

Numerical investigation of the impact of irregular pipe wall build-up on velocity in the water distribution system

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ABSTRACT

This paper presents novel equations for different wall build-up types in old pipes developed for the estimation of real flow velocities in a water distribution system (WDS). This is important when estimating the contamination propagation rate in an operational WDS. Because the inner surface geometry of old rough pipes is complex, real velocities cannot be estimated by conventional WDS models. Therefore, a computational fluid dynamics (CFD) model was used to analyse the flow dynamics in old rough pipes with even and uneven roughness build-up. The developed equations for the velocity correction coefficients derive from the results of numerical simulations. Numerically obtained velocity fields show that in some cases, pipe wall build-up can affect the effective flow section significantly and the flow velocity can be underestimated by more than two times. Therefore, different velocity correction equations have to be used for different types of pipe wall build-up.

Key words | computational fluid dynamics, contamination propagation rate, pipe wall build-up, roughness, water distribution system modelling

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HIGHLIGHTS

- Velocity correction coefficients for old water distribution pipes were developed.
- Velocity correction is significant when estimating the contamination propagation rate using water distribution system models.
- Existing models can underestimate the flow velocity in old pipes by more than two times.

INTRODUCTION

For decades, the water distribution systems (WDS) have been considered as resilient and reliant infrastructures, offering quality service at relatively low cost. On the other hand, we are entering into an era where the average life span of pipes laid down from the late 1800s to the 1950s is running out (e.g. [AWWA 2001](#); [Kanakoudis & Tolikas 2001](#); [Lee & Meehan 2017](#); [Barton *et al.* 2019](#)). [Kanakoudis & Tolikas \(2001\)](#) reported that taking into account the hydraulic, economic and social aspects, the optimum replacement time for cast iron pipes is between 69 and 89 years, depending on the pipe diameter. This exceeds the

estimations by [Barton *et al.* \(2019\)](#) who investigated the data of pipe brakes in 308 water companies in the USA and Canada. They concluded that the average life span of metal water pipes is about 50 years, independent of the type of metal. Studies conducted on the breaking data of copper and galvanized steel pipes have shown that the average lifespan of small diameter service line pipes can be as low as 35 years ([Lee & Meehan 2017](#)). The result of infrastructure ageing is an increase in structural failures (pipe bursts and rate of leakages) and operational shortcomings (increased energy consumption, water quality issues,

low pressure, etc.). The deviations from design operating conditions may indicate to the deviations of pipe characteristics inherent to old rough pipes.

The pipe wall build-up is not only time-dependent but is affected by a number of factors – pipe cross-section, material, water quality and flow velocity (Kändler 2002; Kanakoudis 2004; Vreeburg & Boxall 2007; Vreeburg *et al.* 2009). Design of the WDS has been dominated by the fire-fighting demand and the intuitive need for looping, leading to low velocities and long residence time. A field study carried out in Tallinn revealed that the pipe cross-sections of metal pipes (steel and cast iron) were reduced on average by 10% due to wall build-up. The specimens were 25–100 years old with diameters from 75 to 200 mm (Kändler 2002). Corrosion caused carrying capacity failures due to the pipe wall build-up may prevent WDS from constantly ensuring customer water needs with satisfying quality (Kanakoudis 2004). The side effect of corrosion and low velocity can be discolouration of the water, which has been the main customer complaint about the water quality in the UK (Vreeburg & Boxall 2007). The formation and growth of particles in the WDS is a complex process, but the sedimentation of the particles is related to the hydraulic conditions of the network. It was shown that at low velocities, the sediment settling will take place in the lower half of the horizontal pipe, while at higher velocities, the settling will cover the entire pipe wall (Vreeburg & Boxall 2007). Vreeburg *et al.* (2009) suggested that the daily maximum velocity in the WDS should be at least 0.4 m/s to prevent particles from accumulating and causing pipe wall build-up.

