AQUA

Water Infrastructure, Ecosystems and Society

AQUA — Water Infrastructure, Ecosystems and Society Vol 71 No 7, 862 doi: 10.2166/aqua.2022.066

Water quality reliability based on an improved entropy in a water distribution system

Yumin Wang [a],*, Jianguo Zhu [b] and Guangcan Zhu [a]

[a] School of Energy and Environment, Southeast University, #2 Sipailou Street, Nanjing City, Jiangsu Province 210096, China
[b] Jiangsu Institute of Urban Planning and Design, Nanjing City, Jiangsu Province 210019, China
*Corresponding author. E-mail: wangyumin@seu.edu.cn

YW, 0000-0001-6074-3824; JZ, 0000-0001-5430-6441

ABSTRACT

In this paper, information entropy was proposed to measure water quality reliability in a water distribution system (WDS), which had been applied to evaluate hydraulic reliability in the WDS. In the water quality reliability evaluation, residual chlorine is a representative of water quality, and a first-order decay model was usually adopted. The water quality reliability (R) based on water quality entropy (WQE) and improved water quality reliability (R_d) based on improved water quality entropy (IWQE) were proposed and compared for three networks. The method was developed based on the EPANET toolkit and MATLAB environment. The results indicated that flow entropy (FE) is strongly related to WQE, and improved flow entropy (IFE) is also strongly related to IWQE. In addition, R_d can reflect the effect of pipe velocity, whereas R can only reflect the effects of pipe flow and the WDS layout. The novelty of this paper is to develop the entropy with consideration of the pipe velocity to measure water quality liability as a surrogate index, which can reduce the calculation load and can be applied to a nonlinear system. The proposed water quality reliability evaluation method based on information entropy can help design, analyze, and improve the water quality in the WDS.

Key words: improved water quality entropy (IWQE), water distribution system (WDS), water quality entropy (WQE), water quality reliability (WQR)

HIGHLIGHTS

- Flow entropy (FE), diameter-sensitive flow entropy (DSFE), water quality entropy (WQE), and diameter-sensitive water quality entropy (DSWQE) were discovered.
- Entropy was applied to water distribution systems (WDSs).
- The difference between WQE and multiplier for WDSs was analyzed.
- Water quality reliability analyses based on WQE and DSWQE were compared.
- The relationship between entropy and characteristics of WDSs was discovered.

ABBREVIATIONS

WDS	water distribution system
IFE	improved flow entropy
IWQE	improved water quality entropy
EPS	extended period simulation
MCS	Monte Carlo Simulation
WQR	water quality reliability
FE	flow entropy
WQE	water quality entropy

1. INTRODUCTION

Reliability analysis of the water distribution system (WDS) is one of the most important aspects to guarantee undisrupted, enough, and safe water supply to consumers even during abnormal conditions (Tanyimboh *et al.* 2011). The reliability of the WDS is concluded into the following three groups: mechanical reliability, hydraulic reliability, and water quality reliability (WQR) (Kansal *et al.* 1995; Zhao *et al.* 2010). Substantial researches focused on two aspects of mechanical reliability and hydraulic reliability (Setiadi *et al.* 2005; Tanyimboh *et al.* 2011; Islam *et al.* 2014). Few research works put emphasis on the reliability of water quality (Zhao *et al.* 2010; Islam *et al.* 2014; Beker & Kansal 2022). However, the water quality in the WDS deteriorated with physical, chemical, and biological processes by traveling through pipe systems, which makes the research on WQR more essential and necessary. Since residual chlorine is one of the best indicators to represent water quality in the WDS, it is usually taken as the indicator to analyze the WQR in the WDS (Shafiqul Islam *et al.* 2014).

The main method to evaluate the WQR in the WDS is the simulation-based approach, which measures the probability that a failed WDS can supply the consumers with acceptable water quality (Gheisi *et al.* 2016). By taking account of the variation in water demand and chlorine, Monte Carlo Simulation (MCS) was applied to evaluate residual chlorine availability to represent WQR (Zhao *et al.* 2010; Li *et al.* 2013). Although MCS can obtain accurate reliability estimation, it is computationally inefficient due to a very large number of realizations. To improve the computational efficiency of the MCS, the first-order reliability method was proposed (Tolson *et al.* 2001). However, the first-order reliability method based on Taylor series expansion is more suitable for a linear system. As such, information entropy technology was introduced to analyze the reliability of the WDS, which is termed as flow entropy (FE) to measure the redundancy and flow uniformity of a WDS (Awumah *et al.* 1990, 1991). Thereafter, maximum FE for the WDS with single and multiple sources was proposed (Tanyimboh & Templeman 1993; Yassin-Kassab *et al.* 1999). The method can reduce the computational load since complex stochastic sampling and iteration procedures are not required (Liu *et al.* 2014). FE is proved to be strongly related to hydraulic reliability (Tanyimboh & Templeman 2000; Setiadi *et al.* 2005; Santonastaso *et al.* 2018). Sensitivity analysis between entropy and hydraulic reliability proved that the design of the WDS based on maximum entropy flow was more reliable than other designs (Tanyimboh & Setiadi 2008). Comparison among surrogate indicators of statistical entropy, network resilience, resilience index, and modified resilience index indicated that statistical entropy outperformed other indices (Tanyimboh *et al.* 2011). As such, FE was used for surrogate indicators of hydraulic reliability, and integrated into an optimization design based on reliability (Creaco *et al.* 2014; Singh & Oh 2015). Although entropy technology had been proposed and applied for more than two decades (Awumah *et al.* 1991; Yassin-Kassab *et al.* 1999; Singh & Oh 2015), it had not been widely used for measuring the WQR of the WDS. Tsalli entropy was proposed to measure the WQR of 26 layouts based on a WDS with eight nodes and a source, and a comparison between the Tsalli entropy and Shannon entropy was performed. The results indicated that the Shannon entropy can reflect the effect of loop number better than the Tsalli entropy (Wang & Zhu 2021). As such, the Shannon entropy was selected to measure the hydraulic reliabiltity and WQR. FE, improved flow entropy (IFE), water quality entropy (WQE), and improved water quality entropy (IWQE) were applied and compared for measuring the hydraulic reliability and WQR, respectively. The reason for introducing IFE and IWQE is that the flow velocity has significant effects on pipe flow and nodal water quality (chlorine concentration). For three WDSs, the relationships among FE, IFE, WQE, and IWQE were performed, respectively. In addition, the WQR R based on WQE and R_d based on IWQE were obtained, and the variation of R and R_d with the demand multiplier for three WDSs was analyzed. The novelty of this paper is to develop the entropy to measure WQR as a surrogate index, which can reduce the calculation load and can be applied to a nonlinear system.

In this paper, first, based on FE, the concepts of IFE, WQE, and IWQE were proposed. Second, WQR R and R_d were defined based on WQE and IWQE, respectively, and applied to three networks with various flows and layouts, respectively. Finally, the discussions and conclusions were drawn.

2. METHOD

In this paper, the method proposed was described as shown in Figure 1. Details were described as follows.

2.1. Flow entropy/Improved flow entropy

The entropy concept introduced by Shannon (1948) is expressed by Equation (1) as follows.

$$\frac{S}{K} = -\sum_{i=1}^{M} P_i \ln P_i \tag{1}$$

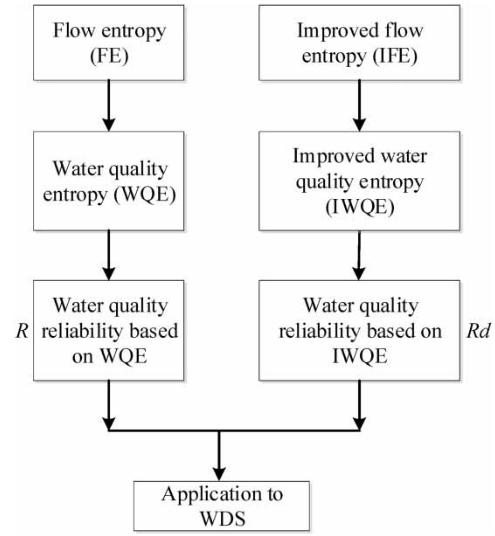

Figure 1 | Schematic diagram of the method.

where S is the entropy, K is an arbitrary positive constant, which is usually set to be 1.0, P_i is the probabilistic associated with event i, $i = 1, 2, 3, \ldots, M$, which satisfies the condition of $\sum_{i=1}^{M} P_i = 1$, M is the number of events.

