The period–luminosity relation for type II Cepheids in globular clusters

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ABSTRACT
We report the result of our near-infrared observations (JHKs) for type II Cepheids (including possible RV Tau stars) in galactic globular clusters. We detected variations of 46 variables in 26 clusters (10 new discoveries in seven clusters) and present their light curves. Their periods range from 1.2 d to over 80 d. They show a well-defined period–luminosity relation at each wavelength. Two type II Cepheids in NGC 6441 also obey the relation if we assume the horizontal branch stars in NGC 6441 are as bright as those in metal-poor globular clusters in spite of the high metallicity of the cluster. This result supports the high luminosity which has been suggested for the RR Lyr variables in this cluster. The period–luminosity relation can be reproduced using the pulsation equation \( P \sqrt{\rho} = Q \) assuming that all the stars have the same mass. Cluster RR Lyr variables were found to lie on an extrapolation of the period–luminosity relation. These results provide important constraints on the parameters of the variable stars.

Using Two Micron All-Sky Survey (2MASS) data, we show that the type II Cepheids in the Large Magellanic Cloud (LMC) fit our period–luminosity relation within the expected scatter at the shorter periods. However, at long periods (\( P > 40 \) d, i.e. in the RV Tau star range) the LMC field variables are brighter by about one magnitude than those of similar periods in galactic globular clusters. The long-period cluster stars also differ from both these LMC stars and galactic field RV Tau stars in a colour–colour diagram. The reasons for these differences are discussed.

Key words: stars: Population II – stars: variables: other – globular clusters: general – infrared: stars.

1 INTRODUCTION
Type II Cepheids (hereafter T2Cs) are variables in the Cepheid instability strip, but belong to older populations than classical Cepheids. (see Wallerstein 2002, and references therein, for a review). They reside in globular clusters, the thick disc, the bulge and the halo, but not in the thin disc or spiral arms. Based on their periods, they are often separated into BL Her stars (\( P < 7 \) d), W Vir stars (\( 7 < P < 20 \) d) and RV Tau stars (\( P > 20 \) d). The main feature of RV Tau stars is alternating deep and shallow minima. However, the classification and the nature of RV Tau stars are ambiguous. Several authors suggested so-called RV Tau stars include some heterogeneous types of variables. Whilst six objects in globular clusters have been claimed to be RV Tau stars, some authors doubted this classification from both the photometric (Zsoldos 1998) and the spectroscopic point of view (Russell 1998). In this paper, we will not make a strict distinction between RV Tau stars and other T2Cs in clusters.
From previous studies of T2Cs in globular clusters, it is known that they obey period–luminosity relation (PLR) in the visible (BVJ). Harris (1985) and McNamara (1995) claimed the slope of the PLR steepens for periods longer than about log $P = 1$. On the other hand, Pritzl et al. (2003) did not find such a feature for the variables in the globular clusters NGC 6388 and 6441. As Pritzl et al. (2003) noted, many studies of the T2Cs were based on old photographic data, and we need more investigations with modern CCD photometry. In the near-infrared, no studies have so far been reported.

Studies of variable stars in the near-infrared have become more numerous in recent times. For example, many papers have been published on the infrared properties of RR Lyr variables (e.g. Clement et al. 2001; Castellani, Caputo & Castellani 2003). RR Lyr variables also lie in the Cepheid instability strip but are fainter than T2Cs. One of the important motivations for studies of RR Lyr variables is their application as distance indicators. Whilst a larger number of investigations have been devoted to their absolute visible magnitudes, studies in the infrared have some advantages. Longmore, Fernley & Jameson (1986) and Longmore et al. (1990) discovered a well-defined PLR in the near-infrared for the first time. It was suggested that the near-infrared relation is less affected by metal abundance than the visible one, making the near-infrared one a promising distance indicator. This led to further works (Butler 2003; Dall’Ora et al. 2004; Storm 2004; Del Principe et al. 2005). Extensive theoretical studies of the RR Lyr PLR have also been carried out (Bono et al. 2001, 2003; Catelan, Pritzl & Smith 2004; Di Criscienzo, Marconi & Caputo 2004).

In this paper, we report the result of our near-infrared observations for T2Cs in globular clusters and present their PLR in $JHK$, filters. We also compare the PLR with that of RR Lyr variables and that of candidate T2Cs in the Large Magellanic Cloud (LMC).

2 OBSERVATIONS AND RESULTS

2.1 Observations

Data for T2Cs were obtained during our project to observe variables of various types in globular clusters. We used the Infrared Survey Facility (IRSF) 1.4-m telescope and the Simultaneous 3-Colour Imager for Unbiased Survey (SIRIUS) near-infrared camera constructed by Nagoya University and the National Astronomical Observatory of Japan, and sited at the Sutherland station of the South African Astronomical Observatory. Images of a $7.7 \times 7.7$ arcmin$^2$ field of view are obtained simultaneously in $JHK_s$. The seeing size was typically 1.5 arcsec. For details of the IRSF and SIRIUS, see Nagashima et al. (1999) and Nagayama et al. (2003).

