Kinematics of the ultracompact helium accretor AM Canum Venaticorum

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Accepted 2006 June 20. Received 2006 June 14; in original form 2006 April 28

ABSTRACT
We report on the results from a five-night campaign of high-speed spectroscopy of the 17-min binary AM Canum Venaticorum (AM CVn), obtained with the 4.2-m William Herschel Telescope on La Palma.

We detect a kinematic feature that appears to be entirely analogous to the ‘central spike’ known from the long-period, emission-line AM CVn stars GP Com, V396 Hya and SDSS J124058.03−015919.2, which has been attributed to the accreting white dwarf. Assuming that the feature indeed represents the projected velocity amplitude and phase of the accreting white dwarf, we derive a mass ratio \( q = 0.18 \pm 0.01 \) for AM CVn. This is significantly higher than the value found in previous, less direct measurements. We discuss the implications for AM CVn’s evolutionary history and show that a helium star progenitor scenario is strongly favoured. We further discuss the implications for the interpretation of AM CVn’s superhump behaviour, and for the detectability of its gravitational-wave signal with the Laser Interferometer Space Antenna (LISA).

In addition, we demonstrate a method for measuring the circularity or eccentricity of AM CVn’s accretion disc, using stroboscopic Doppler tomography. We test the predictions of an eccentric, precessing disc that are based on AM CVn’s observed superhump behaviour. We limit the effective eccentricity in the outermost part of the disc, where the resonances that drive the eccentricity are thought to occur, to \( e = 0.04 \pm 0.01 \), which is smaller than previous models indicated.

Key words: accretion, accretion discs – binaries: close – stars: individual: AM CVn – novae, cataclysmic variables.

1 INTRODUCTION
AM Canum Venaticorum (AM CVn) was found in a survey of faint, blue objects by Humason & Zwicky (1947). It was observed to have peculiarly broad and shallow helium absorption lines, but no hydrogen (Greenstein & Matthews 1957). The star was shown to be a possible ultracompact white dwarf (WD) binary by Smak (1967), who discovered photometric variations on a 18-min period; quickly thereafter, Paczynski (1967) noted that it would be a prime example of a binary whose evolution is expected to be governed by gravitational-wave radiation, and which could serve as an excellent test for the existence of gravitational radiation. The interpretation as an interacting binary analogous to the cataclysmic variables was first proposed by Warner & Robinson (1972) upon their discovery of rapid flickering in the light-curve. The discussion as to the true orbital period of the system was finally put to rest by Nelemans, Steeghs & Groot (2001a) who discovered a kinematic ‘S-wave’ feature in time-resolved spectra of AM CVn, thereby proving that the orbital period is 1028 s, while the main photometric signal at 1051 s is to be interpreted as a ‘superhump’.

Here, we present phase-resolved spectra of AM CVn with significantly higher spectral resolution and signal-to-noise ratio (S/N) than those previously used by Nelemans et al. (2001a). Moreover, our current data set fully samples the proposed 13.37-h accretion disc precession period (Patterson, Halpern & Shambrook 1993; Skillman et al. 1999) for the first time, allowing for a characterization of the spectroscopic appearance of the system as a function of orbital period, precession period and superhump (beat) period.

2 OBSERVATIONS AND DATA REDUCTION
We obtained phase-resolved spectroscopy of AM CVn on 2005 March 20–24 with the William Herschel Telescope (WHT) and the Intermediate dispersion Spectrograph and Imaging System (ISIS) spectrograph. The observations consist of 2159 spectra taken with
Table 1. Summary of our observations.

<table>
<thead>
<tr>
<th>Date</th>
<th>UT</th>
<th>Exposures</th>
<th>Typical seeing</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005/04/20</td>
<td>20:52–05:06</td>
<td>523</td>
<td>0.8</td>
</tr>
<tr>
<td>2005/04/21</td>
<td>22:46–05:15</td>
<td>114</td>
<td>0.8</td>
</tr>
<tr>
<td>2005/04/22</td>
<td>20:41–05:04</td>
<td>585</td>
<td>1.1</td>
</tr>
<tr>
<td>2005/04/23</td>
<td>21:03–04:57</td>
<td>370</td>
<td>1.4</td>
</tr>
<tr>
<td>2005/04/24</td>
<td>20:35–05:01</td>
<td>567</td>
<td>0.8</td>
</tr>
</tbody>
</table>

the 1200B grating, covering \(\sim 4300–5100 \text{Å}\); the exposure time was 30–40 s depending on sky transparency and airmass. The sky varied from clear to opaque due to high clouds, but the median seeing was quite good, about 0.8 arcsec, giving an effective resolution of about 0.7 Å or 45 km s\(^{-1}\).

The chosen slit width was dependent on the seeing and transparency, and varied from 1.0 to 1.4 arcsec. The detector was the standard EEV12 chip in the spectrograph’s blue arm, windowed and binned by a factor of 2 in the spatial direction to increase the read-out speed. The read-out speed was kept ‘low’ to minimize the read-out noise. Each night, an average bias frame was created from 20 individual bias exposures, and a normalized flat-field frame was constructed from 20 incandescent lamp flat-fields.

All spectra were extracted using the IRAF implementation of optimal (variance-weighted) extraction. The read-out noise and photon gain, necessary for the extraction, were calculated from the bias and flat-field frames, respectively. HeNeAr arc exposures were taken every hour during the night to correct for instrumental flexure; all dispersion solutions were interpolated between the two arc exposures nearest in time. Arcs were taken before and after every rotator adjustment needed to keep the slit at near-parallactic angle. In each arc exposure, a total of about 40 arc lines could be fitted well with a Legendre polynomial of order 4 and 0.04 Å rms residuals. The rms residuals due to flexure are estimated to be smaller than this; the total arc drift was measured to be 0.5 Å peak-to-peak over an entire night. All spectra were transformed to the heliocentric rest frame prior to the analysis.

