Models of young powerful radio sources

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ABSTRACT

Observations of compact symmetric double sources suggest that these objects with physical scales of order tens of parsecs to kiloparsecs are young radio active galactic nuclei. There is, in general, a striking similarity between the structures of these compact objects and the structures of large evolved radio galaxies although the latter are two to three orders of magnitude larger. This has led to the use of self-similar models of the evolution of radio sources as a framework for interpreting observational data. However, the assumptions on which the self-similar models are based become increasingly less valid on the physical scales which are probed by the observations of the smallest sources. In this paper, the dynamics of sources on these small scales is examined and a model developed which extends earlier work in a self-consistent way to small physical scales. The limit of applicability of the models is identified as is the transition from an early evolutionary phase to the self-similar phase of expansion.

Key words: hydrodynamics – galaxies: active – galaxies: jets.

1 INTRODUCTION

Understanding young radio-loud active galactic nuclei (AGN) is important both as a tool to study the onset of AGN activity and also the effects the expanding radio source may have on its host galaxy. Young radio sources are often selected on the basis of their spectral shape: gigahertz-peaked spectrum (GPS) and compact steep spectrum (CSS) radio sources are commonly believed to represent the earliest stages of the evolution of powerful radio sources (e.g. Bicknell et al. 2003; Snellen et al. 2004; Polatidis & Conway 2003; Gugliucci et al. 2005). Young sources are also selected on the basis of morphology and are then called Compact Symmetric Objects (CSO; Wilkinson et al. 1994). The radio structures of these sources are very similar to the large-scale Fanaroff–Riley type II (FR II) sources although they are two or three orders of magnitude smaller in physical size. These observations have been successfully explained by self-similar models for the evolution of the radio source. However, on scales smaller than a kiloparsec the assumptions on which the self-similar models are based become increasingly less valid. In this paper, the dynamics of sources on these scales is examined and a model developed which extends earlier work in a self-consistent way to small physical scales.

Self-similar expansion was suggested by Begelman (1996) who conjectured, based on analysis of the results of numerical simulation, that the ratio of hotspot to cocoon pressure should be a constant factor. This conjecture is verified in the detailed simulations of Carvalho & O’Dea (2002a,b). The approach of Begelman has been extended by Bicknell and co-workers (e.g. Bicknell, Dopita & O’Dea 1997) to allow for expansion losses and to address the interaction of the source with its environment (e.g. Bicknell, Saxton & Sutherland). Kaiser & Alexander (1997, hereafter KA) and Kaiser, Dennett-Thorpe & Alexander (1997) extended the earlier analysis of Falle (1991) to develop a fully self-consistent model for self-similar evolution of FR II sources. In their analysis, the cocoon itself confines the jet thereby ensuring a constant ratio between hotspot and cocoon pressure. Alexander (2000) analysed the evolution from kpc-to-Mpc-scales and showed that the expansion remains self-similar in passing from the atmosphere of the host galaxy to the surrounding medium. The evolution of sources on subkpc scales must be more complicated than the simple self-similar models predict. A characteristic length-scale exists which depends on the external density and energy and mass-flow rates within the jet (Falle 1991, KA). This length-scale must play a role in the description of the source evolution on small physical scales.

In this paper, I extend the analysis of KA into a regime in which the self-similar analysis is no longer valid. The basic physical assumptions on which the KA models are based are, however, retained. In Section 2, I introduce the basic physical model and discuss the regimes which must be considered. In Section 3, I derive a set of model equations and find approximate solutions in limiting cases. The transition to the self-similar regime is considered in Section 4. Finally in Section 5, I discuss the dynamical evolution of radio sources in the light of this analysis.

2 PRELIMINARY CONSIDERATIONS

From here on, I adopt the notation used in Fig. 1. Subscripts j, c, h, s and x refer to the jet, cocoon, hotspot, swept-up gas and external gas, respectively. The model developed by KA had a number of
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assumptions about the nature of the jet. The most important of these assumptions are as follows.

(i) The source is characterized by an initially un-collimated jet of constant opening angle ($\theta$), mass-flow rate ($M$) and jet kinetic power ($Q$); furthermore the internal energy flux of the jet can be neglected with respect to the kinetic power.

(ii) All bulk velocities are sufficiently small to permit a non-relativistic description of the gas dynamics although a relativistic equation of state for the jet and cocoon material is permitted.

(iii) The Mach number of the jet with respect to both the internal sound speed of the jet material and the external gas is large.

(iv) The source expands into an isotropic medium characterized by a smoothly varying density profile [simply taken to have the form of a power law $\rho_x = \rho_0(r/a_0)^{-\beta}$].

In this paper, these assumptions about the basic nature of the jet and surrounding medium will be retained.