Our main targets in the project were red variables with long periods (100 d or more) (Matsunaga et al. 2006), so that we observed each globular cluster only once at a night. Generally, the clusters were observed once a month between April and August each year from 2002 to 2005 and on some additional occasions. The 15 or more observations obtained over this long period enable us to investigate basic properties of T2Cs. The survey targeted 145 clusters located south of about +30° declination.

2.2 Photometry and variability detection

The raw data were reduced in the following way. We obtained scientific images in $JHK_s$ filters for each night using pipeline software (Y. Nakajima, private communication). This involved dark subtraction, flat-fielding, elimination of hot pixels and combination of dithered images.

For each filter, one of the best images (weather condition and seeing) was selected as a reference frame among $N$ images from the repeated observations for a globular cluster. Photometry was performed on $N$ images with DOPHOT software (Schechter, Mateo & Saha 1993). In order to standardize the magnitudes, we compared the photometric results of the reference frames with the Two Micron All-Sky Survey (2MASS) point-source catalogue (Curti et al. 2003). We found no effect of a colour term and a typical standard deviation of about $\pm 0.1$ mag in the difference between the magnitudes in the 2MASS catalogue and ours. Colour terms were thus ignored and a constant was added to fit our instrumental magnitudes to those in the 2MASS catalogue. Note that we could use a large number of objects (say 200 or more) in these comparisons so the mean difference of our final magnitudes from the 2MASS system will be small ($<0.01$ mag).

The photometric results for the remaining $N - 1$ images were compared with those of the reference frame, and we collected differences for all the detected objects. We present examples of these comparisons in Fig. 1. Variable stars stand out from the general scatter in these plots. We estimated photometric errors as a function of magnitude by taking standard deviations in boxes of size 0.25 mag or of larger size to include at least 50 objects and by smoothing the deviations. The sizes of the estimated errors ($\pm 1\sigma$) are drawn as solid curves in Fig. 1. Since some stars have larger errors due to special conditions, such as being in a crowded region, we adopted the errors from the DOPHOT software output if they exceeded the errors just discussed. We adopted a 3σ cut to distinguish between variable and non-variable stars. Celestial coordinates of any detected variable were determined by fitting to stars in the 2MASS catalogue.

Figure 1. An example of the magnitude comparisons between the reference data (MJD = 53581.11808) and three repeated observations. These data are for the cluster NGC 104 (47 Tuc). Magnitudes on the x-axis are instrumental ones before the standardization. Solid curves indicate the size of the error in each magnitude range. See the text for details of the analysis.

The astrometrical precision is expected to be better than 0.5 arcsec in most cases.

2.3 Period determination and the selection condition

For any object whose variation was detected in our analysis, we applied the phase dispersion minimization method in order to determine a pulsation period (Stellingwerf 1978). Even in the case of the long period stars (possible RV Tau stars), we did not discriminate between possible deep and shallow minima since the number of the minima around which we observed was not large and the differences in the infrared are small. We discuss as T2Cs in this paper, variables with the following characteristics:

(i) the light curve shows clear periodicity of $1 < P (< 100$ and
(ii) the location in the colour–magnitude diagram ($J - K_\alpha$ versus $K_\alpha$) is bluer than the red giant branch.

In addition, we include in the discussion known T2Cs whose variations were detected even if our data are not sufficient to determine the periods. For most of the T2Cs, periods are well determined from our data, and those obtained from the data in the three filters agree with each other. In the case of previously known T2Cs, our periods are consistent with the earlier results (see Clement et al. 2001). The earlier optical periods are often based on better sampled data than ours, so that we generally adopt published periods. For five objects listed in Table 1, however, our data differ from the previous periods and we adopted our own values. All five objects have relatively long periods (W Vir stars or RV Tau stars). Clement, Hogg & Yee (1988) pointed out that some of these objects show rather random changes of period (Wehlau & Bohlender 1982). However, our observations were not optimized to study such effects.

2.4 Table of T2Cs

Table 2 lists 46 T2Cs obtained in our analyses. We followed the numbering system of variables by Clement et al. (2001) and the updated catalogue at their web page.1 We discovered 10 new variables, and gave them successive numbers after the ones in the Clement’s catalogue. For each variable, Table 2 lists the celestial coordinate (RA and Dec.), the period $P$, the date of phase zero $\phi_0$, mean magnitudes, amplitudes and the number of observations $N_{\text{obs}}$. The flag ‘n’ indicates that the object is newly discovered. The mean magnitudes are taken from the mean of maximum and minimum magnitudes and the amplitudes are defined as the minimum-to-maximum variation. Table 3 lists the individual observations. Only the first few observations are shown. The full table is given in the online version of the paper only. In this table, 99.99 is listed when we failed to obtain the magnitude. This was usually because the object was fainter than the limiting magnitude of the frame which depends on phase and the condition of the frame. We put a superscript “*” to the number of observations $N_{\text{obs}}$ in Table 2 for an object with the missing measurement(s). Fig. 2 plots the light curves in $K_\alpha$ (against modified Julian Dates on the left-hand side and against phases folded according to the periods on the right-hand side). By fitting a sine curve to each light curve in $K_\alpha$, we determined phases so that the maximum light of the fitted sine curve occurs at phase zero. The value $\phi_0$ listed in Table 2 is the first date of phase zero after MJD 53000 (2003 December 27).

