Networked Control System (NCS) is a control system having one or more control loops that are closed via a communication network. The network is used to support the exchange of information between the distributed components of the NCS such as sensors, controllers and actuators. The use of a communication network in NCS considerably reduces the wiring, especially when the control application contains a large number of nodes. Consequently, the maintenance and the deployment become easier.

The automotive industry is an important application field of NCS. The design process of automotive applications is subject to conflicting criteria including cost, safety, reliability and performance. Consequently, these NCS applications are developed on target platforms with limited CPU speed, memory and communication bandwidth. Therefore optimal allocation of calculation and communication resources, operated by scheduling algorithms, are of prime importance. It has been shown that a more efficient use of the computation or communication resources could be attained when the problems of control and resource allocations are considered jointly [3,7,11,12].

To guarantee the overall control performance in such systems, it is important to evaluate the quantity of information that the communication channels can transfer. Thus, for the analysis of NCS, information theoretic approaches are especially useful, and the related concepts and results from this theory can be analyzed and applied. The results
in [16] and [19], for example, show the limitation in the communication rate for the existence of controllers, encoders, and decoders to stabilize discrete-time linear feedback systems.

In this paper, the problem of the optimal static scheduling of controller signals in a distributed control over deterministic real-time networks is addressed. In general and as presented in [5] one should design the control laws and at the same time decide on the control/sensor signals scheduling. A solution approach can be obtained by decomposing this problem into two distinct sub-problems. The first sub-problem formulation aims at obtaining a static or off-line scheduling of control/sensor signals based on the structural properties of the system and the communication/information constraints. This sub-problem can be formulated as an optimal control and sensor signals scheduling problem whose solution consists of a periodic scheduling of a subset of control/sensor signals to be sent from controller/sensor node to the actuator/controller node in each sampling period. The second sub-problem modifies the periodic or static control scheduling function based on the information given by the state of the system. In this manner, not only the feedback control signals but also the scheduling function is modified with respect to the system state.

For the clarity and consistency of presentation, we will focus on the first sub-problem mentioned above considering the system as a communication channel and with the objective of maximizing its capacity with respect to control/sensor signal scheduling. As metrics, we use the controllability and observability Gramian and their related norms as information metrics [1,10,20].

**Relation Between System Information and Controllability Gramian**

**Shannon Entropy**

Shannon proposed a measure of uncertainty in a discrete distribution based on the Boltzmann entropy of classical statistical mechanics. This measure, called the entropy, is defined as follows

\[
H = -\sum_i p_i \log_2 (p_i)
\]

where \( p_i \) is the probability of the alternative \( i \).

The term entropy is associated with the uncertainty or randomness, and information is used to reduce this uncertainty. One of the design objectives in a system control loop is to minimize the effect of uncertainty on system performance. That is, we can easily meet the design objectives if we can reduce the uncertainty by obtaining the relevant information and utilizing it properly.

**Mutual Information** \( I(X;Y) \), entropies \( H(X), H(Y) \) and joint entropy \( H(X,Y) \) are related as:

\[
I(X;Y) = H(X) + H(Y) - H(X,Y)
\]

\[
I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)
\]

where \( H(X) \) is the uncertainty that a random variable \( X \) has about \( Y \), \( H(Y) \) is the uncertainty that the random variable \( Y \) has about \( X \), \( H(X|Y) \) is the conditional entropy of \( X \) knowing \( Y \) or conditioned on \( Y \) and \( H(X,Y) \) is the uncertainty that \( X \) and \( Y \) hold in common.

