ABSTRACT

Can music supplant mathematics for planning environmental systems? The author’s research and experience as presented here would indicate as much. First, an optimization problem is formulated where the number of ecological reserves is to be minimized while conserving all species in a region. Next, the author describes a music-inspired optimization algorithm called “harmony search” by focusing on the analogy between music performance and problem optimization. Finally, computational results are shown.

Modern societies are highly economy-oriented. Governments are very careful to reconcile budgeted income and expenditures. This is especially true where environmental systems must be planned so as to maintain biodiversity within budgetary limitations.

Figure 1 shows a region (Oregon state, U.S.A.) recently studied for environmental system planning purposes [1]. Within a limited budget, the state government seeks to maximize the biodiversity in the region. One way to maintain this biodiversity is to construct ecological reserve areas.

As seen in Fig. 1, the entire region is divided into hexagonal land parcels, and each parcel can become an ecological reserve. We can ask how many parcels should be selected to conserve all species in this region. In order to cost-effectively preserve all species in the region, we have to first determine from surveys what species live in each parcel. The relevant ecological data [1] for Oregon are shown in Fig. 2, where the x-axis represents each land parcel and the y-axis represents species living in the parcel. There are a total of 441 possible candidate land parcels and 426 species in the region. Also in Fig. 2 we observe horizontal lines, which denote that certain species live in a site that groups neighboring parcels.

To find the best solution for preserving all species out of numerous candidate solutions in this example of ecological
diversity planning, the problem is first formulated in the form of optimization with an objective function and certain constraints. One optimization structure can be verbally stated: The objective is to minimize the total number of land parcels to be developed for ecological reserves; the constraint is to preserve all the species in the region. This can be mathematically expressed as follows:

\[
\begin{align*}
\text{Minimize} & \quad \sum_{j=1}^{J} x_j, \\
\text{subject to} & \quad \sum_{j \in M_i} x_j \geq 1, \quad \text{all } i \in I.
\end{align*}
\]

where \(j\) and \(J\) are the index and set of land parcels, respectively; \(i\) and \(I\) are the index and set of species, respectively; \(M_i\) is the set of parcels \(j\) that include species \(i\); and \(x_j\) is a binary variable for parcel selection (it equals 1 if parcel \(j\) is selected, and 0 otherwise). The ecological diversity planning model described in Equations (1) and (2) is called a species set covering problem (SSCP) [2]. However, a more complicated model, called a maximal covering species problem (MCSP), is also used to plan ecological diversity [2].

**MUSIC-INSPIRED METHODOLOGY**

In order to solve the above-mentioned ecological optimization model or to find the minimal number of ecological reserves that cover all the species in the region, we can intuitively use a combinatorial method. For this problem, there exists a total of 441 candidate ecological reserves. Thus, the size of this combinatorial problem can be \(2^{441} \approx 5.7 \times 10^{132}\), because any land parcel \(j\) can become the reserve (\(x_j = 1\)) or not (\(x_j = 0\)). The problem size is truly huge, and traditional mathematical methods could not easily tackle this problem due to so-called combinatorial explosion, or the curse of dimensionality.

Instead of using traditional enumeration-based mathematical methods, researchers have recently turned their interests toward new-paradigm techniques inspired by various natural or artificial phenomena such as genetics, metallic annealing processes and the behaviors of ants, bees, fireflies and cuckoos [3,4]. They have then applied these algorithms to various real-world problems [5].

Music is also useful material for devising new algorithms to solve the ecological optimization problem, because a certain analogy can be drawn between music improvisation and problem optimization [6]: Just as there are certain numbers of musicians (for example, saxophonist, bassist and guitarist, as shown in Fig. 3) in music improvisation, there are certain numbers of decision variables (for example, \(x_1, x_2, \text{ and } x_3\) as in Fig. 3) in problem optimization; just as each musician can play a certain range of musical notes based on his or her musical instrument (for example, Do, Re and Mi for saxophonist, as shown in Fig. 3), each variable can have a certain range of values (for example, 100mm, 200mm and 300mm for \(x_j\)); just as a saxophonist may choose Do out of {Do, Re, Mi}, a bassist Sol out of {Sol, La, Si} and a guitarist Mi out of {Mi, Fa, Sol}, \(x_i\) can have 100mm, 500mm and 300mm, which together make a vector (100mm, 500mm, 300mm) at a certain iteration; just as the harmony (do, sol, mi) makes an audience feel good or bad based on audience aesthetics, the vector (100mm, 500mm, 300mm) makes a design appear good or bad based on an objective function such as Equation (1); and just as music quality can be improved practice by practice, solution quality can be improved iteration by iteration. I developed this music-math analogy, as described by another researcher in an online article titled “Optimization and All That Jazz”:

When jazz musicians play together they select the musical notes in their instruments to give the best overall harmony with the rest of the group. This often happens in stages. One musician will play a tune; others will remember it and work out what notes they want to play along with it. As time progresses, and remembering the music they have played before, they move note by note to blend together, making more and more beautiful music. In effect they are doing an optimization. Each musician has to choose the right notes for their instrument in the right order to make the best group harmony possible [7].