A number of studies have analysed the growth of the pipe roughness over time. Colebrook & White (1937) suggested that the pipe roughness grows linearly and the growth rate depends mostly on the pH of the water flowing inside the pipe. The reported values of the roughness growth rate ranged from 0.066 to 0.63 mm/yr. The latter is similar to the results reported by Willims & Hazen (1920). The range is in line with numerous investigations in the US (Walski *et al.* 1988); however, it is significantly underestimating the growth rate reported by Echávez (1997) for galvanized iron pipes (2.13 mm/yr) that are concurrently less often used in WDS compared to cast iron. Rapid growth rate will result in a rapid reduction in the flow area and growth of the

flow velocity. This is particularly important when estimating changes in the water quality in a WDS. The reason is that underestimation of the flow velocity will lead to considerable errors in the estimations of the propagation rate of the contaminated zones in a WDS (Boxall *et al.* 2004; Annus & Vassiljev 2015).

The numerical model of a WDS is commonly developed using nominal pipe diameters. Therefore, during the calibration of such a model, the pipe wall build-up is compensated by adjusting the roughness value (Lansey *et al.* 2001) or by adjusting the water consumption at the nodes (Kanakoudis & Gonelas 2015a, 2015b). The latter can be done only when the consumption in the nodes is not measured. The adjustment of the pipe roughness value is justified for surfaces whose geometry is easily described (Jimenez 2004). However, in real WDS, the shape of the inner wall can vary significantly over the pipe length, so that the pipe cross-section is not homogeneous. The irregular wall roughness elements greatly complicate the flow dynamics (Christensen *et al.* 2011), indicating that the usage of nominal pipe diameters in the modelling process is not always justified.

The roughness height, as well as the shape of the inner pipe surface, may significantly vary in old pipes, resulting in complex geometries. Annus *et al.* (2020) analysed the influence of irregular pipe wall roughness on the flow velocity in WDS old pipes. Three types of roughness build-up were investigated: Type 1 – evenly distributed roughness (roughness height <10 mm); Type 2 – unevenly distributed roughness (roughness height up to 0.5D) and Type 3 – sediment settling in the lower part of the pipe. It was shown by Annus *et al.* (2020) that in pipes Type 1 and Type 3, the dependence between the relative roughness and the increase in the estimated velocity (so-called velocity correction coefficient) was similar. In Type 2 pipes, the velocity correction coefficient is dependent both on the shape and the number of local disturbances that reduce the effective pipe diameter. Measurements in Tallinn WDS have revealed that in some cases, the exponent describing the dependence between the pressure drop and the flow rate can be larger than 2, indicating that the actual flow behaviour is complex and simple expressions will lead to misjudgement of flow parameters. The main goal of this paper was to analyse the reason for such results. It is

proposed herein that when measurements at nodes reveal abnormal exponent values, pipe sections with irregular roughness elements may be present. This paper analyses the factors influencing the exponent based on computational fluid dynamics (CFD) simulations and presents an equation to estimate real flow velocities in a WDS containing pipes with uneven surface roughness.

METHODS

Existing network calibration data

Analysis of the WDS containing old rough pipes shows that in some cases, the exponent n describing the dependency between the pressure drop Δp and the flow rate Q is larger than 2 (i.e. $n > 2$ in $\Delta p = k \cdot Q^n$, where k is a constant characterizing the pipe). Figure 1 shows the values of the exponents in selected measurement points in a part of Tallinn WDS. These measurements were performed during collecting data for the calibration of the WDS. Pressures were measured in 18 points and flow rate in 13 points. Additional measurements were conducted at pumping stations supplying the WDS. For pressure measurements, portable data loggers of the ‘Sensus’ company with strain gauge pressure sensors were used. Absolute heights of all sensors were measured with high accuracy using geodetic levelling. Water flow rates were measured using turbine and electromagnetic

flow meters that had impulse output for using data. Some results of the calibration are presented in Vassiljev et al. (2015). Seventeen measurement points are shown in Figure 1. The exponent values are larger than 2 in three points.

Due to high costs and constructional restrictions, it is not possible to identify the effective pipe diameters in an operational WDS precisely. Pipe replacements in the operational WDS in Tallinn have revealed that the wall build-up has resulted in an uneven decrease of the diameter along the pipe length, sometimes acting as local obstruction (Kändler 2002). Local obstructions increase energy losses and affect the flow dynamics, making its description by a simple expression impossible. These local losses (Type 2 pipes) generate additional pressure drop that is dependent on the flow rate. In EPANET, the increase in the pressure drop is usually described with the increase of pipe roughness. This can lead to unrealistic pipe roughness values and underestimation of real flow velocities (Annus & Vassiljev 2015; Annus et al. 2020). Therefore, it has to be reckoned that the ageing process of pipes is expressed both in the increase of pipe roughness and the decrease of pipe diameter. Only under these conditions, it can be assumed that the modelled flow velocities are in a reasonable range and the model is reliable for analysing extreme conditions (e.g. fire flows) or changes in the water quality. The next section describes Type 2 pipe models and calculation methods to determine the actual flow velocity in the WDS containing this type of old rough pipe.