Information value degrades over time and entropy value increases over time in general. Consider a random variable \( x \in \mathbb{R}^m \) of continuous type with the associated entropy given by
where \( p(x) \) is the probability density function of \( x \). The conditional entropy of order \( n \) is defined as

\[
H(x_k|x_{k-1}, \ldots, x_{k-n}) := -\int_{\mathbb{R}^n} p(.) \ln p(.) \, dx
\]  

The conditional entropy in (4) is a measure of the uncertainty about the value of \( x \) at time \( k \) under the assumption that its \( n \) most recent values have been observed. By letting \( n \) go to infinity, the conditional entropy of \( x_k \), assuming the limit exists, is defined as,

\[
H_c(x_k) := \lim_{n \to \infty} H(x_k|x_{k-1}, \ldots, x_{k-n})
\]  

### Information Induced by Controllability/Observability Gramian

In general, from the viewpoint of open-loop systems, when the system is unstable, the system amplifies the initial state at a level depending on its unstable poles [17]. Hence, we can say that in systems having a larger number of unstable poles, the signals contain more information about the initial state. Suppose that we have a feedback control system in which the control signal is sent through a network with limited bandwidth. We will consider the case where the state of the system is measurable and the sensors can send the state of the system without error. Under these conditions we may write,

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

Here \( K_c \), \( u^*(t) \) and \( u^e(t) \), represent, respectively, the feedback controller gain, the applied control input and control error due to quantization noise of the limited bandwidth network. In the sequel we are supposing that the control signal errors are caused by Gaussian White Noise which may be given by

\[
u_k^e(t) = \sqrt{D_k} \delta(t)
\]

So we may write:

\[
\dot{x}(t) = (A - BK_c)x(t) + Bu^e(t) = A_c x(t) + Bu^e(t)
\]

The feedback system (6) is a stable one and perturbed by quantization errors or noise due to bandwidth limitations.

**Lemma:** The controllability Gramian matrix \( W_c \) of system (6) can be related with the information-theoretic entropy \( H(x,t) \) as follows [14]:

\[
H(x,t) = \frac{1}{2} \ln \{ \det W_c(D, t) \} + \frac{n}{2} (1 + \ln 2\pi)
\]

The equation (8) gives the average a priori uncertainty of the state \( x \) at time \( t \) for an order \( n \) of the system. In (8), the matrix \( D \) is a diagonal positive definite matrix with \( D_i \) being the \( i \)th diagonal element which is the amplitude of unit impulse inputs. The matrix:

\[
W_c(D, \tau) = \int_{0}^{\tau} e^{A_c \tau} B D B^T c A_c \tau dt
\]

is the reachability/controllability Gramian for a system modeled as (6). Based on equation (8) we can write the entropy reduction as,

\[
\Delta H(x,t) = \frac{1}{2} \Delta \ln \{ \det W_c(D,t) \}
\]

showing that entropy reduction, which is the same as uncertainty reduction, is dependent only on the controllability Gramian. Therefore,

\[
\Delta H(x,t) = H(x(t_1), t_1) - H(x(t_2), t_2) = \frac{1}{2} \ln \{ \det W_{c1}(D_1, t_1) \} - \frac{1}{2} \ln \{ \det W_{c2}(D_2, t_2) \} = \frac{1}{2} \ln \left\{ \frac{\det W_{c1}(D_1, t_1)}{\det W_{c2}(D_2, t_2)} \right\}
\]

Using the above expression along with the concept of mutual information being the difference of the entropy and the residual conditional entropy, i.e.,

\[
I(X;U) = H(X) - H(X|U)
\]

we can conclude that Mutual Information \( I(X;U) \) between the state \( X \) and control input \( U \) denoted simply by Shannon Information \( I_{sh} \) is given by this \( \Delta H \) which can be expressed further as,

\[
\Delta H(x,t) = \det(W_{c1} W_{c2}^{-1}) = e^{2(\Delta H)} = e^{2I_{sh}}
\]

We may conclude that the uncertainty reduction, which is directly related to \( \Delta H(x,t) \), will reduce the variance of the state with respect to the steady state if \( \Delta H(x,t) \) converges to zero. The only influence we have on the control signal is related to that of feedback gain, coding of control signals as well as scheduling of control signals to be chosen such that the norm of Gramians, represented by \( \det(W,(D_o, t)) \), converge rapidly to their infinity norm \( \det(W,(D_o, \infty)) \). One partial illustration of this fact is the zooming coding of control signals which consists of trading coding accuracy with respect to system state. Reducing state uncertainty near steady-state amounts to increasing the number of bits used to code the control.
signal and reducing the sampling period in order that the number of bits transmitted per sampling period remains constant and so respecting the bandwidth constraints [4].