Some of the known variables listed in Table 2 with long periods, say $P > 20$ d, were not classified as T2Cs (or RV Tau stars) in the previous work. For example, NGC 6254 V1 was classified as a semiregular-type variable by Clement, Hogg & Wells (1985). It is difficult to separate the light curves of these stars from those of red variables. However, our sample is clearly defined (see previous section) and the stars we consider as T2Cs are all bluer in $J - K_\alpha$ than in the giant branch of the clusters.

Among about 80 known, or suspected, T2Cs in clusters, about half are not included in this work. Some of them were not targeted in our observations because they are located too far north (Dec. > 30°) or too far from the cluster centre for the field of view of our camera. The others are either too faint for useful photometry or blended with a neighbouring red giant.

2.5 Chances of the contamination of field T2Cs

In Section 2.7, we show that our T2Cs define a narrow PLR. It is therefore unlikely that any of them are cluster non-members. However, it is of interest to make some estimate of the chance of encountering a field T2C in our survey.

In the General Catalogue of Variable Stars (Kholopov 1998), there are 178 variables listed as CW (W Vir and BL Her) and 126 variables listed as RV (RV Tau). Considering their distribution over the sky, we divide them into three groups according to galactic coordinates ($l, b$): the halo ($|b| > 10^\circ$), the bulge ($|l| < 10^\circ$ and $|b| < 10^\circ$) and the disc ($|l| > 10^\circ$ and $|b| < 10^\circ$). The number $N$ and the corresponding density $\sigma (\text{str}^{-1})$ of the variables are listed in Table 4 for each group. Unfortunately, the list of field T2Cs is not based a complete, uniform, survey. Recently, Kubiaik & Udalski (2003) presented the result of a T2C survey with the Optical Gravitational Lensing Experiment (OGLE) data base. They found 54 T2Cs in about 11 square degrees of the Galactic bulge. This corresponds to the density of $\sigma = 1600 (\text{str}^{-1})$, which is larger than the value listed in Table 4 by a factor of 3. The solid angle of a field of view of the SIRIUS camera is $5.0 \times 10^{-6}$ (str), and we observed 43 globular clusters within the bulge region. Therefore, the expected number of field T2Cs in our survey is small, less than 0.4, even in the high field density of the bulge. Since many of the clusters discussed in this paper are in much lower density environments than the bulge, the expected number of field interlopers is much less than this and can be neglected.

2.6 Parameters for globular clusters

Now we turn to absolute magnitudes of T2Cs to combine those in different globular clusters into a period–luminosity diagram. The distance moduli we adopt are based on the magnitudes of horizontal branches of the clusters. We adopted the relation

$$M_V (\text{HB}) = 0.22 [\text{Fe/H}] + 0.89$$  

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1 http://www.astro.utoronto.ca/%7Ecclement/read.html

from Gratton et al. (2003), who calibrated the relation by using the main-sequence fitting method for three clusters. Relations similar to this have also been derived by others. We adopted the values listed in the table compiled by Harris (1996), for the metal abundance [Fe/H], the apparent magnitude of horizontal branch (VB), and the reddening (E(B - V)). We used the version released in 2003 February, updated in his web page, except in the case of HP 1 for which we assumed the values in the version released in 1997 May (see Section 2.8). For the reddening corrections, we used $R_V = 3.1$ and the following extinction law,

$$A_J/E(B - V) = 0.866, \quad A_H/E(B - V) = 0.565, \quad A_K_s/E(B - V) = 0.365,$$

(2)


It is known that NGC 5139 (ω Cen) has a metallicity spread and contains a population as metal rich as [Fe/H] ~ -0.6. However, in this cluster, a large population of horizontal branch stars and RR Lyr stars belong to metal-poor populations and the metallicity distribution peaks at around [Fe/H] ~ -1.6 (Sollima et al. 2006). Adopting [Fe/H] ~ -1.6, the distance modulus ($m - M_V$) is derived to be 13.62, which agrees with the value obtained from an eclipsing binarity.
binary in the cluster (Thompson et al. 2001). NGC 6441 is another cluster for which we need to take special care. It has a peculiar horizontal branch and contains blue HB stars and RR Lyr stars in spite of its high metallicity (Pritzl et al. 2003). We adopted \( [\text{Fe/H}] = -2.0 \) for RR Lyr stars in this cluster as Pritzl et al. (2003) did, and we inserted it into equation (1). We will give more detailed discussion in Section 2.8. Our adopted metallicities, reddenings, \( V(\text{HB}) \)s and distance moduli are listed in Table 5.