Based on the entropy concept, *Tanyimboh* and *Templeman* proposed the concept of conditional entropy, which is expressed by Equation (2) as follows.

$$S = S_0 + \sum_{i=1}^{N} pf_i \times S_i \tag{2}$$

where S is the FE of the system, S_0 is the FE of sources, S_i is the FE of node i, N is the number of nodes in the WDS, and pf_i is the ratio of flows into node i (T_i) to the total flows supplied by all sources, which is expressed by $pf_i = T_i/T_0$, T_0 is the total flow for all the sources.

The FE of the water source S_0 is expressed by Equation (3) as follows.

$$S_0 = -\sum_{i=1}^{N_0} \left(\frac{Q_{0i}}{T_0}\right) \ln\left(\frac{Q_{0i}}{T_0}\right) = -\sum_{i=1}^{N_0} pf_{0i} \times \ln pf_{0i} \tag{3}$$

where Q_{0i} is the flow at the source node i, $T_0 = \sum_{i=1}^{N_0} Q_{0i}$, N_0 is the number of source nodes, and $pf_{0\mathrm{i}} = Q_{0i}/T_0$.

The FE of the node i (S_i) is expressed by Equation (4a) as follows.

$$S_i = -\sum_{j=1}^{NJ_i} \left(\frac{q_i^j}{T_i}\right) \times \ln\left(\frac{q_i^j}{T_i}\right) = -\sum_{j=1}^{NJ_i} (pf_i^j \times \ln pf_i^j) \tag{4a}$$

where q_i^j is the flow from node j to node i, $T_i = \sum_{j=1}^{NJ_i} q_i^j$, NJ_i is the number of nodes that flow into node i, and $pf_i^j = q_i^j/T_i$. When only one pipe flows into node i, the FE of node i (S_i) is equal to 0.

The nodal IFE is expressed by Equation (4b) as follows.

$$S_d i = -\sum_{j=1}^{NJ_i} \left(\frac{q_i^j}{T_i}\right) \times \ln\left(\frac{q_i^j}{T_i}\right) = -\sum_{j=1}^{NJ_i} \frac{C}{v_i^j} (pf_i^j \times \ln pf_i^j) \tag{4b}$$

Based on nodal IFE, the pipe with lower velocity, i.e., larger diameter under definite flow and less flow under definite diameter, results in larger entropy values. Larger pipe diameters and less flow can enhance the system's capability of coping with abnormal conditions, and decrease the probability of pipe bursts.

Equation (2) can be expressed by Equation (5) as follows.

$$S = -\sum_{i=1}^{N_0} pf_{0\mathrm{i}} \times \ln pf_{0\mathrm{i}} + \sum_{i=1}^{N} pf_i \times \left[-\sum_{j=1}^{NJ_i} (pf_i^j \times \ln pf_i^j) \right] \tag{5}$$

Although S characterizes the flow uniformity of the WDS, it remains to be identical when the layouts and the number of loops and pipes in the WDS are identified. However, the pipe velocity has a significant impact on reliability. As such, IFE is proposed, which is expressed by Equation (6) as follows.

$$S_d = -\sum_{i=1}^{N_0} pf_{0\mathrm{i}} \times \ln pf_{0\mathrm{i}} + \sum_{i=1}^{N} pf_i \times \left[-\sum_{j=1}^{NJ_i} \frac{C}{v_i^j} (pf_i^j \times \ln pf_i^j) \right] \tag{6}$$

where S_d is the IFE, v_i^j represents the velocity of pipe from node j to the node i, and C is a velocity constant of 1 m/s, as FE is a dimensionless quantity.

The definition of the IFE improved the FE by strengthening the relationship between reliability and entropy. As such, the IFE not only considers the (non) uniformity of flows in a network but also accounts for the impact of pipe velocity on entropy and reliability.

2.2. Water quality entropy/Improved water quality entropy

Similar with the FE, in this paper, the concept of the WQE was proposed and expressed by Equation (7) as follows.

$$E = E_0 + \sum_{i=1}^{N} p_i \times E_i \tag{7}$$

where E is the overall WQE, E_0 is the WQE of sources, E_i is the WQE of node i, p_i is the weight of node i, which can be expressed by $p_i = Q_i C_i / \sum Q_i C_i$, where C_i is chlorine concentration at node i.

The WQE of water source E_0 is expressed by Equation (8) as follows.

$$E_0 = -\sum_{i=1}^{N_0} \left(\frac{Q_{0i} C_{0i}}{W_0} \right) \times \ln \left(\frac{Q_{0i} C_{0i}}{W_0} \right) \tag{8}$$

where C_{0i} is the chlorine concentration at the source node i, W_0 is the total chlorine content for all the sources, $W_0 = \sum_{i=1}^{N_0} Q_{0i} C_{0i}$.

In terms of $p_{0\mathrm{i}} = Q_{0i} C_{0i} / W_0$, Equation (8) can be simplified as Equation (9) as follows.

$$E_0 = -\sum_{i=1}^{N_0} p_{0i} \times \ln p_{0i} \tag{9}$$

The WQE at the node i termed as E_i is expressed by Equation (10) as follows.

$$E_i = -\sum_{j=1}^{NJ_i} \left(\frac{q_i^j C_i^j}{W_i} \right) \times \ln \left(\frac{q_i^j C_i^j}{W_i} \right) \tag{10a}$$

where C_i^j is the chlorine concentration at the end of the pipe connecting from node j to node i, $W_\mathrm{i} = \sum_{j=1}^{NJ_i} q_i^j C_i^j$. When only one pipe flows into node i, the WQE of node i (E_i) is equal to 0.

The nodal IWQE termed as E_{di} is expressed by Equation (10b) as follows.

$$E_{di} = -\sum_{j=1}^{NJ_i} \frac{C}{v_i^j}\left[\left(\frac{q_i^j C_i^j}{W_i}\right) \times \ln\left(\frac{q_i^j C_i^j}{W_i}\right)\right] \tag{10b}$$

In the WDS, chlorine complies with the first-order decay model, which is usually expressed by Equation (11) as follows.

$$C_i^j = C_j \exp\left(-k_0 t_i^j\right) = C_j \exp\left(-k_0 \frac{L_i^j}{v_i^j}\right) = C_j \exp\left[-\frac{k_0 L_i^j \pi (d_i^j)^2}{4q_i^j}\right] \tag{11}$$

where C_j is the chlorine concentration at node j, k_0 is the chlorine decay coefficient, and t_i^j, L_i^j, v_i^j, d_i^j are the travel time, length, velocity, and diameter of pipe from node j to node i, respectively.

In terms of $\eta_i^j = \exp[-k_0 L_i^j \pi (d_i^j)^2 / 4q_j^i]$, Equation (11) can be simplified as Equation (12) is expressed as follows.

$$C_i^j = C_j \eta_i^j \tag{12}$$

Combining Equations (10) and (12), we can deduce Equation (13) is expressed as follows.

$$E_i = -\sum_{j=1}^{NJ_i} \left(\frac{q_i^j C_j \eta_i^j}{W_i}\right) \times \ln\left(\frac{q_i^j C_j \eta_i^j}{W_i}\right) \tag{13}$$

Next, Equation (13) can be simplified as Equation (14) in terms of $p_i^j = q_i^j C_j \eta_i^j / W_i$,

$$E_i = -\sum_{j=1}^{NJ_i} p_i^j \times \ln p_i^j \tag{14}$$

Accordingly, the WQE termed as E is obtained by the combination of Equations (7), (9), and (14), which can be expressed by Equation (15) as follows.

$$E = -\sum_{i=1}^{N_0} p_{0\mathrm{i}} \times \ln p_{0\mathrm{i}} + \sum_{i=1}^{N} p_i \times \left[-\sum_{j=1}^{NJ_i} (p_i^j \times \ln p_i^j)\right] \tag{15}$$

For a specific pipe, lower velocity leads to longer residence time (i.e., water age), which is not beneficial for water quality due to chlorine decay. From Equations (10) to (14), it can be found that nodal WQE E_i decreased under the condition of lower velocity connected with node i. As such, nodal WQE E_i reflects the effect of pipe velocity on chlorine decay.

