Formula for determining the size of the air tank in the long-distance water supply system

Xuyun Chen, Jian Zhang, Nan Li, Xiaodong Yu, Sheng Chen and Lin Shi

ABSTRACT

In the long-distance water supply system, the air tank can effectively protect the water hammer when the pump stops, and the shape parameters of the air tank determine the protective effect. Based on the theory of rigid water body and harmonic vibration, this paper derived the calculation formulas for the surge and bottom pressure changing process of the air tank in the system with and without friction and impedance and put forward the theoretical method to estimate the air tank volume and established the relationship between the operating parameters and the volume. Combined with the actual water supply project, under different working conditions, the theoretical calculation results and numerical simulation results were compared and analyzed. The results showed that the theoretical calculation results of the system with friction and impedance had a better fitting performance than the numerical simulation results, and the operating parameters of the air tank derived after considering the influence of friction and impedance were accurate. This method can simplify the selection process of air tank body parameters. At the same time, the shape optimization of the air tank considering friction and impedance can be improved by 40–50% compared with the results of ignoring friction and impedance.

Key words | air tank, friction, long-distance water supply, volume, water hammer protection

Xuyun Chen
Jian Zhang
Xiaodong Yu (corresponding author)
Sheng Chen
Lin Shi
College of Water Conservancy and Hydropower Engineering,
Hohai University,
Nanjing 210098,
China
E-mail: *yuxiaodong_851@hhu.edu.cn*

Nan Li
Design Institute, Shanghai Municipal Engineering Design Institute (Group) Co., Ltd,
Shanghai 200092,
China

HIGHLIGHTS

- The calculation formulas of surge and bottom pressure in air tank considering friction and impedance are derived.
- The theoretical estimation method of the volume of the air tank is proposed.
- The relationship between the operating parameters and the volume of the air tank is established.

INTRODUCTION

A long-distance water supply project is an effective way to redistribute water resources and improve water resource utilization. Due to the complex topography and geological conditions, most long-distance water supply projects need to be pressurized by pumping stations to deliver water from low-lying areas to high-lying areas (Ren *et al.* 2019). The most crucial issue in the water supply system is pipeline safety.

When the pump suddenly loses power during pumping, the pressure of the pump drops sharply, and the pressure-reducing wave quickly transmits to the downstream. If the initial internal pressure is small, the pipeline pressure will drop to be equal to the vaporization pressure, and liquid column separates, which in turn induces the bridging of the water hammer. The vast pressure generated by bridging the water hammer may cause severe damage to the entire water supply system, which leads to the interruption of water supply and heavy losses (Wylie *et al.* 1993). Therefore, it is necessary to study measures to protect the water

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hammer in the water supply system of the pumping station. At present, air tank, surge chamber, one-way surge tank, and air valve are used to protect the pump-stopping water hammer. Compared with other protective measures, the air tank can protect negative water hammer more effectively and is frequently used in engineering (Kim *et al.* 2015).

Many researchers have provided diagrams for the preliminary design of air tank parameters. Graze & Horlacher (1986) compiled the air tank size design drawing to protect the rising main pipeline from the transient pressure caused by the pump trip event. Ruus & Karney (1997) established an air container volume chart with differential orifice plates, which significantly reduced the volume of the air container. Stephenson (2002) plotted the volume of the air container under different conditions in the pumping station and found that the appropriate diameter of the inlet and outlet pipes can reduce the volume of the air tank. Martino *et al.* (2004) introduced the pioneering work of Evangelisti. In this work, the author developed a simple graphic for the design of a surge vessel for protecting pipelines based on the incompressible flow theory.

However, the assumptions of these charts and the selection of parameters lack integrity, such as the rigid column model, and it is impossible to accurately determine the specific shape parameters of the air tank (Martino & Fontana 2012).

With computer technology rapidly developing, numerical simulation has been widely used in optimizing air tank parameters. Izquierdo *et al.* (2006) proposed a neural network (ANN) model to minimize the volume of an air container based on system parameters. Jung & Karney (2006) considered the transient situation of the water distribution network and used genetic algorithms and particle swarm optimization to optimize the selection and location of hydraulic protection devices in the system. Ramalingam & Lingireddy (2014) constructed and combined a neural network framework with genetic algorithms to obtain the optimal size of the air container in the pipe network system. Sun *et al.* (2016) studied the influence of the connection structure on the volume of the container and adopted the sequential quadratic programming method to optimize the design of air container parameters. Shi *et al.* (2019) designed a new structural air container and provided new ideas for the optimization of air tank body parameters. By adjusting the air tank position, Wang *et al.* (2019) discussed the optimization effect of the installation position on the air tank size. Besides, many scholars have found that the volume of the air tank can also be effectively reduced by adding other protective measures (Li *et al.* 2013; Miao *et al.* 2017).