2.7 Period–luminosity relation

The distance moduli and reddenings discussed in the last section were used to derive absolute magnitudes, and we obtained period–luminosity diagrams in three filters (Fig. 3). Linear regressions to the T2C data (filled circles) yield,

\[
M_J = -2.23 (\pm 0.05) \log P - 1.2 - 3.54 (\pm 0.03),
\]

(3)

\[
M_H = -2.34 (\pm 0.05) \log P - 1.2 - 3.94 (\pm 0.02),
\]

(4)

\[
M_K_s = -2.41 (\pm 0.05) \log P - 1.2 - 4.00 (\pm 0.02),
\]

(5)

with residual standard deviations of 0.16, 0.15 and 0.14 mag, respectively.

Arp (1955) and Nemec, Nemec & Lutz (1994) claimed that there were fundamental-mode and first-overtone-mode pulsators forming separate parallel sequences in the T2C PLR. On the other hand, McNamara (1995) doubted the existence of any overtone pulsators. Fig. 3 shows there is no evidence for more than one mode of pulsation. As already mentioned, some papers claimed that the slope of the PL in the optical gets steeper for T2Cs at around \( \log P = 1 \) (Harris 1985; McNamara 1995), whilst Pritzl et al. (2003) did not find such an effect. As Fig. 3 shows there is no evidence for other than a linear relation in the near-infrared over the whole period range.

2.8 Comments on some clusters

2.8.1 HP 1

The two T2Cs in HP 1 are of nearly the same magnitudes and periods strongly suggesting a common distance and making it unlikely that they are field stars. However, if we use the reddening in the 2003 version of the Harris catalogue \( E(B-V) = 0.74 \) derived by Davidge (2000) from infrared observations of field stars, these stars lie above the PLR by amounts which depend on wavelength. This wavelength dependence is symptomatic of an incorrect reddening correction. On the other hand, using the reddening from the 1997 version of the Harris catalogue places the star on the PLR at all wavelengths. This latter reddening \( E(B-V) = 1.19 \) was obtained by Ortolani, Bica & Barbuy (1997) from a comparison of the \( (V-I) \) colour of the RGB with that of NGC 6752. Other authors have also found reddenings larger than that derived by Davidge (i.e. \( E(B-V) = 1.44 \), Armandroff & Zinn 1988; 1.88, Minniti 1995). We have therefore used the Ortolani value of the reddening.

2.8.2 NGC 6441

Despite the relatively high metallicity usually adopted for NGC 6441 \( ([\text{Fe/H}] = -0.53;\) Harris 1996), Rich et al. (1997) discovered it to have a blue horizontal branch as does the related cluster NGC 6388. These clusters have many RR Lyr variables (Layden et al. 1999; Pritzl et al. 2003, and references therein), and these RR Lyr stars resemble those in Oosterhoff II clusters (metal poor) rather than those in Oosterhoff I ones (relatively metal rich). Nevertheless, Clementini et al. (2005) have recently reported that RR Lyr stars in NGC 6441 are not of low metallicity. Pritzl et al. (2000) suggested that they comprise a new Oosterhoff group and they found circumstantial evidence that they may be at least as bright as those in the very metal-poor clusters. In view of these results, we have followed Pritzl et al. (2003) and used an absolute magnitude for the HB of this cluster equivalent to that of one with \( [\text{Fe/H}] = -2.0 \). There is very little doubt that the T2Cs belong to NGC 6441 since they are still four more T2Cs observed optically, but in the crowded cluster centre not observed by us, besides the two discussed here. They have (optical) magnitudes consistent with the ones we have studied (Pritzl et al. 2003). In so much as the T2Cs in NGC 6441 fall on our PL relation at our adopted distance, they support the high luminosities for the RR Lyr variables as discussed by Pritzl et al. (2000).
Figure 2. Light curves in $K_s$. Right-hand side: plotted against MJDs. Left-hand side: plotted against phases folded by the period indicated at the top of the right-hand panel.

3 DISCUSSION

3.1 Metallicity effect on the PLR

First, we discuss the metallicity effect on the zero-point of the PLR, by comparing deviations from the PLR (3)–(5) with the metallicity for each object. We simply adopted the metallicity of the globular cluster, in which a T2C is found, as the metallicity of the T2C. The relation in the $K_s$ filter is shown in Fig. 4. The distance moduli we used equation (1) have of course a metallicity dependence by themselves. A linear regression for the data in Fig. 4 has a slope of $-0.10(\pm 0.06)$ which is hardly significant. It would be reduced to $-0.02$ if we adopted the slope of 0.30 for equation (1) derived earlier by Sandage (1993). Note that adopting the latter slope makes negligible difference (less than 1 per cent) to our PLR slopes.