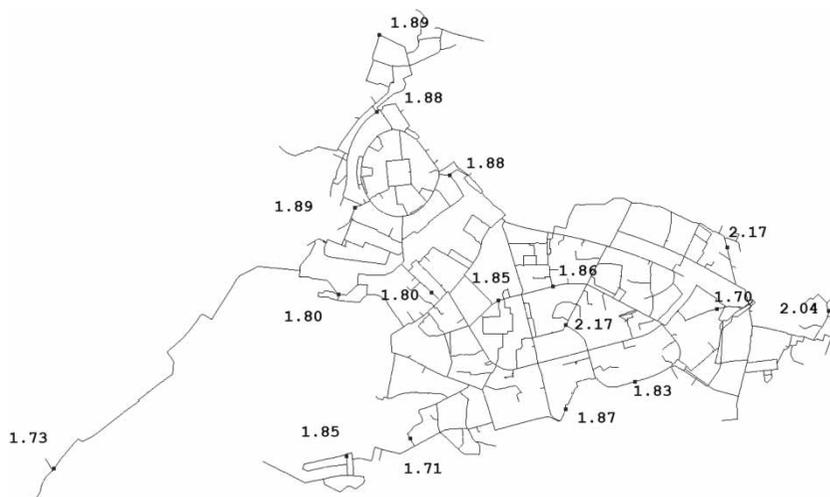


Figure 1 | Exponents of dependence between the measured pressure drop and the flow rate in Mustamäe-Õismäe district, Tallinn.

Numerical procedure

CFD model was used to analyse the flow dynamics in a pipe where the diameter decreases unevenly along the pipe length (referred to as Type 2 pipe in [Annus *et al.* \(2020\)](#)) ([Figure 2](#)). In Type 2 pipes, the changes in average velocity are dependent both on the shape and the number of local obstructions that reduce the effective flow section. Therefore, CFD models are used as the complexity of the flow dynamics needs more attention to improve the estimation of real flow velocities in WDS.

To analyse the flow dynamics, a random pattern was created ([Figure 3](#)) and ‘bruised’ into the pipe wall at two and four locations over the pipe length to mimic the irregular inner surface of a typical old rough pipe ([Figures 4 and 5](#)). The surface pattern was applied to the test pipe both on the top and bottom side. The CFD calculations were performed at six different pressure drops between the pipe segment inlet and outlet ranging from 50 to 2,500 Pa. The range of average flow velocities used in this study was 0.13...1.3 m/s. The corresponding flow rates were calculated at each pressure drop while the pipe surface sand-grain roughness was set to $e = 1$ mm.

In [Annus *et al.* \(2016\)](#), the performance of the Reynolds-averaged Navier–Stokes (RANS) turbulence model in a complex pipe was analysed by comparing the measured and the modelled velocity and kinetic energy distributions

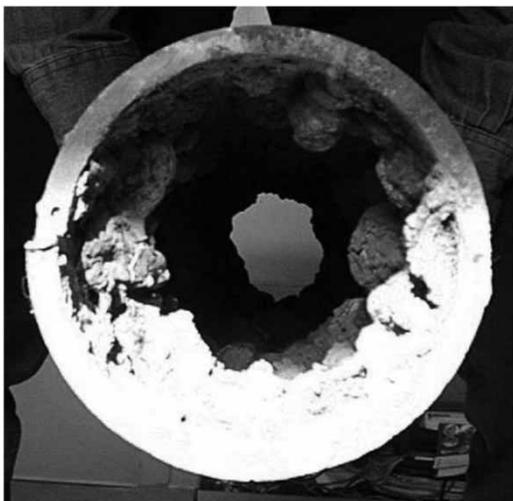


Figure 2 | Typical pipe wall build-up with unevenly distributed roughness – Type 2 pipe ([Kändler 2002](#)).