The constants of the problem define a characteristic length-scale which plays a central role in the analysis of the evolution of the radio source:

$$L_1 \sim \left( \frac{\rho_0^3 a_0^2 Q}{M^3} \right)^{1/(2\beta-2)}.$$

As we shall see, this length-scale is small compared to the size of the host galaxy and therefore it is appropriate to determine $L_1$ in a constant density atmosphere with $\beta = 0$ and density $\rho_0 = \rho_0$. $L_1$ can then be re-written by noting that the constant mass-flow rate and kinetic power are simply related by $Q = \frac{1}{2} M v_j^2$ where $v_j$ is the (constant) jet speed in the un-collimated flow:

$$L_1 \propto \left( \frac{Q}{\rho_0 v_j^2} \right)^{1/2} = 2\sqrt{2} \left( \frac{Q}{\rho_0 v_j^2} \right)^{1/2}.$$

where the factor of $2\sqrt{2}$ is chosen for later convenience. In terms of typical values for the jet power, speed and external density, this characteristic length-scale is given by

$$\frac{L_1}{pc} = 25 \left( \frac{Q}{10^{39}W} \right)^{1/2} \left( \frac{n_x}{3 \times 10^5 \text{m}^{-3}} \right)^{-1/2} \left( \frac{v_j}{c} \right)^{-3/2},$$

where $n_x$ is the number density of protons in the external gas and the fiducial value of $3 \times 10^5 \text{m}^{-3}$ is chosen to be typical of the warm-ionized interstellar medium in a galaxy. Clearly, this is small compared to the extent of the host galaxy. It is important to consider the physical meaning of this length-scale. For a maximum extent of the source $D$, the cross-sectional area of the un-collimated jet is $D^2\Omega$, where $\Omega$ is the solid angle of the jet ($\Omega \approx \pi\theta^2$). The jet kinetic power is then given by $Q = \frac{1}{2} \rho_1 v_j^2 \Omega D^2$, where $\rho_1$ is the density of the jet at a distance $D$, and

$$L_1^2 = 4\Omega D^2 \frac{\rho_1}{\rho_0}.$$

Since the mass-flow rate and jet velocity are constant, $\rho_1$ at the end of the jet falls as $1/D^2$. When the source size is comparable to $L_1$, the jet density is given by

$$\rho_1 = \frac{\rho_0}{4\Omega}.$$

Therefore for scales less than $L_1/2\Omega^{1/2} \sim L_1$, the jet density exceeds the external density and vice versa. The evolution of the source will be very different on scales very much larger than $L_1$, very much smaller than $L_1$, and on scales comparable to $L_1$.

The situation when the source size is very much larger than $L_1$ is precisely the case considered in KA. In this limit, we do not expect the evolution of the source to depend on $L_1$. An equivalent statement is that the mass swept up by the radio source greatly exceeds the mass injected by the jet and therefore the constant $M$ cannot enter into the solution. From the remaining dimensional quantities (the constants $Q$, $\rho_0 a_0^2$, the source size $D$ and the time $t$), we find from dimensional analysis that

$$D \propto \left( \frac{Qt}{\rho_0 a_0^2} \right)^{1/(5-\beta)} = c_1 a_0 \left( \frac{t}{\tau} \right)^{\frac{3}{3(5-\beta)}},$$

where $\tau = (a_0^2 \rho_0/Q)^{1/3}$ (Fall 1991, KA). KA went on to show that both the bow shock and cocoon expand self-similarly in this regime and that the jet comes into pressure balance with the cocoon and is approximately collimated during its passage through the lobe. This feedback is the crucial factor in obtaining self-similar expansion. The cocoon is able to collimate the jet in this way since the cocoon pressure balances the sideways ram-pressure of the jet (i.e. the component of the momentum flux of the jet which is perpendicular to the axis of the jet). An oblique, or re-collimation, shock is driven into the jet.

Conversely, the limit when the source size is less than $L_1$ corresponds to the situation when the jet is overdense with respect to the surrounding medium and the jet then propagates approximately ballistically. Assuming the jet velocity is supersonic with respect to the external gas, the expanding jet will be preceded by a bow shock. A jet shock, and hence hotspot, will be formed provided the jet speed is supersonic in the instantaneous rest frame of the advancing contact surface between jet material and the external gas: the contact surface expands at speeds only slightly less than the jet speed. This condition for the formation of a jet shock is in fact met as I will discuss in Section 5. During this ballistic phase, almost all the energy passing along the jet goes into the work done by the bow shock plus the stored internal energy of the hotspot.