Similar with IFE, the IWQE termed as E_d was proposed, and expressed by Equation (16) as follows

$$E_d = -\sum_{i=1}^{N_0} p_{0\mathrm{i}} \times \ln p_{0\mathrm{i}} + \sum_{i=1}^{N} p_i \times \left[-\sum_{j=1}^{NJ_i} \frac{C}{v_i^j} (p_i^j \times \ln p_i^j)\right] \tag{16}$$

Based on IWQE, the impact of pipe velocity has two sides. One side is that within Equation (16), lower velocity results in larger entropy values. Under definite pipe flow lower velocity means larger pipe diameter, which can reduce the occurrence of abnormal conditions, probability of pipe bursts, and water quality failure, IWQE improved WQE by strengthening the relationship between WQR and WQE. The other side is that in Equation (16), under definite pipe diameter lower velocity, means less flow, which results in longer residence time (i.e., water age) from node j to node i, and WQE at node i becomes lower since chlorine concentration at the end of pipe connecting from node j to node i (C_i^j) decreases due to the process of chlorine residual decay. The definition of WQE considers the effect of the chlorine decay and WDS layout, while the definition of IWQE considers also considers the effects of pipe velocity.

2.3. Water quality reliability

From the viewpoint of water quality, the WDS is reliable when each water source, each node, and each pipe carry the same amount of chlorine, respectively (Wang & Zhu 2021). Assume chlorine contents in all the pipes are the same, we can obtain Equations (17a)–(17c) are expressed as follows.

$$p_{0i} = \frac{1}{N_0} \tag{17a}$$

$$p_i = \frac{1}{N} \tag{17b}$$

$$p_i^j = \frac{1}{NJ_i} \tag{17c}$$

The WQE is converted to $E_{\max}$, which is expressed by Equation (18) as follows.

$$\begin{aligned} E_{max} &= -\sum_{i=1}^{N_0} \frac{1}{N_0} \times \ln \frac{1}{N_0} + \sum_{i=1}^{N} \frac{1}{N} \times \left[-\sum_{j=1}^{NJ_i} \left(\frac{1}{NJ_i} \times \ln\left(\frac{1}{NJ_i} \right) \right) \right] \\ &= \ln N_0 + \frac{1}{N} \sum_{i=1}^{N} \ln\left(NJ_i\right) \end{aligned} \tag{18}$$

Accordingly, the ratio of E and E_d to E_{max} can be surrogate indices for assessing the WQR of the WDS, which are expressed by Equations (19a)–(19b) as follows.

$$R = \frac{E}{E_{\max}} \tag{19a}$$

$$R_d = \frac{E_d}{E_{\max}} \tag{19b}$$

where R is WQR based on the WQE, and R_d is WQR based on the IWQE. The computation of R and R_d in the WDS was performed in MATLAB environment by calling EPANET toolkit. The water quality modeling in the WDS requires extended period simulation (EPS). In the EPS water quality simulation or EPANET, the quality time step is 5 minutes, which is much shorter than the hydraulic simulation time step of 1 h. The assumed initial chlorine concentrations were assigned to nodes, which does not happen in real WDS. The program was operated for 72 h to avoid the effect of initial chlorine concentrations on R and R_d, and the obtained values of WQR from the 49th hour to the 72nd hour were analyzed. The bulk decay and wall decay are considered to follow the first-order decay regulation in simulations.

3. CASE STUDY

3.1. Case 1

A primary network with a single source, three loops, and 34 pipes is shown in Figure 2, which had been adopted by many researchers (Suribabu 2010). Node 1 is a source node with a fixed hydraulic head of 100 m, and the elevations for all the nodes are taken as zero. The nodal demand multiplier is shown in Table 1. The detailed pipe information can be found in the literature (Sivakumar *et al.* 2016). The bulk chlorine decay coefficient and the wall chlorine decay coefficients are −1.00 and −0.55/day, respectively. The initial chlorine concentration at the source node and demand nodes are 0.8 and 0.5 mg/L, respectively.

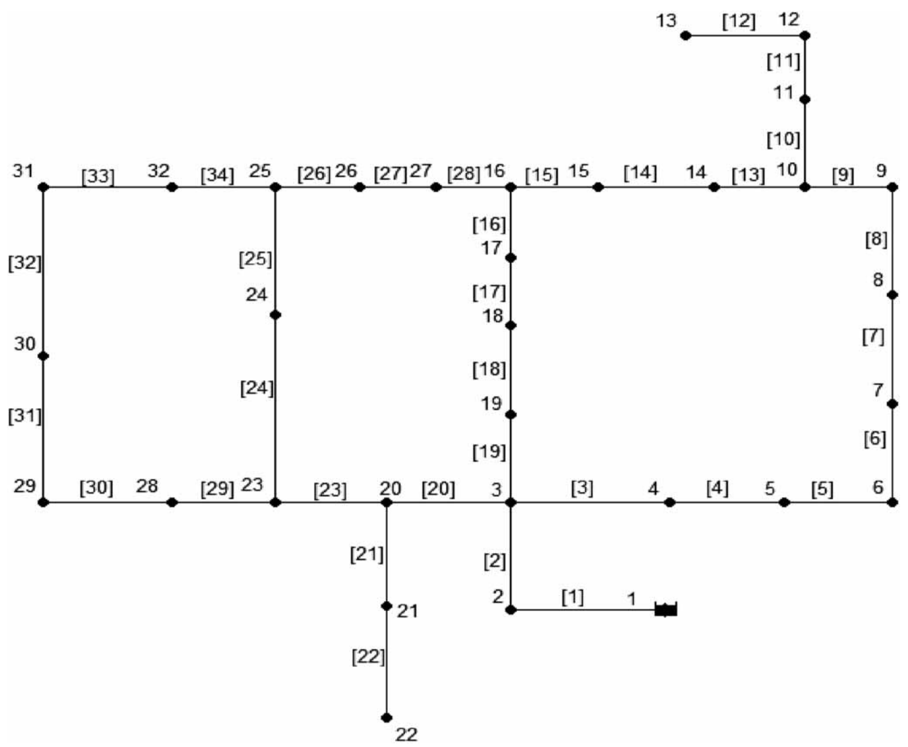

Figure 2 | Layout of Case 1.

3.2. Case 2

A network with two sources, 27 nodes, and 40 pipes is shown in Figure 3, which had been adopted by other researchers (Islam *et al.* 2011; Islam *et al.* 2014). Water is supplied by gravity from two elevated reservoirs with a total head of 90 and 85 m, respectively. The detailed nodal demands, elevations, and pipe information can be found in literature works (Islam *et al.* 2011, 2014). The water demand multiplier, the bulk chlorine decay coefficient, the wall chlorine decay coefficient, the initial chlorine concentrations at source nodes, and the initial chlorine concentrations at demand nodes are the same as Case 1.

3.3. Case 3

A network (Anytown network) with three source nodes of 10, 65, and 165 is shown in Figure 4 (Walski *et al.* 1987). The elevations of nodes 10, 65, and 165 are 10 ft (3.05 m), 215 ft (65.575 m), and 215 ft (65.575 m), respectively. The WDS has 19 connection nodes and 40 pipes. The pump has a shutoff head value of 300 ft (91.5 m), a maximum flow rate of

Table 1 | Hourly water demand multiplier for Case 1

Hour	Multiplier	Hour	Multiplier	Hour	Multiplier
1	0.41	9	1.30	17	1.06
2	0.38	10	1.24	18	1.27
3	0.38	11	1.28	19	1.21
4	0.45	12	1.16	20	1.15
5	0.83	13	1.14	21	0.87
6	0.99	14	0.87	22	0.71
7	1.53	15	0.89	23	0.60
8	1.46	16	0.87	24	0.41

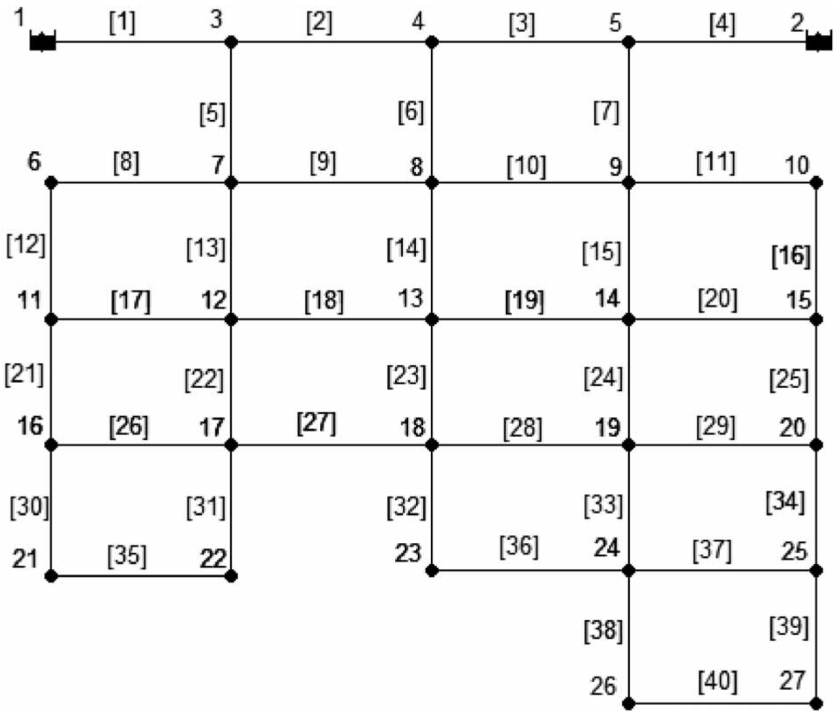

Figure 3 | Layout of Case 2.

