The Stochastic Synthesis of Iannis Xenakis

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STOCHASTIC MUSIC

In 1954, Iannis Xenakis introduced the use of probability distributions in musical composition in order to control the orchestral sound masses of Pithoprakta. In 1956, he named this music Stochastic Music and enthusiastically set about investigating its possibilities.

There were many reasons why Xenakis was interested in the use of probability functions in music. For him, they presented:
- a solution to what he called “the impasse of serial music”:
  The composers [Stockhausen, Boulez and others] thought they were orthodox serialists but that was only true on paper. In reality they had mass events which they should have listened to in an unbiased manner. On the level of conscious thinking they should have introduced such notions as average density, average duration, colours and so on [1].
- a technique for creating and articulating sound masses inspired by the musical aspects of natural events: “collisions of hail or rain with hard surfaces” or “political crowds of dozens or hundreds of thousands of people” [2].
- an opportunity to incorporate concepts from modern science into the field of music composition. For example, the use of probability distributions in kinetic theory (Boltzmann and Maxwell) to determine the energy of a given quantity of gas: “I followed Maxwell’s approach step by step: what he did with the molecules I did with the sounds” [3].
- the problem: “What is the minimum of logical constraints necessary for the construction of a musical process?” [4]

THE STOCHASTIC MUSIC PROGRAM

In 1962, Xenakis started using computers to accelerate the numerous calculations required by his stochastic approach to composition; he wrote the Stochastic Music Program (ST Program) in the FORTRAN programming language. This program employs interlinked probability functions to determine the global structure (e.g. length of sections) and the note parameters (e.g. pitch, duration) of a composition.

At the very same time, Xenakis was speculating about the possibility of using stochastic techniques to synthesize sounds:

Although this program gives a satisfactory solution to the minimal structure, it is, however, necessary to jump to the stage of pure composition by coupling a digital-to-analogue converter to the computer. The numerical calculations would then be changed into sound, whose internal organization had been conceived beforehand [5].

COMPOSING SOUND WITH MUSICAL PROCEDURES

Since the appearance of computers with digital to analog converters in the late 1950s, some composers have been interested in synthesizing sound through the manipulation of individual digital samples. In this process, amplitude and duration values are obtained through musical procedures and not based on any acoustical model. This approach, often referred to as non-standard synthesis [6], reflects a willingness to explore the sound synthesis possibilities unique to computers.

Three non-standard synthesis strategies that appeared during the 1970s are: “New Proposals in Microsound Structure” by Iannis Xenakis, SAWDUST by Herbert Brün [7] and SSP (Sound Synthesis Program) by Gottfried Michael Koenig [8]. These approaches have the following goals in common: to unify the macrostructure and the microstructure of compositions, to use synthesis techniques idiomatic to computers and to open an experimental field in sound synthesis.

NEW PROPOSALS IN MICROSOND STRUCTURE

It was during his tenure at Indiana University in Bloomington, from 1967 to 1972, that Xenakis first used a computer for stochastic sound synthesis. In 1972, he continued these experiments at the Centre d’Etudes de Mathématique et Automatique Musicales (CEMAMu) in Paris. However, in 1977, with the advent of the Unié Polyagogique Informatique du CEMAMu (UPIC) system [9], Xenakis postponed his stochastic synthesis research until the late 1980s [10].

Xenakis’s first concrete ideas about stochastic synthesis were published in Formalized Music [11], in the manifesto-like chapter “New Proposals in Microsound Structure.”

In it, he starts by rejecting:
- Fourier analysis as the basis for sound synthesis:
Now, the more the music moves toward complex sonorities close to “noise,” the more numerous and complicated the transients become, and the more their synthesis from trigonometric functions becomes a mountain of difficulties, even more unacceptable to a computer than the permanent states. It is as though we wanted to express a sinuous mountain silhouette by using portions of circles [12].

- “pure” electronic sounds: “Any electronic music based on such sounds only, is marked by their simplistic sonority” [13].
- serialism in electronic music: “The serial system, which has been used so much by electronic music composers, could not by any means improve the result, since it itself is too elementary” [14].

Instead, he advocates:
- mixing “pure” electronic sounds with “concrete” sounds: Only then “could electronic music become really powerful” [15].
- the use of stochastic processes to efficiently produce sonorities with “numerous and complicated” transients: “It seems that the transient part of the sound is far more important than the permanent part in timbre recognition and in music in general” [16].
- an approach in which sound synthesis is performed only in the time domain; starting directly from the sound pressure curves, defining them by means of stochastic variations: “We can start from a disorder concept and then introduce means that would increase or reduce it” [17].