A very different situation exists when the source size is comparable to and larger than $L_1$. The jet remains un-collimated with a constant opening angle and therefore the jet density continues to decrease. When the source is of size $L_1$, the jet density is comparable to the external density and quickly becomes small compared to the external density once the source is a few times of $L_1$ in size. We now expect a cocoon to form: the cocoon, rather than hotspot, dominates.
the energetics of the expansion once the stored energy in the cocoon greatly exceeds the stored energy in the hotspot. In Section 3, I will develop a model for this phase of the evolution and determine the point at which this condition is met. The evolution of the source will also depend explicitly on $L_1$, or equivalently $M$ or $v_i = \sqrt{2Q/M}$. A necessary condition for the cocoon to drive a re-collimation shock is that the cocoon pressure must exceed this sideways momentum flux at the end of the jet. For the un-collimated jet, the sideways ram-pressure of the jet perpendicular to the jet axis is given by $\rho_j v_j^2 \sin^2 \theta \propto D^{-2}$ since the density decreases as $D^{-2}$. For self-similar expansion, KA found that the cocoon pressure varies with source size as $p_c \propto D^{-(4+\beta)/3}$. For all $\beta < 1$, the cocoon pressure increases more slowly with decreasing source size than the sideways ram-pressure of the jet – there must therefore be a stage in the early development of the source where the cocoon pressure is insufficient to collimate the jet. I will show below that this occurs on a scale of several times $L_1$. The evolution of the source on small scales will therefore be that of a source in which the jet propagates as an un-collimated flow of constant opening angle, mass-flow rate and kinetic power. This situation closely resembles that originally considered by Scheuer (1974). In Section 3, I will develop the governing equations for the evolution of the source.

3 DEVELOPMENT OF THE MODEL

3.1 Model equations

In this section, I develop a model for the dynamical evolution of the radio source in the regime where the physical size, $D$ is comparable to, or larger than, $L_1$. For $D \ll L_1$, the source size is of the order of a few parsecs or less and on these scales relativistic effects are almost certainly important and the assumptions used here are very unlikely to apply. For $D \ll L_1$, we are in a regime in which the cocoon is just beginning to form and the hydrodynamics of the source is complex. I therefore concentrate on the regime $D \gg L_1$; this is an interesting regime as it probes a range of physical sizes accessible to current radio imaging.

An important simplifying assumption is to assume that the sound speeds in the cocoon are sufficiently large that the pressure throughout the cocoon is uniform. This assumption can also be adopted for the hotspot. The energy passing up the jet goes into stored energy within the radio source plus doing $p \, dV$ work on the environment. Applying the first law of thermodynamics in the form

$$Q \, dj = \left[ d(\alpha V) + p_c \, dV_c \right] + \left[ d(\alpha V) + p_h \, dV_h \right]$$

gives for the energy balance equation for the radio source:

$$Q = \frac{1}{\gamma - 1} V_c \frac{dp_c}{dr} + \frac{\gamma_c}{\gamma - 1} p_c \frac{dV_c}{dr} + \frac{1}{\gamma - 1} V_h \frac{dp_h}{dr} + \frac{\gamma_c}{\gamma - 1} p_h \frac{dV_h}{dr}$$

(6)

where the equation of state for the cocoon and hotspot are assumed to be the same.

Since the Mach number of the jet with respect to both itself and the external gas are assumed to be large, the jet terminates in a shock giving rise to a hotspot; a bow shock is also driven into the external gas. There must be pressure balance at the contact surface between the material which has passed up the jet and the shocked external gas. I define the hotspot pressure to be the (possibly time-averaged) pressure over the blunt contact surface at the end of the jet. Provided the jet is cold, a strong jet shock will always form. In the instantaneous rest frame of the contact surface, we find that:

$$p_h = \alpha_j \frac{2}{\gamma_j + 1} \rho_j (v_j - D)^2$$

$$= p_s$$

$$= \alpha_x \frac{2}{\gamma_x + 1} \rho_x D^2$$

(7)

The factors $\alpha_j$ and $\alpha_x$ allow for the possibly complex flow between the two shocks and the contact surface. For a simple planar geometry, KA applied the Bernoulli equation along a streamline from the post-shock gas to the stagnation point on the contact surface and hence show that $\alpha(\gamma) = \frac{1}{\gamma \gamma - 1} \gamma^2 (\gamma-1)$, which is a factor very close to unity for all reasonable values of $\gamma$.

To make further progress, additional assumptions are required. A common assumption is that the cocoon is overpressured with respect to the external gas. This is likely to be a reasonable approximation for the early stages of the evolution of powerful radio sources. Alexander (2002) and Kaiser & Cotter (2002) discussed the behaviour of sources when this approximation is no longer valid which they assume to occur after a self-similar phase of expansion.