The calculation of the volume of the air tank depends on different operating parameters, which have many variables and affect each other. The numerical simulation trial calculation method has high accuracy and reliability, but it also has some blindness in the engineering design stage (Zaki & Elansary 2011). In work on the surge protection of water distribution systems by Jung & Karney (2009), it is concluded that the search space is not significantly limited, even modern computer systems cannot cope with the computational challenges brought by a strict and systematic search for the best system protection in most practical field applications. In order to avoid this situation, Miao *et al.* (2018) derived an approximate theoretical formula for determining the parameters of the air tank based on the characteristic line method. However, in the process of deducing the formula, some conditions are so ideal that the calculated results are conservative. In the actual operation of the pipeline system, the influence of pipe friction and the impedance of the air tank cannot be ignored (Ghidaoui *et al.* 2005; Martino & Fontana 2012; Salem *et al.* 2017).

Therefore, based on the rigid water body and simple harmonic motion theory, this paper introduced and deduced the approximate analytical solution of the size of the air tank in the actual operation process. In the process of derivation, the volume parameters of the air tank were optimized by taking into account the influence of pipe friction and the impedance of the air tank.

BASIC EQUATION OF THE AIR TANK OPERATION

Figure 1 shows a pressurized water supply system with air tank protection measures. The air tank was arranged close to the water pump. If the elasticity of the water body in the water supply system is not taken into account, a series of basic equations of the air tank during operation can be obtained when the pump unit is in pumping power failure condition (Chaudhry 2014).

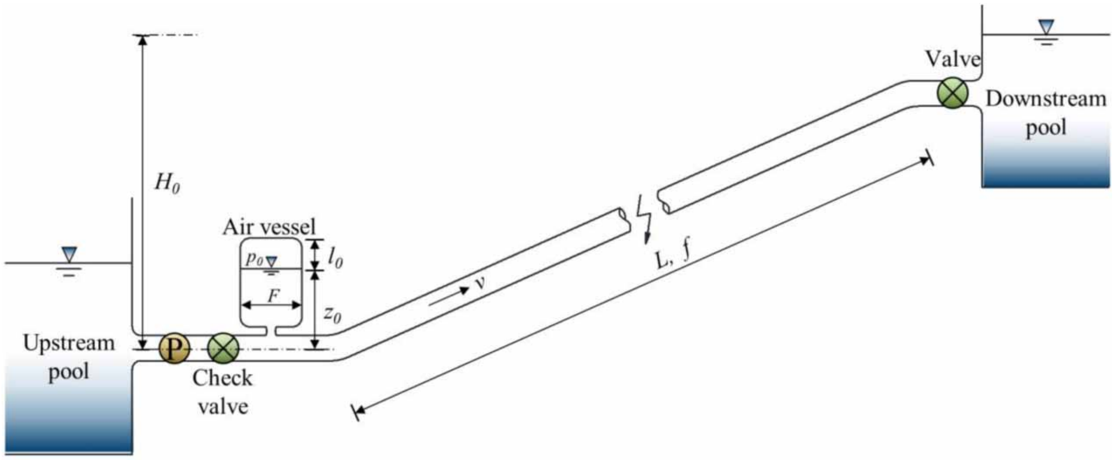

Figure 1 | Layout of the water supply system with air tank protection after the pump.

Dynamic equation

$$\rho gf\ [z_0 - z + (p - p_a) - \beta|Q_{st}|Q_{st} - \alpha|Q|Q - H_0] = \rho Lf\frac{dv}{dt} \tag{1}$$

Simplify the formula (1) to be:

$$z_0 - z + (p - p_a) - \beta|Q_{st}|Q_{st} - \alpha|Q|Q - H_0 = \frac{L}{g}\frac{dv}{dt} \tag{2}$$

