Calibration of a full-disc longitudinal magnetogram at the Huairou Solar Observing Station

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ABSTRACT
Based on a spectral profile analysis of a filter magnetograph and the application of the astro-nomical data analysis technique, a full-disc distribution of Stokes $I$-to-$V$ crosstalk in the solar magnetism and activity telescope (SMAT) has been acquired quantitatively. We produce a correction template to remove the full-disc Stokes $I$-to-$V$ crosstalk. This template is important for the accuracy of the full-disc longitudinal magnetogram. Then, we propose an observational calibration method under the weak-field approximation on the basis of the scanning spectrum of the filter magnetograph. The following main results have been obtained. (i) For the original Stokes $V$ images, there is about $-1.67 \times 10^{-3}$ of Stokes $I$ crosstalk into $V$ at all wavelength positions, and this crosstalk presents itself as a horse-saddle surface distribution on the full disc. (ii) A sample of the calibration curve at different wavelength positions is given. (iii) After being calibrated into magnetic flux density, the Stokes $I$-to-$V$ crosstalk has an amplitude of about 50 G, but introduces negative and positive spurious magnetic fields into the blue and red wing magnetograms, respectively.

Key words: techniques: polarimetric – Sun: photosphere – Sun: surface magnetism.

1 INTRODUCTION
The measurement of the magnetic field is an important way to understand the activity of the Sun. The measurement of the full-disc magnetic field reveals the large-scale evolution of the solar magnetic field and the solar dynamo process. At present, there are many instruments that provide observations of the full-disc solar magnetic field. These can be classified into two types:

(i) non-vector type, such as the Michelson Doppler imager (MDI) aboard the Solar and Heliospheric Observatory (SOHO; Scherrer et al. 1995), the mean magnetic field of the Sun-as-a-star observed by the Wilcox Solar Observatory (Scherrer et al. 1977) and the full-disc spectromagnetograph of the Kitt Peak observatory (Jones et al. 1992);

(ii) vector type, such as the solar magnetic activity research telescope (SMART) at the Hida Observatory (Ueno et al. 2004), the vector spectromagnetograph instrument (VSM) belonging to the project of the Synoptic Optical Long-Term Investigation of the Sun (SOLIS; Keller et al. 2003; Henney, Keller & Harvey 2006; Henney et al. 2009), the helioseismic and magnetic imager (HMI) aboard the Solar Dynamics Observatory (SDO; Scherrer 2002; Borrero et al. 2006; Norton et al. 2006) and the solar magnetism and activity telescope (SMAT) at the Huairou Solar Observing Station of the National Astronomical Observatories of China.

The SMAT began to operate formally at the end of 2007. Its scientific objective is to monitor the fast variation of the photospheric vector magnetic field and the $H\alpha$ chromospheric activities on a global scale (Zhang et al. 2007). In the following sections, we discuss a systematic polarization in the SMAT. This is a problem to which we have not paid special attention since the Huairou Solar Observing Station began to develop the Lyot filter early on (Hu & Ai 1986). That is to say, the instrument polarization in the filter magnetograph may have non-uniform spatial distribution, and this non-uniform instrument polarization becomes distinguished in a wide field-of-view (FOV) magnetograph. Su & Zhang (2007) discussed an instrument polarization in one dimension along a circle on a full-disc magnetogram. Wang, Su & Zhang (2008) further acquired a two-dimensional spatial distribution for the instrument polarization, but without connecting the non-uniform polarization with crosstalk. In the present paper, we introduce the scanning spectrum of a filter magnetograph to analyse this instrument polarization quantitatively. A successfully refined ‘template’ is produced for the SMAT to remove it in Stokes $V$. The layout of this paper is as follows. In Section 2 we describe the data and polarization analysis. In Section 3, we give a profile of the Stokes $V$ calibration through observing. We give our conclusions in Section 4.

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2 DATA AND POLARIZATION ANALYSIS

2.1 Instrument and measurement

The SMAT observes the solar magnetic field using one of its telescopes, which is made up of a 10-cm lens, a Lyot filter, a polarization analyser and a CCD camera. All these parts are put into an axial symmetric telecentric optical system. Another telescope in the SMAT is the 20-cm Hα telescope. A detailed description, including the optical design of the telescope and the layout of the birefringent filter for the SMAT, can be found in Zhang et al. (2007). As an example of the polarization measurement of a video magnetograph, the Stokes $V$ is measured by differencing two measurements with the retardation setting of $\phi_1 = +\lambda/4$ (for $0.5I - 0.5V$) and $\phi_2 = -\lambda/4$ (for $0.5I + 0.5V$). Here, $\lambda$ is the centre wavelength of the filter transmission peak. To achieve the necessary signal-to-noise ratio in the measurement, we typically integrate 256 5-ms exposures, cycling through two retardation settings ($+\lambda/4$, $-\lambda/4$) repeatedly, for one set of measurements of the Stokes $0.5I - 0.5V$ and $0.5I + 0.5V$ parameters. This retardation is implemented by crystals of the KD*P modulator (Ai & Hu 1986b). The KD*P modulator requires about 1000 V ac voltage and may have spatial retardance variation in a wide FOV. Other modulators, based on liquid crystal variable retarders, are also being developed at the Huairou Station (Zhang et al. 2005). When we integrate 256 5-ms exposures, an integrated Stokes $V$ image can be acquired within 5 s. The Stokes $V$ images used in this study belong to the wavelength scanning data set, which is acquired by tuning up the filter transmission peak from Fe I 5324.19 − 0.4 Å to Fe I 5324.19 + 0.6 Å with a 5-mÅ step and 201 wavelength positions. The observing time of this data set is 2008 December 31, during the end of solar cycle 23.