![Fig. 3. Analogy between music improvisation and problem optimization. (© Zong Woo Geem)](image-url)
Actually, when a musician selects one note in this music-inspired algorithm, called “harmony search,” it can use one operator out of three: random selection, memory consideration or pitch adjustment. In random selection, the musician can randomly play any one note out of the total playable range of its instrument, as shown in Fig. 4(a). In memory consideration, the musician can randomly play any one note out of its memory, where historically good notes are stored, as shown in Fig. 4(b). In pitch adjustment, the musician can play a neighboring note next to any note stored in its memory, as shown in Fig. 4(c).

These three operations in music improvisation can also be expressed mathematically as follows [8]:

$$\frac{\partial f}{\partial x_i} = \frac{1}{K} \cdot P_{\text{Random}} \cdot \frac{n(x_i, (k))}{HMS} \cdot P_{\text{Memory}} \cdot \frac{n(x_i, (k \pm m))}{HMS} \cdot P_{\text{Pitch}} \quad (3)$$

Equation (3) represents that any note can be played by one operation out of three (random selection, memory consideration and pitch adjustment), and the total probability of selecting a note is the summation of the three individual probabilities. The above equation can be explained intuitively: When a musician starts improvisation with five candidate notes (do, re, mi, fa, sol), the probability of selecting any note out of five is equal (20% each). However, as improvisation progresses, a certain note (for example, fa) can be preferred by the musician, and its probability of being selected increases to a much higher value (for example, 45%), calculated by the equation, which aggregates the probabilities to select the note fa in random selection, memory consideration and pitch adjustment. Here, the random selection is to select the note fa randomly out of total range; the memory consideration is to select the note fa stored in musician’s preferred memory, and the pitch adjustment is to select the note fa after a note mi or sol was first selected from the musician’s preferred memory.

This expression of a musician’s behavior is called stochastic partial derivative, and this novel derivative has even overcome the weakness of traditional calculus-based derivative! For an example in water supply network design, while a calculus-based approach such as the Lagrange multiplier method gets stuck in a local optimum solution [9], phenomenon-mimicking algorithms, including harmony search, search for a global optimum solution [10]. In addition, the harmony search algorithm overcomes the drawbacks of calculus-based methods when a function is difficult to differentiate, step-wise or discontinuous.

When compared with traditional calculus-based methods such as the Lagrange multiplier and BFGS methods, this music-inspired method does not require complex gradient derivatives and convergence-guaranteed initial vectors. It can efficiently consider discrete variables and avoid local optima.

Various applications of the harmony search algorithm, which is based on this derivative, include music composition, Sudoku puzzling, tour planning, Internet routing, structural design, water network design, dam scheduling, vehicle routing, etc. Extensive examples can be found in several books I have recently edited [11–13].

In application to music composition, harmony search found the best polyphonic line to be added to an original Gregorian chant line in the form of Organum (an early form of polyphonic music). The objective of this optimization was to find the polyphonic line that would maximally satisfy the aesthetics of an audience in the Medieval period, when people preferred perfect interval (perfect fourth or perfect fifth) rather than major or minor third. This preference was quantified, and the quantified aesthetic function was optimized using the harmony search algorithm.

There have actually been numerous theoretical endeavors relating music and math: Iannis Xenakis (composer, architect and music theorist) applied stochastic processes and game theory to music [14]; Dmitri Tymoczko (composer and music theorist) proposed the interesting idea that chords can be represented as dots in an orbifold [15]; Callender et al. mathematically classified music sequences in a new paradigm [16]; and Richard Cohn collected ideas from Neo-Riemannian theory, which relates harmonies directly to each other without referring to a tonic [17].

**Computational Results**

I applied the harmony search algorithm to the Oregon ecological diversity planning problem. There are a total of 441 musicians (= variables = land parcels), and each musician (variable $x_i$) can have one note (value 0 or 1) out of two. Here, the musician can randomly choose one note (0 or 1) from a total range (0, 1) in random selection operation, or randomly choose one note that is stored in memory (called Harmony Memory) in memory consideration operation, or slightly adjust an original note into the neighboring note in pitch adjustment operation. Since there are only two candidate values (0 and 1) in this ecological planning problem, the third operation can be eliminated. The results showed that with the development of as few as 24 land parcels, ecological reserves could in theory act to conserve all 426 species in the region [18]. Figure 5 is one of the theoretically optimal solutions.