Flow continuity equation:

$$Q = fv = Q_p + Q_{st} = Q_p + F\frac{dz}{dt} \tag{3}$$

Gas multi-party process equation:

$$p_0 l_0^m = p(l_0 + z)^m \tag{4}$$

where Z_0 is the initial stable running water level of the air tank (m), and z is the vector value of changing water level in the tank. The initial stable running water level was taken as the datum, and the orientation was negative (m); F is the cross-sectional area of the air tank (m^2); f is the cross-sectional area of the pipe (m^2); p and p_0 are the absolute pressure and initial absolute pressure in the upper air chamber of the air tank (m), respectively; p_a is the local atmospheric pressure (m); L is the length of the pipe between the air tank and the downstream outlet (m); H_0 is the downstream water level (m); Q_p is flow amount at the pump outlet (m^3/s); Q is the flow amount behind the tank (m^3/s); Q_{st} is the inflow amount and outflow amount of the air tank, where the outflow amount is positive (m^3/s); v is the velocity of water behind the tank (m/s); α is the total head loss coefficient of the water diversion pipe behind the tank (s^2/m^5); β is the impedance coefficient of the connecting pipe between the air tank and the main pipeline (s^2/m^5); l_0 is the initial height of the upper air chamber of the air tank (m); and m is the polytropic index of the gas in the air tank, usually taken as $1 \le m \le 1.4$, where m is taken as 1.2.

ESTIMATION OF AIR TANK OPERATING PARAMETER

Without friction and impedance

Equation (1) established the water movement equation of the air tank and the pipeline behind the tank after the pump was suddenly cut off. To obtain an explicit solution of the change process of the surge in the air tank and the bottom pressure to reflect the influence of various parameters on the protective performance of the air tank, Equation (2) needs to be further simplified. If the water in the pipeline system is regarded as an ideal fluid, the relevant head loss, including the head loss along the pipeline, the local head loss, and the impedance loss of the air tank connection pipe, can be ignored. Therefore, Equation (2) can be simplified as follows:

$$z_0 - z + (p - p_a) - H_0 = \frac{L}{g}\frac{dv}{dt} \tag{5}$$

Under the condition of constant flow and stable operation, Equation (5) can be changed into:

$$z_0 + p_0 = H_0 + p_a \tag{6}$$

When the air tank protection measures are installed after the pump, once a pumping power failure accident happens to the pump unit, the air tank will quickly fill the water in front of the pump to form a backflow. In order to ensure the water hammer protection performance of the air tank, the back valve of the pump should be quickly closed to prevent backflow. At the time, Q_p can be approximately equal to 0, so Equation (3) can be changed into:

$$Q = fv = Q_{st} = F\frac{dz}{dt} \tag{7}$$

Tayler expansion of Equation (4) and the omission of high-order small quantities can be obtained:

$$p = \frac{p_0 l_0^m}{(l_0 + z)^m} \approx p_0\left(1 - \frac{mz}{l_0}\right) \tag{8}$$

By substituting Equations (6)–(8) into Equation (5), it can be simplified and obtained:

$$\frac{d^2z}{dt^2} + \omega_0^2 z = 0 \tag{9}$$

where $\omega_0^2 = (gf/LF)\sigma$ and $\sigma = (mp_0/l_0) + 1$.

Equation (9) is a simple harmonic oscillation equation in differential form. Therefore, if the water elasticity and the friction resistance in the water supply system are not considered, the water level in the air tank will be changed simply by harmonic vibration when the pump unit stops pumping in an accident. Therefore, the surge period in the air tank is as follows:

$$T = 2\pi\sqrt{\frac{LF}{gf}}/\sqrt{\sigma} \tag{10}$$

According to the initial conditions, the surge amplitude of the air tank can be solved as follows:

$$\Delta Z^* = |v_0|\sqrt{\frac{Lf}{gF}}/\sqrt{\sigma} \tag{11}$$

The variation amplitude of water pressure at the bottom of the air tank is as follows:

$$\Delta H^* = |v_0|\sqrt{\frac{Lf}{gF}}\sqrt{\sigma} \tag{12}$$

The change process of the surge in the tank is:

$$z(t) = z_0 + \Delta Z^* \sin(\omega_0 t + \pi) \tag{13}$$

The change process of water pressure in the tank bottom is:

$$H(t) = H_0 + \Delta H^* \sin(\omega_0 t + \pi) \tag{14}$$

where v_0 is the initial flow rate under stable operating conditions.

By analyzing the influence of the shape parameters of the air tank on the water hammer protection performance, it can be indicated that the larger the initial air chamber height and cross-sectional area of the air tank, the smaller the maximum pressure along the pipeline, which means the initial volume of the air tank plays a key role.

It is essential and tedious work to determine the optimal air tank shape parameters in the numerical simulation of the transient process of the pump shutdown. In order to provide some theoretical basis for optimizing the air tank shape and solving the overpressure problem of the high-head pump station, an approximate theoretical formula for determining the optimal shape parameters of the air tank can be derived from above. Without considering all the friction in the water conveyance system, inequality can be established according to Equation (12):

$$\Delta H^* = |v_0|\sqrt{\frac{Lf}{gF}}\sqrt{\sigma} \leq \varphi H_0 \tag{15}$$

where φ generally takes 0.3–0.5 (Miao *et al.* 2018).