In developing their self-similar model, KA used dimensional arguments to find an explicit expression for $p_h$ which they then show is consistent with the cocoon confining the jet as discussed above. For small scales, an explicit relationship between the cocoon volume and pressure is required. Scheuer (1974) introduced such an approach by making the approximation that the bow shock to the sides of the cocoon remained close to the contact surface. This approximation enables the sideways expansion of the cocoon to be determined explicitly by equating the cocoon pressure to post-shock conditions of the bow shock (or alternatively the sideways ram-pressure of the cocoon). By making the same approximation, solutions for the present case can also be found in much the same way as in Scheuer (1974).

Equation (7) may be solved to find the hotspot advance speed $D$:

$$D \left[ 1 + \frac{\gamma_j + 1 \alpha_x}{\gamma_x + 1 \alpha_j} \left( \frac{\rho_h}{\rho_j} \right)^{1/2} \right] = v_j$$

(8)

but $\rho_j = 2Q/A_j v_j^3 = 2Q/D^2 v_j^3$, hence

$$\left( \frac{\rho_h}{\rho_j} \right)^{1/2} = \frac{\Omega \rho_x v_h^3}{2Q} D = 2Q^{1/2} D \frac{L_1}{L}$$

(9)

Introducing the constant $K_1$:

$$K_1 = 2Q^{1/2} \left( \frac{\gamma_j + 1 \alpha_x}{\gamma_x + 1 \alpha_j} \right)^{1/2}$$

(10)

the differential equation for $D$ becomes

$$\frac{D}{L_1} \left[ 1 + K_1 \frac{D}{L_1} \right] = v_j$$

(11)

which can be integrated to give

$$\frac{D}{L_1} = \frac{1}{K_1} \left[ \frac{2K_1 v_j L_1}{L_1} + 1 \right]^{1/2} - 1$$

(12)

This result holds, within the context of the model, for all $D$. For $D \ll L_1$, we find $D \approx v_j t$, that is, the jet expands ballistically (Section 2), whereas for $D \gg L_1$, we find $D^2 \approx 2L_1 v_j t / K_1$ in agreement with Scheuer (1974).
The cocoon is taken to have cylindrical symmetry and the sideways (radial) expansion is assumed to be given by simple ram-pressure confinement \( \rho c^2 \sim \rho_0 \), where \( \rho_0 \) is the cocoon pressure. The radius of the cocoon at a distance \( z \) from AGN is then given by:

\[
r = \int_{r=0}^{\rho_c} r \, dr = \int_{\rho=0}^{\rho_c} \left( \frac{\rho_c}{\rho_0} \right)^{1/2} \, d\rho
\]

and the cocoon volume follows by integration along the extent of the cocoon

\[
V_c = \int_0^D \pi r^2 \, dz.
\]

To complete the model specification, a description for the hotspot is required. The hotspot pressure follows from the shock conditions at the jet shock:

\[
p_h = \frac{2\alpha_2}{\gamma_2 + 1 \left[ (K_1 D/L_1) + 1 \right]} p_0 v_j^2.
\]

The volume of the hotspot is more difficult to define – the high-pressure region at the termination of the jet will be determined by the detailed flow of plasma as it enters the cocoon. The extent of the hotspot perpendicular to the source is likely to scale as the radius of the jet at the jet shock which is of the order of \( D^{1/2} \). The distance between the jet shock and the contact surface is determined by the complex flow near the hotspot. Hence, I assume a volume for the hotspot given by:

\[
V_h \approx c_0 \Omega^{3/2} D^{2+\delta} L_1^{1-\delta}.
\]

where \( c_0 \) and \( \delta \) are constants; we expect \( c_0 \ll 1 \) and \( 0 \leq \delta \leq 1 \). The set of equations (6)–(16) provide a complete description of the dynamical model. The jet is characterized by its kinetic power, \( Q \), speed \( v_j \), and solid angle, \( \Omega \); the external atmosphere in this case is characterized completely by its density, \( \rho_0 \).

### 3.2 Approximate solution

I now seek an approximate solution of this set of equations and begin by considering the two limiting cases \( D \gg L_1 \) and \( D \ll L_1 \). At both of these limits, we are in regimes in which the physical model no longer applies, however, it is still valid to seek an algebraic solution and then to consider the range of \( D \) for which this solution is physically acceptable. For \( D \ll L_1 \), we are in the ballistic regime as already discussed and for large \( D \) there is a transition to the self-similar regime.