2.2 Mixture of Stokes $I$ and $V$ in the filter magnetograph

Because a detector such as a CCD camera is not sensitive to polarization but the intensity of photons, the directly measurable quantity in the filter magnetograph is not Stokes $V$ but the mixture of Stokes $I$ and $V$, as

$$S_\pm = 0.5I \mp 0.5V,$$

and

$$V = S_+ - S_- \quad (I = S_+ + S_-).$$

The parameters used follow the convention of Ai & Hu (1986a). Therefore, Stokes $I$ and $V$ are functions $I(x, y, t, \lambda)$ and $V(x, y, t, \lambda)$ in which $\lambda$ is the centre wavelength position of the filter transmission peak. Fig. 1 shows an example of full-disc Stokes $I$ and $V$ images measured by the SMAT. Their filter transmission peak is centred on the blue wing $\sim 50$ mÅ from the spectral line core. The Stokes $I$ and $V$ in Fig. 1 are normalized by their respective maximum values, and the grey-scale bars are shown at the bottom of the figure. There are a few dirty points and vertical stripes on the Stokes $I$ image in Fig. 1, but these disappear in the right image of Fig. 1. Conventionally, the circular polarization for the filter magnetograph is defined by the differencing of two video-state filtergrams.

![Figure 1](https://example.com/figure1.jpg)

**Figure 1.** The Stokes $I$ and $V$ images on 2008 December 31. The left image is a filtergram measured in the blue wing. The obvious points and stripes on this Stokes $I$ image are some dirty dust, which do not appear in the polarization images. The right image is a Stokes $V$ image measured in the same wavelength position. Their observing wavelengths are marked at their respective lower-left corners. The two normalizing values we used in these two grey-scales correspond to their maximum values in Stokes $I$ and $V$ images, respectively. The region in which our solar centre spectrum is sampled is enlarged in the lower-right corner of the right image.
as

\[ \frac{V_{\text{obs}}}{I_{\text{obs}}} = \frac{S_c - S_e}{S_c + S_e}. \]  

(3)

Then, the effect of the flat frame is largely ruled out by the division operation. However, the instrument and measurement do not strictly abide by equation (1). For the SMAT,

\[ S_c = (0.5 + c_1)I + 0.5V, \]  

(4)

and

\[ S_e = (0.5 + c_2)I - 0.5V, \]  

(5)

are more close to the observational results on the condition that

\[ c_1 \approx c_2 \text{ and } \vert c_1 + c_2 \vert \ll 1. \]

As a consequence,

\[ \frac{V_{\text{obs}}}{I_{\text{obs}}} \approx \frac{V}{I} + (c_1 - c_2), \]  

(6)

is a corrected version of equation (3), where \( V_{\text{obs}}/I_{\text{obs}} \) and \( V/I \) are the total measuring signal and the real solar signal, respectively. Here, the essential term \( c_1 - c_2 \), which is mainly discussed in this paper, is presented directly. However, it is a difficult process to find and understand the \( c_1 - c_2 \) term. In the following, we denote it as \( M_{41} \), which is the standard expression for \( I \)-to-\( V \) crosstalk in the Mueller notation (see, for example, the introductions to astronomical polarimetry provided by Keller 2002), and so on. According to equation (6), \( M_{41}(x, y, \lambda, t) \) can be understood preliminarily as a two-dimensional matrix distribution, which may vary with time and the centre wavelength of the filter transmission, although \( M_{41}(x, y) \) is our final answer. The following two subsections illustrate this problem step by step with spatial integration and wavelength integration methods.

2.3 Two types of spatial integration scanning spectra of the filter magnetograph

Traditionally, the crosstalk among four polarization parameters, the Stokes \( I, Q, U \) and \( V \) parameters, should be expressed as \( I_{\text{obs}} = \mathbf{M} \cdot \mathbf{I} \), the Mueller matrix multiplied by the Stokes vector (Stenflo 1994, p. 315; Keller 2002, p. 23). When we only consider the crosstalk in the Stokes \( V \) measurement, we have a general expression from the fourth line of \( I_{\text{obs}} = \mathbf{M} \cdot \mathbf{I} \),

\[ V_{\text{obs}} = V + M_{41}I + M_{42}Q + M_{43}U, \]  

(7)

where Stokes \( I_{\text{obs}} \approx I \) and \( M_{41} \approx 1 \) are assumed. As is usual for the measurement of the filter magnetograph, the observed Stokes \( V \) is normalized by the intensity, and then spatially averaged within the unresolved pixel or ‘man-made defined’ area. In the following integration, \( S_0 \) expresses the integral area and also the normalizing denominator:

\[ \int_{S_0} \frac{V_{\text{obs}}}{S_0} \frac{dS}{S_0} = \int_{S_0} \left( \frac{V}{I} + M_{41} + M_{42} \frac{Q}{I} + M_{43} \frac{U}{I} \right) \frac{dS}{S_0}. \]  

(8)

When equation (8) is applied to the wavelength scanning data set mentioned in Section 2.1, the spatial coordinate variables \((x, y)\) disappear after integration. Hence, on the right-hand side of equation (8) are three types of spectra: the circular polarization spectrum from the Sun, the crosstalk spectrum from intensity and the crosstalk spectrum from linear polarization. However, these are not of the same magnitude in many cases. Because the intrinsic degree of polarization \((Q^2 + U^2 + V^2)^{1/2}/I \) is usually \( \ll 1 \) on the Sun, except in sunspots and spatially resolved magnetic flux tubes, it is the crosstalk from Stokes \( I \) into the other Stokes parameters that is the dominating effect (Stenflo 1994, p. 326).