Generally, the absolute pressure p_0 of the initial gas in the air tank is much greater than the initial gas chamber height l_0, so $\sigma = (mp_0/l_0) + 1 \approx (mp_0/l_0)$, taking σ into Equation (15) can get the initial gas chamber volume estimation formula:

$$V_a = Fl_0 \geq \frac{v_0^2 Lfmp_0}{g\varphi^2 H_0^2} \tag{16}$$

where $p_0 = p_a + H_0 - l_{w0}$, l_{w0} is the initial water depth of the air tank, that is, the initial running water level minus the elevation of the centerline of the pipe at the location of the air tank.

Considering friction and impedance

Equations (10)–(14) are the calculation formulas of the operating parameters of the air tank deduced without considering the friction of the water pipeline and the impedance of the connecting pipe of the air tank. To a certain extent, it can intuitively reflect the pressure fluctuation period and pressure fluctuation of the system and the influence of various parameters on the protection performance of the air tank. In fact, due to the friction of the pipeline, the surge wave will continue to attenuate during the transmission process (Salem *et al.* 2017). Therefore, the amplitude of the internal water pressure, which changes along the pipeline behind the tank, will be lower than that at the bottom of the air tank. However, it is feasible to evaluate whether the extreme pressure value of the water pipeline behind the tank meets the design protection requirements according to the amplitude of the water pressure change in the bottom of the air tank, and it has a certain safety margin.

If the friction resistance of the pipeline in the water supply system and the impedance of the connecting pipe of the air tank are considered, Equations (2) and (6)–(8) can be obtained as follows:

$$z_0 - z + \left(p_0 - \frac{p_0 mz}{l_0} - p_a\right) - H_0 - (\alpha+\beta)F\frac{dz}{dt} = \frac{L}{g}\frac{dv}{dt} \tag{17}$$

Further sorted out:

$$\frac{d^2z}{dt^2} + \frac{fg}{L}(\alpha+\beta)\frac{dz}{dt} + \frac{fg}{FL}\left(\frac{mp_0}{l_0}+1\right)z = 0 \tag{18}$$

Equation (18) is a second-order linear homogeneous differential equation with constant coefficients. The characteristic root of the characteristic equation is complex conjugate root:

$$\lambda_{1,2} = \frac{-(\alpha+\beta)\frac{fg}{L} \pm \sqrt{4\left(\frac{mp_0}{l_0}+1\right)\frac{fg}{FL} - (\alpha+\beta)^2\frac{f^2g^2}{L^2}}\,i}{2}$$

Therefore, the general solution of Equation (18) is:

$$z = \mathrm{e}^{(-(\alpha+\beta/2)\cdot(fg/L))t}\left(C_1\cos\frac{\sqrt{4\left(\frac{mp_0}{l_0}+1\right)\frac{fg}{FL} - (\alpha+\beta)^2\frac{f^2g^2}{L^2}}}{2}t + C_2\sin\frac{\sqrt{4\left(\frac{mp_0}{l_0}+1\right)\frac{fg}{FL} - (\alpha+\beta)^2\frac{f^2g^2}{L^2}}}{2}t\right) \tag{19}$$

where $z_{(t=0)} = 0$, $C_1 = 0$, and $C_2 = -|v_0|\sqrt{(Lf/gF)}/\sqrt{\sigma}$, so the change process of the surge wave in the air tank is:

$$z(t) = z_0 + \left(|v_0|\sqrt{\frac{Lf}{gF}}/\sqrt{\sigma}\right)\mathrm{e}^{(-(\alpha+\beta/2)\cdot(fg/L))t} \sin\left(\frac{\sqrt{4\left(\frac{mp_0}{l_0}+1\right)\frac{fg}{FL} - (\alpha+\beta)^2\frac{f^2g^2}{L^2}}}{2}t + \pi\right) \tag{20}$$

The pressure change process at the bottom of the air tank is:

$$H(t) = H_0 + \left(|v_0|\sqrt{\frac{Lf}{gF}}\sqrt{\sigma}\right)e^{(-(\alpha+\beta/2)\cdot(fg/L))t} \sin\left(\frac{\sqrt{4\left(\frac{mp_0}{l_0}+1\right)\frac{fg}{FL}-(\alpha+\beta)^2\frac{f^2g^2}{L^2}}}{2}t+\pi\right) \tag{21}$$

where $e^{(-(\alpha+\beta/2)\cdot(fg/L))t}$ is the attenuation term, the fluctuation period is: $4\pi/\sqrt{4((mp_0/l_0)+1)(fg/FL)-(\alpha+\beta)^2(f^2g^2/L^2)}$. Due to the friction of the system, the pressure at the bottom of the air tank has a maximum value during the first fluctuation period. Therefore, the position of the extreme point can be obtained by calculating the first derivative of t.