I begin by making a further approximation which will be valid for some range of \( D \); how important is the hotspot as an energy reservoir? The stored energy within the hotspot scales as:

\[
U_h \sim p_h V_h \sim \frac{1}{[(K_1 D/L_1) + 1]^{\gamma_2/2}} p_0 v_j^2 \Omega^{3/2} D^{2+\delta}.
\]

It follows that the \( U_h \propto D^2 \) for \( D \gg L_1 \). The total energy injected after a time \( t \) is \( E_t = Q t \), and writing

\[
t = \frac{L_1}{2K_1 v_j} \left( \frac{K_1 D}{L_1} + 1 \right)^2 - 1,
\]

we see that \( E_t \propto D^2 \) for \( D \gg L_1 \) and \( E_t \propto D \) for \( D \ll L_1 \). For \( D \gg L_1 \), the energy stored in the hotspot increases less quickly than the total energy input, and therefore there is a value of the source size beyond which it is valid to make the approximation that the hotspot can be neglected as an energy reservoir. For \( D \ll L_1 \), the energy stored in the hotspot can never be neglected.

The energy balance described by equation (6) is therefore approximated by neglecting the contribution from the hotspot which will be valid physically for some range of \( D \) to be determined. The energy balance then becomes

\[
Q = \frac{1}{\gamma_2 - 1} \frac{p_h}{c_0} \frac{d \rho_c}{dr} + \frac{\gamma_2}{\gamma_2 - 1} \rho_c \frac{d V_c}{dt}.
\]

The limit \( D \gg L_1 \) corresponds to that by Scheuer (1974). In this case, \( D \approx L_1 v_j / K_1 D \) and \( D \approx (2L_1 v_j / K_1)^{1/2} \), where

\[
\begin{align*}
\dot{Q} &\approx \frac{Q}{v_j L_1^{3/2}} D^{-3/2}, \\
p_c &\approx \alpha K_1 \frac{Q}{v_j L_1^{3/2}} D^{-3/2},
\end{align*}
\]

and the constants are given by:

\[
\begin{align*}
a_1 &= \left( \frac{32\pi}{63} \right)^{1/2} \left( \frac{\gamma_2 - 1}{2\gamma_2 - 1} \right)^{1/2} \left( \frac{\alpha_1 (\gamma_1 + 1)}{\alpha (\gamma_1 + 1)} \right)^{3/4}, \\
a_2 &= \left( \frac{63}{2\pi} \right)^{1/2} \left( \frac{\gamma_2 - 1}{2\gamma_2 - 1} \right)^{1/2} \left( \frac{\alpha_1 (\gamma_1 + 1)}{\alpha (\gamma_1 + 1)} \right)^{-1/4}.
\end{align*}
\]

These correspond precisely to the expressions found by Scheuer (1974) in his model A.

The algebraic solution of the problem for \( D \ll L_1 \) can be found in a similar fashion except that now \( D \approx v_j t \) and I find in this case

\[
V_c = \alpha_3 L_1 D^2,
\]

\[
p_c = \alpha_4 \frac{Q}{v_j L_1},
\]

\[
a_3 = \left( \frac{\pi}{30} \right)^{1/2} \left( \frac{\gamma_2 - 1}{2\gamma_2 - 1} \right)^{1/2},
\]

and

\[
a_4 = \left( \frac{\pi}{12} \right)^{1/2} \left( \frac{\gamma_2 - 1}{2\gamma_2 - 1} \right)^{1/2}.
\]

I have not been able to find a solution to the model equations in the general case, but a good approximate solution can be found as follows. The two limiting cases can be combined into single continuous expressions by postulating for \( V_c \) and \( p_c \) a solution of the form:

\[
V_c = \alpha \Omega^{3/2} L_1 \left( \frac{D}{L_1} \right)^{2 \beta} \left( \frac{\gamma + D}{L_1} \right)^{3/2}
\]

and

\[
p_c = \kappa \Omega^{-1/4} \frac{Q}{\gamma_1 L_1} \left( \frac{D}{L_1} \right)^{1-\beta} \left( \frac{\gamma + D}{L_1} \right)^{-1/2}.
\]
Physically, we expect the cocoon volume and pressure to be a continuous and smooth and continuous function of the source size $D$ and therefore I adopt this form for an approximate solution. Matching this form to the limiting cases required $\alpha = a_1, \kappa = a_2, \beta = (a_3/a_1)^{3/2} \Omega^{-1/2}$ and $\lambda = (a_2/a_3)^{2} \Omega^{-1/2}$. It is then straightforward (although tedious) to show that this approximate solution when substituted into the energy equation (17) is accurate to better than 3 per cent for all $D$.

For numerical estimates, I will assume: a relativistic equation of state for the cocoon $\gamma_j = 4/3$; for the external gas $\gamma_x = 5/3$; a jet opening angle of $10^\circ$ giving $\Omega \approx 0.1$.

This solution is valid for source sizes such that $p_0 V_j$ significantly exceeds $\rho_0 V_c$. It is straightforward to verify that the stored energy in the cocoon exceeds that in the hotspot by an order of magnitude for $\delta \approx 0$. I conclude this solution is a reasonable physical model for all $D > L_1$.