3.2 Extension of the PLR to RR Lyr region

We found that RR Lyr variables also obey the PLR (3)–(5). Plus symbols in Fig. 3 indicate RR Lyr variables in NGC 6341 (M92) taken from Del Principe et al. (2005). We adopted a distance modulus of 14.65 mag obtained in the same manner as for other clusters (equation 1). This is the only cluster with RR Lyr observations at all three wavelengths. A comparison can be made at $K_s$ for a...
number of other clusters. As shown in Fig. 5, both the slope and the zero-point of the PLR agree satisfactorily with that of the RR Lyr variables in all cases. The data for the RR Lyr variables are from Longmore et al. (1990), Butler (2003), Storm (2004) and Del Principe et al. (2005). We averaged the results for the eight globular clusters in Longmore et al. (1990). Although their magnitudes are $K$ (not $K_s$) in various photometric system, the differences are negligible (less than 0.01 mag; see Carpenter 2001, for example). Theoretical studies also provided sufficiently close PLRs. For example, the slope in Bono et al. (2001) was $-2.07$ and that in Catelan et al. (2004) was $-2.35$. These works showed that there is a small metallicity dependence of the zero-point ($\sim 0.17 \log Z$ in $K$), but the effect is not clear in the observational results and must be small (Longmore et al. 1990 derived the metallicity-dependent term as 0.04[Fe/H]). These results carry the implication that stars with the same age and probably the same mass within the instability strip obey the same PLR. We will discuss this in the next section.

Some comments should be provided about NGC 6341 V7 (Del Principe et al. 2005), which has a period of $\log P = 0.0259$ and is deviant from the PLR (the triangle in Fig. 3). Unfortunately, NGC 6341 lies too north to be observed by us. Kopacki (2001) reported that this star is a BL Her star, but it is apparently brighter than expected from our PLR. Although we need to confirm its membership, the location of NGC 6341 in the halo indicates that this star belongs to the cluster (see Section 2.5). In that case, this star may be the second anomalous Cepheid in globular clusters after NGC 5466 V19. An
anomalous Cepheid is a more massive variable with $0.3 < P(d) < 2$ and is brighter than RR Lyr variables by about one magnitude (Zinn & Dahn 1976; Cox & Proffitt 1988). This star needs further investigation, including a confirmation of cluster membership.

3.3 Reproduction of the PLR

Here, we discuss the PLR by using the $P \sqrt{P} = Q$ relation. The relation can be written as

$$M_{\text{bol}} = -3.33 \log P - 1.67 \log M - 10 \log T_{\text{eff}}$$

$$+ M_{\text{bol,}\odot} + 10 \log T_{\text{eff,}\odot} + 3.33 \log Q,$$

where $M$ is the mass in units of the solar mass and $T_{\text{eff}}$ is the effective temperature (e.g. McNamara 1995). Two linear relations,

$$\log T_{\text{eff}} = -0.058 \log P + 3.81,$$

$$\log Q = 0.24 \log P - 1.39,$$

are adopted from McNamara & Pyne (1994) to derive the PLR. We express the bolometric correction as

$$M_{\text{bol}} - M_{\odot} = \alpha_{\odot} \log T_{\text{eff}} + \beta_{\odot}.$$

Using these relations, equation (6) can be expressed in the form,

$$M_{\text{bol}} = -(1.95 - 0.058 \alpha_{\odot})(\log P - 1.2) - 1.67 \log M$$

$$- (2.70 + 3.74 \alpha_{\odot} + \beta_{\odot}).$$
Table 4. The density of T2Cs based on the General Catalogue of Variable Stars (Kholopov 1998). The regions are separated by galactic coordinate (see the text). 'CW' and 'RV' are the classified types in the catalogue for the combination of BL Her and W Vir (CW) and for RV Tau (RV).

<table>
<thead>
<tr>
<th>Region</th>
<th>CW</th>
<th>RV</th>
<th>CW+RV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Halo</td>
<td>85</td>
<td>8.2</td>
<td>47</td>
</tr>
<tr>
<td>Bulge</td>
<td>53</td>
<td>438</td>
<td>25</td>
</tr>
<tr>
<td>Disc</td>
<td>40</td>
<td>19.4</td>
<td>54</td>
</tr>
<tr>
<td>All sky</td>
<td>178</td>
<td>14</td>
<td>126</td>
</tr>
</tbody>
</table>

Table 5. Parameters for globular clusters. The metallicity [Fe/H], the colour excess $E(B - V)$, and the magnitude of the horizontal branch $V(\text{HB})$ were adopted from Harris (1996). The distance modulus $(m - M)_0$ was estimated from equation (1), except for NGC 6441 (see the text).

