Frampton’s Demon

A Mathematical Interpretation of Hollis Frampton’s Zorns Lemma

CLINT ENNS

Zorns Lemma, a 1970 experimental film by Hollis Frampton (1936–1984), is a complex and fascinating film that has a labyrinthine structure alluding to a mathematical reading of the film as a visual metaphor for mathematician Max Zorn’s (1906–1993) famous axiom Zorn’s Lemma. The film, in addition to Ernie Gehr’s Serene Velocity (1970), Michael Snow’s Wavelength (1967), Paul Sharits’s T,O,U,C,H,I,N,G (1968) and Tony Conrad’s The Flicker (1965), belongs to the tradition of American structural film, a “cinema of structure wherein the shape of the whole film is predetermined and simplified, and [wherein] it is that shape that is the primal impression of the film” [3]. In the extensive literature written about Zorns Lemma, there have been many different interpretations offered; however, none of the readings has provided a satisfactory mathematical interpretation of the film. After first providing a brief overview of Zermelo’s

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Grosseteste’s *On Light, or the Ingression of Forms*, an 11th-century text discussing the nature of the universe, at a rate of one word per second.

**THE AXIOM OF CHOICE AND ITS EQUIVALENTS**

Mathematically, Zorn’s Lemma explicitly states:

Let \((P, <)\) be a nonempty, partially ordered set and let every chain in \(P\) have an upper bound. Then \(P\) has a maximal element [9].

It seems when Frampton discussed Zorn’s Lemma, he was actually referring to one of its many equivalent statements. Frampton explicitly used the following statement:

Every partially ordered set contains a maximal fully ordered subset [10].

When the word *fully* is replaced by the correct, mathematically loaded term *totally*, it can be shown that Frampton’s statement is equivalent to the Hausdorff maximality principle [11], a result that was first proved by Felix Hausdorff in 1914 [12]. Furthermore, it can also be shown that the Hausdorff maximality principle is equivalent to Zorn’s Lemma [13]. Finally, Zermelo’s Axiom of Choice is equivalent to Zorn’s Lemma and is therefore equivalent to Frampton’s version of Zorn’s Lemma [14]. In other words, Frampton’s version of Zorn’s Lemma is mathematically equivalent to Zermelo’s Axiom of Choice.

The Axiom of Choice was first formulated by Zermelo in 1904 and formally states the following:

For every family \(F\) of non-empty sets, there is a function \(f\) defined on \(F\) such that \(f(S) \in S\) for each set \(S\) in \(F\) [15].

In other words, for each set in \(F\) there is a function that *chooses* an element of that set. For this reason, the function \(f\) is referred to as a *choice function* on \(F\). It is worth noting that invoking the Axiom of Choice was at one time considered controversial among mathematicians, since “the Axiom of Choice can be used to prove theorems which are to a certain extent ‘unpleasant’, and even theorems which are not exactly in line with our ‘common sense’ intuition” [16].

Here is one way to conceptualize the Axiom of Choice. Suppose we are given bins each containing at least one object. The Axiom of Choice states that it is always possible to select exactly one object from each bin. Of course, only in special cases do we actually need to evoke such a powerful tool. For instance, in cases where there are only a finite number of bins, we do not need to use the Axiom of Choice, since there is an obvious procedure for selecting objects from the bins. That is, we can enumerate the bins and then select an object from the first bin, the second bin and so on. As there are only a finite number of bins, this procedure will eventually end. The selection process becomes a bit trickier, however, in the case of an infinite number of bins. Such a scenario was the impetus for a famous aphorism attributed to Bertrand Russell that states “to select one sock from each of infinitely many pairs of socks requires the Axiom of Choice; but for shoes the Axiom is not needed” [17]. In the case of infinitely many shoes, you can always select the left (or, equivalently, the right) shoe. In the case of infinitely many socks, however, it is not possible to assume that such a choice function exists since there is no intrinsic way of distinguishing between socks. In other words, when the choice is truly arbitrary, one must invoke the Axiom of Choice.

**A MATHEMATICAL INTERPRETATION OF ZORNS LEMMA**

Before providing a mathematical interpretation of Zorn’s Lemma, it is worth observing that Frampton did not plan for his film to be interpreted this way; however, he did encourage such a reading. Frampton states:

I did not set out to provide a cinematic demonstration of a mathematical proposition. On the other, I don’t mind that the work should respire that possibility [18].

One of the standard interpretations of this film involves viewing it in terms of “cuts.” There is evidence indicating that Frampton was interested in exploring this concept in *Zorns Lemma*, as indicated in a 1964 letter to his friend Reno Odlin:

The excercmment [sic] of the fully ordered set constitutes a cut. Where there are several possible cuts, the set of all cuts constitutes the maximal ordered set [19].

Melissa Ragona has suggested that the sculptor Carl Andre might have first introduced Frampton to thinking about mathematical “cuts” in a filmic context based on a discussion that the two had in the early 1960s regarding the mathematical concept known as the Dedekind Cut. Ragona goes on to suggest that this idea sparked Frampton’s ‘interest in analyzing the ‘cut’ in film as the ‘cut’ in the order of film’ [20].