The average spectrum was corrected for instrumental response using spectrophotometric standard star Feige 34.

A summary of all observations is given in Table 1.

3 RESULTS

3.1 Average and phase-binned spectra

The grand-average spectrum of AM CVn is shown in Fig. 1. It shows several broad neutral helium absorption lines seen before in AM CVn’s spectrum, and clear emission of He\(\text{II}\) 4686. The absorption wings extend to about 1300 km s\(^{-1}\) (half width at half-minimum). See Table 2 for a list of equivalent widths for the strongest absorption lines.

The phase-binned (trailed) spectrum reveals several additional, weaker features that are too smeared out in the average spectrum to be detected. See Figs 2 and 3. Among these are the ‘S-wave’ emission features in several lines, already found by Nelemans et al. (2001a). It includes the metal line Mg\(\text{II}\) 4481; already present but not mentioned in Nelemans et al. (2001a). Two very weak S-wave features flank the (also very weak) He\(\text{I}\) 5048 line, which we identify with Si\(\text{II}\) 5041 and 5056.

The most remarkable is a weak and narrow emission feature slightly offset to the red of He\(\text{I}\) 4471. It is clearly offset in phase from the presumed bright spot feature, and has a much lower amplitude (see Fig. 3). As such it has everything in common with the ‘central spike’ feature that was first detected in GP Com (Smak 1975) and has since become a common (though poorly understood) feature in long-period, emission-line AM CVn stars (Ruiz et al. 2001; Roelofs et al. 2005). As in the aforementioned systems, the phase of the low-velocity component (relative to the bright spot) is consistent with the expected phase of the accreting WD, and its relatively low velocity amplitude is difficult to reconcile with any emission site in the accretion disc. It is also intrinsically redshifted through some as yet unknown mechanism (cf. Marsh 1999; Morales-Rueda et al. 2003). Based on these three properties, we interpret the feature as the perfect analogue of the central spike in GP Com, V396 Hya and SDSS J124058.03–015919.2.

Figure 1. Average spectrum of AM CVn. The wavelength-dependent instrumental response has been corrected for, but the overall received flux is strongly affected by clouds; no attempt has been made to correct for this.

Figure 2. Phase-binned, average-subtracted spectrum of AM CVn around the He\(\text{I}\) 4387 and He\(\text{I}\) 4921 lines.
Figure 3. Phase-binned, average-subtracted spectrum of AM CVn around the He I 4471 line. The bright spot trails around zero velocity (component a), in phase with the bright spot from the nearby Mg II 4481 line (b). In addition there is a weak component moving around 40 km s$^{-1}$ with a semi-amplitude of 92 ± 5 km s$^{-1}$, and offset in phase from the bright spot (c). We propose that this is the ‘central spike’ of the He I 4471 line.

3.2 Doppler tomography

3.2.1 The central spike

The weak, sinusoidal emission feature we interpret as the central spike in the He I 4471 line can be made more tangible in a linear back-projection Doppler tomogram (Marsh & Horne 1988). We begin by phase binning our 2159 spectra in ∼200 bins using the known orbital period $P_{\text{orb}} = 1028.7322$ s (Harvey et al. 1998; Skillman et al. 1999). The two bright spots close to the spike – from the He I 4471 line itself and from the nearby Mg II 4481 line, see Fig. 3 – are masked out to prevent them from dominating the (back-projected) spectra. Each phase-binned spectrum is then normalized by fitting a polynomial and dividing the spectrum by it, and finally all phase bins are divided by the column-averaged spectrum so that we are left with the components of the spectrum that vary on the orbital period, apart from the bright spots which were masked out earlier.

The next step is determining the central spike’s rest wavelength or, equivalently, its intrinsic (rest) velocity $\nu$ relative to the He I 4471 rest wavelength. We proceed in the same manner as in Roelofs et al. (2005), that is, we make Doppler tomograms for a range of trial wavelengths and determine the wavelength at which the spectra auto-correlate best. As in Roelofs et al. (2005), the height of the presumed central spike nicely peaks at a certain wavelength, which we take as the rest wavelength of the central spike. See Table 3 and the corresponding Doppler tomogram in Fig. 4.

The final step is determining the central spike’s velocity amplitude and phase, and their associated errors. We employ the bootstrap method used in Roelofs et al. (2006). In this Monte Carlo process, we make a large number ($10^3$) of Doppler tomograms with the recipe described above, using a random selection of 2159 spectra taken from our data set, and allowing for replacement. We determine the emission centroid of the central spike in each tomogram, make a 2D histogram of all the $K_X$ and $K_Y$ values obtained, and fit a 2D Gaussian to this distribution. The width of this Gaussian is taken as a measure for the error on the central spike’s coordinates in $K_X$ and $K_Y$ space. See Table 3 for the results.

3.2.2 The bright spot

We follow the same procedure as for the central spike, but now masking out only the bright spot of the Mg II 4481 line. We again construct an ensemble of $\sim 10^3$ Doppler tomograms via the bootstrap

Table 3. Velocities of the bright spot and central spike components of the He I 4471 line, after aligning the central spike with the negative $K_Y$ axis.