For the first spatial integration type, we choose a \( 22 \times 22 \text{arcsec}^2 \) solar region in which there is a plage region and no sunspots (\( S_0 \), the enlarged picture in the lower-right part of Fig. 1). As the transverse field is weak for this region, the last two crosstalk terms are neglected but the Stokes \( I \) crosstalk term remains itself in equation (8). Then, the spatial averaged spectrum of this region is decomposed into two parts as follows:

\[ \int_{S_0} \frac{V_{\text{obs}}}{S_0} \frac{dS}{S_0} = \int_{S_0} \left( \frac{V}{I} + M_{41} \right) \frac{dS}{S_0} = \left\langle \frac{V}{I} \right\rangle + \left\langle M_{41} \right\rangle. \]  

(9)

Here, the symbol \( \langle \rangle \) means the average inside area \( S_0 \). The first part \( (V/I)_{S_0} \) is a spectrum antisymmetric to the spectral line centre (Fig. 2a). The second part \( (M_{41})_{S_0} \approx -1.67 \times 10^{-3} \) is a constant along the wavelength dimension (the dashed line in Fig. 2a). The standard deviation of the mean value \((\sigma_{\text{mean}} = \sigma/\sqrt{N})\) is adopted as the error bar definition of the scatter points \((V/I)_{S_0}\) in Fig. 2(a), where \( \sigma \) is the standard deviation. The second term \((M_{41})_{S_0}\) just makes the spectrum \((V/I)_{S_0}\) shift upward or downward in Fig. 2(a), and it does not change the shape of the Stokes \( V \). Because this circular polarization signal has such a baseline (dashed line in Fig. 2a), we call \((M_{41})_{S_0}\) the ‘polarization zero-point’ of the solar region. The polarization zero-point problem has also been mentioned by Ulrich et al. (2002), Demidov (1996) and Stenflo (1994, p. 290). However, both of the meaning of and the reason for the ‘zero-point’ are not completely the same.

For the second spatial integration type, the integration area is enlarged to the full-disc (\( S_0 \) denotes the full-disc area). In this case, the opposite polarities in circular and linear polarization from the Sun compensate each other, and the three terms containing \( V, Q \) and \( U \) should be zero in equation (8) after the full-disc average. The only surplus term is the Stokes \( I \) crosstalk if \( M_{41} \) retains the plus or minus sign in the FOV. Of course, we must point out that the Sun has a non-zero mean field if the integration area is a projected disc having the normal line along the line of sight rather than a close surface (Scherrer et al. 1977). However, it is assumed here that the SMAT could not measure the solar mean field of the full-disc Stokes \( V \), as its magnetic sensitivity does not reach \( 10^{-4}\text{I}_c \). Here, we use the convention of Lin, Penn & Tomczyk (2000) to express the polarization sensitivity. In our data reduction, we find the obvious integration average term

\[ \int_{S_0} \frac{V_{\text{obs}}}{S_0} \frac{dS}{S_0} = \int_{S_0} M_{41} \frac{dS}{S_0} = \langle M_{41} \rangle_{S_0}, \]  

(10)

where \((M_{41})_{S_0}\) is also \(-1.67 \times 10^{-3}\). The antisymmetric Stokes \( V \) profile disappears in Fig. 2(b) as a result of the mixture of opposite polarities. That is to say, the polarization zero-point of the solar centre in the FOV is very close to the polarization zero-point of the full-disc average in the FOV. Therefore, the full-disc spatial average of \( V_{\text{obs}}/I \) almost becomes a horizontal straight line in Fig. 2(b). The error bar displayed in Fig. 2(b) is \( 10\sigma / \sqrt{N} \), which is 10 times the error bar displayed in Fig. 2(a). This is because averaging the data set on the full disc has less statistical error than averaging the data set in the local region.

2.4 Quantitative analysis of the scanning spectrum averaged in the solar centre

The purpose of using the scanning spectrum is to analyse the instrument’s systematic signal not the Sun itself. This spectrum is acquired in solar minimum and there is no solar eruptive event during the scanning process. Hence, the lower activity and relative quiet solar minimum is not a bad time for us. However, this scanning
Figure 2. Two types of spatial integration scanning spectra from 01:09 to 01:58 UT on 2008 December 31. The left curve is spatially sampled from the enlarged region in Fig. 1. The right curve is the full-disc spatial average of the scanning spectrum. The horizontal axis in the two pictures begins from $-0.6 \, \text{Å}$, but the scanning points begin from $-0.4 \, \text{Å}$. This scanning range is the maximum range for the present SMAT. The dashed lines in the two pictures give the level of polarization zero-point. The polarization zero-point, $\langle M_{41} \rangle_{S_{c}}$ in our equation (9), in Fig. 2(a) is coincidentally equal to $\langle M_{41} \rangle_{S_{f}}$ in our equation (10), in Fig. 2(b). The error bar definitions are $\sigma/\sqrt{n}$ in Fig. 2(a) and $10\sigma/\sqrt{n}$ in Fig. 2(b), where $\sigma$ is the standard deviation and $n$ is the number of CCD pixels used during the spatial average.

Figure 3. Asymmetry of the solar centre scanning spectrum on 2008 December 31. The scattered diamond points are the original data from the solar centre scanning spectrum. The dashed line is a fitted curve from these scattering points. The four parameters (the blue lobe area $A_{b}$, the red lobe area $A_{r}$, the blue lobe amplitude $a_{b}$ and the red lobe amplitude $a_{r}$) are calculated on the basis of the fitted curve.
above results support the idea that the turbulent convection of fluid is coupled with the magnetic flux tube on a small scale. If the elementary flux tubes exist on the Sun with a size of 0.3 arcsec, or smaller, our solar centre scanning spectrum acquired in 50 min is averaged within a region of 22 × 22 arcsec², which contains more than 6850 elementary flux tubes. Therefore, our results for the Stokes V asymmetry should be considered as the large-scale average of many elementary flux tubes, and this statistical property may persist during the scanning process. However, considering the scanning time and the effect of the terrestrial atmosphere on the seeing, we just provide the asymmetry parameters of the scanning spectrum as a reference.