In the last part of the chapter, Xenakis proposes seven methods for stochastic microsound synthesis [18]:

1. Amplitude or duration values obtained directly from a probability distribution (e.g. uniform, Gaussian, exponential, Poisson).
2. Combination of a random variable with itself by means of a function (e.g. addition, multiplication).
3. The random variables are functions of other variables (e.g. elastic forces, centrifugal forces) or of other random variables (e.g. random walks).
4. The random variables move between two elastic barriers that reflect excessive values back into the barrier range.
5. The parameters of a probability function as variables of other probability functions.
6. Combinations of probability functions (e.g. linear, polynomial). Composite functions (e.g. modulation).
7. Categorization of probability functions through at least three kinds of criteria (e.g. stability, curve characteristics).

**Polytope de Cluny**

Xenakis first used the results of his experiments in stochastic synthesis in *Polytope de Cluny* (1972), a 31-min multimedia work set in the Roman baths of Cluny in Paris, consisting of music (recorded in seven channels and distributed over 12 loudspeakers), 600 flashbulbs and three lasers (redirected with 400 programmable mirrors). He was also proud to be the first in France to use digitally synthesized sounds [19].

Decorrelated stochastic synthesis opens the work (Fig. 1) (just after a brief introduction in the third channel, intended for when the audience entered the performance space), and it is present for about 6 minutes, sometimes in the foreground and sometimes receding to the background, as it is inhabited by the other sounds: ceramic windchimes, thumb pianos, low stringed instruments bowed with extreme overpressure and other sounds sources that are hard to identify. All the sonorities have a very rich spectrum and are full of buzzes, rattles and distortion.


From the preface to the score of *N’Shima*:

The melodic patterns of N’Shima are drawn from a computer-plotted graph as a result of Brownian movement (random walk) theory that I introduced into sound synthesis with the computer in the pressure versus time domain [20].

**Dynamic Stochastic Synthesis (1977): La Légende d’Eer**

In 1977, Xenakis composed *La Légende d’Eer*, which was the musical component of *Le Diatope*, a multimedia work also including 1,680 flashbulbs, 4 colored lasers (reflected by 400 programmable mirrors) and a pavilion constructed from red vinyl stretched over a metal frame [21].

Most of the sound materials used in *La Légende d’Eer* are very similar to the ones used in *Polytope de Cluny*, although a greater prominence is given to synthetic sounds: analog and digital (stochastic) (Fig. 2).

Reverberation and filtering were applied to some of the stochastic sounds present in the section that starts at 32'58", in the first four channels.

In this piece, Xenakis started using a new technique for stochastic synthesis that he named Dynamic Stochastic Synthesis. This technique had its origin in the methods presented in "New Proposals in Microsound Structure," and introduced an important conceptual development: the waveform as the basic unit to be varied stochastically at each iteration.

In this model, waveforms are constructed by linearly interpolating a set of breakpoints. Each breakpoint is defined by a pair of duration and amplitude values. At every repetition of the waveform, these values are varied stochastically using random walks: Any probability distribution can be employed to determine the size and direction of the steps. There are as many pairs of duration and amplitude random walks as there are breakpoints in the waveform [22].

The fluctuation speed of a parameter is directly proportional to the step size of its random walks: the smaller the steps, the slower the rate of change in that parameter. Depending on their speed, the perception of these fluctuations in duration and amplitude can be located on a continuum ranging from slow glissandi and subtle variations in timbre to noise.

Each random walk is forced to remain within a predefined space by means of two elastic barriers that reflect excessive values back into the barrier range. These barriers provide control over the frequency and amplitude of the waveform: the larger the space between a pair of barriers, the larger the variation that is possible in that parameter (i.e. the bigger the potential size of glissandi and the amplitude of the waveform); if the two elastic barriers of the duration random walks are set to the same value and the amplitude values fluctuate slowly, then gradual and independent variations in the amplitude of the overtones of a fixed pitch are heard.

Previously, Xenakis worked with individual duration and amplitude values that were either independent or dependent on the preceding value (e.g. random walks). The new approach evidences Xenakis’s interest in having a finer control over the periodicities (duration) and symmetries (amplitude) of stochastic waveforms. According to Xenakis, this control would allow him to modulate

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from white noise to a square wave, with "melodies, symphonies, natural sounds" in between [23]: "Following these principles, the whole gamut of music past and to come can be approached" [24].

In my opinion, the above statement is extremely unrealistic. Nonetheless, the Dynamic Stochastic Synthesis model is a very important development and is capable of producing rich and lively sounds that would be difficult to obtain through other sound synthesis techniques.