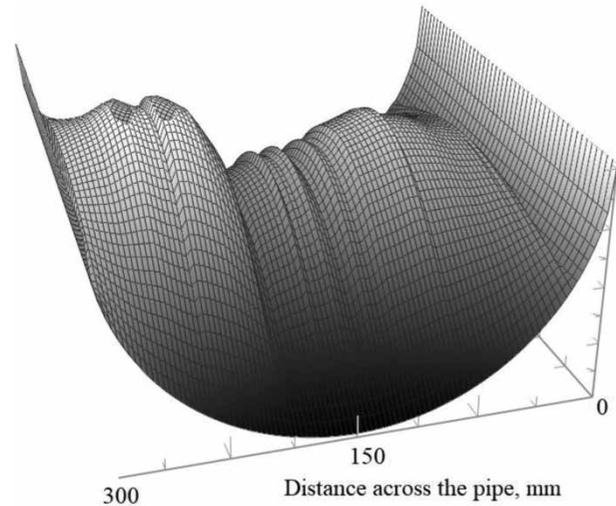


Figure 3 | 3D visualization of random surface pattern.

over a complex geometry pipe segment length and perimeter. The standard $k-\omega$ turbulence model by [Wilcox \(1998\)](#) showed the best qualitative and quantitative correlation. In this study, we used the experimentally validated solution algorithm with the $k-\omega$ turbulence model.

In addition to the turbulent flow regime, the fluid was assumed incompressible, isothermal and Newtonian. In the validation process, numerical simulation results obtained from the stationary and nonstationary solution algorithms were compared. It was concluded that after a sufficiently large number of iterations (1,500 to 5,000, depending on the initial conditions), the flow becomes steady; nonstationary and stationary models gave similar results. A solver from OpenFOAM v1218 (OpenCFD Ltd) employing the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm ([Caretto *et al.* 1973](#)) was used to solve continuity and momentum equations.

In the current study, the pipe had a nominal diameter of 300 mm and a length of 1,500 mm for both configurations of the irregular pipe inner surface (i.e. two and four irregular roughness elements). Results for Type 2 pipe with 100 and 200 mm diameter are shown in [Annus *et al.* \(2020\)](#). The computational domains ([Figures 4 and 5](#)) were discretized by a hybrid grid containing a mixture of structured and unstructured grid portions. In the vicinity of local complex geometry, a more refined grid was applied. The pipe with two irregular roughness elements was meshed with

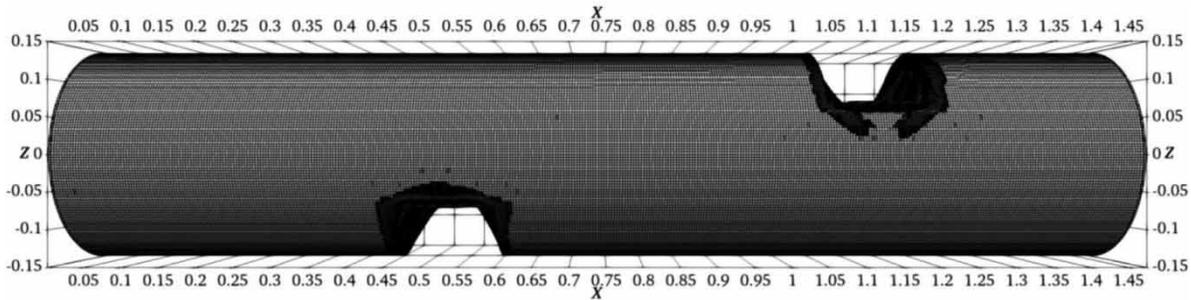


Figure 4 | Computational domain of the 1.5 m long pipe with two irregular roughness elements.