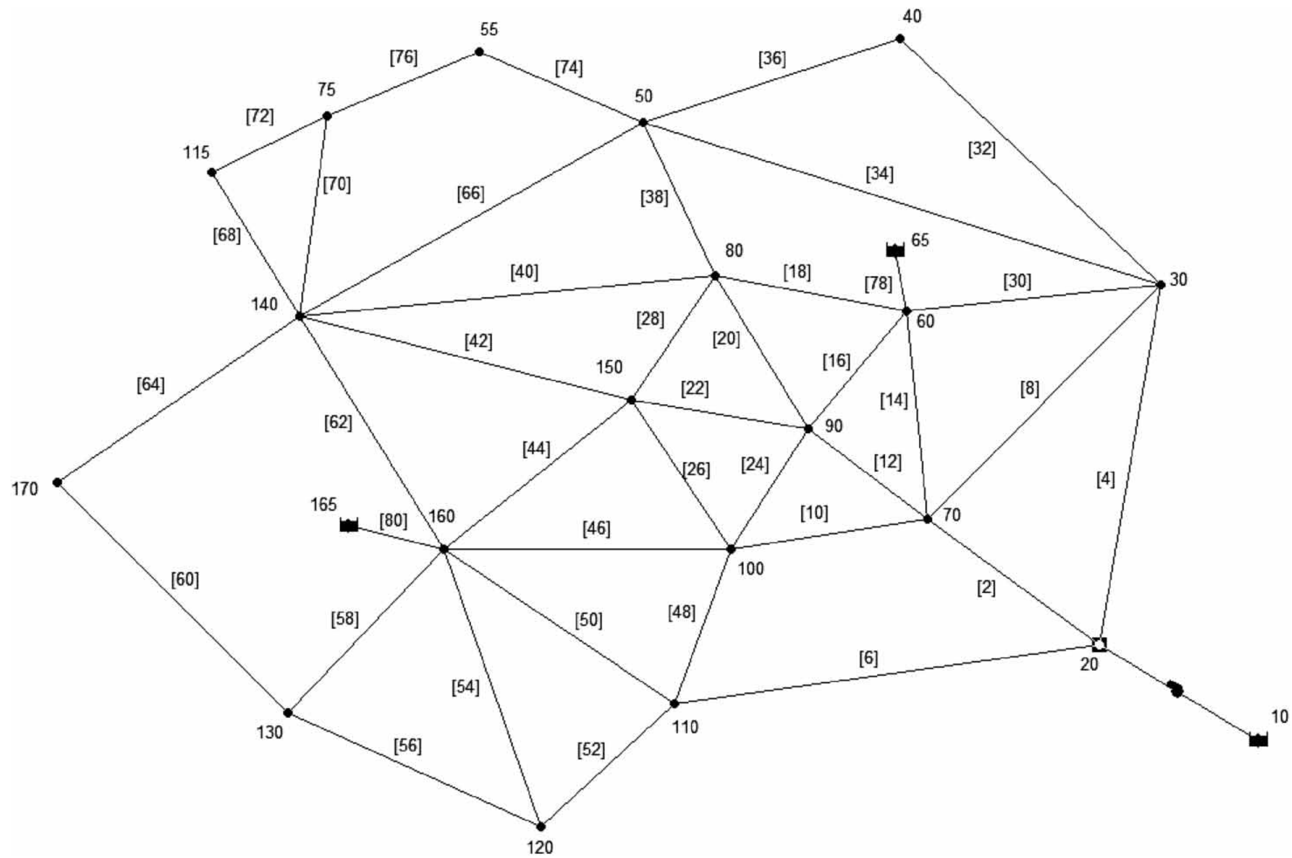

Figure 4 | Layout of the Anytown network (Case 3).

8,000 GPM (0.506 m^3/s). Detailed information of pipe length, diameter, and nodal elevation can be found in the literature (Walski *et al.* 1987). The water demand multiplier, the bulk chlorine decay coefficient, the wall chlorine decay coefficient, the initial chlorine concentrations at source nodes, and the initial chlorine concentrations at demand nodes are the same as Case 1.

4. RESULTS AND DISCUSSION

4.1. Application to Case 1

To analyze the variation of nodal inflow, chlorine concentration, FE, IFE, WQE, IWQE, R, and R_d, the average node degree (AND) is introduced. The AND is a basic measure of the connectivity reflecting the overall topological similarity of the network to perfect grids or lattice-like structures. The average value of the node degree distribution can be expressed as $< k > = 2m/n$, where $<k>$ is AND, m is the number of pipes, and n is the number of nodes in the WDS. The values of AND are 2.2, 3.2, and 4.2 for Case 1, Case 2, and Case 3, respectively, which are related to the nodal chlorine concentrations due to the mixing process that occurred at nodes.

For Case 1, the values of nodal inflow range from 405 to 1,631 L/s, and the values of nodal chlorine concentration range from 0.9 to 1.1 mg/L (shown in Figure 5(a)). The variation trend of average nodal inflow with time is in accordance with the variation of demand multiplier, while the variation trend of average nodal residual chlorine concentration is contrary to the variation of demand multiplier. The results indicated that with the increase of the demand multiplier, the average nodal inflow increases and the average nodal residual chlorine concentration decreases. With the increase of demand multiplier, the increase of nodal inflow is easy to be understood, whereas the variation of average nodal chlorine concentration is