2.5 Two-dimensional spatial distribution of the Stokes I-to-V crosstalk

From the above analysis, we know that \( M_{41}(x, y, \lambda, t) \) does not change with wavelength in the two types of scanning spectrum. \( M_{41}(x, y, t) \) discussed in this paper is also non-stochastic, in fact. Except for the gross magnitude of \( M_{41}(x, y) \), we have not found its spatial distribution in Section 2.3, because the spatial information is removed by integration in order to strengthen the information in the wavelength domain. Now, the wavelength integration method can be used to solve this problem. Ronan, Mickey & Orrall (1987), Lites, Rutten & Berger (1999) and Lites et al. (2008) have defined the wavelength-integrated Stokes V formula. The absolute area of the blue and red lobes are added together, and then normalized by the area below the Stokes I spectrum. In doing so, they significantly enhance the sensitivity to weak Zeeman polarization in the presence of measurement noise, and thus avoid the problems of convergence and non-uniqueness that arise in inversions of noisy profiles (Lites et al. 2008). In this study, we would rather acquire the non-stochastic instrument polarization ‘noise’ than be interested in the polarization of the quiet-Sun internet work that they have studied. Therefore, the purpose of our data reduction is opposite to theirs. One different operation in our data reduction is that we do not use ‘the absolute integration’ but we integrate the observational Stokes V directly. According to Fig. 3, it is obvious that the Lyot filter of the SMAT has good spectral quality in the range from 5324.19–0.2 Å to 5324.19+0.2 Å, as the filter transmission is designed around the spectral-line core and the signal-to-noise ratio is high near the peak of the Stokes V profile. We integrate both sides of equation (7) as

\[
M = \int_{\lambda_0-0.2}^{\lambda_0+0.2} V_{\text{obs}} \, d\lambda \\
= \int_{\lambda_0-0.2}^{\lambda_0+0.2} (V + M_{41} I + M_{42} Q + M_{43} U) \, d\lambda \\
\approx M_{41} \int_{\lambda_0-0.2}^{\lambda_0+0.2} I \, d\lambda + \int_{\lambda_0-0.2}^{\lambda_0+0.2} (M_{42} Q + M_{43} U) \, d\lambda.
\]

Here, the approximation is a result of the neglect of the solar original circular polarization, which is generally cancelled each other in this symmetric integration from the blue wing to the red wing (whether the small solar magnetic structure disappears or not is the criterion in our data processing). Because of the Stokes asymmetry discussed in Section 2.4, the wavelength range of the integration should not be from 5324.19–0.4 Å to 5324.19+0.4 Å. However, if the wavelength range of the integration is too small, then the signal-to-noise ratio is not enough. In the following, the normalizing denominator is defined as

\[
N = \int_{\lambda_0-0.2}^{\lambda_0+0.2} I \, d\lambda,
\]

which is also different from Lites et al. (1999, 2008) where the continuum intensity is used. Then, the function \( M_{41}(x, y) \) becomes

\[
M_{41} \approx \frac{M}{N} = \frac{\int_{\lambda_0-0.2}^{\lambda_0+0.2} (M_{42} Q + M_{43} U) \, d\lambda}{N} \\
\approx \frac{M}{N},
\]

where the transverse field \( Q \) and \( U \) crosstalk to \( I \) is normalized by \( N \) and neglected, as

\[
\left| \int_{\lambda_0-0.2}^{\lambda_0+0.2} (M_{42} Q + M_{43} U) \, d\lambda \right| < 10^{-4}
\]

is generally preserved well because of the low spatial resolution properties of the SMAT and this extremely quiet time of solar minimum (between the end of solar cycle 23 and the beginning of 24). The final result is displayed in Fig. 4 in grey-scale (Fig. 4, left) and in a three-dimensional display (Fig. 4, right). The distribution of \( M_{41}(x, y) \) looks like a horse-saddle on which its amplitude is from -0.61 × 10⁻⁴ to -2.72 × 10⁻³ and its average value is -1.67 × 10⁻³ according to Fig. 4. This distribution has not been acquired accurately in Wang et al. (2008). Because the spectral profile analysis developed here was not applied in Wang et al. (2008), an accurate value of \( M_{41}(x, y) \) could not be acquired in that work. Now, Fig. 4 is a product that can be used as the ‘correction template’. In Section 3.3, the effect of this ‘correction template’ is shown for a different data set.

Furthermore, a natural question is why \( M_{41}(x, y) \) has a distribution in the FOV and how this polarization comes from the intensity. Hence, the basic measurement theory of the video magnetograph mentioned in Section 2.2 (equations 3–5) can be applied to the wavelength scanning data set as

\[
\frac{\sum_{\lambda_0-0.2}^{\lambda_0+0.2} V_{\text{obs}}}{\sum_{\lambda_0-0.2}^{\lambda_0+0.2} I_{\text{obs}}} = \frac{\sum V + (c_1 - c_2) \sum I}{(1 + c_1 + c_2) \sum I} \\
\approx \frac{\sum V}{\sum I} + (c_1 - c_2) \\
\approx (c_1 - c_2) = M_{41},
\]

on the condition that \( c_1 + c_2 \ll 1 \) and \( \partial(c_1, c_2)/\partial\lambda \approx 0 \). \( M_{41}(x, y) \) in the SMAT should originate from the difference of two modulation states. That is to say, the modulation state is not completely symmetric even if the light entering into the telescope is non-polarized natural light. We suppose that the main reason for this crosstalk is because of the design of the crystals used in the KD*P modulator, which does not seriously consider the wide FOV design, as we have no experience of the full-disc vector magnetic field measurement before the SMAT project.

