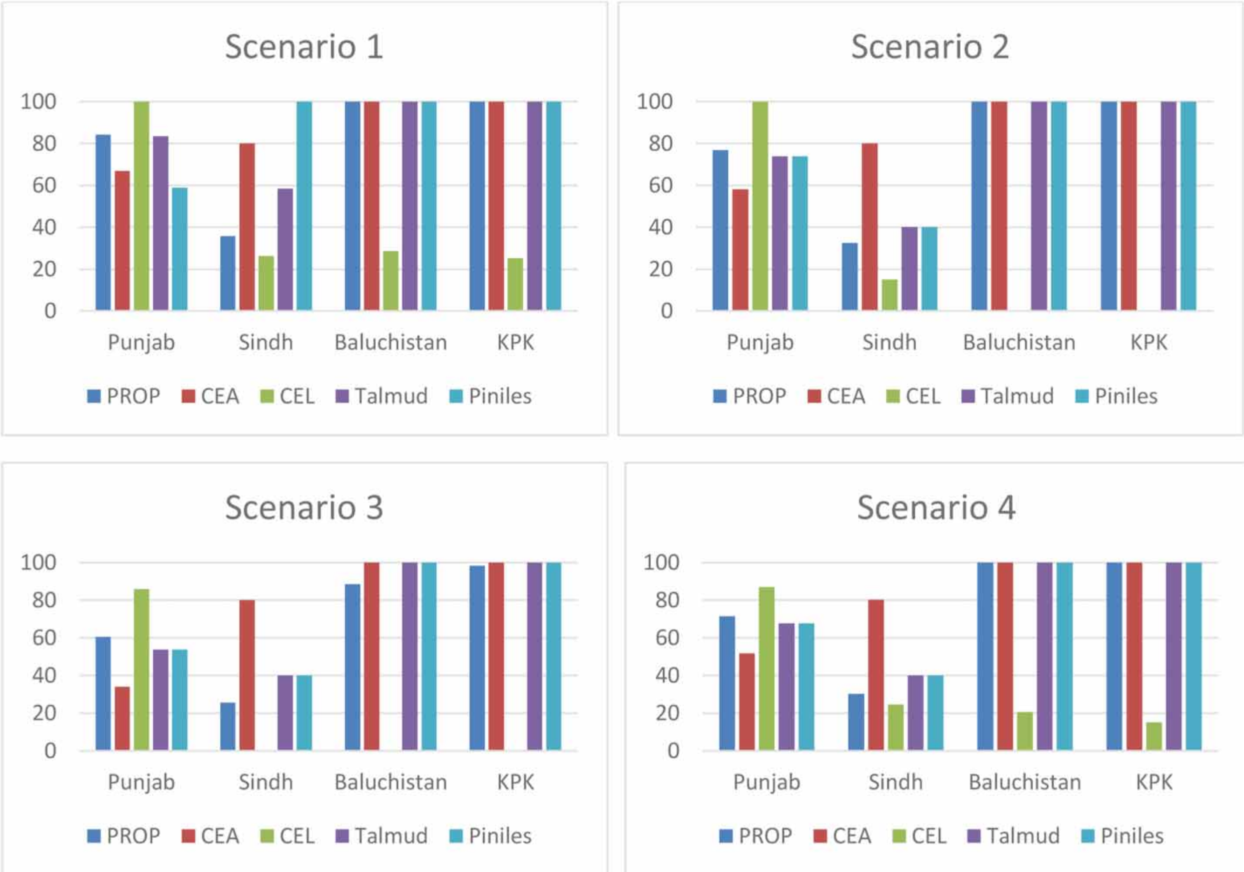

Figure 5 | Division rule solutions using priorities (percentage with respect to individual water demand).

the negative aspects of climate change. However, other measures will be required as well, such as increasing water use efficiency, improving water resource management (Granit & Joyce 2012).

Due to the complex nature of transboundary water allocation, we cannot be certain that the simple bankruptcy rules and the weighted bankruptcy rules will be able to solve all the issues related to shared water resource allocation. Water sharing is viewed differently by the people living in different regions, hence their appreciation of the resource and the values attributed to the various functions of the water as a result of cultural, climatic, and economic circumstances.

CONCLUSIONS

The water scarcity issue can be a cause of conflict among riparian countries, states, or provinces. This paper examined the utility of bankruptcy rules in addressing the supply–demand gap in shared rivers. Five bankruptcy rules were used in this study to resolve the conflict between the provinces of Pakistan over the allocation of water. Apart from water scarcity, the uncertain and stochastic nature of rivers and the increasing water demands due to climate change makes the water sharing mechanism more complex and challenging. The water allocation mechanism described in this study uses the bankruptcy game (BG) technique, which is a branch of cooperative game theory. This approach can be a useful tool for decision-making when it comes to the sharing of water resources under critical scarce conditions for complex water resource systems having competing water demands. Using the five bankruptcy rules, the water distribution was done among the four provinces of Pakistan under four different critical scenarios. Also, the bankruptcy rules were applied again, which included the water allocation priorities favoring the users which have a higher agricultural productivity. From the results obtained, it can be seen that the simple bankruptcy rules and weighted bankruptcy rules can be important tools for decision-makers to allocate water in critical scarcity and uncertain conditions between the different water users. Although appropriate vision is provided by the allocation rules for the conflict management of transboundary water resources, the distribution of water among riparians can be a complex problem that cannot be solved only by mathematical methods; therefore, water diplomacy and negotiation between the provinces of Pakistan are suggested, which would help them to develop a consensus and reach an agreement. This method can help policy-makers to facilitate negotiation in managing conflict and dispute over water resources allocation problems. It is a tool to create more options that may assist riparian countries when negotiations are tedious. However, some fine-tuning may still be necessary. Further studies may address the limitations of this study and consider some additional influential factors such as the impacts of climate change, reliable relative weights of states, and socio-political aspects of the basin as well as the effects of external powers.

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