**DYNAMIC STOCHASTIC SYNTHESIS (1977): METHOD**

1. Select the number of breakpoints for the waveform. For example, 3 breakpoints (Fig. 3a).
2. Select the waveform’s minimum and maximum frequencies and convert them to duration in number of samples. For example, using a sampling rate of 44100 Hz:
   
   \[306 \text{–} 735 \text{ Hertz} = 120 \text{–} 60 \text{ samples}\]

3. Divide the minimum and maximum number of samples by the number of breakpoints:
   
   \[60/3 = 20\]
   \[120/3 = 40\]

   These values are the barriers for the duration random walk of each breakpoint.

4. For the continual generation of steps for all duration random walks: select a probability distribution, its parameters and the \(z\) number that will be the minimum and maximum size for these steps.

5. An initial duration is given to each breakpoint: values taken from stochastic or trigonometric functions, zeroes, etc. (Fig. 3b).

6. A maximum amplitude is selected and this \(z\) value is the barrier for the amplitude random walk of each breakpoint.

7. For the continual generation of steps for all the amplitude random walks: select a probability distribution, its parameters and the \(z\) number that will be the minimum and maximum size for these steps.

8. An initial amplitude is given to each breakpoint: values taken from stochastic or trigonometric functions, zeroes, etc. (Fig. 3c).

9. Breakpoints are linked by linear interpolation. At each repetition, the last breakpoint of the current waveform is connected to the first breakpoint of the next variation of the waveform (Fig. 3d).

This technique is described in the chapter "Dynamic Stochastic Synthesis" of *Formalized Music* [25] and is often mistaken to be the explanation for the dynamic stochastic synthesis algorithm implemented in the early 1990s as part of the GENDY program.

Also, it is important to remember that, for Xenakis, this method was just an arbitrary starting point that he used in *La Légende d’Eer* [26].

**DYNAMIC STOCHASTIC SYNTHESIS (1991): MORE THOROUGH STOCHASTIC MUSIC**

In an interview that took place in the mid-1990s, Xenakis said:

During my initial tests, I realized that probabilities could yield rich sonic results, but you have to control them—they are like wild horses! I have been working like a labourer to obtain interesting things from the [GENDY] program. I have been obliged to throw away many experimental results and keep only those that interested me [27].

It was not until the late 1980s that Xenakis continued with his research on...
stochastic synthesis [28]. He wrote a program that implemented an extended version of the dynamic stochastic synthesis algorithm used in *La Légende d’Er*. This program was written in the BASIC programming language, with the assistance of Marie-Hélène Serra, and was called GENDY (a portmanteau constructed from the French words *génération* and *dynamique*).

The only difference between the new implementation of the algorithm and the previous one is the use of second-order random walks. A second-order random walk consists of three elements: a probability distribution and two random walks. The probability distribution generates the step sizes of the primary random walk; the successive positions of the primary random walk are the step sizes of the secondary random walk. The successive positions of the secondary random walk are the values of the second-order random walk [29].

A second-order random walk with elastic barriers behaves very differently from a first-order one:

- A first-order random walk oscillates around an equilibrium position that changes arbitrarily over time (Fig. 4).
- A second-order random walk gravitates around one of its two barriers: If the position of its primary random walk is positive, it gravitates around the upper barrier and vice versa (Fig. 5).


1. Select the number of breakpoints for the waveform, for example: 3.
2. Select the waveform’s minimum and maximum frequencies and convert them to duration in number of samples. For example:
   
   
   368–735 Hertz = 120–60 samples

3. Divide the minimum and maximum number of samples by the number of breakpoints:
   
   
   $60/3 = 20$
   
   $120/3 = 40$

   These values are the barriers for the secondary duration random walk of each breakpoint.

4. Select a ± number that will be the minimum and maximum size for the primary duration random walks, which will give the step sizes for the secondary duration random walks.

   Usually, this number is equal to or smaller than the size of the space between the barriers of the secondary duration random walks. For example: a number between 0.0 and 20.0, if the secondary duration random walks have their barriers at 20.0 and 40.0.

   Secondary random walk barriers: 20–40

   Primary random walk barriers: -10–10

   I have found it more convenient to think of frequency intervals in terms of proportions rather than in terms of number of samples. Therefore, in order to have a more pertinent control of the size of the primary duration random walk space, I would suggest calculating it as a ratio to the secondary duration random walk space. For example:

   Secondary random walk barriers: 20–40

   Secondary random walk space: 20

   Primary random walk space as a ratio: 0.5

   Primary random walk barriers: -10–10

5. For the continual generation of steps for the primary duration random walks: select a probability distribution and the ± number that will be the minimum and maximum size for these steps (it is recommended to calculate this value as a ratio to the primary random walk space).