3 CALIBRATION THROUGH OBSERVING PROFILE

3.1 Calibration of magnetograph, weak-field approximation and convolution of instrument transmission profile

In this section, we calibrate the Stokes V image and the crosstalk term \( M_{41}(x, y) \) to gauss. The magnetograms are more easily understood by solar physicists if the magnetograms are described by
gauss and angles rather than the Stokes parameters. The solution of radiative transfer for polarized radiation has been derived soundly by many authors in theory (Unno 1956; Rachkovsky 1962; Landi & Landi 1973; Jefferies, Lites & Skumanich 1989). However, the interpretation of the results of observing the solar magnetic field is not satisfied in some circumstances. This interpretation is an inver-\textsc{sp}se problem, which inevitably depends on theoretical models and assumptions. The filter magnetograph has more ambiguity in the calibration process than the spectrometer magnetograph. Jefferies & Mickey (1991) discussed the effective range of the weak-field approximation (WFA). Wang, Ai & Deng (1996a,b) calibrated the nine-channel solar magnetic field telescope at the Huairou Solar Observing Station using two methods. For the first method, they convolved the Kitt Peak spectrometer’s standard spectral-line profiles (removing the instrument profile) with the formulated filter transmission profile as the applicable calibration profile (semi-observational calibration). For the second method, they used the known solar differential rotation results to calibrate the velocity field observed by their filter magnetograph, and acquired the derivative of the intensity profile. Then, the derivative of the intensity profile could be used for the calibration of Stokes $V$. The second method can be called ‘observation revised cross-calibration’. Moon, Deuk Park & Yun (1999) applied the deconvolution profile of the scanning intensity profile to the Stokes $V$ calibration. For the simplicity of deconvolution, the filter transmission profile was approximated as Gaussian by Moon et al. (1999). Chae et al. (2007) calibrated the $V$ images of the narrow-band filter imager (NFI) aboard the \textit{Hinode} satellite by means of the calibrated magnetic field observed by the spectro-polarimeter (SP) aboard \textit{Hinode} (cross-calibration). For the sunspot umbra magnetic field, a correction term $\beta_1$, which revises the deviation from the WFA and was calibrated by the SP, was introduced in Chae et al. (2007). Wang et al. (2009) compared magnetograms of active regions observed by the MDI and SP with the ‘cross-comparison’ method (cross-calibration). They found that the MDI level-1.8 magnetograms recalibrated in 2008 had successfully removed the centre-to-limb variation. Ulrich et al. (2009) introduced a calibration formula containing the saturation correction factor $\beta(\lambda)$ in which the deviation from the WFA was given by the term containing the second-order derivatives of the Stokes $V$ profile.

Under the WFA, the shape of $S'$, where $S$ is defined in equation (2) and $'$ denotes convolution by the instrument transmission profile, is very similar to the shape of $S''$. These move in parallel a splitting distance from each other. A geometrical explanation for this ‘weak-field splitting’ is illustrated by the schematic diagram in Fig. 5(b). According to the geometrical relation in Fig. 5(b), we have

$$V' = -\Delta\hat{B} = \Delta S' = S' - S''$$

$$\frac{\text{WFA}}{\Delta\lambda \ll \Delta\lambda_D} 0.5I'(\lambda - \Delta\lambda' \cos \gamma) = 0.5I'(\lambda + \Delta\lambda' \cos \gamma),$$

and

$$\Delta\hat{C} = 2\Delta\lambda' \cos \gamma,$$

where the WFA contains the assumption that the shapes of the two modulated profiles are almost the same and decided by the shape of Stokes $I'$. The mathematical derivative and limit can be mapped into the physical WFA as

$$\frac{\partial I'}{\partial \lambda} = \lim_{\Delta\lambda' \to 0} \frac{2\Delta\hat{B}}{\Delta\hat{C}} = \lim_{\Delta\lambda' \to 0} \frac{-2V'}{2\Delta\lambda' \cos \gamma} \approx -\frac{V'}{\Delta\lambda' \cos \gamma}.\tag{18}$$

Therefore, it follows that

$$\Delta\lambda' \cos \gamma \approx -V' \left(\frac{\partial I'}{\partial \lambda}\right)^{-1},\tag{19}$$

where the filling factor is not mentioned. In fact, we cannot fully disentangle the intrinsic field strength only with one spectral-line

**Figure 4.** $M_{41}$ crosstalk correction template. The right picture is a three-dimensional display of the left grey-scale image of our $M_{41}$ template. The upward direction is the north projection of the solar polar axis because the solar $p$-angle is corrected in this template. Before calibration, this template has the same effect at all wavelength positions. After calibration, the template has different effects at different wavelength positions, generally about 50 G. For the data set on different days without changing or removing any hardware, one template from a set of excellent scanning spectrum can be used for all other data sets.
observation (Stenflo 1973; Socas-Navarro et al. 2008). The line shift amount is related to the magnetic field strength by the standard Zeeman formula

\[ \Delta \lambda = 4.67 \times 10^{-13} g^2 \lambda^2 B, \quad (20) \]

where \( \lambda \) is in Å and \( B \) is in G. Combining equations (19) and (20), we then acquire

\[
\Delta \lambda \cos \gamma = 4.67 \times 10^{-13} g^2 \lambda^2 B \cos \gamma \\
\approx -\frac{S'_e - S'_i}{S'_e + S'_i} \left[ \frac{1}{S'_e + S'_i} \frac{\partial (S'_e + S'_i)}{\partial \lambda} \right]^{-1}, \quad (21)
\]

and

\[ B_1 \approx - \left( 4.67 \times 10^{-13} g^2 \lambda^2 \frac{\partial \ln I'}{\partial \lambda} \right)^{-1} \frac{V'}{T}. \quad (22) \]

Equation (22) has no essential difference from the equation in Ulrich et al. (2009) \((B_{\text{ave}} \text{-hand in their equation})\). They introduced a factor \( \beta(\lambda) \) on the right-hand side of equation (22), with which they included the term containing the third-order derivatives of \( I'(\lambda) \) in Taylor expansion (see Stenflo 1994, p. 258), in order to correct the calibration deviated from the WFA. The Taylor expansion terms of \( I'(\lambda) \) containing zero and second power of \( \Delta \lambda \) are subtracted in \( V' = S'_e - S'_i \).