$$H'(t) = 0 \tag{22}$$

Therefore, the moment of the peak in the first fluctuation period:

$$t_1 = \frac{\arctan\dfrac{\dfrac{\sqrt{4\left(\frac{mp_0}{l_0}+1\right)\frac{fg}{FL}-(\alpha+\beta)^2\frac{f^2g^2}{L^2}}}{2}}{\dfrac{\alpha+\beta}{2}\cdot\dfrac{fg}{L}}+\pi}{\dfrac{\sqrt{4\left(\frac{mp_0}{l_0}+1\right)\frac{fg}{FL}-(\alpha+\beta)^2\frac{f^2g^2}{L^2}}}{2}} \tag{23}$$

Since $H(t_1)$ is an implicit solution about F, a simple trial calculation is required to find the cross-sectional area of the air tank that meets the requirements for pump overpressure protection.

$$H(t_1) - \varphi H_0 \approx 0 \tag{24}$$

where φ generally takes 0.3–0.5 according to the relevant specifications. The partial safety factor is taken as 0.3. H_0 is the pressure at the bottom of the air tank under constant flow conditions (including the head loss of the pipeline) (m).

CASE STUDY

A high-lift pressurized water supply project is shown in Figure 2. Table 1 lists the parameters of the water supply system. As can be seen from Table 1, the total length of the water pipeline is about 10,400 m. A single tube of nodular cast iron pipe with a diameter of 1.2 m is used for water supply. The designed water supply flow is 1.30 m^3/s, and the actual lift of the pump station pump is 165.57 m.

Based on the characteristic line method, a numerical model of the hydraulic transient of the pipeline system was established (Wylie et al. 1993; Chaudhry 2014). In this case, the pipeline should not have negative pressure, and the maximum pressure should be below 247.00 m to ensure the stability and safe operation of the system.

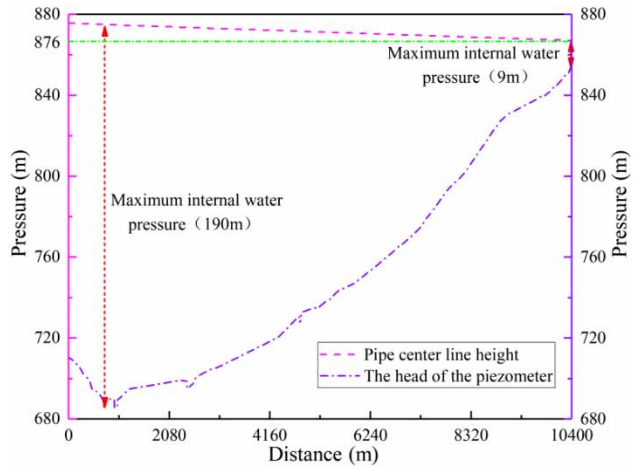

Figure 2 | Schematic diagram of the long-distance water supply system.

Table 1 | System parameters

Water level of suction sump (m)	711.0	Quantity of pumps (m)	3
Water level of outlet sump (m)	867.0	Rated flow (m^3/s)	0.43
Quantity of pipes	1	Rated head (H) (m)	167
Pipe length (km)	10.4	Rated speed (N) (r/min)	1,480
Pipe diameter (m)	1.2	Rated motor power (T) (kW)	1,250
Flow discharge (Q) (m^3/s)	1.27	Flywheel moment (GD^2) (kg m^2)	448

Without protection

Here, when a pump trip occurs, there are no protective measures. Due to the large inertia of water, the pressure and flow in the pipeline are significantly reduced. In this case, the pressure changed after the pump, and the pipeline pressure enveloping as shown in Figure 3.

As shown in Figure 3(a), when the pump tripped, the pressure after the pump started to drop. At $t = 21$ s, the pressure after the pump reached the minimum. It can be seen from Figure 3(b) that the maximum pressure of the pipeline was higher than the maximum pressure control standard of the pipeline (251.05 > 247.00 m).

In addition, the minimum pressure along the pipeline was much lower than the vaporization pressure ($-67.34 \ll -10.00$ m). At number 4 + 752 m, the minimum pressure along the pipeline became negative. When the pressure dropped below 10 m, the water body had actually vaporized, so the pressure drop below 10 m will not exist in practice. It is only used to characterize the severity of the pressure dropping. At this time, in order to ensure the safe operation of the water supply system, reasonable protective measures must be set.