4 THE TRANSITION PHASE

I now consider the transition to self-similar evolution. KA showed that it is the self-re-confinement of the jet by the cocoon pressure which leads to self-similar expansion of the radio source. The cocoon will be able to drive a re-collimation shock into the jet when the cocoon pressure is comparable to the transverse momentum flux of the jet. The analysis of the previous section shows that for large source size, $D$, the cocoon pressure decreases as $D^{-3/2}$. The transverse momentum flux of the jet is of the order of $\rho_1 v_j^2 \theta^2$, where $\theta$ is the half-angle of the conical jet – since $v_j$ and $\theta$ are constant this decreases as $D^{-1}$ as the source expands, that is the transverse momentum flux decreases faster than the cocoon pressure. The source must therefore evolve until a size is reached such that the cocoon pressure can re-collimate the jet – I now obtain an expression for when this occurs.

The collimation condition is that the jet pressure at positions in the jet after the re-collimation shock must equal the cocoon pressure. The re-collimation shock must be an oblique shock; a detailed analysis of the geometry of the shock given in Falle (1991) and Komissarov & Falle (1998); here I use the approximate result as used by KA (see also Falle 1991), which treats the re-collimation shock as an oblique shock which makes an angle $\theta$ to the source axis - that is, equal to the half-angle of the jet (Fig. 2). Assuming that the re-collimation starts at a distance $z_1 = \alpha D$ from the AGN then the post-shock jet pressure will be given approximately by:

$$p_l = \frac{2}{\gamma_j + 1} \rho_0(z_1) v_j^2. \quad (28)$$

In this approximation, the start of the re-collimation shock must be when $\alpha \leq 1/2$. The jet density at $z_1$ is $\rho_j(z_1) = 2Q/\rho_0 v_j^3$; since $\theta^2 = \Omega/\pi$, we find

$$p_l = \frac{2}{\gamma_j + 1} \frac{2Q}{\pi a_1^2 D^2 v_j}. \quad (29)$$

Equating this to the expression for the cocoon pressure found above (equation 27), we obtain after a little algebra a quadratic in $D/L_1$ which has the solution:

$$\frac{D}{L_1} = 2 \left( \frac{6}{7\pi} \right)^{2} a_2^{-2} \alpha^{-4} \Omega^{1/2} \left( 1 + \sqrt{1 - \frac{49\pi^2 a_1^2}{36a_2^2 \Omega}} \right) \approx 8 \left( \frac{\alpha}{1/3} \right)^{-4} \Omega^{1/2} \left( \frac{\Omega}{0.1} \right)^{1/2} \times \left( 1 + \sqrt{1 + 0.6 \left( \frac{\alpha}{1/3} \right)^{-4} \left( \frac{\Omega}{0.1} \right)^{-1}} \right) \approx 16 \left( \frac{\alpha}{1/3} \right)^{-4} \Omega^{1/2} \left( \frac{\Omega}{0.1} \right)^{1/2}, \quad (30)$$

where the standard parameters introduced in Section 3 have been used. Assuming $\alpha \approx 1/3$, the re-collimation of the jet starts when $D \sim 16 L_1$. The fully self-similar evolution will be fully developed on larger scales when more than one shock structure fits within the length of the jet. The hydrodynamics of this stage are complicated, but the strong dependence on $\alpha$ in this expression suggests that the fully self-similar phase is unlikely to be reached until perhaps $D \sim 100 L_1$. The requirement that $D \gg L_1$ in the self-similar phase of the source evolution is clearly satisfied. Furthermore, for any reasonable value of $L_1$, this transition occurs well within the host galaxy justifying the assumption that the external density if constant is the above analysis.

The subsequent evolution of the source during the self-similar phase is very similar to that described by KA and Kaiser et al. (1997), except that the initial conditions are determined during the re-collimation phase. For completeness, I now outline this analysis. As discussed in Section 2, dimensional arguments show that for the self-similar phase $D \propto t^{3/5 - \beta}$: it is useful to write this result in the form.