My statement in the previous section that music quality...
can be improved practice by practice, just as solution quality can be improved iteration by iteration, can be elaborated as follows: As computational iteration proceeds, the number of land parcels needed to conserve all 426 species in this region decreases from an original 26 parcels to 25 parcels and finally to 24 parcels. Or, when the number of finally selected parcels is fixed at 24 parcels, the number of species these parcels conserve increases from an original 421 species (when parcels 9, 24, 44, 85, 93, 120, 135, 181, 188, 222, 268, 274, 289, 297, 321, 324, 345, 347, 357, 375, 401, 424, 428 and 438 are selected) to 424 species (when parcels 9, 24, 27, 52, 55, 120, 121, 136, 147, 169, 252, 278, 289, 314, 319, 321, 324, 345, 375, 386, 395, 402, 428 and 438 are selected) and finally to 426 species (when parcels 9, 11, 24, 26, 55, 75, 102, 114, 120, 136, 147, 169, 222, 224, 289, 314, 319, 324, 345, 357, 367, 375, 428 and 440 are selected).

CONCLUSIONS

This paper introduces a music-inspired optimization algorithm, called harmony search, which was applied to an environmental planning problem. This jazz-improvisation-mimicking algorithm overcame the drawbacks (complex derivative requirement, convergence-guarantee initial vector requirement, local optima plunging, etc.) of traditional mathematical techniques. The results of my previous research demonstrated that this algorithm can successfully obtain an optimal planning strategy to conserve all species in the region within a limited budget.

References

Unedited references as provided by the author.

7 Computer Science for Fun, "Optimization and All that Jazz": <www.cs4fn.org/music/optimisation.php>.

Zong Woo Geem is an assistant professor at Gachon University (formerly adjunct faculty member at Johns Hopkins University) and the inventor of the music-inspired harmony search algorithm. He is also a concert soloist who has appeared at various venues, including the National Cathedral and National Shrine in Washington, D.C.
Leonardo seeks articles in the following areas of special interest

Art and Atoms
*Guest Editor: Tami I. Spector*

The modern world of chemistry is vast and its connection to art strong. From nanocars and extraterrestrial materials to DNA origami and biofuels, chemistry—like art—expresses its transformative, material essence. Chemistry's unique connection to art is the focus of this special section.

Full call for papers: <http://leonardo.info/isast/journal/calls/artandatoms.html>
Author guidelines: <http://leonardo.info/isast/journal/editorial/edguides.html>
Submissions: <leonardomanuscripts@gmail.com>

Re-Imagining the Moon
*Guest Editor: Sundar Sarukkai*

The moon has profoundly influenced the human imagination over the centuries, in the domains of myths, religion, art and science. This special section aims to publish articles from a variety of disciplines exploring various social and cultural aspects related to the moon as well as those that engage with the relation between the moon and the artistic and scientific imaginations.

Author guidelines: <http://leonardo.info/isast/journal/editorial/edguides.html>
Submissions: <leonardomanuscripts@gmail.com>

Environment 2.0: Through Cracks in the Pavement
*Guest Editor: Drew Hemment*

In urban environments we are separated from the consequences of our actions as surely as the tarmac of the road cuts us off from the earth beneath. But between the cracks in the pavement, another world flourishes—local activism, recycling, environmental collectives, permaculture, urban gardening. Leonardo solicits texts that document the works of artists, researchers, and scholars involved in the exploration of sustainability in urban environments.

Full call for papers: <http://leonardo.info/isast/journal/calls/environment-2.0_call.html>
Author guidelines: <http://leonardo.info/isast/journal/editorial/edguides.html>
Submissions: <leonardomanuscripts@gmail.com>

ArtScience: The Essential Connection
*Guest Editor: Robert Root-Bernstein*

What is the value of artistic practices, techniques, inventions, aesthetics and knowledge for the working scientist? What is the value of scientific practices, techniques, inventions, aesthetics and knowledge for the artist? When does art become science and science, art? Or are these categories useless at their boundaries and intersections? Artists, scientists, artist-scientists and researchers of all sorts are invited to explore such questions in the pages of Leonardo.

Full call for papers: <http://leonardo.info/isast/journal/calls/artsciencecall.html>
Author guidelines: <http://leonardo.info/isast/journal/editorial/edguides.html>
Submissions: <leonardomanuscripts@gmail.com>