<table>
<thead>
<tr>
<th>Feature</th>
<th>$\nu$ (km s$^{-1}$)</th>
<th>$K_Y$ (km s$^{-1}$)</th>
<th>$K_X$ (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bright spot</td>
<td>0 ± 5</td>
<td>330 ± 8</td>
<td>−336 ± 8</td>
</tr>
<tr>
<td>Central spike</td>
<td>40 ± 4</td>
<td>−92 ± 4</td>
<td>0 ± 4</td>
</tr>
</tbody>
</table>

Figure 4. Linear back-projection Doppler tomogram of the redshifted, low-velocity central spike. Its velocity amplitude comes out at 92 ± 5 km s$^{-1}$. Its phase has been aligned with the negative $K_Y$ axis, the conventional zero-phase of the accreting star.
next section, we also plot the velocities of a free-falling accretion disc radius (\(R_{\text{KY}}\)) along with the measured values. The results are displayed in Fig. 6.

The velocities and phases at the stream–disc impact point match with the ballistic stream trajectory. The labels represent the accretion disc radius as a fraction of \(R_{L,1}\).

Fig. 5 shows the velocities of the central spike and the bright spot, with errors, to show their relative phases and amplitudes, after aligning the central spike with the negative \(K_Y\) axis. Looking ahead at the next section, we also plot the velocities of a free-falling accretion stream, as well as the Keplerian velocities along the trajectory of the ballistic accretion stream, for a binary of mass ratio \(q = M_2/M_1 = 0.18\).

3.3 The mass ratio of AM CVn

Using the velocity amplitudes and phases of the central spike and the bright spot, we can constrain the mass ratio and the effective accretion disc radius in AM CVn. To this end, we solve the equation of motion for a free-falling stream of matter through the inner Lagrangian point, based on the results of Lubow & Shu (1975), and we see whether the resulting accretion stream and/or accretion disc velocities and phases at the stream–disc impact point match with the measured values. The results are displayed in Fig. 6.

Keplerian velocities in the bright spot imply very small effective accretion disc radii (\(R \sim 0.3R_{L,1}\), with \(R_{L,1}\) being the distance from the centre of the accretor to the inner Lagrange point), and very large mass ratios (\(q \sim 0.5\)). In particular, the disc would not come anywhere near the 3:1 resonance radius. This resonance is commonly thought to drive the superhump behaviour that is observed in outbursting dwarf novae as well as nova-like systems such as AM CVn (Hirose & Osaki 1990; Whitehurst & King 1991).

For purely ballistic stream velocities in the bright spot, the best-fitting accretion disc radius is 5–10 per cent smaller than the maximum accretion disc radius that can be contained within the primary star’s Roche lobe. In this case, the disc would clearly extend past the 3:1 resonance radius. If we assume purely ballistic stream velocities, we obtain a mass ratio \(q = 0.18 \pm 0.01\). This constitutes a lower limit.

If we allow for mixing between the ballistic stream and the Keplerian disc velocities, the mass ratio could be slightly larger than this. See Fig. 6. The maximum mass ratio for which the bright spot would remain outside the 3:1 resonance radius is \(q \leq 0.22 \pm 0.01\), which corresponds to a (best-fitting) mix of about 80 per cent stream and 20 per cent disc velocities. A mass ratio of \(q \leq 0.25\) is commonly quoted as the requirement for a binary to be able to excite its 3:1 resonance and develop the corresponding superhumps (Whitehurst 1988; Hirose & Osaki 1990; Whitehurst & King 1991).

Two values for the mass ratio \(q\) from the literature are overplotted in Fig. 6. Both these values are derived from the fractional superhump period excess \(\epsilon = (P_{\text{sh}} - P_{\text{orb}})/P_{\text{orb}} = 0.0216\) observed in AM CVn (e.g. Skillman et al. 1999), combined with theoretical-numerical or empirical relations between \(\epsilon\) and \(q\). The mass ratio derived from AM CVn’s kinematics is significantly larger than these predictions. The present data indicate a lower limit of \(q = 0.18 \pm 0.01\), compared with \(q = 0.10\) as obtained from the latest empirical \(\epsilon(q)\) relation for hydrogen-rich cataclysmic variables by Patterson et al. (2005) (P05). The value \(q = 0.087\) found by Nelemans et al. (2001a) (NSG01), from fits to numerical accretion disc simulations as taken from Warner (1995), deviates even more. In the remainder of this paper, we shall conservatively use \(q = 0.18 \pm 0.01\), which corresponds to pure stream velocities in the bright spot.

3.4 Stroboscopic Doppler tomography

The permanent superhump phenomenon, exhibited by AM CVn, is usually explained in terms of a precession of a tidally deformed (i.e. non-circular) accretion disc. After exactly one orbital cycle, the orientation of the two stars with respect to the observer is restored, but during the orbit the eccentric disc has advanced slightly so that it takes slightly more than one orbital period to restore the orientation of the two stars with respect to the accretion disc. This beat period is then assumed to be responsible for the main photometric signal, which is usually observed to be a few per cent longer than the orbital period.

The model requires that there be a precessing accretion disc with sustained eccentricity. Furthermore, Skillman et al. (1999), among others, find the superhump cycle to be extremely stable over many thousands of orbital periods, with the superhump period always within 0.1 s of 1051.2 s and the superhump waveform remaining very stable throughout the years.\(^1\) This implies that the shape of the disc is (semi-)stationary rather than chaotic. It further implies that the superhump phase will drift by at most 0.04 of a cycle over our 100-h observing baseline, if we adopt their average superhump period of 1051.2 s. These two results open up the possibility of combining the large number of superhump cycles in our data set to reconstruct the appearance of the system as a function of superhump phase.