<table>
<thead>
<tr>
<th>Cluster</th>
<th>[Fe/H]</th>
<th>$E(B - V)$</th>
<th>$V(\text{HB})$</th>
<th>$(m - M)_0$</th>
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<tbody>
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<td>0.01</td>
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<td>15.57</td>
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<td>0.60</td>
<td>17.30</td>
<td>14.86</td>
</tr>
<tr>
<td>HP 1</td>
<td>−1.50</td>
<td>1.19</td>
<td>18.60</td>
<td>14.36</td>
</tr>
<tr>
<td>Terzan 1</td>
<td>−1.30</td>
<td>2.28</td>
<td>21.40</td>
<td>13.73</td>
</tr>
<tr>
<td>NGC 6441</td>
<td>−0.53</td>
<td>0.47</td>
<td>17.51</td>
<td>15.60</td>
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<tr>
<td>NGC 6453</td>
<td>−1.53</td>
<td>0.66</td>
<td>17.53</td>
<td>14.93</td>
</tr>
<tr>
<td>NGC 6569</td>
<td>−0.86</td>
<td>0.55</td>
<td>17.52</td>
<td>15.11</td>
</tr>
<tr>
<td>NGC 6626</td>
<td>−1.45</td>
<td>0.40</td>
<td>15.55</td>
<td>13.74</td>
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<tr>
<td>NGC 6749</td>
<td>−1.60</td>
<td>1.50</td>
<td>19.70</td>
<td>14.51</td>
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<tr>
<td>NGC 6779</td>
<td>−1.94</td>
<td>0.20</td>
<td>16.16</td>
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</tr>
<tr>
<td>NGC 7078</td>
<td>−2.26</td>
<td>0.10</td>
<td>15.83</td>
<td>15.13</td>
</tr>
<tr>
<td>NGC 7089</td>
<td>−1.62</td>
<td>0.06</td>
<td>16.05</td>
<td>15.33</td>
</tr>
</tbody>
</table>

Figure 3. PLR in $JHK_s$ filters for T2Cs (filled circles). Linear regressions (to filled circles) are shown as solid lines. The data for RR Lyr variables in NGC 6341 are also plotted as plus symbols taken from Del Principe et al. (2005). The triangle at log $P = 0.026$ is the data for NGC 6341 V7. See the discussion in Section 3.2 for RR Lyr variables and NGC 6341 V7.

Here, we introduce the zero-point of the relation at log $P = 1.2$, respectively [i.e. $M_\lambda = \mu_\lambda(\log P - 1.2) + \eta_\lambda$]. The mass term (log $M$) has no dependence on the period, $\mu_\lambda$ equals to $- (1.95 - 0.058 \alpha_\lambda)$. However, Bono, Caputo & Santolamazza (1997) predicted that the mass of T2Cs varies from 0.59 to 0.52 M$_\odot$, decreasing with increasing period from 1 to 10 d. This period dependence increases $\mu_\lambda$ by 0.08 compared with the case of the constant mass.

We obtained $\alpha_\lambda$ and $\beta_\lambda$ in Table 6 from the model atmospheres listed in table 1 of Bessell, Castelli & Plez (1998). They listed both the models with overshooting (Table 1) and those without overshooting (Table 2), but the difference between the two sets with the constants of $M_\text{bol,\odot} = 4.75$ and $T_\text{eff,\odot} = 5780$ K. We define $\mu_\lambda$ and $\eta_\lambda$ as the dependence on the period $(dM_\lambda/d \log P)$ and the zero-point of the relation at log $P = 1.2$, respectively [i.e. $M_\lambda = \mu_\lambda(\log P - 1.2) + \eta_\lambda$]. If the mass term (log $M$) has no dependence on the period, $\mu_\lambda$ equals to $- (1.95 - 0.058 \alpha_\lambda)$. However, Bono, Caputo & Santolamazza (1997) predicted that the mass of T2Cs varies from 0.59 to 0.52 M$_\odot$, decreasing with increasing period from 1 to 10 d. This period dependence increases $\mu_\lambda$ by 0.08 compared with the case of the constant mass.

We obtained $\alpha_\lambda$ and $\beta_\lambda$ in Table 6 from the model atmospheres listed in table 1 of Bessell, Castelli & Plez (1998). They listed both the models with overshooting (Table 1) and those without overshooting (Table 2), but the difference between the two sets...
Figure 4. The relation between the metallicity and the deviation from the PLR in $K_s$.

Figure 5. Comparison of the slope and the zero-point of the PLR of T2Cs (this work) and those of RR Lyr variables in references. L90 shows the result of Longmore et al. (1990), B03 Butler (2003), S04 Storm (2004) and DP05 Del Principe et al. (2005).

Table 6. The relation between the adopted bolometric correction ($\alpha_\lambda$, $\beta_\lambda$) and the PLR (slope $\mu_\lambda$ and zero-point $\eta_\lambda$). Observational values for $JHK_s$ are from the present paper, and those for $V$ and $I$ filters are from Pritzl et al. (2003).