For the spectral line \( \text{Fe} I \) 5324.19 Å \((g^2 = 1.5)\) at a temperature of 6000 K, the Doppler width is about 24 mÅ, which corresponds to the magnetic flux density of 1200 G. Jefferies et al. (1989) proposed that the ‘WFA requires the Zeeman splitting is less than the Doppler width and the sufficient condition is less than half of the Doppler width’. Jefferies & Mickey (1991) applied the approximation that ‘an accuracy of ~20 per cent for field strengths as great as 3500/g_l G’ where \( g_l \) is the Lande factor and the filter transmission peak is centred on the line wings. For the line we used the WFA’s upper limit of 600–1200 G according to Jefferies et al. (1989), and 2300±470 G according to Jefferies & Mickey (1991). We have compared the SMAT magnetograms with the MDI magnetograms and the VSM magnetograms (we do not intend to present the scatter-point plots in this paper). It is found that the ‘non-linear regime’ of the split points begins at about >800 G. Hence, the WFA of the SMAT is close to the condition given by Jefferies et al. (1989). Of course, deviation from the WFA could be revised by cross-calibration (Chae et al. 2007; Wang et al. 2009). However, this approach depends on the belief that one type of instrument is always more accurate than another.

Finally, we should point out three problems. First, the directly measurable quantity in the filter magnetograph is the intensity difference of two modulated polarization states, \( \Delta S' = S'_e - S'_i \). The quantity \( \Delta S' \) is equal to the shadowed area, which is normalized by the equivalent width of the filter transmission profile, between the curve \( S_e \) and the curve \( S_i \) at some wavelength position (illustration in Fig. 5a). \( \Delta S' \) is not necessarily equal to \( \Delta S \). Secondly, the linear relation between the splitting distance \( \Delta \lambda \) and the magnetic field strength \( B \) is preserved well even for a sunspot field of several thousand gauss, only if the spectral-line splitting still belongs to the Zeeman effect regime. Thirdly, the deduction from observation quantity \( \Delta S' \) to the Zeeman splitting \( \Delta \lambda \) is the calibration purpose for the filter magnetograph. It is proved in Appendix A that the Zeeman splitting \( \Delta \lambda \) can take place with \( \Delta \lambda' \) when the higher-order residual error is neglected.

In our opinion, both the true solar intensity profile and the intensity profile convoluted by the instrument transmission profile can be applied to the calibration process. If the latter is used, then \( \Delta S' \) should be used together. If the true solar intensity profile is used for the calibration, then \( \Delta S \) should be used. The mixture using the
Figure 6. The solid line in Fig. 6(a) is the solar centre spectrum of the spectrometer from atlas data. The dotted line is our solar centre scanning spectrum on 2008 February 27. Fig. 6(b) contains two calibration curves, which are calculated according to equation (22), corresponding to the two curves in Fig. 6(a). Please note that the calibration coefficients in the red wing are all negative and we just plot their absolute values here. The normalization value for the dotted line in Fig. 6(a) is 0.87, not the real continuum value. However, we take this value just for the convenience of comparing with the atlas spectrum. The dotted calibration curve in Fig. 6(b) is not affected by this value at all.

true solar intensity profile (or some deconvolution profile) with \( \Delta S' \) in the calibration process will result in the underestimation of the calibration coefficients of the magnetogram.

3.2 Observational sample and calibration curve

We emphasize the general calibration formula by using all the quantities measured by the filter magnetograph. In the calibration equation (22), using the scanning spectrum or using the spectrometry spectrum, the results become very different. Fig. 6(a) is the normalized Stokes \( I \) profile in the vicinity of the spectral-line core. The solid line has been downloaded from the Spectral Atlas in Bass2000 (http://bass2000.obspm.fr/solar_spect.php) and the dotted line has been scanned by the SMAT. In fact, not all the scanning profile is as good as this dotted line in Fig. 6(a). This is because the design of the SMAT has not included the automatic scanning profile function, and the solar luminosity or instrument state will change during the manual wavelength scanning. The two terms \( \partial \ln I'/\partial \lambda \) and \( V'/I' \) are not affected by this because the ratio largely removes the gain table problem, but the variations from the Sun itself during the scanning still remain. Using the deconvolution profile or the profile from the standard Spectral Atlas in the calibration process (replacing \( \partial \ln I'/\partial \lambda \) by \( \partial \ln I/\partial \lambda \)) in equation (22) might result in the underestimation of the magnetic field as the deconvolution profile is steeper than the scanning profile. Corresponding to the two Stokes \( I \) curves in Fig. 6(a), their calibration coefficients at different wavelength positions are plotted in Fig. 6(b), where the absolute values of the calibration coefficients are plotted to make it easier to compare the blue and red wings. Please note that our normalization coefficient for the dotted curve in Fig. 6(a) is a constant about 0.87 at 0.3 Å, but this normalization coefficient does not affect the dotted calibration curve in Fig. 6(b). It is obvious that the calibration coefficients from the scanning profile are larger than the calibration coefficients from the profile of the Spectral Atlas. In fact, the calibration coefficients given in this paper are about 30 per cent larger than the calibration coefficients given previously by Su & Zhang (2004) for the SMAT. This difference comes from the different intensity profiles used or created by different authors.