**OPTIMAL OFF-LINE SCHEDULING OF CONTROL/SENSOR SIGNALS**

Consider the continuous-time LTI plant described by:

\[
\begin{align*}
\dot{x}_c(t) &= A_c x_c(t) + B_c u_c(t) \\
y_c(t) &= C_c x_c(t)
\end{align*}
\]

where \( x_c(t) \in \mathbb{R}^n \), \( u_c(t) \in \mathbb{R}^m \) and \( y_c(t) \in \mathbb{R}^p \). The transpose of matrix \( A \) will be denoted by \( A' \). The plant contains \( m \) distinct actuators and \( p \) distinct sensors which are spatially distributed. The sensors and actuators are connected to the main controller through a limited bandwidth communication network. The network bandwidth is limited in the sense that it can carry at most \( b_r \) measures and \( b_w \) controls.

In order to derive a digital control law, a discrete-time representation of system (11) in the sampling period \( T_s \) is considered:

\[
\begin{align*}
x(k+1) &= A x(k) + B u(k) \\
y(k) &= C x(k)
\end{align*}
\]

where \( x(k) = x_c(kT_s) \), \( u(k) = u_c(kT_s) \) and \( y(k) = y_c(kT_s) \).

In [5,8], the description of the constraints affecting the transmission of the control commands to the actuators was done using the notion of Scheduling Function. Communication constraints may be formally described by introducing two vectors of Booleans \( \sigma(k) \in \{0,1\}^{b_r} \) and \( \delta(k) \in \{0,1\}^{b_w} \), defined for each sampling instant \( k \).

**Definition 1**

The vector \( \sigma(k) \) defined by

\[
\begin{align*}
\sigma(k) &= 1 \text{ if } y_i(k) \text{ is read by the controller at instant } k, \\
\sigma_i(k) &= 0 \text{ otherwise.}
\end{align*}
\]

is called sensors-to-controller scheduling vector at instant \( k \).

**Definition 2**

The vector \( \delta(k) \) defined by

\[
\begin{align*}
\delta(k) &= 1 \text{ if } u_i(k) \text{ is updated at instant } k, \\
\delta_i(k) &= 0 \text{ otherwise.}
\end{align*}
\]

is called controller-to-actuators scheduling vector at instant \( k \).

The vector \( \sigma(k) \) indicates the sensor signals that the controller may read at instant \( k \). In a similar way, \( \delta(k) \) indicates the control inputs to the plant that the controller may update at instant \( k \).

The plant (11), the analog-to-digital (A/D) and digital-to-analog (D/A) converters, the communication bus and the controller are schematically depicted in **Figure 1**. In this figure, \( \eta(k) \in \mathbb{R}^{b_r} \) represents the vector of partial measurements that the controller receives (through the communication bus) at the sampling period \( k \). In a similar way, vector \( v(k) \in \mathbb{R}^{b_w} \) represents the vector of partial control commands that the controller may send to the actuators (through the limited bandwidth communication bus) at the sampling period \( k \), as given in **Figure 2**. Blocks D/A and A/D respectively represent the digital-to-analog and analog-to-digital converters.

As illustrated in [5], a time invariant model of a discrete periodic system may be obtained using the lifting technique. The time invariant modeling may be seen as a down sampled representation of the system with periodicity \( T_S \) called hyperperiod. In the sequel we will assume that the scheduling functions of control and sensor signals, \( \delta^{(T_S-1)} \) and \( \sigma^{(T_S-1)} \) respectively, have the same length \( T_S(T_S = T_S) \). So, we may write the following LTI state-space representation of the periodic system \( \mathcal{S} \):