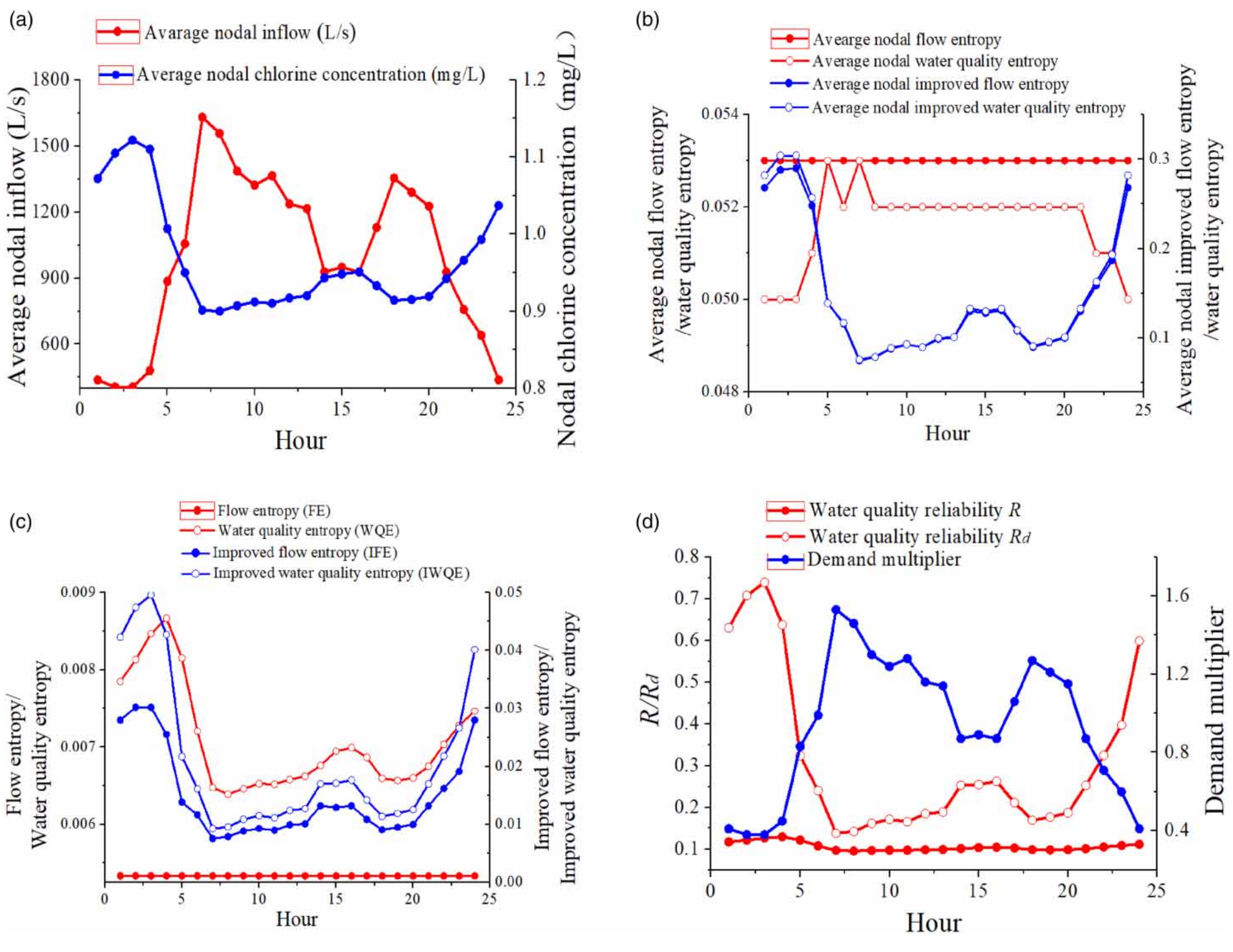

Figure 5 | Variation of nodal inflow, chlorine concentration, FE, IFE, WQE, IWQE, WQR (R), and WQR (R_d) for Case 1.

more complex. The pipe flow increased with the increase of the demand multiplier, which led to the decay time of chlorine shortened and residual chlorine increased. However, the nodal chlorine concentration has a relationship with not only the chlorine decay process but also the pipes' flow and number connecting with the node, which may increase or decrease the nodal chlorine concentration. For Case 1 with the relatively larger nodal flow and relatively less AND value, the nodal chlorine concentration decreased with the increase of the demand multiplier.

In Figure 5(b), the values of average nodal FE remain to be the same value of 0.053, and the average nodal IFE values range from 0.076 to 0.304 larger than the values of average nodal FE. The value of average nodal WQE almost unchanged ranging from 0.05 to 0.053 with little variation, and the average nodal IWQE values range from 0.075 to 0.290, which are much larger than average nodal WQE. The average nodal FE and WQE have a relationship with the layout of the WDS, which reflect the uniformity of nodal flow and chlorine concentration in the WDS. In addition, the values of average nodal IFE have a negative relationship with the demand multiplier, which is different from the nodal inflow. Since the average nodal IFE is affected by pipe velocities, which increase with the increase of nodal inflow. The values of average nodal IWQE also have a negative relationship with the demand multiplier, which is the same with chlorine concentration. The reason is that the average nodal IWQE is affected by the decay process, the WDS layout, and pipe flows. With the increase of demand multiplier, the decaying process leads to the increase of average nodal IWQE, and the WDS layout and pipe flow leads to the increase or decrease of average nodal IWQE. The effect of the WDS layout and pipe flow is larger than the decay process, which leads to the values of average nodal IWQE having negative relationship with the demand multiplier.

In Figure 5(c), the values of FE remain to be the same value of 0.005, and the values of IFE range from 0.008 to 0.03. The values of WQE range from 0.006 to 0.009, while the IWQE values range from 0.009 to 0.05. The values of FE, IFE, WQE, and IWQE are less than average for nodal FE, IFE, WQE, and IWQE. In addition, the variation trends of FE, IFE, and IWQE are the same with the variation trends of average nodal FE, IFE, and IWQE, while the variation trends of WQE are almost contrary to the variation trends of average nodal WQE. The reason is that in Equations (5), (6), (15), and (16), the values of FE, IFE, WQE, and IWQE are not only affected by average nodal FE, IFE, WQE, and IWQE, but also affected by flow and residual chlorine distribution represented by pf_i and p_i. The values of four entropy measures decreased in the order of IWQE $>$ IFE $>$ WQE $>$ FE. In Case 1, the demand multiplier only affects IFE WQE, and IWQE, while having no effects on FE. The values of IFE and IWQE also have a negative relationship with the demand multiplier. The reason is the same with the reason for average nodal IFE and IWQE.

In Figure 5(d), the values of R and R_d for Case 1 are obtained by performing Equations (18) and (19), which range from 0.095 to 0.129 and from 0.138 to 0.739, respectively. The values of R and R_d have a negative relationship with demand multipliers, which are the same with the variation trends of WQE and IWQE. When the demand multiplier is the greatest at the 7th hour (1.53) with higher user demands, the R and R_d reach almost the lowest value of 0.097 and 0.138, respectively. When the water demand multiplier is the lowest during periods from the 2nd hour to the 3rd hour (0.38) with less user demands, the R and R_d reach almost the highest value of 0.126 and 0.739, respectively. The values of R and R_d are also affected by two aspects. One aspect is that when the water demand multiplier increases, the water age becomes shorter which causes the rise of R and R_d in the WDS. The other aspect is that when the water demand multiplier is higher, the pipe flow increases, which may increase or decrease the values of R and R_d for a specific WDS layout. For Case 1, the effect of pipe flow on R and R_d is greater than the decay process, which leads to the negative relationship between R/R_d and the demand multiplier. In addition, the value of R_d is greater than R which proved that R_d can reflect the effect of the water demand multiplier more significantly than R.

4.2. Application to Case 2

For Case 2, the values of nodal inflow range from 144 to 449 L/s, and the values of nodal chlorine concentration range from 0.9 to 1.0 mg/L (shown in Figure 6(a)). Similar with Case 1, the variation of average nodal inflow with time is also in accordance with the variation of demand multiplier, while the variation of average nodal residual chlorine concentration is also contrary to the variation of demand multiplier. For Case 2 with a relatively larger nodal flow and AND the value of 3.2, the effect of flow is also larger than the effect of the decay process, which leads to the nodal chlorine concentration decreased with the increase of the demand multiplier.

In Figure 6(b), the values of average nodal FE and IFE range from 0.226 to 0.283 and from 0.268 to 0.856, respectively, which are larger than Case 1. The value of average nodal WQE and IWQE range from 0.225 to 0.281 and from 0.266 to 0.849, respectively, which are also larger than Case 1. The results indicated that the uniformity of nodal flow and chlorine