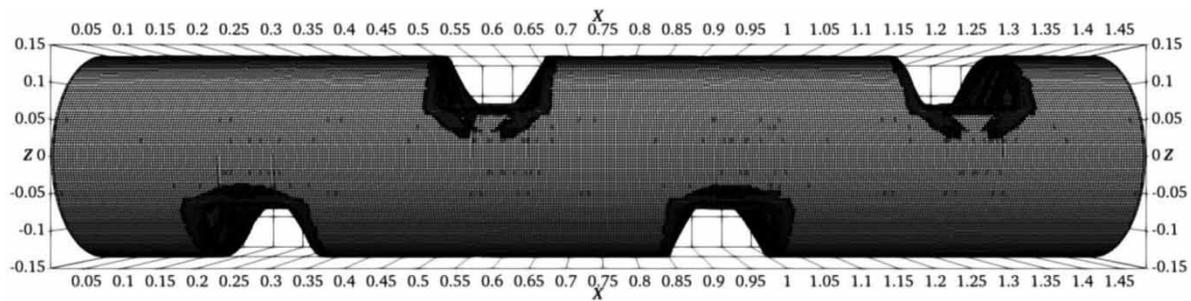


Figure 5 | Computational domain of the 1.5 m long pipe with four irregular roughness elements.

2.475×10^6 finite volumes, of which 2.333×10^6 were hexahedral. The pipe with four irregular roughness elements was meshed with 2.642×10^6 finite volumes, of which 2.404×10^6 were hexahedral. The convergence criteria for x -, y - and z -velocity components, ω and k were set to 0.0001.

Grid sensitivity

Grid sensitivity study was conducted for a more complicated geometry with eight irregular roughness elements and

100 mm nominal diameter, resulting in a smaller computational domain and therefore allowing higher numbers of finite volumes to be tested. Figure 6 presents simulation results for three mesh refinement levels. It can be seen that the difference in the calculated flow rate value for grids with 1.45 and 2.77 million elements is 1%. It was concluded that 2.77 million elements are sufficient and further refinement is not necessary. For the pipe with the nominal diameter 300 mm, the grid element size was scaled accordingly.

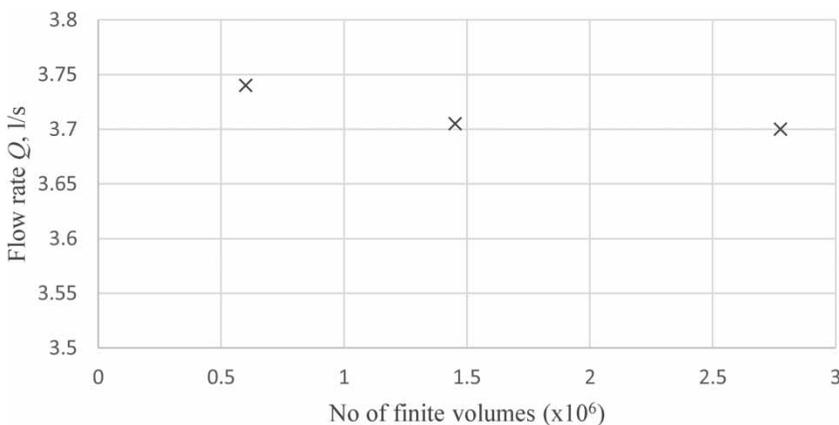


Figure 6 | Calculated flow rate for three grid sizes.

RESULTS AND DISCUSSION

CFD calculation results for different numbers of irregular roughness elements are given in Table 1. The exponent for the pressure drop is higher than 2 at two irregular roughness elements. In other cases, the exponent is lower than 2. It has to be reckoned that the exponent is calculated using the flow rate (not average velocity). At an average velocity (calculated using the estimated effective flow section), the exponent would be close to 2 for all configurations of irregular roughness elements.

Analyses show that in some pipe wall configurations, local disturbances have to be taken into account as they change the function between the pressure drop and the flow rate. Numerical investigations revealed that the complex pipe inner wall shape leads to complex flow dynamics and interactions between the developing vortices near the deformed wall elements. Analysis of the developing vortices shows that at some pipe configurations, the size of the eddies changes at different pressure drops, and in some cases, the size is constant (Table 2). This depends on the distance between the irregular roughness elements. Similar to

Table 1 | Exponent of dependence $\Delta p = f(Q)$ for different numbers of irregular roughness elements

Number of irregular roughness elements	Exponent
0	1.95
2	2.27
4	1.95

Table 2 | Dependence between the pressure drop and the size of vortices (% of computational domain volume) at different numbers of irregular roughness elements

Pressure drop (Pa)	Volume of vortices (%)	
	Number of irregular roughness elements 2	Number of irregular roughness elements 4
2,500	22	24
1,500	22	24
900	21	24
500	16	24
200	14	24
50	12	24

corrugated pipes, if the distance between the deformations is small enough, the vertical structures trapped between the deformations have a minimal influence on the core flow (Vijiapurapu & Cui 2007, 2010). The effective flow section of a pipe is equal to the free space between the irregular roughness elements, excluding the volume of the near-wall vortices. The interaction between the near-wall vortices and the jet-like flow at the pipe axis is low (Annus *et al.* 2019).