However, the Sun has plenty of features that have different Stokes \( I \) profiles. Therefore, the single wavelength position observing mode could not reach the best magnetic sensitivity for all pixels. Thus, the transmission peak should be placed slightly further away from the line core, when the strong sunspot field is measured, to avoid the Zeeman splitting line getting close to or even surpassing the centre of the filter transmission peak. Even the MDI, which samples the signal at five wavelength positions (one filtergram near continuum, two filtergrams centred on the wings and another two filtergrams centred about the line core), still has magnetic saturation. Liu, Norton & Scherrer (2007) concluded that the low intensity in sunspot umbrae coupled with the MDI 15-bit onboard algorithm is the main cause of the saturation seen in MDI magnetograms. The saturation problem in our magnetograms might be produced by the complicated filter transmission profile, which is not narrow enough (FMHW 0.25 mÅ) and has a non-zero side lobe. In principle, the limited sampling calibration curves could not calibrate the whole observing data set. To solve this problem, we might develop two methods in the future. The first is to build a relatively complete and fast-sampling Stokes \( I \) data base for the filter magnetograph, in which the calibration
curves are classified according to the Sun’s features. The second depends on the technical progress of the filter magnetograph. Deng & Zhang (2009) mentioned the eight-channel birefringent filter in the future Chinese Space Solar Telescope (SST) project, in which the eight filters will be fixed on eight bandpasses of one spectral line. If this multichannel idea could be combined with the line ratio method in order to disentangle the filling factor problem (Stenflo 1994, p. 276), then the number of terminal CCD cameras will be increased to 16. In our opinion, it is worth paying more attention to and putting more investment into the real and accurate intensity profile for the filter magnetograph. The more believable the intensity profile, the better the calibration results. The outlook for the filter magnetograph is still promising.

3.3 Correction template

In the previous sections, the main crosstalk term has been calculated and a longitudinal calibration method has been proposed. In this subsection, we calibrate the \( I \)-to-\( V \) crosstalk into gauss and illustrate the correction effect. In Fig. 7, we choose the magnetograms from different wavelength positions: \(-12\) mÅ, \(+12\) mÅ, \(-7\) mÅ and \(+7\) mÅ. The observing times are also different: 2008 December and 2009 June. The SMAT hardware was not changed from 2008 December to 2009 June (for other times, we did change the KD*P conducting glass at least twice). The original magnetograms are shown in the upper panel of Fig. 7 and their corresponding corrected magnetograms are shown in the lower panel. We just use one correction template for all of these. The correction effect in Fig. 7 is strengthened as we limited the grey-scale of all the magnetograms to \( \pm 50\) G. The uncorrected magnetograms appear dark (blue wing) or bright (red wing) because the \( M_{41} \) matrix is negative and the calibration coefficients change sign from blue wing to red wing.

Table 1 lists some calibration coefficients at 10 wavelength positions, and also the minimum and maximum \( I \)-to-\( V \) crosstalk correction corresponding to these observing wavelength positions. From this table, it is clear that \( M_{41} \) is the same matrix for all wavelengths, but ‘the crosstalk magnetic field’ is different because of different calibration coefficients. Generally, the magnetic field error due to \( M_{41} \) is a systematic error and its amplitude is about 50 G. The larger the calibration coefficient ‘crosstalk’, the more spurious the magnetic field into magnetograms.

4 DISCUSSION AND CONCLUSIONS

In this paper we are concerned with two topics. One topic is the Stokes \( I \)-to-\( V \) crosstalk. We separately employ the integration of spatial domain and wavelength domain to discuss the \( I \)-to-\( V \) crosstalk. We analyse the information in one domain at the expense of losing the information in the other. Therefore, using integration is just a compromise in the case of low signal-to-noise ratio. Strictly speaking, the instrument polarization of the telescope system should be analysed on the laboratory platform through the illumination of the particular instrument with spatially ‘flat’, spectrally ‘flat’ and polarization ‘flat’ light. However, it is not a simple problem and it is hard for us to achieve now. The other topic in this paper is the calibration of the longitudinal magnetic field based on the scanning spectrum. Both topics contain one idea that we try to improve the magnetogram calibration for the filter magnetograph by increasing its applicable spectral information.

Other important problems about the full-disc longitudinal magnetogram measured by the filter magnetograph are not mentioned in this paper. One of these is the spatial distribution of calibration coefficients on a full disc. We only use one calibration coefficient (not a matrix) on the full disc in this paper. It is correct for a...
local FOV magnetogram, but incorrect for a wide FOV magnetogram. Comparing the $I$-to-$V$ crosstalk corrected magnetograms of the blue wing with that of the red wing in the lower panel of Fig. 7, it is clear that the red wing and blue wing network fields obviously and potentially appear in different disc regions. When the movie from the blue wing to red wing of different wavelength positions is displayed, it is easy to understand that this problem is a result of solar rotation. Hence, the further process of the magnetogram should remove the effect of full-disc Doppler rotation (solid rotation and differential rotation) in the process of full-disc calibration. Then, magnetograms of three to five wavelength positions should be combined together to acquire a new magnetogram. Therefore, the final time resolution will become lower. The full-disc transverse field problem could be considered when the longitudinal field problem is solved to some extent. We may discuss or solve these topics in our future work.

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APPENDIX A: EQUIVALENT SPLITTING UNDER THE WFA

We prove a conclusion given in Section 3.1 that the Zeeman splitting $\Delta \lambda$ deduced from the true solar Stokes $I$ profile is equal to the splitting $\Delta \lambda'$ deduced from the Stokes $I$ profile convoluted by the instrument transmission profile if we limit this discussion to the

Table 1. The SMAT calibration coefficients of the longitudinal magnetic field. Units are gauss.