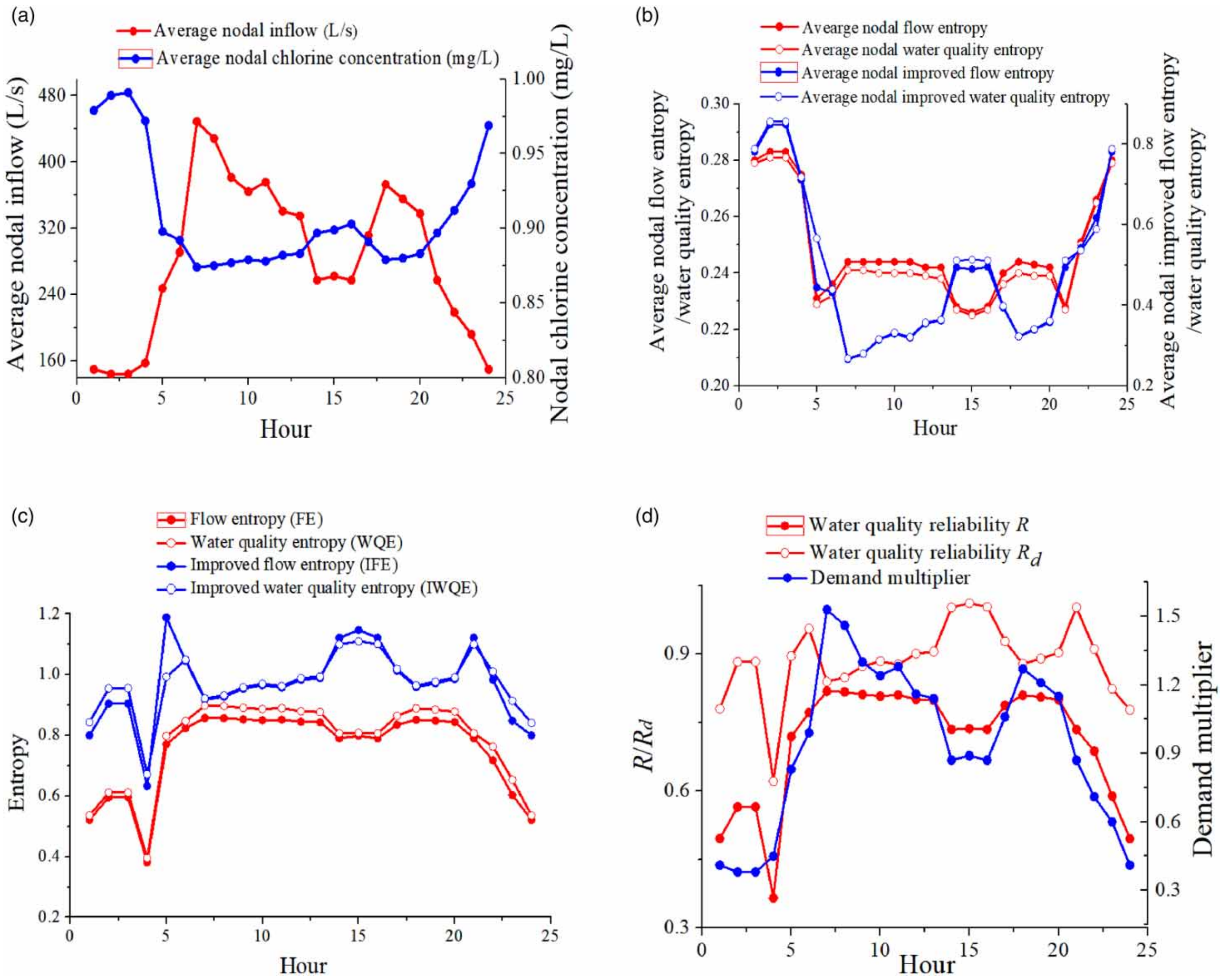

Figure 6 | Variation of nodal inflow, FE, IFE, chlorine concentration, WQE, IWQE, WQR (R), and WQR (R_d) for Case 2.

concentration in Case 2 is better than Case 1. In addition, due to the effect of pipe velocity, the values of average nodal IFE and IWQE are greater than average nodal FE and WQE, respectively. The average nodal FE and WQE have a negative relationship with demand multipliers except for the times from the 6th hour (0.99) to the 20th hour (1.15), and the average nodal IFE and IWQE have a negative relationship with demand multiplier. The results indicated that at times with relatively greater demand multipliers for Case 2 with AND value larger than Case 1 the WDS layout has an effect on the average nodal FE and WQE values. In addition, both WDS layout and pipe velocity have effects on the average nodal IFE and IWQE.

In Figure 6(c), the values of FE and IFE range from 0.382 to 0.858 and from 0.633 to 1.189, respectively. The values of WQE and IWQE range from 0.395 to 0.898 and from 0.672 to 1.110, respectively, which are larger than Case 1. The variation trends of FE, IFE, WQE, and IWQE are in accordance with the variation trends of average nodal FE, IFE, WQE, and IWQE except for the times with the lower multipliers from the 21st hour (0.87) to the 5th hour (0.83). The results indicated that the effects of pf_i and p_i are more significant at times with lower multipliers. Different from Case 1, the values of four entropy measures decreased in the order of IFE > IWQE > WQE > FE. For Case 2 the demand multipliers affect not only IFE, WQE, and IWQE, but also FE. The values of FE and WQE have a positive relationship with the demand multiplier, while the values of IFE and IWQE have a negative relationship with the demand multiplier except for the times from the 21st hour to the 5th hour. The results indicated for Case 2 with the relatively larger flow and AND, the decay process, the WDS layout, and distribution of flow and residual chlorine have effects on the values of FE and WQE, and the pipe velocity as well as effects on the values of IFE and IWQE.

In Figure 6(d) the values of R and R_d for Case 2 range from 0.366 to 0.818 and from 0.621 to 1.011, respectively. The values of R and R_d in Case 2 are much higher than Case 1. The variation trends of R and R_d are also the same with the variation trends of WQE and IWQE. The relationship between R as well as R_d and water demand multiplier is different from Case 1. The value of R has a positive relationship with the demand multiplier while R_d has a negative relationship with the demand multiplier except for the times from the 21st hour to the 5th hour.

4.3. Application to case 3

For Case 3, the values of nodal inflow range from 47 to 125 L/s, and the values of nodal chlorine concentration range from 0.5 to 0.7 mg/L (shown in Figure 7(a)). Different from Case 1 and Case 2, the variation of average nodal inflow and average nodal residual chlorine concentration are all in accordance with the variation of the demand multiplier. The reason is that the residual chlorine concentration is affected by both decay and mixing processes. In Case 3 with a relatively smaller nodal inflow and relatively larger AND value of 4.2, the residual chlorine concentration is affected primarily by the decay process, which leads to the residual chlorine concentration increasing with the increase of the demand multiplier.

In Figure 7(b), the values of average nodal FE and IFE range from 0.469 to 0.538 and from 1.257 to 4.027, respectively. The value of average nodal WQE and IWQE range from 0.425 to 0.503 and from 0.957 to 3.520, respectively. The values of average nodal IFE and average nodal IWQE are larger than average nodal FE and WQE. The average nodal FE, IFE, WQE, and IWQE are larger than Case 1 and Case 2. The results indicated that the uniformity of nodal flow and chlorine concentration

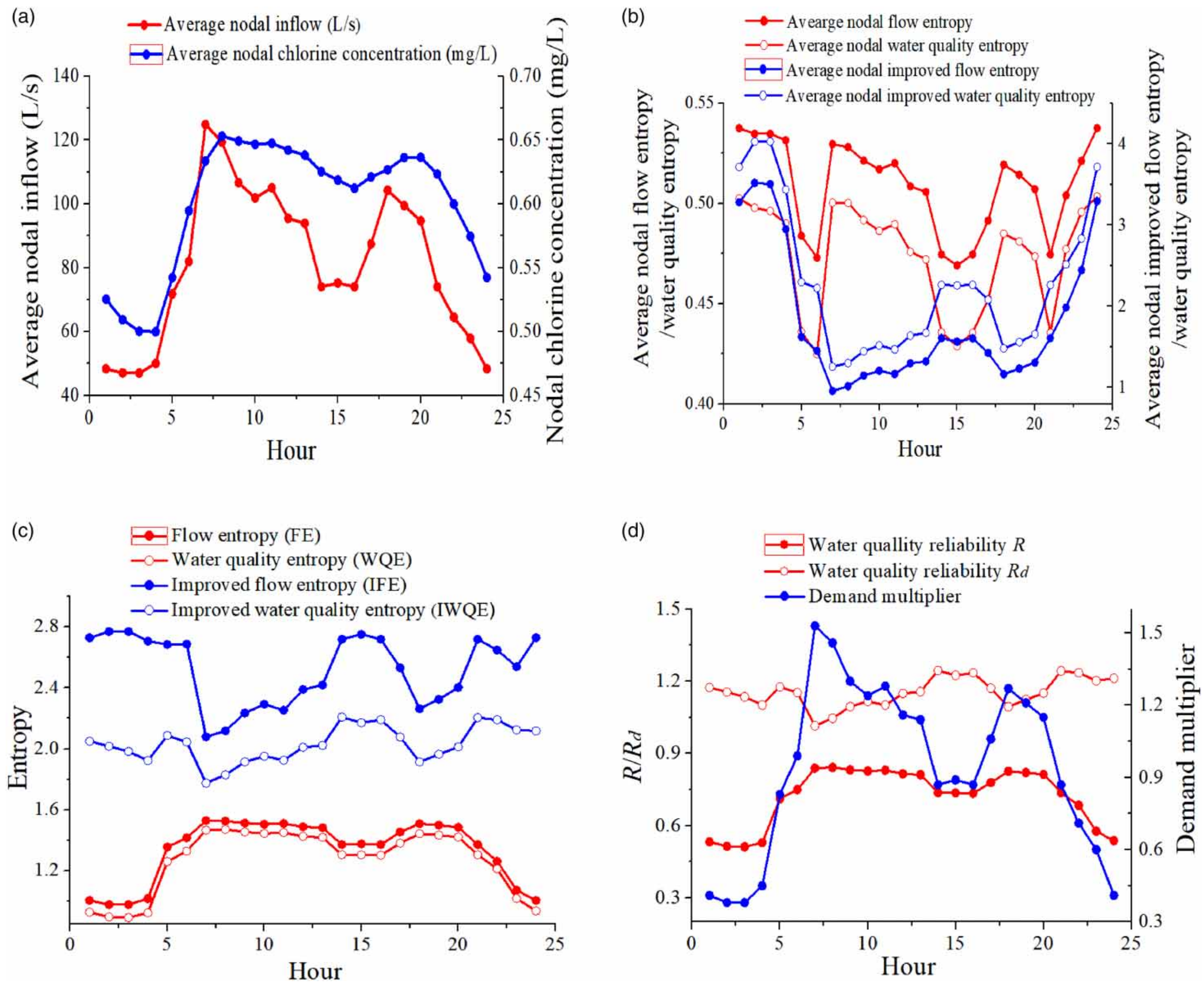

Figure 7 | Variation of nodal inflow, FE, IFE, chlorine concentration, WQE, IWQE, WQR (R), and WQR (R_d) for Case 3.

in Case 3 is better than Case 1 and Case 2 due to the relatively larger AND value of 4.2. Similar with Case 2, the average nodal FE and WQE have a negative relationship with demand multipliers except for the times from the 6th hour (0.99) to the 20th hour (1.15), and the average nodal IFE and IWQE have a negative relationship with demand multiplier. The results indicated that at times with relatively greater demand multipliers for Case 3 with AND value larger than Case 1 and Case 2 the pipe flow has an effect on the average nodal FE and WQE values. In addition, both pipe flow, the WDS layout, and pipe velocity have effects on the average nodal IFE and IWQE.