<table>
<thead>
<tr>
<th>Filter</th>
<th>$\alpha_\lambda$</th>
<th>$\beta_\lambda$</th>
<th>Equation (10)</th>
<th>Observation $\mu_\lambda$</th>
<th>$\eta_\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$+2.1 \pm 1.0$</td>
<td>$-7.06$</td>
<td>$-1.85$</td>
<td>$-1.94$</td>
<td>$-1.64 \pm 0.05$</td>
</tr>
<tr>
<td>$I$</td>
<td>$-2.0 \pm 0.5$</td>
<td>$9.11$</td>
<td>$-2.08$</td>
<td>$-2.77$</td>
<td>$-2.03 \pm 0.03$</td>
</tr>
<tr>
<td>$J$</td>
<td>$-4.75 \pm 0.1$</td>
<td>$19.40$</td>
<td>$-2.23$</td>
<td>$-3.34$</td>
<td>$-2.23 \pm 0.07$</td>
</tr>
<tr>
<td>$H$</td>
<td>$-7.0 \pm 0.1$</td>
<td>$27.66$</td>
<td>$-2.36$</td>
<td>$-3.75$</td>
<td>$-2.34 \pm 0.06$</td>
</tr>
<tr>
<td>$K_s$</td>
<td>$-7.2 \pm 0.1$</td>
<td>$28.47$</td>
<td>$-2.37$</td>
<td>$-3.81$</td>
<td>$-2.41 \pm 0.06$</td>
</tr>
</tbody>
</table>

has negligible effect on our results (up to 0.03 mag). Whilst these models are computed with the solar metallicity, Sandage, Bell & Tripicco (1999) computed model atmospheres for Cepheids between $[\text{Fe/H}] = 0.0$ and $-1.7$. Their results show that the slopes of the log $T_{\text{eff}}$–bolometric correction relation (equation 9) are within the uncertainty of our adopted values, and the zero-points get slightly smaller for the lower metallicity (about 0.1-mag difference between $[\text{Fe/H}] = 0.0$ and $-1.7$). In $JHK_s$ filters, we would expect any effect of metallicity on equation (9) to be less than in $V$ and $I$.

The third and fourth columns in Table 6 show the predicted slope $\mu_\lambda$ and zero-point $\eta_\lambda$ in case of constant mass. They are approximately consistent with the counterparts obtained from the observational data (the fifth and sixth columns). The observational values in $V$ and $I$ filters are taken from Pritzl et al. (2003) and those in $JHK_s$ filters are obtained by us. Fig. 6 shows the relation between $\alpha_\lambda$ and the slope of the PLR ($\mu_\lambda$). The solid line shows the relation according to equation (10) in case of the constant mass, and the broken one is the one shifted by 0.08 considering the mass dependency mentioned in the text. Filled circles show the observational results.

Table 7. 2MASS magnitudes for candidate T2Cs in the LMC. Star IDs are from Alcock et al. (1998).

<table>
<thead>
<tr>
<th>Star ID</th>
<th>$P$</th>
<th>$J$</th>
<th>$H$</th>
<th>$K_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3812.61</td>
<td>9.387</td>
<td>15.453</td>
<td>14.970</td>
<td>15.222</td>
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<tr>
<td>80.6469.135</td>
<td>10.509</td>
<td>15.749</td>
<td>15.483</td>
<td>15.125</td>
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<tr>
<td>80.6590.137</td>
<td>11.442</td>
<td>15.347</td>
<td>14.824</td>
<td>14.373</td>
</tr>
<tr>
<td>3.7332.39</td>
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<td>15.868</td>
<td>15.380</td>
<td>15.178</td>
</tr>
<tr>
<td>81.9006.64</td>
<td>14.337</td>
<td>15.053</td>
<td>14.647</td>
<td>14.469</td>
</tr>
<tr>
<td>47.2611.589</td>
<td>14.469</td>
<td>15.793</td>
<td>15.326</td>
<td>15.183</td>
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<tr>
<td>2.5877.58</td>
<td>14.855</td>
<td>15.857</td>
<td>15.389</td>
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<td>78.6609.38</td>
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<td>14.281</td>
<td>13.823</td>
<td>13.463</td>
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<tr>
<td>78.8586.2363</td>
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<td>13.668</td>
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<td>13.089</td>
<td>12.635</td>
<td>12.093</td>
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<td>47.2496.8</td>
<td>56.224</td>
<td>13.124</td>
<td>12.709</td>
<td>12.512</td>
</tr>
</tbody>
</table>

Figure 6. The relation between the adopted bolometric correction ($\alpha_\lambda$) and the slope of the PLR ($\mu_\lambda$). The solid line indicates the relation according to equation (10) in case of the constant mass, and the broken one is the one shifted by 0.08 considering the mass dependency mentioned in the text. Filled circles show the observational results.
It seems rather clear that longer period LMC variables (data because they are based on single-epoch observations. However, there are some uncertainties of about T2Cs in the LMC (crosses). Also plotted (filled circles) are the T2Cs and the slope of a linear fit to the entire period range is about -1.7 in V, which is close to the observational value (-1.64). Since our discussion is based on very simple scheme, more detailed work on both the theoretical and observational side is desirable.

It is worth noting that T2Cs and RR Lyr variables may be unique in that they comprise a group of variables with almost a constant mass obeying PLRs. For instance, in the cases of classical Cepheids and Mira variables the mass increases with increasing period. It is therefore interesting that our PLRs can be reproduced by the simple scheme with a constant mass and no need to adopt a mass–luminosity relation.