Comparison of protective effects of air tanks under different conditions

Based on the engineering example above, the protection study was carried out. The air tank was set at the pile number 0 + 000 m (that is, the head end of the water pipeline), and the numerical simulation calculations were carried out with and without friction. The relevant parameters of the water supply system are listed in Table 2.

In the case of no friction, the relevant parameters were substituted into the formula $V_a = Fl_0 \geq (v_0^2 Lfmp_0/g\varphi^2 H_0^2)$ to estimate the initial gas chamber volume required by the air tank, and the cross-sectional area of the air tank was determined to be 46.60 m^2 by $F = V_a/l_0$. With the consideration of

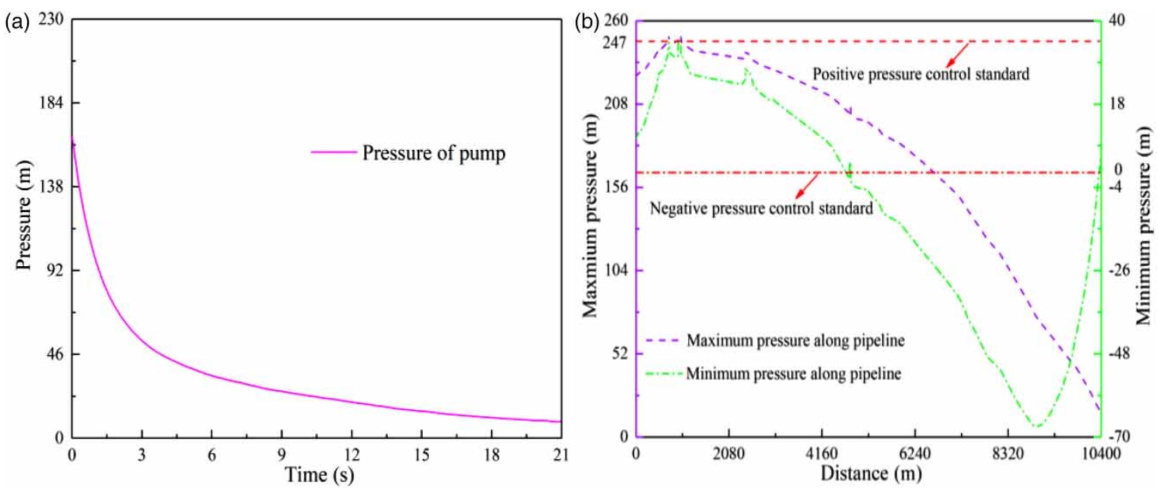

Figure 3 | (a) Pressure change after the pump is powered off and (b) pipeline pressure envelope without protection.

Table 2 | Statistics of initial parameters of the water supply system

Program	Pipe head loss coefficient α	Constant flow air tank bottom pressure (m)	Initial operation water level (m)	Initial air chamber height (m)	Initial gas absolute pressure (m)	Impedance hole area (m^2)
No friction	/	156.50	715.00	3.00	162.00	/
With friction	5.12	165.19	715.00	3.00	170.69	0.50

friction, the parameters above were put into the trial calculation program for trial calculation, and the cross-sectional area of the air tank most consistent with Equation (24) was 26.00 m^2.

By substituting the solved air tank parameters into the water supply system for numerical simulation, the results are listed in Table 3 and Figure 4.

The numerical calculation results showed that the maximum pressure of the two schemes did not exceed the pipeline maximum pressure control standard of 247.00 m, and the minimum pressure was greater than 8 m, which met the pipeline without negative pressure with a certain margin. Therefore, the two protection schemes can effectively protect the water hammer pressure drop caused by pump loss of power.

It should be noted that the cross-sectional area of the air tank calculated by the formula without considering friction was 46.6 m^2, the cross-sectional area calculated by considering the friction test was 26.0 m^2, and the height of the air chamber and the total height of the initial water depth in the tank were both 7.50 m.

The air tank volume calculated by the formula without considering friction was 349.5 m^3, and the volume calculated after considering friction was 195.0 m^3, which was only 55.8% of the former. When the pressure standard for water hammer protection control was consistent, the calculation formula considering friction can significantly reduce the volume of the air tank, and the volume reduction value reached 154.5 m^3.

Figures 5 and 6 show the comparison between the results of numerical simulation and theoretical calculation.