\[
\begin{align*}
\tilde{x}_i(q + 1) &= \tilde{A}_i \tilde{x}_i(q) + \tilde{B}_i \tilde{v}_i(q) \\
\tilde{y}_i(q) &= \tilde{C}_i \tilde{x}_i(q), \quad (i \in \{0, \ldots, T_S - 1\})
\end{align*}
\]
It is clear that $A_\delta$ and $B_\delta$ are dependent on the value of parameter $\delta$ which is the current period index value giving the subset of controller and sensor signal transmitted through the network. In case of a set of real-time task execution, it gives the task currently executed by the processor. So we have $T_\delta$ realizations of the LTI state-space equations (13). We may see the set of $T_\delta$ LTI realizations as competing systems aiming to obtain communication resources based on the performances they may provide with respect to the current state. Each of these realizations is a periodic one with period $T_\delta$ dependent on the scheduling of control/sensor signals. Since computing real-time optimal solution is an NP-hard problem, the control design problem may be split into two levels. The first level or the off-line design has to consider only the dynamic model characteristics of the systems and the network bandwidth constraints. It has to give us a periodic scheduling of control/sensor signals and the corresponding feedback control gains based on the induced periodic system (13). The second level problem has to modify the real-time control/sensor signals to handle the perturbations affecting our system based on the energy criteria consistent with the degree of controllability/observability.

In the following subsection we will give the solution of the first level problem or off-line optimal scheduling design. For clarity of our presentation we will introduce at first the concept of degree of reachability and observability.

### Reachability/Observability of NCS

As aforementioned, control design of an NCS with bandwidth resource constraints requires specifying the structure of the feedback control law and associated scheduling function $\delta(k)$, $\sigma(k)$, $k \in \{0, 1, \ldots, T_\delta - 1\}$ of the control and sensor signals. It is natural that we have to apply (send through the network) the control signals with greatest impact on the system, relative to the current state and the target state. This physical fact is closely related to the degree of reachability ([15], [2]) or the minimal energy control needed to transfer the current state of the system $x$ to the target state $x_T$. The dual concept is the degree of observability or the observation energy produced by the observed state $x_o$. Before providing the expressions of these energies and control law design, we recall some results concerning the reachability and observability and related Gramians [15], [2].

**Definition 3** ([18])

A linear discrete-time system is called $l$-step reachable (resp. $l$-step observable) if $l$ is a positive integer such that the system is reachable (resp. observable) on $[i, i + l]$, for any $i \in N$.

**Lemma 1** ([6], [2]) The minimal energy required to transfer the initial state $x$, to the desired state $x_T$ in $T_f$ sampling periods or the transient reachability function is given by:

$$F_r = (x_d - (A_\delta^T)^{T_f} x_0)^T W_r^{-1}(T_f)(x_d - (A_\delta^T)^{T_f} x_0)$$

and its optimal control solution is:

$$u(k) = B_\delta^T (A_\delta^{T_f - k}) W_r^{-1}(T_f)(x_d - (A_\delta^T)^{T_f} x_0)$$

From this Lemma we have, simultaneously, the function which measures the degree of controllability/reachability of our system and the open-loop optimal control signal realizing it. Naturally, for $T_\delta$ different state-space realizations (13), the degree of reachability and the associated control signals will be different. The smaller this required energy is, the more important the actuator action is on the system state.

**Lemma 2** ([6], [2]) The transient observability function is given by:

$$F_o = x_o^T W_o(T_f) x_0$$

Denote by $N = HT_\delta$ the horizon of periodic system $A_\delta, B_\delta, i \in \{0, \ldots, (T_\delta - 1)\}$, with $H$ a positive integer and $N \geq T_f$. Hence, $H$ measures the horizon of our study in the number of hyperperiods ($T_\delta$).