\[ \frac{D}{L_1} = 2 \left( \frac{6}{7\pi} \right)^{2} a_2^{-2} \alpha^{-4} \Omega^{1/2} \left( 1 - \frac{49\pi^2 a_1^2}{36a_2^2 \Omega} \right) \approx 8 \left( \frac{\alpha}{1/3} \right)^{-4} \Omega^{1/2} \left( \frac{\Omega}{0.1} \right)^{1/2} \times \left( 1 + \sqrt{1 + 0.6 \left( \frac{\alpha}{1/3} \right)^{-4} \left( \frac{\Omega}{0.1} \right)^{-1}} \right) \approx 16 \left( \frac{\alpha}{1/3} \right)^{-4} \Omega^{1/2} \left( \frac{\Omega}{0.1} \right)^{1/2}. \quad (30) \]
form:

\[ D = c_1 L_1 \left( \frac{t - t_1}{\tau_1} \right)^{3/(5 - \beta)}, \]

where \( t_1 \) is a constant required to match the boundary conditions at re-collimation and \( \tau_1 \) is a characteristic time defined by \( \tau_1 = L_1 / v_1 \). It is easy to show that this characteristic time differs from \( \tau \) above and used by KA by a constant numerical factor. As in Section 2.1, the hotspot pressure is defined to be the pressure at the contact surface and the hotspot pressure can be written as in equation (7) by considering the flow from the jet shock and bow shock to the contact surface giving in the present context

\[ p_h = \frac{2}{\gamma_x + 1} \alpha_x D^2 p_0 \left( \frac{D}{d_0} \right)^{-\beta} \]

(31)

\[ = \frac{2}{\gamma_x + 1} \alpha_x \left( v_j - D^2 \rho_j \right), \]

(32)

where \( \rho_j \) is the jet density after the re-collimation shock [\( \rho_j = (\gamma_x + 1) \rho_j / (\gamma_x - 1) \)]. From equation (11) that \( D \sim 16L_1 \), we see from the previous two sections has extended the model of KA into the regime in which the self-similar evolution of the source begins to escape the host galaxy and expand into the intracluster cocoon. The analysis of the previous two sections has extended the model of KA into the regime in which the self-similar evolution of the source begins to escape the host galaxy and expand into the intracluster cocoon. The flow of gas in the jet is assumed to be transonic with respect to the external gas and the jet itself so that a jet shock forms. The jet shock is approximately stationary in the instantaneous rest frame of the contact surface at the end of the source and therefore the fluid in the jet has a velocity relative to the jet shock of \( v_j - D \). As shown in Section 2, the jet density equals the external density when the source size is equal to \( L_1 / 2\Omega^{1/2} \). Therefore the jet density within the un-collimated flow can be written in terms of the external density \( \rho_j = 4\Omega \rho_j (D/L_1)^2 \); the sound speed of the jet can be written as \( c_j = \gamma_j \rho_j / (4\Omega \rho_j) (D/L_1)^{-2} \). The Mach number of the jet fluid as it enters the jet shock is

\[ v_j - D \sim \frac{K_i D/L_1}{1 + K_i D/L_1} \frac{v_j}{\sqrt{\gamma_j \rho_j / \rho_x}} \sqrt{4\Omega L_1 / D}. \]

If a strong jet shock is able to form at some stage of the source evolution it will form at all earlier stages. It follows that the source will always have a well-defined hotspot and the jet material will be thermalized at the strong jet shock.

As the source size approaches \( L_1 \), a significant cocoon forms as the shocked jet material leaves the hotspot. The above analysis assumes that the pressure of the cocoon material exceeds the pressure in the external gas. If this condition is not met then the evolution of the source is markedly different – not only does the cocoon come into pressure balance with the external gas, but also the pressure in the external gas is responsible for driving a re-collimation shock into the jet and a self-similar stage of evolution is not achieved. This situation will be considered in a forthcoming paper.

The evolution of the source on scales greater than \( L_1 \) but before the jet is re-collimated, is similar to Scheuer’s Model A (Scheuer 1974), however, of course this stage of the evolution is on physical scales very much less than Scheuer originally envisaged. The aspect ratio, \( \beta \), defined to be the ratio of the length to the cocoon radius at \( D/2 \) is given approximately by

\[ \beta \approx 3.4 \left( \frac{\Omega}{0.1} \right)^{5/8} \left( \frac{D}{L_1} \right)^{1/4} \].

The aspect ratio decreases slowly as the source grows. This weak dependence of the aspect ratio on source size means that the source will appear to expand approximately self-similarly even before it reaches the true self-similar regime.