It can be seen from Figure 5 that, without considering friction, the theoretically calculated extreme pressure values were 203.80 and 109.20 m, respectively, and the numerically simulated extreme pressure values at the bottom of the air tank were 191.90 and 121.88 m, respectively. The errors of the maximum and the minimum pressure value were 6 and 11%, respectively. The extreme pressure value of numerical simulation obviously lagged behind that of the theoretical calculation, and the pressure at the bottom of the air tank was not attenuated because the pipeline friction was ignored. It can be seen that the results of the numerical simulation were poorly fitted to the results calculated by the theoretical formula. During the first fluctuation period, the theoretically calculated maximum pressure at the bottom of the air tank was too enormous when the minimum pressure at the bottom was small. In the design of the air tank protection scheme, it is necessary to increase the volume of the air tank to meet the pressure control standard. Therefore, the volume of the air tank estimated by Equation (16) was too large, and the safety margins for positive and negative pressure protection were both high.

It can be seen from Figure 6 that under the condition of considering friction resistance and impedance, the theoretical calculated extreme pressure values were 211.46 and 214.44 m, respectively, and the numerical simulation extreme pressure values at the bottom of the air tank were 110.15 and 107.18 m, respectively, and the pressure extreme value errors were less than 3%. Moreover, the pressure fluctuation period and attenuation degree at the bottom of the air tank were the same, and the calculation results of the theoretical formula and the numerical simulation results were in good agreement. The theoretically calculated pressure extreme value had a certain safety margin relative to the numerical simulation results. Compared with the case without friction, it can be found that the optimization of the air tank body parameter in the calculation example was improved by 44.2%. It can be seen that the cross-sectional area of the air tank calculated by the theoretical formula deduced in this paper has certain rationality, and this method can accurately determine the operating parameters of the air tank.

Table 3 | Numerical simulation results

Program	Cross-sectional area of the air tank (m^2)	Volume of the air tank (m^3)	Maximum pressure (m)	Minimum pressure (m)
No friction	46.6	349.5	233.02	9.01
With friction	26.0	195.0	215.98	8.97

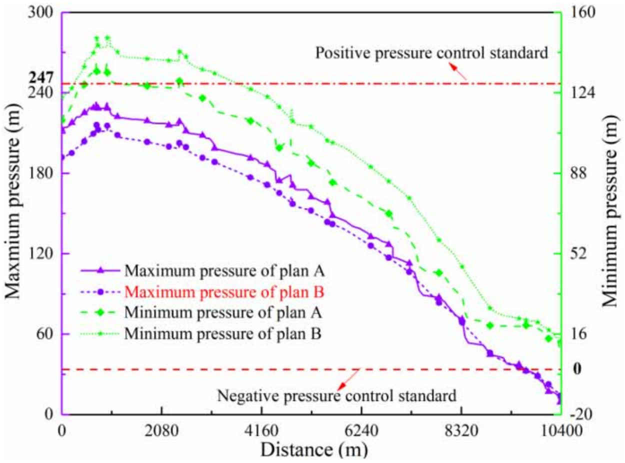

Figure 4 | Pressure change process at the bottom of the air tank.

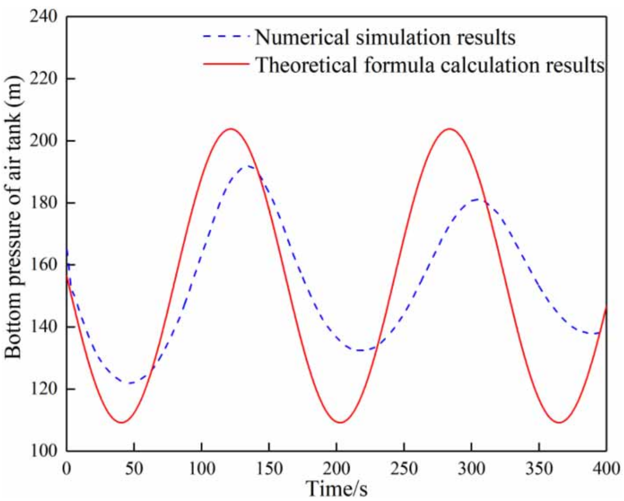

Figure 5 | Pressure change process at the bottom of the air tank (without considering friction and impedance).

DISCUSSION

In order to study the influence of pipeline friction and air tank resistance on water hammer protection, the relationship of air tank operating parameters under friction and resistance was established. Subsequently, taking the actual project as an example, the formula of the operating parameters of the air tank was verified by numerical simulation. In this section, the effects of pipeline friction and impedance will be discussed.