\(^1\) Note that the superhump waveform is double humped; most of the power is actually in the first harmonic at 525.6 s.
The application of Doppler tomography as an instrument for testing the eccentric disc model is as follows. As the secondary star goes around the slowly precessing and non-circular disc, the effective radius at which the accretion stream impacts the disc will vary. Since the bright spot is caused by the free-falling stream of matter from the inner Lagrange point crashing into the edge of the accretion disc, a variation in effective accretion disc radius will lead to differences in the gas velocities of both the stream and the disc matter at the impact point. Observationally, we then expect the bright spot to show excursions in velocity space, as a function of the relative orientation of the eccentric disc and the secondary star, i.e. as a function of superhump phase.

In order to measure these excursions, we must divide our spectra in a number of bins corresponding to different superhump phases, and subsequently phase fold the spectra contained in each bin on the orbital period. The method bears resemblance to the ‘stroboscopic Doppler tomography’ method employed by Marsh & Duck (1996) to disentangle the orbital and WD spin periods in the magnetic accretor FO Aqr, except that we now have the orbital period of the binary and the precession period of the accretion disc.

We argue that this is in fact quite a sensitive method for measuring variations in the effective radius of the accretion disc. If we consider the simplest case of an eccentric, elliptical accretion disc, a simple estimate for the ratio of the projected apastron and periastron velocity amplitudes $v_{a,p}$ as a function of the eccentricity $e$ is

$$\frac{v_{a}}{v_{p}} \approx \frac{1 + e}{1 - e} \approx 1 + 2e, \quad e \ll 1.$$  \hfill (1)

A bright spot of average velocity amplitude $\pi \approx 450$ km s$^{-1}$, combined with a reasonable spectral resolution and S/N that allow the bright spot to be measured to 10 km s$^{-1}$ accuracy, gives an eccentricity sensitivity of $e \sim 0.01$. In practice, the excursion of the bright spot in a Doppler tomogram will be larger and the eccentricity sensitivity will correspondingly be better than this.

As an example, we show in Fig. 7, by solving numerically the equation of motion for a ballistic test mass in the binary potential, how we expect the bright spot to vary as a function of superhump phase, in a binary of mass ratio $q = 0.18$, with an elliptical accretion disc of semimajor axis $R = 0.65R_{L,1}$ and eccentricity $e = 0.2$. The elliptical group of spots represent Keplerian accretion disc velocities in the bright spot, the group sticking out to the left-hand side represent ballistic accretion stream velocities, as a function of superhump phase (one full cycle shown).
order to explore the possible effects of such variations, consider the pressure of the accretion stream $p_s$,

$$
p_s = p_{\text{static}} + p_{\text{dynamic}} \approx p_{\text{dynamic}} \approx \frac{1}{2} \rho v^2, \tag{2}
$$

where $\rho$ and $v$ are the density and velocity of a fluid element along the stream, respectively, and $p_{\text{static}}$ can be neglected since $k_B T \ll m_{\text{He}} v^2$ in the free-fall regime (by definition, one could say), where $v$ is of the order of $10^2 \text{ km s}^{-1}$ and the temperature $T$ of the stream, set by the surface temperature of the donor star, is of the order of $10^4 \text{ K}$. If we assume a steady ballistic flow of gas, where the scaleheight of matter in the flow remains fairly constant in the region just outside the disc (Lubow & Shu 1976) since the matter does not have time to maintain hydrostatic equilibrium perpendicular to the flow, then conservation of mass gives

$$
\rho \propto v^{-1} \quad \text{(along streamline)} \tag{3}
$$

so that

$$
p_s \propto v \quad \text{(along streamline)}. \tag{4}
$$

The pressure along the accretion stream thus increases linearly with velocity. In case of a ‘soft’ edge to the accretion disc (that is, a slowly increasing disc pressure $p_d$ from the disc edge inwards) this could have the effect of increasing the ratio of observed periastron and apastron bright spot velocities for given $e$, as the stream impacts slightly deeper into the disc rim at periastron due to its higher pressure than it does at apastron. Thus, if one employs the radius at which the bright spot occurs as the effective in situ accretion disc radius, and one allows the disc edge to be soft, an eccentricity measurement based on the bright spot excursion amplitude could lead to an overestimate of $e$, and thus to an upper limit for $e$.

### 3.5 Eccentricity of the accretion disc of AM CVn: the bright spot

Figs 8 and 9 show the results from our stroboscopic Doppler tomograms. In Fig. 8, we plot eight Doppler tomograms that together cover one full superhump period. In order to improve on the S/N of the individual maps, we combined the data (in velocity space) of four lines that show a clear bright spot in the trailed spectra, namely He i 4387, He i 4471, Mg II 4481 and He i 4921 (Figs 2 and 3). Fig. 9 shows the measured excursions of the bright spot as a function of superhump phase, for 16 superhump phase bins, connected by a line, where the bright spot centroids have been determined in the same way as in Section 3.2. Here, the 16 phase bins oversample the superhump cycle by a factor of 2, so that each bin contains 1/8th of the data as in Fig. 8.

Based on the observed bright spot excursion amplitude of less than 100 km s$^{-1}$ (see Fig. 9), we constrain the effective eccentricity of the outermost part of the accretion disc to $e = 0.04 \pm 0.01$, where we define the ‘effective’ eccentricity as that which would cause a simple elliptical disc to give rise to the same bright spot excursion amplitude in velocity space. The error on this value can be rather small because the method of measurement is insensitive to the accretion disc’s semimajor axis (or average radius), and to whether the bright spot represents stream or disc velocities (see also Fig. 7). In case the bright spot velocity is a variable mix of the disc and stream velocities, the derived eccentricity can be an overestimate.