### 3.4 Comparison with T2Cs in the LMC

Alcock et al. (1998) reported 33 candidate T2Cs with 8 < P(d) < 100 based on the massive compact halo object (MACHO) data base in the LMC. We searched for their near-infrared magnitudes in the 2MASS point-source catalogue (Curti et al. 2003), and found 27 matches among 33 objects (Table 7). Fig. 7 shows the PLR in JHK, for the T2Cs in the LMC (crosses). Also plotted (filled circles) are the T2Cs in globular clusters. The absolute magnitudes for the LMC objects were obtained with an assumed distance modulus of 18.50 mag. There are some uncertainties of about ±0.3 mag in using the 2MASS data because they are based on single-epoch observations. However, it seems rather clear that longer period LMC variables (P > 40 d) are brighter than the counterparts in globular clusters. The LMC variables at shorter periods fit the PLR of globular clusters within the uncertainties. This feature is also seen in the log P–V diagram (fig. 9) in Pritzl et al. (2003), but their conclusion is somewhat uncertain because they do not have their own data for the cluster variables with P > 40 d [compare the panels (a) and (d) of their fig. 9].

Figure 7. The PLR for the globular cluster sample (filled circles) and the LMC candidates (crosses).

### Figure 8. A colour–colour diagram for variables with P > 20 d: filled circles for globular cluster stars, crosses for the LMC stars from Alcock et al. (1998) and triangles for galactic field stars (Lloyd Evans 1985). The colours of the globular cluster stars and the LMC stars were corrected for the reddening, while those of the galactic field stars were not (see the text).

A mass difference could be one of the reasons for the LMC RV Tau stars being about one magnitude brighter than the globular cluster PLRs. Massive variables are expected to be brighter according to equation (10). A difference of about 1 mag corresponds to an increase in mass by a factor of about 4. Considering that the masses of the globular cluster variables are about 0.5–0.6 M☉, this would lead to a mass larger than the Chandrasekhar limit for the LMC RV Tau stars. This is too large if these variables are post-asymptotic giant branch (AGB) stars which have already gone through their major mass-loss phase (e.g. Jura 1986; Pollard & Lloyd Evans 1999). In that case, some other parameter is necessary to explain the difference in absolute magnitude.

Fig. 8 shows a colour–colour diagram for the variables with P > 20 d in globular clusters (filled circles) and the LMC (crosses) also for the galactic field RV Tau stars (triangles) taken from Lloyd Evans (1985). The colours of our cluster objects and the LMC objects were corrected for reddenings, while those of the galactic field objects were not because Lloyd Evans (1985) did not give any estimate of the reddenings. More than half of the galactic field objects have the galactic latitudes of |b| > 5° so that the reddening effect on the colour is expected to be not large (E(H – K) < 0.1). The large excesses of the K – L colours reported by Lloyd Evans (1985), which should be smaller than E(H – K) in the case of interstellar reddening, also support that the objects are intrinsically red. In Fig. 8, the globular cluster sample occupies a rather limited region whilst many of the LMC and the local stars spread to redder regions. This is too large if these variables are post-asymptotic giant branch (AGB) stars which have already gone through their major mass-loss phase (e.g. Jura 1986; Pollard & Lloyd Evans 1999). In that case, some other parameter is necessary to explain the difference in absolute magnitude.

The only globular cluster RV Tau star which has been claimed to have an infrared excess is NGC 6626 V17. Nook & Cardelli (1989) found an excess at 10 μm in this star. However, NGC 6626 V17 which has (H – K) = 0.12 and (J – H) = 0.48 lies with the other cluster stars in Fig. 8, and it also lies on our PLRs. We conclude that RV Tau stars in the LMC belong to a different family of variables from the T2Cs of the same periods in globular clusters. Whether they define a PLR is not clear.
4 SUMMARY

We have shown from our near-infrared observations of T2Cs in globular clusters that they define linear PLR at \( JHK \), with little scatter. There is no evidence for a change of the slope at around \( \log P = 1 \), such as was suggested at \( V \) in some early papers. An extrapolation of our infrared relation is shown to fit globular cluster RR Lyr variables. Both the slopes and the zero-points of the infrared PLRs can be successfully reproduced by a simple application of the pulsation equation at constant mass. The T2Cs and the RR Lyr variables in clusters therefore seem to form an interesting family of stars all with closely the same mass and showing a common PLR.

2MASS \( JHK \), magnitudes for W Vir stars and RV Tau stars in the LMCs show that, within the uncertainties, W Vir stars with \( P < 20 \) (d) obey the same PLR as those for the globular cluster T2Cs. However, RV Tau stars with \( P > 40 \) (d) are brighter than variables of the same periods in globular clusters. The reason for this is unclear, but the distribution in the \((H - K_s) - (J - H)\) diagram also shows differences between the two groups. RV Tau stars in the LMC are generally redder than those in globular clusters as are RV Tau stars in the galactic field.

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SUPPLEMENTARY MATERIAL

The following supplementary material is available for this article online:

Table 3. The released table of light variation.

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