In my interpretation of *Zorns Lemma*, I am less concerned with the act of cutting and more concerned with the act of substitution. In order to interpret Zorn’s Lemma as a visual demonstration of the Axiom of Choice, let us first think of the letters as enumerating the bins. For the first substitution, that is, the first application of the Axiom of Choice, the family of sets consists of the set of all the words that begin with \(A\), the set of all the words that begin with \(B\), the set of all the words that begin with \(C\), and so on. By applying the Axiom of Choice, a word is selected from each of the above *finite* sets. Through each iteration, different words are selected, demonstrating the arbitrary nature of the choice. It is important to note that evoking the Axiom of Choice is excessive in this case since there are only a finite number of letters and a finite number of words to choose from. In fact, this was observed by Frampton:

Most words (not all, but most) were from the environment; they’re store signs and posters and things like that, and one finds out very quickly that very many words begin with \(c\) and \(s\), and so forth; very few begin with \(x\) or \(q\), or what have you. One quickly begins to run out of \(q\)’s and \(x\)’s and \(z\)’s and \(z\)’s [21].

For the second substitution, the family of sets consists of the set of all moving images of pages being turned in a book (replacement for \(A\)), the set of all moving images of an egg frying (replacement for \(B\)), the set of all moving images of a red ibis flapping its wings (replacement for \(C\)), et cetera [22]. By applying the Axiom of Choice, a moving image is selected...
from each of the above infinite sets. Once again, invoking the Axiom of Choice is superfluous since there are only a finite number of substitutions to be made, despite the fact that each of the moving image sets is infinite.

In theory, we can imagine a case in which the Axiom of Choice is needed. For instance, consider the infinite family of sets that consists of all cinematic images. Each of these moving image sets would contain infinitely many elements, since it is possible to shoot an image from any given angle at any given time. By applying the Axiom of Choice to the family of sets that consists of all cinematic images, one obtains a film. In fact, this is one of the main concepts that Frampton explored in his unfinished film cycle Magellan, an idea that I explore in the next section.

CONSEQUENCES OF THIS INTERPRETATION

If we view Zorn's Lemma as a demonstration of the Axiom of Choice, we find that the artist decides on the choice function. In other words, Frampton chose the shot, the content within shots, the sequence in which each shot is shown, the duration of the shot, etc. In fact, in Zorns Lemma, Frampton even chose “to incorporate deliberately a series of kinds of errors” [23]. Applying the Axiom of Choice to the family of sets that is all cinematic images provides us with a conceptual procedure for eliminating what Frampton refers to as “nominal subjectively, ‘thumbprint’ procedures” [24]—one of the goals of Frampton’s most ambitious projects, Magellan—an unfinished 36-hour film cycle.

This interpretation can also be read as the personification of an abstract concept, an idea that interested Frampton, as demonstrated by his homage to Scottish physicist James Clerk Maxwell in his 1968 film Maxwell’s Demon. Maxwell’s Demon is a thought experiment in which a demon sits between two chambers, allowing only hot molecules to pass, causing one of the chambers to heat up and the other to cool down. In essence, the demon acts as a molecular bouncer that allows the Second Law of Thermodynamics to be violated. Although it has been shown that Maxwell’s Demon is a physical impossibility, it still provides an interesting example of the personification of an abstract concept. In fact, the Axiom of Choice has even been referred to as the “mathematicians’ Maxwell’s Demon” [25]. In other words, through Zorns Lemma, Frampton is assuming the role of the mathematical Maxwell’s Demon, since he is deciding on the choice function.

Zorns Lemma was Frampton’s attempt to bridge the gap between mathematics and cinema. Mathematics is often thought of as the underlying structure of the universe, and Frampton’s unfinished film cycle Magellan can be seen as an attempt to use cinema to create an epistemological model of the universe, a cinematic Library of Babel, “more poignant” [26] than its text-based predecessor. Although Frampton realized it was impossible to create an “infinite film,” he believed he could “generate a grammatically complete synopsis of it” [27]. At first glance this may all seem ridiculous; however, physicists are using mathematics to develop an epistemological model of the universe. Zorns Lemma provides a theoretical framework for constructing an “infinite film,” namely, by invoking the Axiom of Choice. Although Frampton didn’t believe he was creating a cinematic demonstration of a mathematical principle, this interpretation demonstrates that many of the concepts being worked through in Zorns Lemma formed a conceptual basis for Magellan. The ultimate strength of Zorns Lemma is that it successfully creates a link between mathematical reasoning and filmic reasoning—one of the many reasons this film remains one of Frampton’s most discussed.

References and Notes

2 Note the spelling difference between Zorn’s axiom and the title of Frampton’s film.
4 A 16mm print of Zorns Lemma is still available for rent from the Film-Makers Cooperative in New York.
6 Gidal [5].
7 Gidal [5].
11 The Hausdorff maximality principle: In any partially ordered set, every totally ordered subset is contained in a maximal totally ordered subset.
13 For an explicit proof that the Hausdorff maximality principle is equivalent to Zorn’s Lemma see Kelley [12] 33.
14 For an explicit proof that Zorn’s Lemma is equivalent to the Axiom of Choice see Jech [9] 10.
CLINT ENNS is a video artist and filmmaker from Toronto, Ontario, whose work primarily deals with moving images created with broken and/or outdated technologies. His work has been shown both nationally and internationally at festivals, alternative spaces and microcinemas. He has an M.Sc. in mathematics from the University of Manitoba and an M.A. in cinema and media from York University, where he is currently pursuing a Ph.D. in cinema and media studies. His writings and interviews have appeared in Millennium Film Journal, INCITE Journal of Experimental Media and Spectacular Optical.

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