From Lemma 1 and Lemma 2 we can see that the inverse of the reachability Gramian is the weighted matrix of optimal transfer energy criterion. It is dependent on the systems dynamics, the periodic scheduling of control signals and the initial states. The smaller the required transfer energy is, the greater the controllability we have on the system state via the scheduled control signals. The same analysis may be done for the observability Gramian and its dependence on the scheduling of measurements or sensed signals. Denote by $S^r$ and $S^o$ the sets of all periodic scheduling functions of length $T_\delta - 1$ and $T_\delta - 1$ respectively, then, we can arrive at the optimal scheduling of control and measurement signals by the solution of the following two optimization problems:

**Problem Formulation 1**

**Optimal Degree of Reachability**

$$\max_{(\delta^{T_\delta - 1}) \in S^r} \{A_{\min}(W_r(T_f, \delta^{T_\delta - 1}))\}$$

$$\bar{x}_i(q + 1) = \tilde{A}(\delta^{T_\delta - 1}) \bar{x}_i(q) + \tilde{B}(\delta^{T_\delta - 1}) \bar{v}_i(q)$$

$$\bar{v}_i(q) = -\bar{K}(i) \bar{x}_i(q) + \bar{v}_s(q)$$

**Problem Formulation 1**

**Optimal Degree of Observability**

$$\max_{(\delta^{T_\delta - 1}) \in S^o} \{A_{\min}(W_o(T_f, \delta^{T_\delta - 1}))\}$$

$$\bar{x}_i(q + 1) = \tilde{A}(\delta^{T_\delta - 1}) \bar{x}_i(q) + \tilde{B}(\delta^{T_\delta - 1}) \bar{v}_i(q)$$

$$\bar{v}_i(q) = -\bar{K}(i) \bar{x}_i(q) + \bar{v}_s(q)$$
Since reachability and observability are dual concepts we treat Problem - 2 like the one in Problem - 1 but in the dual system. Substituting $\tilde{A} = \tilde{A}_1, \tilde{B} = \tilde{C}_1, \tilde{C} = \tilde{B}_1$, in Problem - 1 we can formulate as follows:

**Problem Formulation 2**

**Optimal Degree of Observability**

$$\max \{\sigma_{r-1} \in S_o\} \{\Lambda_{\min}(W_o(T_f, \sigma^{T_o-1}))\}$$

$$\tilde{x}_o(q + 1) = \tilde{A}_1(\sigma^{T_o-1})\tilde{x}_o(q) + \tilde{C}_1(\sigma^{T_o-1})\tilde{v}_o(q)$$

$$\tilde{y}_o(q) = \tilde{B}_1(\sigma^{T_o-1})\tilde{x}_o(q)$$

where $\Lambda_{\min}(W_o)$ and $\Lambda_{\min}(W_o)$ are the respective eigenvalues of matrices $W_o$ and $W_o$ with minimal moduli.

These two problems are integer programming problems whose complexity depends on the system order and bandwidth constraints. An algorithm based on incremental values of $\sigma^{T_o-1}$ and $\sigma^{T_o-1}$ and detection of its induced periodicity on a given maximal length offers satisfying results and reduces significantly the complexity with respect to the method proposed in [5].

**An Example**

Let us consider the periodic scheduling of three real-time control tasks, whose open-loop models are given by:

$$S_1 = \{A_1, B_1, C_1, D_1\}, S_2 = \{A_2, B_2, C_2, D_2\}, S_3 = \{A_3, B_3, C_3, D_3\},$$

with

$$A_1 = \begin{bmatrix} 0.5 & 0 \\ 1 & 0 \end{bmatrix}; A_2 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}; A_3 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix};$$

$$B_1 = B_2 = B_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; C_1 = C_2 = C_3 = \begin{bmatrix} 0 & 1 \end{bmatrix};$$

$$D_1 = D_2 = D_3 = \begin{bmatrix} 0 \end{bmatrix};$$

The open-loop eigenvalues of the systems are respectively: $\{0.5 \ 0\}, \{-1 \ 0\}$ and $\{1 \ 0\}$ which means that the second system is the only open-loop stable system. So, naturally, the order of importance of the resource allocation is given by the ordered set $\{S_1, S_2, S_3\}$. This order is consistent with the information rate definition given in [9,13,17,19,21,22]. In order to fit with...
REFERENCES


the NCS model presented above we construct the overall open-loop model as a diagonal one formed by the three given open-loop models. So the overall system has three actuators and three sensors and we will suppose that only one controller or sensor signal can be transmitted through the network. We discretize the overall system with the same sampling period and calculate feedback gains placing the feedback poles of controller/observer at the same place on the complex plane for the three subsystems.