<table>
<thead>
<tr>
<th>Calibration coefficient</th>
<th>Blue wing</th>
<th>$-0.15 , \AA$</th>
<th>$-0.12 , \AA$</th>
<th>$-0.10 , \AA$</th>
<th>$-0.05 , \AA$</th>
<th>$-0.02 , \AA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{41}$ correction</td>
<td>24 806</td>
<td>19 728</td>
<td>16 402</td>
<td>14 316</td>
<td>30 568</td>
<td></td>
</tr>
<tr>
<td>Red wing</td>
<td>$+0.02 , \AA$</td>
<td>$+0.05 , \AA$</td>
<td>$+0.10 , \AA$</td>
<td>$+0.12 , \AA$</td>
<td>$+0.15 , \AA$</td>
<td></td>
</tr>
<tr>
<td>$M_{41}$ correction</td>
<td>$-26 764$</td>
<td>$-12 386$</td>
<td>$-12 249$</td>
<td>$-16 393$</td>
<td>$-21 197$</td>
<td></td>
</tr>
</tbody>
</table>
WFA. For simplicity, we take $2 \cos \gamma = 1$, which has no effect on the final conclusion.

\[ I'(\lambda) = I \ast T = T \ast I = \int_{\lambda_1}^{\lambda_2} T(\lambda_0) I(\lambda - \lambda_0) d\lambda_0, \quad (A1) \]

where we use the commutative theorem of convolution (Bracewell 2000, p. 117)

\[ \Delta I(\lambda) = I(\lambda + \Delta \lambda_H) - I(\lambda), \quad (A2) \]

\[ \Delta I'(\lambda) = \int_{\lambda_1}^{\lambda_2} T(\lambda_0) \left\{ [I(\lambda + \Delta \lambda_H - \lambda_0) - I(\lambda - \lambda_0)] - \Delta \lambda_H \frac{\partial I}{\partial \lambda} \right\} d\lambda_0, \quad (A3) \]

When the higher-order small quantities $o(\Delta \lambda)$ are neglected, the difference and the derivative have the following relation

\[ \Delta \lambda_H - \Delta \lambda_H^* = \Delta I \left( \frac{\partial I}{\partial \lambda} \right)^{-1} - \Delta I' \left( \frac{\partial I'}{\partial \lambda} \right)^{-1} \]

\[ = \left[ \Delta I \frac{\partial I'}{\partial \lambda} - \Delta I' \frac{\partial I}{\partial \lambda} \right] \left[ \frac{\partial I}{\partial \lambda} \frac{\partial I'}{\partial \lambda} \right]^{-1}, \quad (A4) \]

where the curve of Stokes $I$ should have a first-order non-zero derivative. The deconvolution of Stokes $I'$ should be considered when it is suspected that the right-hand side of equation (A4) is not zero.

\[ \Delta I \frac{\partial I'}{\partial \lambda} = [I(\lambda + \Delta \lambda_H) - I(\lambda)] \int_{\lambda_1}^{\lambda_2} T(\lambda_0) \frac{\partial I(\lambda - \lambda_0)}{\partial \lambda} d\lambda_0, \quad (A5) \]

\[ \Delta I \frac{\partial I}{\partial \lambda} = \frac{\partial I(\lambda)}{\partial \lambda} \int_{\lambda_1}^{\lambda_2} T(\lambda_0) \left\{ [I(\lambda + \Delta \lambda_H - \lambda_0) - I(\lambda - \lambda_0)] - \Delta \lambda_H \frac{\partial I}{\partial \lambda} \right\} d\lambda_0, \quad (A6) \]

\[ \Delta I' \frac{\partial I}{\partial \lambda} - \Delta I' \frac{\partial I}{\partial \lambda} = \int_{\lambda_1}^{\lambda_2} T(\lambda_0) M d\lambda_0, \quad (A7) \]

\[ M = [I(\lambda + \Delta \lambda_H) - I(\lambda)] \lim_{\Delta \lambda \to 0} \frac{I(\lambda - \lambda_0 + \Delta \lambda) - I(\lambda - \lambda_0)}{\Delta \lambda} \]

\[ - [I(\lambda - \lambda_0 + \Delta \lambda_H) - I(\lambda - \lambda_0)] \lim_{\Delta \lambda \to 0} \frac{I(\lambda + \Delta \lambda) - I(\lambda)}{\Delta \lambda}, \]

where we use the physical approximation $\lim_{\Delta \lambda \to 0} \Delta \lambda \approx \Delta \lambda_H$ to take the place of the ideal mathematical limit, when $\Delta \lambda_H \ll \Delta \lambda$ Doppler is satisfied. Then

\[ M = [I(\lambda + \Delta \lambda_H) - I(\lambda)] \]

\[ \times \lim_{\Delta \lambda \to 0} \frac{I(\lambda - \lambda_0 + \Delta \lambda) - I(\lambda - \lambda_0)}{\Delta \lambda} \]

\[ - [I(\lambda - \lambda_0 + \Delta \lambda_H) - I(\lambda - \lambda_0)] \lim_{\Delta \lambda \to 0} \frac{I(\lambda + \Delta \lambda) - I(\lambda)}{\Delta \lambda}. \]

Hence,

\[ \Delta \lambda_H - \Delta \lambda_H^* = 0. \quad (A8) \]

In equation (A8), we suppose $\Delta \lambda_H \ll \Delta \lambda$ Doppler in physical significance is equivalent to $\Delta \lambda \rightarrow 0$ in mathematical significance (the difference operation taking the place of the differential operation). However, we often choose the approximation of this limit as $\Delta \lambda_H \leq 2/3 \Delta \lambda$ Doppler, which is equal to 800 G for this spectral line in our data reduction. Here, we should explain the existence of the non-zero derivative of the Stokes $I$ profile. The computer code performs numerical differentiation using three-point Lagrangian interpolation. In general, the zero derivative seldom appears in the data process even in the spectral-line centre for which the derivative is very small and the calibration coefficient is very large (the calibration coefficient at the line centre is set to be zero in Fig. 6b). The derivative of the Stokes $I$ profile can become large when the intensity varies significantly within two wavelength sampling intervals (in our case 0.01 Å). This could happen during solar flares when the intensity profile becomes the emission spectrum and closes the saturation value of CCD well depth capacity. Therefore, the above discussion is limited to the absorption spectrum and the WFA. Then, the accuracy of the Stokes $I$ derivative is decided by the accuracy of the intensity measurement and the wavelength resolution.

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