6. A ± maximum amplitude is selected; this ± value is the barrier for the secondary amplitude random walk of each breakpoint.

7. Select a ± number that will be the minimum and maximum size for the primary amplitude random walks, which will give the step sizes for the secondary amplitude random walks.

   Usually, this number is equal or smaller than twice the maximum amplitude.

   Secondary random walk barriers: -0.5–0.5

   Primary random walk barriers: -0.1–0.1

   It is also recommended to calculate this ± size as a ratio to the secondary random walk space.

8. For the continual generation of steps for the primary amplitude random walks: select a probability distribution and the ± number (or ratio) that will be the minimum and maximum size for these steps.

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Fig. 4. A first-order random walk with two elastic barriers. (© Sergio Luque)
9. Breakpoints are linked by linear interpolation. At each repetition, the last breakpoint of the current waveform is connected to the first breakpoint of the next variation of the waveform.

This technique is described in the chapter “More Thorough Stochastic Music” of *Formalized Music* [30] and is the one used by Xenakis in *GENDY3*

**GENDY3 (1991)**

To write a piece with the sounds produced by the GENDY program, Xenakis wrote another program, called PARAG, that treated a number of outputs of the GENDY program as voices and created sections with them.

The number of voices and their parameters seem to have been selected by hand by Xenakis and were hardcoded into the PARAG program. The procedure that he used to obtain these values is not known, but it is probable that he determined them intuitively.

In a page from Xenakis’ sketchbook at CEMAMu [31], he listed the parameters of all the voices for a PARAG section and added references about their resulting pitch characteristics: precise pitch or register or direction (e.g. A6 one quarter-tone high, “very low,” “descending”). This could be an indication of his interest in directly controlling the harmonic content of each section.

The PARAG program also independently broke each of the voices into parts, called *time fields*. The duration of each *time field* was determined using an exponential distribution; and a Bernoulli trial [32] decided if a *time field* was silent or not [33]. The length of each section was determined by the length of the longest of its voices.

In a 1996 interview, Xenakis elaborated on the difficulty of composing with the GENDY and the PARAG programs:

> I am always trying to develop a program that can create the continuity of an entire piece. This is a struggle, because there are always parts that you prefer over others. So you have to change them, to stop the process, start some other one, and then put these two different ones together. This can be taken very far. As I move toward multiple voices, the problem becomes even more complicated…. The final result is an edifice realized almost entirely by probability calculations. It takes time to successfully realize the probability calculations on the architectural level. The work is always intuitive.

You are lost if you base yourself only on the calculations, unless they themselves are also intuitive [34].

Xenakis composed two works with this version of the programs, each about 20 min-utes long: *GENDY301* and *GENDY3* [35].

*GENDY301* premiered in October 1991, at the International Computer Music Conference in Montreal. This work was withdrawn from Xenakis’s catalog.

*GENDY3* premiered in November 1991, at the Journées de Musique Contemporaine in Metz. This work was released on compact disc in 1994:

In spite of the close relation of the two pieces in terms of their genesis, they in fact sound completely different…. [GENDY301] exhibits a wider dynamic range that the other better-known piece, with extremely loud textures entering suddenly on top of narrower-range sonorities. In addition, it contains more breaks of silence. Xenakis never stated his dissatisfaction with the piece, but he may have decided not to release a “family” of works as he had produced in 1962 with the data from his ST program [36].

*GENDY3* consists of a sequence of 11 PARAG sections, of about 2 minutes in
length each. In Formalized Music, Xenakis proposed that an arbitrary chain of PARAG sections could produce an interesting musical composition [37]. In the case of GENDY3, the arrangement of these sections does not give the impression of being arbitrary; adjacent sections are clearly separated from each other by the considered use of contrasting material: changes in register, number of voices, barriers, etc. Each section has a consistent and, to a certain degree, static behavior, as the settings of all the parameters of a section do not vary over time; the abrupt changes from section to section give a sense, or illusion, of progress to the composition.

Most sections of GENDY3 have a combination of fixed pitches, glissandi and noise. The fixed pitches of a section, due to the insertion of silences, create a texture that could be described as a stochastic ostinato. Also, by using the same type of behavior in all the voices, Xenakis created some homogeneous sections, for example:

Section IV (4:58–6:28): noise
Section IX (13:50–15:49): glissandi
Section XI (17:06–18:53): a cluster of fixed pitches

In GENDY3, Xenakis made a stereo file by joining the mono outputs of two runs of the same PARAG program (he added a delay of about 100 milliseconds between them). In Formalized Music, Xenakis proposed a similar method to achieve multichannel output: to compute the same PARAG program, as many times as there are channels, but with a different random seed for each of the amplitude and/or duration random walks [38].