In Figure 7(c), the values of FE and IFE range from 0.979 to 1.530 and from 2.079 to 2.769, respectively. The values of WQE and IWQE range from 0.894 to 1.472 and from 1.775 to 2.208, respectively. The variation trends of FE and WQE are in accordance with the variation trends of average nodal FE and WQE except for the times with the lower multipliers from the 21st hour (0.87) to the 5th hour (0.83), while the variation trends of IFE and IWQE are in accordance with the variation trends of average nodal IFE and IWQE. The results indicated that average nodal FE and WQE were affected by both the values of pf_i and p_i and the flow in the WDS. For Case 2 with a relatively larger flow, the average nodal FE, WQE, IFE, and IWQE were all affected by the values of pf_i and p_i at times with lower multipliers. For Case 3 with the relatively smaller flow, the average nodal FE and WQE were affected at times with lower multipliers, while the average nodal IFE and IWQE were not affected by pf_i and p_i. The values of four entropy measures decreased in the order of IFE > IWQE > FE > WQE. The values of FE and WQE have a positive relationship with the demand multiplier, while the values of IFE and IWQE have a negative relationship with the demand multiplier.

In Figure 7(d), the values of R and R_d range from 0.512 to 0.842 and from 1.015 to 1.245, respectively. The values of WQR are higher than Case 1 and Case 2. The variation trends of R and R_d are also the same with the variation trends of WQE and IWQE. The relationship between R as well as R_d and water demand multiplier is different from Case 1 and Case 2. The value of R has a positive relationship with the demand multiplier while R_d has a negative relationship with the demand multiplier. As such, R can only reflect the effects of the decay process, pipe flow, and the WDS layout, while R_d can reflect the effects of pipe velocity as well.

4.4. Nodal mean WQE and nodal mean IWQE

The mean WQE and IWQE for connection nodes are analyzed as follows. In Case 1, the mean R for node 16, node 27, and node 29 are not equal to 0 with values of 0.549, 0.641, and 0.413, respectively, which only occupied 0.098% of the total number of connection nodes. In Case 2, during the simulation process, the mean WQE for ten nodes including node 3, node 6, node 7, node 10, node 12, node 17, node 22, node 23, node 24, and node 26 are equal to zero shown in (Figure 8(a)). For node 8, node 11, node 14, node 15, node 16, node 19, node 20, and node 27, the WQE is greater than 0.5, which occupied 29.6% of the connection nodes. The regulations for WQE and IWQE for nodes are different, and the greatest WQE is at node 20, while the greatest IWQE is at node 27. The reason is that node 27 is at the end of the WDS with less velocity and pipe

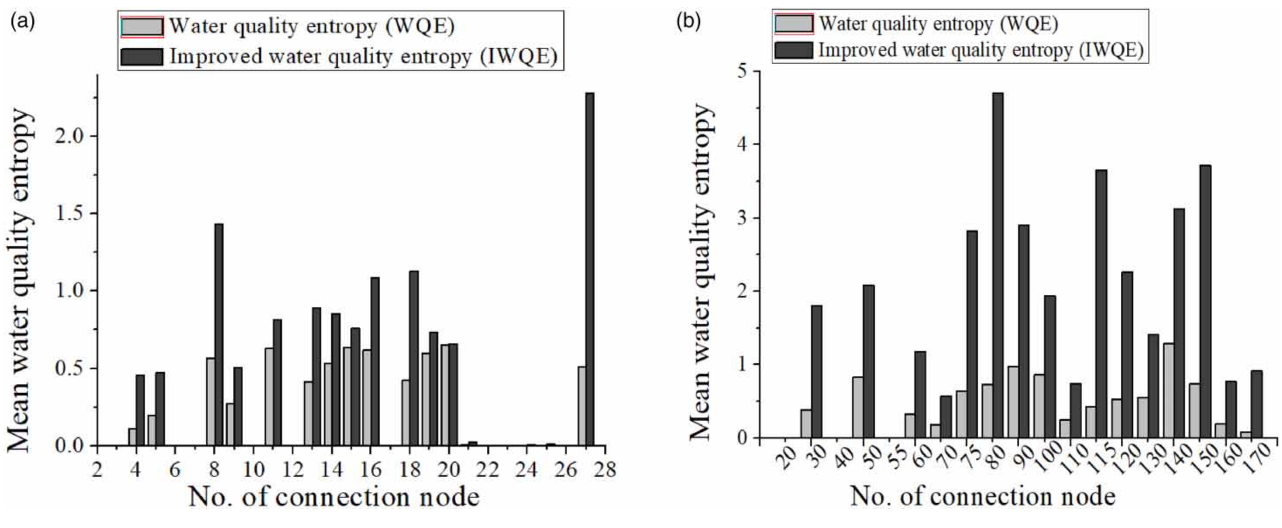

Figure 8 | Mean WQE and IWQE of connection nodes for (a) Case 2 and (b) Case 3.

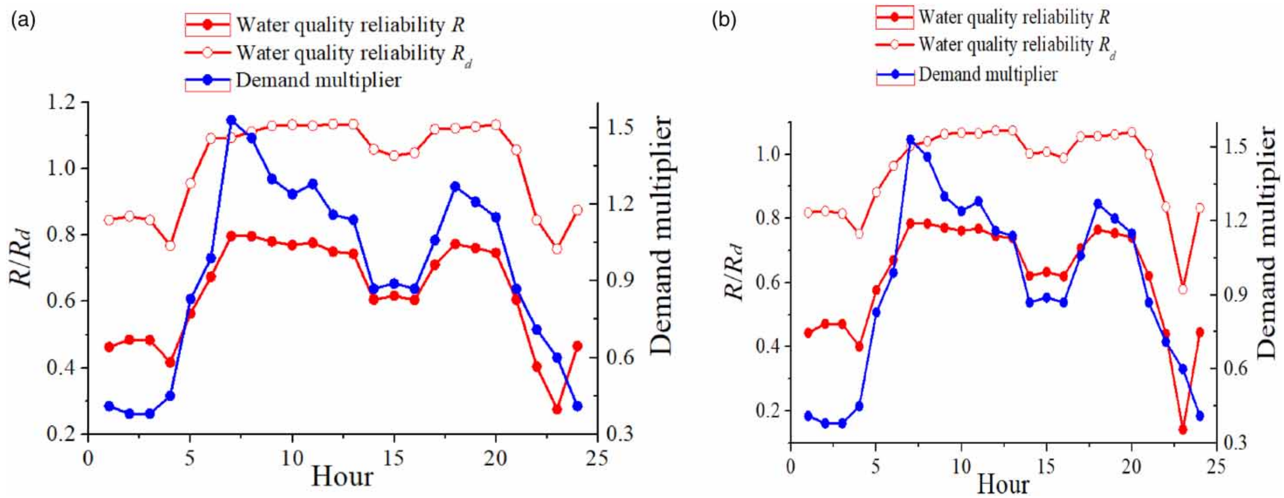

Figure 9 | Sensitivity analysis of WQR for Case 3.

flow, which leads to the greatest IWQE. The result indicated that IWQE can reflect the effect of pipe velocity comprehensively.

The mean WQE of connection nodes of the Anytown network is shown in Figure 8(b). During the simulation process, the mean WQE for three nodes including node 20, node 40, and node 55 is equal to zero. For node 50, node 75, node 80, node 90, node 100, node 120, and node 130, node 140, and node 150, the values of WQE are greater than 0.5, which occupied 47.4% of the total connection nodes. Similar with Case 2, the regulations of WQE and IWQE for nodes are different, and the greatest WQE is at node 140, while the greatest IWQE is at node 80. The reason is that WQE considers the effect of the WDS layout and distribution of flow and residual chlorine, and IWQE also takes into account of pipe velocity.