As the source expands beyond \( L_1 \), the jet remains un-collimated. As a result the jet density, and hence momentum flux decreases as \( D^{-2} \). The cocoon pressure, however, drops approximately as \( D^{-3/2} \). The expanding source therefore enters the regime where the cocoon pressure is large enough to drive a re-collimation shock into the jet. This process begins when the source is of the order of 16 \( L_1 \) in size although it is unlikely to be complete until the source is considerably larger than this size. Once collimated by the cocoon pressure, the source expands in a self-similar manner. Using typical values, the self-similar phase will occur on scales greater than approximately 350 (\( Q/10^{49} \) W)\(^{1/2} \) pc. All of this evolution has occurred within the environment of the host galaxy. As the source expands still further it begins to escape the host galaxy and expand into the intracluster or intergalactic medium. This phase of the source evolution was considered by Alexander (2000) who showed that the source would continue to expand self-similarly even in the changing density environment. The source will continue to expand until either the jet injection is ended or the effect of the swept up gas and external pressure become important (Alexander 2002).
Calculating the observed luminosity during the evolutionary phase discussed here is not straightforward since synchrotron self-absorption must be considered carefully. A proper calculation requires each of the source components (cocoon, hotspot and jet) to be modelled together with projection effects. A detailed calculation of this form will be presented in a forthcoming paper. Simple scaling relationships, however, can be used to predict the evolution of the source luminosity at frequencies where the emission is optically thin. The optically thin radio luminosity is assumed to arise from synchrotron emitting plasma in which the electron and magnetic field energy density are in equipartition. The optically thin radio luminosity is then given by \( L = \rho c^2 \), which again assumes the dominant contribution to the luminosity is from the cocoon. Using the results of Section 3 gives

\[
P_l \propto \left( \frac{D}{L_1} \right)^{1/4} \left[ \left( \frac{a_2}{a_1} \right)^2 \Omega^{-1/2} + \frac{D}{L_1} \right]^{-7/8} \times \left( \frac{a_3}{a_1} \right)^{3/2} \Omega^{-1/2} + \frac{D}{L_1} \right]^{-3/2}.
\]

When \( D \gg L_1 \) this gives \( P_l \propto (D/L_1)^{1/8} \), whereas the self-similar model gives \( P_l \propto (D/L_1)^{2/3} \) (Alexander 2000).

One prediction of the model which can be compared to experimental data is the hotspot radius. The hotspot size perpendicular to the jet axis is assumed to be proportional to the jet radius. Prior to re-collimation the hotspot size perpendicular to the jet axis is therefore just proportional to \( \Omega^{1/2} D \). Once the jet re-collimates and the source is expanding self-similarly the radius of the jet can be found by considering the point where the re-collimation shock begins, \( z_1 \). The jet and cocoon pressures after the re-confinement shock are given by

\[
\rho = \Omega(\rho_1 z_1) \Omega_j \Omega_0 \Omega, \quad \rho_j = M/v_j^2 \rho_1 ,
\]

and since the mass-flow rate is constant it follows that \( r_j \propto D^{2/3} \) and a similar scaling result is predicted for the hotspot size perpendicular to the jet axis. The transition between these two regimes should occur at sizes of a few hundred parsecs which is the typical size at which the jet first re-collimates. The predictions are in very good agreement with the observed trend in hotspot size presented by Perucho & Marti (2003) using the data from Perucho & Marti (2002) and Hardcastle et al. (1998).

6 CONCLUSIONS

A model for the dynamical evolution of powerful radio sources has been developed which extends the earlier self-similar model of KA to the regime where the flow is no longer self-similar. The model is based on the assumption of an initially un-collimated jet with constant mass-flow rate and energy flux.

Two length-scales are identified which both play key roles in the evolution of a radio source. The characteristic length-scale of the problem is \( L_1 \) which is given by

\[
L_1 = 25 \left( \frac{Q}{10^{36} \text{W}} \right)^{1/2} \left( \frac{v_j}{\Omega_1} \right)^{-1/2} \left( \frac{\Omega}{\Omega_1} \right)^{-1/2} \left( \frac{Q}{10^{36} \text{W}} \right)^{-3/2}.
\]

The physical significance of this length-scale is that for source sizes, \( D \), less than \( L_1 \) the jet is overdense with respect to the external gas and for sizes greater than \( L_1 \) the jet is light. For \( D < L_1 \), the source expands ballistically with at best a modest cocoon: the hotspot is the main store of ‘waste energy’ from the jet. Observationally the source appearance will be dominated by its hotspots.

The second length-scale is the source size at which the jet is re-collimated by the pressure of the cocoon and enters the phase of self-similar expansion. This is given by

\[
L_i \approx 16L_1 \left( \frac{\alpha}{1/3} \right)^4 \left( \frac{\Omega}{0.1} \right)^{1/2}
\]

and re-collimation is likely to be complete on scales of several times this size.

The model developed here gives a complete description of the source evolution on scales from \( L_1 \) until the transition to the self-similar phase. This is an important regime. Observationally, it covers sizes from tens of parsec to approximately one kiloparsec where there are a number of observations of compact symmetric double radio sources. From a theoretical standpoint, this range of physical scales is important since it represents the start of the formation of a double radio source in which the energetics are dominated by the cocoon acting as the store for the waste energy of the jet through to the point where the jet is collimated by the pressure of the cocoon. The model is in good agreement with the available observations.

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