I have worked with two techniques to create multichannel stochastic synthesis:

- To have as many amplitude random walks per breakpoint as there are channels. The resulting sound is detailed in timbre and, due to the amplitude decorrelation, it propagates through all the channels, continuously changing the perceived spatial disposition in a diffused and erratic manner.
- To use a second-order random walk per breakpoint to determine the panning movement of the waveform. This technique creates a highly localizable sound that has a controllable spread and conveys a peculiar illusion of depth.

S.709 (1994)

After composing GENDY3, Xenakis extended the GENDY program, adding the possibility of modulating the parameters of the dynamic stochastic synthesis algorithm. With this version of the program, Xenakis created S.709 [39].

S.709 premiered at a concert at La Maison de Radio-France in December 1994 [40]. Its title stands for Sequence 709. Sequence was the name that Xenakis gave to the sections created by the PARAG program [41]. In a radio interview, Brigitte Robindoré, head of musical production at Les Ateliers UPIC, said about S.709: “It’s unedited. It’s unrestrained” [42]. It could then be inferred that S.709 consists of the output of only one PARAG program: it could be a PARAG section of 7 minutes in length. In this piece, the rapid and periodic modulation of the parameters creates voices that are constantly and widely fluctuating in pitch, amplitude and timbre.

In the same radio interview, Robindoré mentions that S.709 “produces quite a polemical reaction in the audience.” This is not surprising; this work is extremely original in its materials and in its construction; it does not resemble any other piece that I have ever heard.

EROD (1997)

In Erod, a 5-min work that was withdrawn after its premiere, waveforms extracted from GENDY samples were treated in the UPIC system and mixed with processed acoustic sounds [43].

Stochastic Concatenation of Dynamic Stochastic Synthesis

After writing an implementation of Xenakis’ 1991 dynamic stochastic synthesis algorithm in the C programming language, as a plugin for SuperCollider, I have been looking for ways of extending this model. The stochastic concatenation of GENDYS (i.e. the dynamic algorithm from 1991) is a procedure that almost immediately started to yield very promising results.

In this technique, a waveform is constructed by concatenating the waveforms of a set of GENDYS, one iteration at a time. For example, Fig. 6 shows a sequential concatenation of two GENDYS.

Conceptually, there is no limit to the number of GENDYS in a set, but I have found that 72 is a reasonable limit.

Any stochastic procedure can be used for selecting GENDYS from a set; the most fruitful ones that I have used so far are: tendency masks, random walks, Markov chains and probability distributions. Each of these procedures gives its own character to the resulting sounds, which range from continuous textures to differentiated arrangements of microsounds to timbres that exhibit interesting behaviors over time.

This approach is very close to SSP’s use of Selection Principles for the creation of the Permutation elements [44].

Conclusion

The musical achievements that Xenakis obtained through his extremely original approach to sound synthesis are awe-inspiring, as is the rigorous thinking that runs through them.

Keeping in mind the somewhat limited timbral space of the non-standard synthesis approach, I believe that, with some luck, it might be rewarding to continue exploring new methods and developing the known ones. After all, I wholeheartedly agree with the following remark by Xenakis: “research in sound synthesis has come to an impasse…. Scientists simply lack imagination in a field which lies outside mathematics or physics” [45].

Most electroacoustic music of today seems to consider clean and simple nondynamic sounds a goal, as if they were not, most probably, devoid of expressive potential. Marginal as Xenakis’ rough electronic sounds might be, it can be argued that they have created some of the most powerful electronic music to date.

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References and Notes

7. A computer program for composing and manipulating waveforms that uses linear change in its transformational procedures.
8. Sound Synthesis Program: a computer program

Lugue, The Stochastic Synthesis of Iannis Xenakis
for sound synthesis in which selection principles (various types of random decisions and direct enumeration) are used to join duration and amplitude values into segments (waveforms) and to determine the order of segments in permutations. The development of this program was started by Koenig and finished by Paul Berg.

9. Unité Polyagogique Informatique du CEMAMu: a computer system with a graphic input device that enables the user to create sounds by drawing lines and shapes.


35. Serra [33] p. 239.


43. Robindoré [42].


Discography
Polytope de Chan (1972) (Mode 98/99).
La Légende d’Eer (1977) (Aurivis Montaigne MO 782058; Mode 148).

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