Firstly, considering the requirements of the same pressure control standard, the pipe friction coefficient and

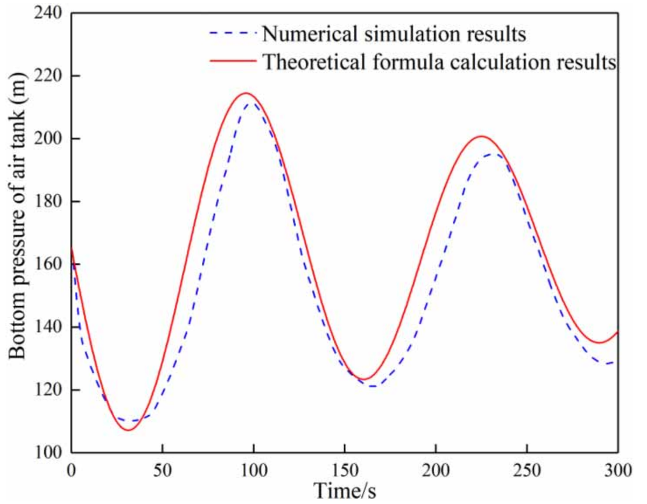

Figure 6 | Pressure change process at the bottom of the air tank (considering friction and impedance).

impedance coefficient were added, and the cross-sectional areas of the air tank derived from different formulas were compared. The sizes of the air tank in these two cases are shown in Table 3. The results showed that the total volume without considering friction was 349.5 m^3, and the total volume considering friction and impedance was 195.0 m^3.

The reason for this volume difference was that the friction of the pipe and the impedance of the air tank had a subtractive effect on the pressure fluctuation of the water hammer. When the water hammer pressure drop traveled downstream through the air tank, the existence of the air tank impedance reduced the dropped penetration pressure, and the pressure amplitude at the bottom of the air tank decreased. The water hammer pressure drop continued to travel downstream. Due to the existence of pipeline friction, the water hammer wave continued to attenuate, the internal water pressure fluctuation amplitude at each point in the pipeline was smaller than the pressure amplitude at the bottom of the air tank, and the amplitude of the water hammer wave changes in different phases presented a decreasing trend. While designing the parameters of the air tank, it is feasible to use the amplitude of the water pressure change at the bottom of the air tank to evaluate whether the extreme pressure value of the water pipeline after the tank meets the design protection requirements and has a certain safety margin. Therefore, when the pressure amplitude at the bottom of the air tank decreased, the volume of the air tank that met the protection requirements also decreases.

Secondly, in previous studies, the choice of air tank volume was mostly based on numerical simulation, which had a certain degree of blindness (Ghidaoui *et al.* 2005). The operating parameter formulas deduced in this paper considering pipeline friction and impedance can be directly used in the selection of air tank parameters. Moreover, by comparing the theoretical formula calculation results with the numerical simulation results, as shown in Figures 5 and 6, it can be found that the formula derived in this paper was more consistent with the numerical simulation results, which was more in line with the actual situation during the operation of the water supply system. Due to the rigid water body, the slight error was assumed by the theoretical formula calculation, while the numerical simulation calculation based on the characteristic line method considered the elastic water body. These errors within 3% can provide a certain safety margin for the air tank design. This formula can provide designers with guidance basis during the design stage and has certain engineering practical significance.

CONCLUSION

Based on the theory of rigid water body and simple harmonic vibration, the estimation formulas of various operating parameters, including surge period, surge amplitude, and bottom pressure change amplitude, of air tanks with and without pipeline friction and air tank impedance were deduced. According to the amplitude of the surge change of the air tank, the volume of the air tank that met the protection requirements was calculated, and an engineering example was introduced for numerical simulation verification. According to the research results, the following main conclusions can be drawn:

Without considering the friction and impedance, there was a large deviation between the calculation results of numerical simulation and formula calculation, the accuracy of the theoretical formula calculation results was low, and the surge wave had no apparent attenuation. In the case of considering friction and impedance, the numerical simulation results fitted well with the theoretical calculation results, and the attenuation degree of surge wave was also equivalent. At the same time, the volume optimization of the air tank can be improved by 40–50% if friction and impedance were taken into account. In addition, the resistance of the air tank connecting pipe and the friction of the pipeline system had a more significant impact on the overpressure protection effect of the pump stop. When the negative pressure protection of the water supply system met the requirements, the area of the impedance should be as small as possible.

Therefore, the operating parameter estimation formula derived after considering the friction and impedance was more in line with the actual operating conditions of the water supply system, which can provide theoretical support for the determination of the air tank size. In addition, it must be pointed out that the derivation of the calculation formula in this study is based on the assumption of the rigid water body and constant friction. In future research, these factors can be discussed further to optimize the body parameters of the air tank.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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