Flow structure in pipes with two and four irregular roughness elements is presented in Figures 7 and 8. In the case of two irregular roughness elements, the eddy size is significantly affected by the change in the pressure drop, and it is evident that the irregular roughness element acts as local obstruction. Calculations reveal that the exponent describing the dependence between the pressure drop and the flow rate is larger than 2 for this configuration.

Velocity field in the pipe with four irregular roughness elements shows no changes in the eddy size at the increase of the pressure drop.

It is proposed that the flow structure can be characterized according to the exponent while calibrating the model of a WDS. Figure 9 presents the dependence between the relative roughness and the velocity correction coefficient for the three types of old rough pipes with diameters of 100, 200 and 300 mm. The results for pipes with diameters of 100 and 200 mm are based on Annus *et al.* (2020).

When the roughness in the EPANET calibration process is large and the exponent is equal to or less than 2, it can be concluded that there are old pipes in the network that have uniform roughness build-up (in the form of reduced cross-section area – Type 1 or changed cross-section shape – Type 3). The velocity correction coefficient should be used taking into account a corrected pipe cross-section area:

$$v_{\text{cor}} = 2.2 \cdot \frac{e_m}{D} + 1.3 \quad (1)$$

where e_m/D is relative roughness and e_m is pipe roughness gained from EPANET for pipe nominal diameter D .

When the roughness in the EPANET calibration process is large (i.e. equal to or larger than the pipe radius) and the exponent is more than 2, then there are Type 2 segments in the pipeline and the velocity should be recalculated

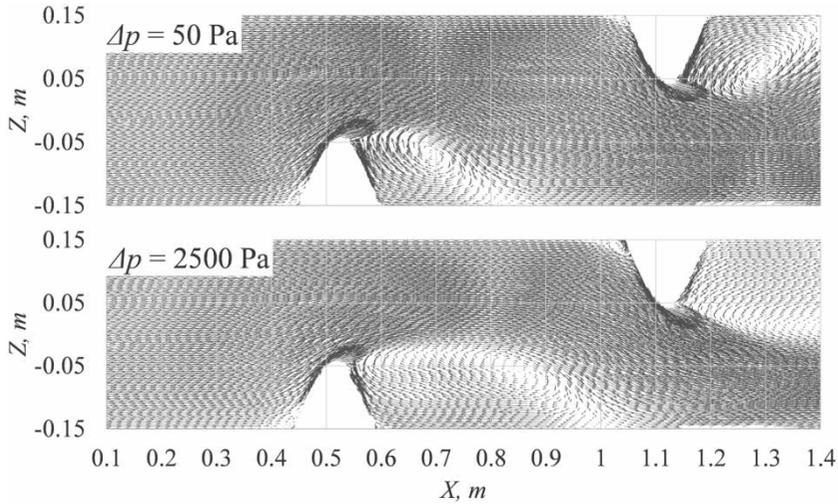


Figure 7 | Comparison of vortex formation behind two irregular roughness elements.