Despite the rather large error bars in the left-hand panel of Fig. 9, at least relative to the small movement of the bright spot, the disc appears to be more complex than the simple ellipse which it is often pictured to be in explaining the superhump phenomenon. Detailed numerical simulations, e.g. by Simpson & Wood (1998), show that the accretion disc in a permanent superhump system like AM CVn is in fact most likely of irregular shape. Not only that, the disc is also predicted to change shape during a superhump period, only to return to its original shape after one full superhump cycle – therefore, the disc could in principle be exhibiting strong changes in radius at the other side of the accreting star, where the mass stream cannot be used to probe its effective radius. The changes in effective radius of the accretion disc might thus be larger than is revealed by the excursions of the bright spot.
3.6 Eccentricity of the accretion disc of AM CVn: integrated disc profile

Fig. 10 shows trailed spectra and Doppler tomograms of He II 4686 as a function of superhump phase. The He II 4686 line was chosen because it clearly shows emission in the average spectrum (Fig. 1), which is commonly used as a tracer of locally enhanced dissipation in accretion discs, for instance in searches for spiral shocks in outbursting dwarf novae. A sinusoid with the phase and amplitude of the central spike – i.e. the presumed motion of the accretor – has been shifted out of the spectra to leave the net kinematic signal of the matter in the disc orbiting the accretor. The ionized helium emission profile is clearly variable on the superhump period: the emission is asymmetric and the degree of asymmetry is changing (see also Fig. 9). These general features agree with the predictions of Simpson & Wood (1998).

It can furthermore be deduced that the He II 4686 emission is strongest where the effective radius of the disc is smallest. The right-hand panel in Fig. 9 shows both the bright spot velocity amplitude and the angle between the bright spot and the centroid of He II 4686 emission, as a function of superhump phase. This angle is given in spatial coordinates – for the transformation of velocity into spatial coordinates we assume roughly ballistic stream velocities for the bright spot based on the results of Section 3.3, and roughly Keplerian disc velocities for the He II 4686 emission. Despite the rather large error bars, there appears to be a trend whereby the maximum bright spot velocity amplitude (which should correspond to the smallest effective disc radius) corresponds to alignment of the bright spot with the He II emission centroid, while the minimum bright spot velocity (i.e. largest effective disc radius) corresponds to anti-alignment.

These findings again agree with the numerical simulations of Simpson & Wood (1998). Their analysis of the changes in internal energy in the disc as a function of superhump cycle, too, showed that energy production was highest where the disc radius was smallest. This may not be that surprising since streamlines of matter in the disc will be squeezed more closely together towards a region of smaller disc radius, causing extra viscous dissipation. We may tentatively conclude that cooling proceeds on a time-scale less than the orbital period of the matter in the disc, which is a few minutes.

4 DISCUSSION

4.1 A central spike in AM CVn?

The mysterious central spike has so far been observed in long period, low mass transfer rate AM CVn stars, where the accreting WD dominates the optical flux. It is thought to originate on (or very close to) the surface of the accreting WD, since the spike perfectly tracks the expected movement of the accreting WD relative to the
bright spot in the accretion disc. In addition, there is no accretion
disc component that is expected to move at such low velocities, and
the phase of the spike relative to the bright spot does not match
the phase of the secondary. See Morales-Rueda et al. (2003) for a
detailed analysis of the central spike in GP Com.

The kinematic feature observed in the He I 4471 line in AM
CVn, as presented here, has all the characteristics for being the
perfect analogy to the central spike in GP Com (Morales-Rueda et
al. 2003), V396 Hya (Steehgs et al., in preparation) and SDSS
J124058.03−015919.2 (Roelofs et al. 2005): (i) it is intrinsically
redshifted; (ii) it agrees in amplitude with the expected velocity
amplitude of the accreting WD (and not with any other binary com-
ponent’s velocity amplitude); and (iii) it agrees in phase with the
expected phase of the accreting WD, relative to the bright spot. We
therefore conclude that it is very likely that it is the central spike.

The mass ratio for AM CVn implied by the central spike is sig-
ificantly higher than previously thought, although there has been a
study by Pearson (2003) suggesting, in fact, a mass ratio $q \sim 0.19–
0.25$ based on a model in which the secondary star is moderately
magnetic. The motivation for that study was to try to reconcile the
mass ratio with the $K$-velocity of the He II 4686 emission line mea-
sured (but discarded) by Nelemans et al. (2001a), which suggested
a mass ratio in this regime. It is clear from this work, however, in
particular Fig. 10, that the measured $K$-velocities of the He II 4686
emission indeed cannot be used reliably as an indicator for the mo-
tion of the accretor, since the apparent kinematic signal could easily
be dominated by the eccentricity of the disc rather than the actual
motion of the primary.

4.2 Implications for AM CVn’s formation channel

One of the long-standing questions regarding the AM CVn stars
is how they are formed. Theoretically, they can be formed in both
double- and single-degenerate configurations: Roche lobe overflow
from a WD on to another (more massive) WD (e.g. Nelemans et al.
2001b), or from a helium-burning star on to a WD (Iben & Tutukov
1991). A third suggested formation channel is Roche lobe overflow
from an evolved main-sequence star on to a WD (Podsiadlowski,
1991). A third suggested formation channel is Roche lobe overflow
from an evolved main-sequence star on to a WD (Podsiadlowski,
Han & Rappaport 2003), where mass transfer starts by the time
hydrogen core burning ends. This latter scenario, however, has the
problem of leaving significant amounts of hydrogen, while our high-
S/N average and phase-binned spectra show exclusively helium and
metals. For the other two formation channels, the main discrimi-
nator is the mass of the secondary star: in the helium-star channel,
one expects a relatively hot and massive, only partially degenerate
secondary, while in the double-degenerate channel, one expects a
‘cold’ degenerate donor star.