Results Obtained

For the case when we have only one system input/output update at each sampling instant of time:

(1) Traces of the Inverse of Reachability Gramians (first column) with respect to the Scheduling Sequences (second column) are shown in Table 1a.

(2) Determinant of the Inverse Reachability Gramian (first column) with respect to the Scheduling Sequences (second column) is as follows:

\[ \det W_r^{-1} = 10^6 \times \begin{bmatrix} 1.8429 & 0.7493 & 0.3046 \\ 4.5329 & 0.2490 & 0.1012 \\ 0.6125 & 1.5065 & 1.2736 \end{bmatrix} \]

(3) Let us denote the eigenvalues, maximal moduli of eigenvalues and minimal moduli of eigenvalues of Inverse Reachability Gramians by \( \Lambda, \max|\Lambda(W_r^{-1})| \) and \( \min|\Lambda(W_r^{-1})| \) respectively. Representing the differences between Max-Min of Moduli of eigenvalue by \( \Delta_{\Lambda_{\max-min}} \), the maximal moduli and the minimal moduli of the eigenvalues of the Inverse Reachability Gramians (first column and second column) with respect to the Scheduling Sequences (fourth column) are as presented in Table 1b.

(4) For a Dual System i.e. \( A=A^T, B=C^T, C=B^T \) and \( D=D^T \) we can treat reachability and observability as dual to each other and hence Inverse of Observability Gramian \( W^{-1}r=\bar{W}^{-1}r \), where \( \bar{W}^{-1}r \) is the inverse of the reachability Gramian of the dual system. Therefore, traces of the Inverses of Observability Gramians (first column) with respect to the Scheduling Sequences (second column) are as given in Table 2a.

(5) Determinant of the Inverse of Observability Gramian (first column) with respect to
### TABLE 1

<table>
<thead>
<tr>
<th>Sequence $S_i$</th>
<th>$N_r$</th>
<th>$\text{tr}(W_r^{-1})(\times10^4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \rightarrow 2 \rightarrow 3 \rightarrow 1$</td>
<td>1</td>
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</tr>
<tr>
<td>$1 \rightarrow 3 \rightarrow 2 \rightarrow 3$</td>
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<td>0.0096</td>
</tr>
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<tr>
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</tr>
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<td>0.0117</td>
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### TABLE 2

<table>
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<tr>
<td>$1 \rightarrow 2 \rightarrow 1 \rightarrow 3$</td>
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</tr>
</tbody>
</table>

### TABLE 1a

Traces of inverse reachability gramians $W_r^{-1}$ w.r.t. scheduling sequences $S_i$.

### TABLE 2a

Traces of inverse observability gramians $W_0^{-1}$ w.r.t. scheduling sequences $S_i$.

### TABLE 1b

Maxima-minima moduli of eigenvalues of inverse reachability gramians $W_r^{-1}$ w.r.t. scheduling sequences $S_i$.

### TABLE 2b

Maxima-minima moduli of eigenvalues of inverse observability gramians $W_0^{-1}$ w.r.t. scheduling sequences $S_i$.

The values in red color correspond to the sequence which is the best one for the criteria (trace, determinant and the min/max of eigenvalues of the inverse of reachability and observability gramians) used for the inverse of reachability and observability Gramians. Those in blue are the second best values and those in green correspond to the third best values. It is shown here that a combination of these criteria can make a reliable metric selection for the best sequence of the message. So, we may say that the Sequence No. 3 is the best one in this example study.

### CONCLUSIONS AND PERSPECTIVES

In this paper we have addressed the problem of optimal control and scheduling of Networked Control Systems over limited bandwidth deterministic networks using some insight on the interplay between the control and information theory. The results obtained for differently chosen criteria are consistent with physical insights and results obtained based on information theory. Slight differences observed in the solution obtained with respect to the metrics used can be related to model approximation induced by discretization and calculation accuracy relative to the metrics used. This last point will be further investigated and reported in future studies.