4.5. Sensitivity analysis

The sensitivity analysis of R and R_d under various optimal designs of pipe diameters for Case 3 (Siew *et al.* 2016) is performed (shown in Figure 9). Compared with Figure 7(d) for Case 3 (Original designs), the values of R and R_d based on WQE and IWQE decreased after optimization with the objectives of minimizing total cost and maximizing system performances. In addition, the variation trend of R based WQE is similar to that of R_d based on IWQE, which is different from R and R_d for the original design of Case 3 (shown in Figure 7(d)). The results indicated that the variation of R_d based on IWQE is associated with the layout of the WDS, while the variation of R based on WQE is almost not affected by the layout of the WDS, which can be explained by the definition of R and R_d. The relationship between optimization layout and WQR assessment should be researched in depth in the future study.

5. CONCLUSIONS

In this paper, based on FE, the concepts of IFE, WQE, and IWQE based on the Shannon entropy were proposed, and R and R_d based on WQE and IWQE were defined and applied to three networks. The results indicated that: (1) Generally, the R_d values have a negative relationship with the demand multiplier for the three cases, while the R values have a negative relationship, partly negative relationship, and positive relationship with the demand multiplier for Case 1, Case 2, and Case 3, respectively; (2) The FE, IFE, WQE, IWQE, R, and R_d increased in the order of Case 1 < Case 2 < Case 3, which is in accordance with the order of AND values for the three cases; (3) The FE has a strong relationship with WQE, and IFE has a strong relationship with IWQE, and the values of IFE and IWQE are much larger than values of FE and WQE; (4) The nodal inflow increase with the demand multiplier, while chlorine decay is affected by decay process, pipes' flow and number connecting to the node. The results indicated that R based on the WQE can only reflect the effects of the decay process, pipe flow, and the WDS layout, while R_d based on the IWQE can reflect the effects of velocity as well. As such, R_d can be a more suitable surrogate indicator for WQR than R.

In this paper, the WQR based on the WQE and IWQE was analyzed and compared based on the traditional demand-driven hydraulic and water quality model. However, it cannot deal with the effects of deficiency in pressure on the water quality through the WDS. In addition, the variation of R_d is different under the various layout of the WDS. The method proposed is difficult to be applied to real WDS with the various layout since in the real WDS, the water quality in the WDS is affected by many factors, such as pipe burst, chlorine decay, and pipe failure. As such, in future research, the WQR under pressure-deficient conditions and under various layouts should be researched and quantified. The proposed method can help analyze the reliability of the WDS from viewpoint of water quality and can help improve the design of WDS.

ACKNOWLEDGEMENT

This work was funded by the Natural Science Foundation of Jiangsu Province (Grant No. BK20191147).

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

CONFLICT OF INTEREST

The authors declare there is no conflict.

REFERENCES

Awumah, K., Goulter, I. & Bhatt, S. K. 1990 Assessment of reliability in water distribution networks using entropy based measures. *Stochastic Hydrology and Hydraulics* **4**, 309–320.

Awumah, K., Goulter, I. & Bhatt, S. K. 1991 Entropy-based reundancy measures in water-distribution networks. *Journal of Hydraulic Engineering* **117**, 595–614.

Beker, B. A. & Kansal, M. L. 2022 Fuzzy logic-based integrated performance evaluation of a water distribution network. *Journal of Water Supply: Research and Technology-Aqua* **71** (3), 490–506.

Creaco, E., Fortunato, A., Franchini, M. & Mazzola, M. R. 2014 Comparison between entropy and resilience as indirect measures of reliability in the framework of water distribution network design. *Procedia Engineering* **70**, 379–388.

Gheisi, A., Forsyth, M. & Naser, G. 2016 Water distribution systems reliability: a review of research literature. *Journal of Water Resources Planning and Management* **142** (11), 04016047.

Islam, M. S., Sadiq, R., Rodriguez, M. J., Francisque, A., Najjaran, H. & Hoorfar, M. 2011 Leakage detection and location in water distribution systems using a fuzzy-based methodology. *Urban Water Journal* **8** (6), 351–365.

Islam, M. S., Sadiq, R., Rodriguez, M. J., Najjaran, H. & Hoorfar, M. 2014 Reliability assessment for water supply systems under uncertainties. *Journal of Water Resources Planning and Management* **140** (4), 468–479.

Kansal, M. L., Kumar, A. & Sharma, P. B. 1995 Reliability analysis of water distribution systems under uncertainty. *Reliability Engineering and System Safety* **50**, 51–59.

Li, X., Sun, Y., Han, X. & Zhao, X. H. 2013 Water quality reliability analysis of water distribution systems based on Monte-Carlo simulation. *Advanced Materials Research* **777**, 401–406.

Liu, H., Savić, D., Kapelan, Z., Zhao, M., Yuan, Y. & Zhao, H. 2014 A diameter-sensitive flow entropy method for reliability consideration in water distribution system design. *Water Resources Research* **50** (7), 5597–5610.

Santonastaso, G., Di Nardo, A., Di Natale, M., Giudicianni, C. & Greco, R. 2018 Scaling-laws of flow entropy with topological metrics of water distribution networks. *Entropy* **20** (2), 95.

Setiadi, Y., Tanyimboh, T. T. & Templeman, A. B. 2005 Modelling errors, entropy and the hydraulic reliability of water distribution systems. *Advances in Engineering Software* **36** (11–12), 780–788.

Shafiqul Islam, M., Sadiq, R., Rodriguez, M. J., Najjaran, H. & Hoorfar, M. 2014 Reliability assessment for water supply systems under uncertainties. *Journal of Water Resources Planning and Management* **140** (4), 468–479.

Shannon, C. E. 1948 A mathematical theory of communication. *Bell System Technical Journal* **24** (3), 379–423.

Siew, C., Tanyimboh, T. T. & Seyoum, A. G. 2016 Penalty-free multi-objective evolutionary approach to optimization of Anytown water distribution network. *Water Resources Management* **30** (11), 3671–3688.

Singh, V. P. & Oh, J. 2015 A Tsallis entropy-based redundancy measure for water distribution networks. *Physica A: Statistical Mechanics and its Applications* **421**, 360–376.

Sivakumar, P., Prasad, R. K. & Chandramouli, S. 2016 Uncertainty analysis of looped water distribution networks using linked EPANET-GA method. *Water Resources Management* **30** (1), 331–358.

Suribabu, C. R. 2010 Differential evolution algorithm for optimal design of water distribution networks. *Journal of Hydroinformatics* **12** (1), 66–82.

Tanyimboh, T. T. & Setiadi, Y. 2008 Sensitivity analysis of entropy-constrained designs of water distribution systems. *Engineering Optimization* **40** (5), 439–457.

Tanyimboh, T. T. & Templeman, A. B. 1993 Maximum entropy flows for single-source networks. *Engineering Optimization* **22** (1), 49–63.

Tanyimboh, T. T. & Templeman, A. B. 2000 A quantified assessment of the relationship between the reliability and entropy of water distribution systems. *Engineering Optimization* **33** (2), 179–199.

Tanyimboh, T. T., Tietavainen, M. T. & Saleh, S. 2011 Reliability assessment of water distribution systems with statistical entropy and other surrogate measures. *Water Science and Technology: Water Supply* **11** (4), 437–443.

Tolson, B. A., Maier, H. R. & Simpson, A. R. 2001 Water distribution network reliability estimation using the first-order reliability method. In *World Water Congress*.

Walski, T. M., Downey Brill, E., Gessler, J., Jeppson, R. M., Lansey, K., Lee, H.-L., Liebman, J. C., Morgan, D. R. & Ormsbee, L. *et al.* 1987 Battle of the network models: Epilogue. *Journal of Water Resources Planning and Management* **113** (2), 191–203.

Wang, Y. & Zhu, G. 2021 Evaluation ofwater quality reliability based on entropy in water distribution system. *Physica A* **584**, 126373.

Yassin-Kassab, A., Templeman, A. B. & Tanyimboh, T. T. 1999 Calculating maximum entropy flows in multi-source, multi-demand networks. *Engineering Optimization* **31** (6), 695–729.

Zhao, Y., Luo, B., Zhuang, B. & Zhao, X. 2010 Hydraulic and water quality reliability analysis of water distribution system. In: *2010 2nd Conference on Environmental Science and Information Application Technology*. pp. 580–583.

First received 9 May 2022; accepted in revised form 5 July 2022. Available online 19 July 2022