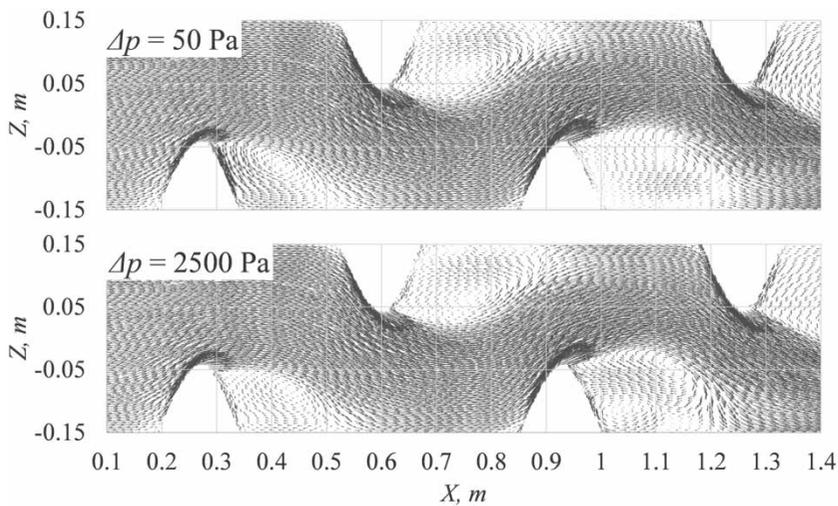


Figure 8 | Comparison of vortex formation behind four irregular roughness elements.

according to

$$v_{\text{cor}} = 0.8 \cdot \frac{e_m}{D} + 1.25. \quad (2)$$

CONCLUSIONS

Measurements on old operational WDS sections have revealed that in some cases, the exponent describing the

relationship between the pressure drop and the flow rate yields values larger than 2. It is proposed herein that this indicates the presence of pipes with irregular roughness elements in-between measurement nodes. Due to high costs and constructional restrictions, it is not possible to identify the effective pipe diameters in operational WDS precisely. Therefore, numerical modelling was applied in the present study to investigate the effect of the shape of an irregular pipe wall on the flow velocity. Based on the computational analysis, velocity correction coefficients

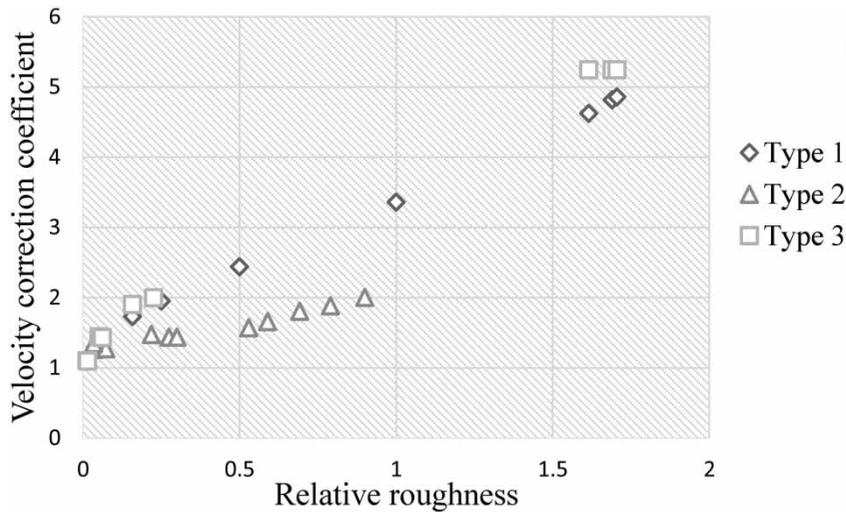


Figure 9 | Relative roughness and velocity correction coefficient.

were estimated for WDS. The study showed that in some old rough pipes, the inner shape is changed such that the irregular roughness elements act as local obstructions. Formation of vortices is dependent on the number of obstructions and distance between them. These vortices reduce the effective flow section and an increase in volume with an increasing flow rate.

It was concluded that different equations hold for the types of wall build-up when finding the real flow velocity estimation. When the exponent is less than or equal to 2, it can be concluded that roughness build-up in the pipeline sections is uniform. Field studies in Tallinn have shown that the exponent in an operational WDS containing old rough pipes may in some cases be larger than 2. In this case, the wall build-up is inconsistent, which in the EPANET calibration process, is expressed by large roughness and an exponent value higher than 2. For the second type, it is shown via velocity field visualization that irregular roughness elements affect the eddy size significantly, and therefore also the effective flow section. In this case, the velocity correction coefficient is dependent on the number of local obstructions and their distance. The inner surfaces of old rough pipes presented here with two and four irregular roughness elements are simplifications. In real old pipes, more complex geometries are possible and the formation of the vortices is not straightforward, possibly resulting in further effective flow section reductions. The analysis here

showed that the flow velocity in old rough pipes can be underestimated by more than two times.

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DATA AVAILABILITY STATEMENT

Data cannot be made publicly available; readers should contact the corresponding author for details.

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