The significantly higher mass ratio found for AM CVn in this
paper has important implications for the nature of the donor star, and
hence for AM CVn’s formation channel, as it suggests a relatively
massive donor. If we combine the orbital period with the mass–
radius relation for a relatively ‘cold’ (core temperature $T_c \leq 10^6$ K),
degenerate helium WD as used in Nelemans et al. (2001b) and
refined recently in Deloye, Bildsten & Nelemans (2005), we get a
secondary mass $M_2 \approx 0.35 M_\odot$ and hence, through our minimum
mass ratio of $q = 0.18 \pm 0.01$, a maximum primary mass of only $M_1
= 0.19 M_\odot$. This is very low; the maximum combined mass of $M_1
+ M_2 = 0.22 M_\odot$ would, for instance, be significantly lower than
the average value $M_1 + M_2 = 0.8 M_\odot$ found in detached WD–WD
binaries (e.g. Nelemans et al. 2005). This casts doubt on the possible
WD nature of the donor star, and thereby on the double-degenerate
formation channel for AM CVn, unless the WD donor was still very
hot upon Roche lobe overflow – that is, unless mass transfer started
shortly after the second common-envelope phase.

Concurrent with the measurement of AM CVn’s mass ratio pre-
icted here, it was discovered that AM CVn’s distance is larger than
expected, through a parallax measurement with the Hubble Space
Telescope’s Fine Guidance Sensors ($\tau = 1.65 \pm 0.30$ mas; Roelofs et
al., in preparation). This means a smaller absolute magnitude $M_v$
and thus, since we expect the accretion disc to dominate the optical
flux, a higher accretion luminosity.

In order to link the observed $M_2$ to the mass of the secondary $M_2$,
we proceed in the same way as in Deloye et al. (2005). We assume
conservative mass transfer that is driven entirely by loss of angular
momentum due to the emission of gravitational waves and, since
accretion proceeds via an accretion disc, we assume that all angular
momentum carried by the transferred matter is fed back to the orbit.

We then have

$$\frac{M_2}{M} = \frac{\dot{J}}{\dot{J}} = \frac{2}{\zeta + 5/3 - 2q},$$

where $\zeta$ is the orbital angular momentum, $J$ is the orbital angular
momentum loss rate

$$\frac{\dot{J}}{\dot{J}} = \frac{32 G^3 M_1 M_2 (M_1 + M_2)}{c^5 a^4}$$

due to the emission of gravitational waves (Landau & Lifschitz
1971) and

$$\zeta = \frac{\log R_2}{\log a}.$$

We take $\zeta = -0.06$ for $M_2 < 0.2 M_\odot$ as in Nelemans et al. (2001b),
based on evolutionary calculations for a mass-losing helium-burning
star by Tutukov & Fedorova (1989). Clearly, $\zeta = 0$ around the point
where helium burning stops, while it may be somewhat closer to $\zeta
= -0.19$ along the semidegenerate part of the track for a secondary
that is more helium depleted, as found by Savonije, de Kool & van
den Heuvel (1986). This represents a $\lesssim 10$ per cent uncertainty
in equation (5).

The last step is to calculate the bolometric luminosity $L$, for which we
use

$$L = \frac{1}{2} M_2 [\Phi(L) - \Phi(R_1)],$$

where $\Phi$ is the common Roche potential, at the inner Lagrange point
$L_1$ and the surface of the accretor $R_1$. This states that the matter in
the disc stays virialized during the accretion process, which should
hold for approximately Keplerian particle orbits.

Fig. 11 shows $M_2$ as a function of $M_2$ for $q = 0.18$, $\zeta = -0.06$ and
a bolometric correction $BC = -3.0$ (see below). The lower limit to $M_2$
is given by the requirement that the system does not eclipse. The
data point $M_2 = 5.0 \pm 0.4$ as determined from our aforementioned
HST parallax is overplotted, which yields a secondary mass $M_2
= 0.125 \pm 0.012 M_\odot$ and, via $q = 0.18 \pm 0.01$, a primary mass
$M_1 = 0.68 \pm 0.06 M_\odot$. The corresponding mass transfer rate is
$\dot{M}_2 = 6.7_1^{+1.9} \times 10^{-9} M_\odot$ yr$^{-1}$, and the inclination of the binary
comes out at $i = 43 \pm 2^\circ$. Table 4 lists the system parameters for
AM CVn in convenient tabular form.

Our results have been corrected for the geometric effect that the
‘apparent absolute magnitude’ (Warner 1995) of an accretion disc
depends on the inclination. A factor

$$\Delta M_2(i) = -2.5 \log \left(\frac{\cos i}{1/2}\right)$$

has been taken into account as the correction from apparent absolute magnitude to absolute magnitude, where the factor 1/2 represents the direction-averaged fraction of the disc area seen by the observer, \(\cos i\). We neglect the (unknown) effect of limb darkening.

In converting the bolometric luminosity \(L_{\text{bol}}\) obtained from equation (8) to an absolute visual magnitude \(M_V\), the bolometric correction is of some importance. AM CVn’s spectral energy distribution from the far-ultraviolet to the optical (Roelofs et al., in preparation; see also Nasser, Solheim & Semionoff 2001) indicates that it is dominated by a blackbody of \(\lesssim 30,000\, \text{K}\). This corresponds to a BC of \(-3.0\), with an estimated error of 0.3. If we calculate the minimum temperature, the accretion disc must have in order to be able to radiate away the required luminosity (for any given \(M_1\)) we know the maximum radiating surface of the disc), we find \(T_{\text{disc}} \gtrsim 30,000\, \text{K}\) for accretion rates \(M_2 \gtrsim 4 \times 10^{-8}\, M_\odot\, \text{yr}^{-1}\). The BC we use is thus self-consistent with the accretion rate we derive; in particular,

it seems unlikely that we are overestimating the mass transfer rate due to an overestimate of the BC.

Summarizing, we have two independent pieces of evidence suggesting a relatively massive donor star – the large mass ratio and the large luminosity. If we compare our results to evolutionary models for helium star and WD secondaries as given by Nelemans et al. (2001b), we see that our derived secondary mass is compatible with the secondary being the semidegenerate core of a formerly helium burning star, while a ‘cold’ degenerate donor star is ruled out. See Fig. 12. The secondary mass \(M_2 = 0.125\, M_\odot\) is close to the turn-off mass for helium burning, which lies at \(M_2 \sim 0.16\, M_\odot\) (Savonije et al. 1986; Tutukov & Fedorova 1989). We conclude that it is very likely that AM CVn formed through an evolutionary channel in which the donor star was, upon the start of Roche lobe overflow, still burning helium. These observations thus provide the first observational evidence that the helium star evolutionary channel contributes to the AM CVn population.

4.3 Implications for AM CVn’s gravitational wave signal

We expect the orbit of AM CVn to be circular, so that the gravitational-wave polarization amplitudes \(A_+\) and \(A_\times\) at twice the orbital frequency, \(f\), become (e.g. Timpano, Rubbo & Cornish 2006)

\[
A_+ = \frac{4 (GM)^{5/3}}{c^3 d^2} (\pi f)^{2/3} (1 + \cos^2 i),
\]

\[
A_\times = -\frac{4 (GM)^{5/3}}{c^3 d^2} (\pi f)^{2/3} \cos i,
\]

where \(M = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}\) is the so-called chirp mass, \(i\) is the inclination of the binary and \(d\) is its distance. Defining the

**Table 4.** The collection of system parameters for AM CVn.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{\text{orb}}) (s)</td>
<td>1028.7322 ± 0.0003 (Skillman et al. 1999)</td>
</tr>
<tr>
<td>(q)</td>
<td>0.18 ± 0.01</td>
</tr>
<tr>
<td>(M_1) ((M_\odot))</td>
<td>0.68 ± 0.06</td>
</tr>
<tr>
<td>(M_2) ((M_\odot))</td>
<td>0.125 ± 0.012</td>
</tr>
<tr>
<td>(M_2) ((M_\odot) yr(^{-1}))</td>
<td>6.7(^{+1.9}_{-1.3}) \times 10(^{-9})</td>
</tr>
<tr>
<td>((C))</td>
<td>43 ± 2</td>
</tr>
<tr>
<td>(d) (pc)</td>
<td>606(^{+135}_{-93}) (Roelofs et al., in preparation)</td>
</tr>
</tbody>
</table>
In short, they conclude that for hydrogen-rich systems the binary. See Patterson et al. (2005), and references therein, for an overview including the very latest empirical work on the $\epsilon(q)$ relation. In short, they conclude that for hydrogen-rich systems $\epsilon(q) = 0.18q + 0.29q^2$.

4.4 The superhump excess–mass ratio relation

It is understood from numerical simulations, and it is observed in outbursting dwarf novae and nova-likes, that there exists a relation between the superhump period excess $\epsilon$ and the mass ratio $q$ of the binary. See Patterson et al. (2005), and references therein, for an overview including the very latest empirical work on the $\epsilon(q)$ relation. In short, they conclude that for hydrogen-rich systems $\epsilon(q) = 0.18q + 0.29q^2$.

For AM CVn, this results in $q = 0.10$ based on the photometry of Skillman et al. (1999). Although it is unknown what the intrinsic spread in $\epsilon(q)$ is, it appears from fig. 9 in Patterson et al. (2005) to be quite low, no more than about 0.01. This means that the $\epsilon(q)$ for AM CVn deviates significantly from our minimum kinematic mass ratio $q = 0.18 \pm 0.01$.

Of course, the superhump excess $\epsilon$ may depend on more parameters than just the mass ratio $q$; it is quite conceivable that the helium nature of AM CVn may be of influence. A recent study by Goodchild & Ogilvie (2006) indicates, for instance, quite a significant dependency of $\epsilon$ on the thickness of the accretion disc, where thicker discs lead to smaller superhump period excesses for a given mass ratio. If this effect should be at play, our results would suggest a physically thicker disc in AM CVn compared to discs in hydrogen-rich systems of the same mass ratio.

Although AM CVn represents only a single data point, it seems advisable to apply the empirical superhump excess–mass ratio relation with caution for such ultracompact binaries. An obvious test would be to obtain the mass ratio in HP Librae, the other known helium nova-like, and check whether it, too, shows a discrepancy with relation (14).

4.5 Size and eccentricity of the accretion disc

In our derivation of the minimum mass ratio $q = 0.18 \pm 0.01$, we have assumed a maximum accretion disc radius equal to the minimum radius of the primary Roche lobe. It is customary to assume a slightly more stringent limit on the maximum accretion disc radius, caused by tidal truncation of the disc. See e.g. Warner (1995) and references therein. For reference, we show in Fig. 14 the often-used tidal truncation radius

$$R_T = \frac{0.6}{1 + q} \left(0.03 < q < 1.0\right)$$

for an inviscid flow, which is indeed smaller than the full minimum Roche lobe radius we assume. However, since this tidal truncation radius depends, in particular, on the unknown viscosity of the matter in the disc (and increasing with viscosity), we have not used it as

$$h = \left[\frac{1}{2} (A_1^2 + A_2^2)\right]^{1/2}$$

and converting to more convenient solar units, one finds

$$h = 2.84 \times 10^{-22} \frac{\cos^4 i + 6 \cos^2 i}{\left(\frac{P_{eh}}{1 \text{ hr}}\right)^{-2/3} \left(\frac{d}{1 \text{kpc}}\right)^{-1}}$$

Filling in the numbers derived above gives $h = 2.1^{+0.4}_{-0.3} \times 10^{-22}$. Fig. 13 shows the estimated strain amplitudes and frequencies of the ‘known’ AM CVn stars. AM CVn itself is the first potential Laser Interferometer Space Antenna (LISA) source which is sufficiently well constrained that we can plot error bars with some confidence, and we see that it stands out significantly above both the instrumental design sensitivity and the estimated confusion-limited Galactic background due to, mainly, detached WD–WD binaries.
a hard upper limit to the disc’s radius. Interestingly, one sees that the best-fitting accretion disc radius we derive for ballistic stream velocities in the bright spot (reproduced in Fig. 14) lies very close to, but slightly above, the tidal truncation radius (15).

The effective eccentricity value $e = 0.04 \pm 0.01$ we derive for the outermost part of the disc, from the stream–disc impact spot, is smaller than typical values $e \sim 0.1–0.2$ found in numerical simulations. Although the eccentricity is not one of the observables that is usually quoted in numerical studies (the resulting superhump periods. Although the eccentricity is not one of the observables that is usually quoted in numerical studies (the resulting superhump period excess is often the only given quantity), it can be inferred that the effective eccentricity we find is significantly smaller than that found by Simpson & Wood (1998) in their simulations of discs in AM CVn stars.

A possible explanation would be that tidal truncation, discussed in the previous paragraph, acts to circularize the disc at the outer edge to some extent by effectively removing particles from the most eccentric orbits there. This tidal truncation process is essentially a competition between viscosity transporting angular momentum outwards and tidal forces ‘dissipating’ this angular momentum increasingly strongly with radius, and predictions about this tidal truncation process would thus be affected by our lack of understanding of the viscosity in accretion discs.

4.6 Spin of the accretor and tidal synchronization

If the central spike originates from the surface of the accreting WD, in particular if it rotates with the accretor, its width puts an interesting constraint upon the accretor’s spin. The central spike is unresolved in our data, and we therefore place an upper limit on $\sim 45 \text{ km s}^{-1}$, equal to our formal spectral resolution, on the full width at half-maximum (FWHM) of the feature. For the system parameters derived for AM CVn, corotation with the orbit would imply a projected equatorial velocity of $\sim 36 \text{ km s}^{-1}$. The central spike we observe is thus consistent with a corotating primary, while significantly faster rotation is ruled out.

We can estimate the tidal synchronization time-scale of the accretor from this constraint. For simplicity, we assume that AM CVn has been steadily accreting matter at a rate $\dot{M}_2 \sim 7 \times 10^{-9} \text{ M}_\odot \text{ yr}^{-1}$ for a long time. As long as the accretor is near corotation with the orbit, the spin-up time-scale $\tau$ of the primary due to the accretion of disc matter can be written as

$$\tau \approx \frac{\omega}{\omega_0} \approx \frac{2\pi}{P_{\text{orb}}} k \sqrt{\frac{M_1 R_1^{3/2}}{G M_2}},$$

(16)

where $k \approx 0.18$ (see Marsh, Nelemans & Steeghs 2004) is the moment of inertia factor ($k = 2/5$ for a solid sphere), and $\omega_0$ is the angular velocity of the accretor. Requiring that tidal synchronization proceeds on time-scales similar to or shorter than the accretion-induced spin-up of the primary, gives a maximum tidal synchronization time-scale $\tau_* \lesssim 2 \times 10^2 \text{ yr}$.

A key question in binary evolution theory is whether two WDs, upon Roche lobe overflow at orbital periods of a few minutes, can manage to stabilize the accretion process and avoid a merger by feeding back angular momentum from the spin-up accretor to the orbit. This has profound implications for the number of systems that may survive the initial phase of mass transfer, when accretion proceeds at a high rate of $\dot{M}_2 \sim 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$ (Marsh et al. 2004).

Although its absolute magnitude is highly uncertain from a theoretical point of view, tidal synchronization is understood to scale with the orbital separation and the radius of the accretor as

$$\tau_* \propto \left(\frac{M_1}{M_2}\right)^2 \left(\frac{a}{R_1}\right)^6,$$

(17)

(see e.g. Marsh et al. 2004). This means that, for a 3-min WD binary in which Roche lobe overflow is about to commence, the tidal synchronization time-scale will be at least a factor of $10^3$ shorter than for AM CVn, or $\tau_* \lesssim 200 \text{ yr}$. This is an interesting result since it is exactly the regime that is needed for an appreciable fraction of binary WD systems to survive the initial phase of mass transfer. See Marsh et al. (2004), in particular their fig. 11.

If the central spike is thus associated with the accreting WD as we think, it implies a tidal synchronization time-scale that is short enough to make the double-degenerate formation channel for AM CVn stars viable (Nelemans et al. 2001b). It would thereby also increase the viability of the direct-impact accretor scenario for the ultracompact binaries RX J0806.3+1527 and V407 Vul (Marsh & Steeghs 2002).

ACKNOWLEDGMENTS

We thank the referee, Jan-Erik Solheim, for valuable comments and suggestions. GHAR and PJG are supported by NWO VIDI grant 639.042.201 to PJG. DS acknowledges a Smithsonian Astrophysical Observatory Clay Fellowship. GN was supported by NWO VENI grant 639.041.405 to GN. TRM was supported by a PPARC Senior Research Fellowship. This work is based on observations made with the WHT operated on the island of La Palma by the Isaac Newton Group, in the Spanish Observatorio del Roque de los Muchachos of the Instituto de Astrofísica de Canarias. We are indebted to the astronomy staff of the Isaac Newton Group for their hospitality during our observing missions.

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