Thermal evolution of a rotating strange star in the colour superconductivity phase

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ABSTRACT

Under the combination effect of recommencement heating due to the spin-down of strange stars (SSs) and heat preservation due to the weak conduction heat of the crust, Cooper pair breaking and formation (PBF) in colour superconducting quark matter arises. We investigate the cooling of SSs with a crust in the colour superconductivity phase including both deconfinement heating (DH) and the PBF process. We find that DH can delay the thermal evolution of SSs and the PBF process suppresses the early temperature rise of the stars. The cooling SSs behave within the brightness constraint of young compact objects when the colour superconductivity gap is small enough.

Key words: dense matter – stars: evolution – stars: neutron – pulsars: general.

1 INTRODUCTION

In the 1990s, a breakthrough came with ROSAT for measuring thermal radiation directly from the surface of pulsars. ROSAT offered the first confirmed detections for such surface thermal radiation from at least three pulsars. Recently, radio pulsars and isolated neutron stars (NSs) have been extensively studied with the new powerful X-ray observatories Chandra and XMM-Newton. There are numerous pulsars for which it is possible to measure the surface temperature (Trümper 2005; Tsuruta 2006). At the same time, more careful and detailed theoretical investigation of various input microphysics has been in progress. Understanding the data and constraining the composition of NS interior matter is hoped for. Standard, enhanced and minimal scenarios are usually proposed as cooling NS theories. These models emphatically show stellar mass dependence of cooling NS behaviours. However, large theoretical uncertainties in the supernucleus density regime may be in trouble. Strange quark matter stars (strange stars, SSs) may exist in accordance with high-density physics. Phenomenological and microscopic studies have confirmed that quark matter at a sufficiently high density, as in compact stars, undergoes a phase transition into a colour superconducting state, which is a typical case of the two-flavour colour superconductivity (2SC) or colour-flavour locked (CFL) phases (Shovkovy 2005; Alford 2004). Theoretical approaches also concur that the superconducting order parameter, which determines the gap $\Delta$ in the quark spectrum, lies between 1 and 100 MeV for baryons densities existing in the interiors of compact stars. Of course, a colour superconducting phase could occur in SSs, and its effect on the cooling of the stars is a significant issue. Blaschke, Klähn & Voskresensky (2000) show that the stars in the CFL phase (CSS hereafter) cool down too rapidly, which disagrees with the data. In those calculations, an important factor, as described below, is ignored.

An SS, both in the normal phase and in the colour superconducting phase, can sustain a tiny nuclear crust with a maximum density below the neutron drip ($\sim 10^{11}$ g cm$^{-3}$) and a mass typically of $M_c \lesssim 10^{-5}$ M$_\odot$ due to the existence of a strong electric field on the quark surface (Alcock, Farhi & Olinto 1986; Usov 2004; Zheng & Yu 2006). The spin-down of the star makes the matter at the bottom of the crust compress. As soon as the density exceeds the neutron drip, the surplus matter in the crust falls into the quark core in the form of neutrons. Consequently, the engulfed neutrons dissolve into quarks, and the released energy during this process leads to so-called deconfinement heating (DH) (Yuan & Zhang 1999; Yu & Zheng 2006).

DH causes extreme changes in the cooling behaviour of CSSs. It delays the cooling of the stars especially for CSSs so the CSSs could be observed until they are $10^4 \sim 10^7$ yr old, which is not ruled out by the X-ray data. For CSSs with crust, however, a hundreds-of-years rise in temperature (Yu & Zheng 2006) may be a matter of debate after the birth of the stars, in the absence of supporting data. As is known, the temperature gradient from the surface to the core in compact stars could be 2 or even more than 2 orders of magnitude. Weak conduction crust makes the deconfinement heat settle inside the quark matter core. Thus the core temperature may reach $10^{10} \sim 10^{11}$ K, just in the vicinity of paring of quarks. The superconducting pair breaking and formation (PBF) is important. The initial estimates in Yu & Zheng (2006) did not properly take into account the neutrino emissivity from this processes.

As present in Schaab et al. (1997) and Jaikumar & Prakash (2001), in superfluid NSs, the superfluid PBF processes accelerate mildly both the standard and the non-standard cooling scenario. Former works have mentioned that just below the critical temperature, $T_c$, the neutrino pair emissivity from the PBF processes of
superconducting neutron pairs greatly exceeds that from the so-called modified Urca processes in NS interiors (Yakovlev, Kaminker & Leventish 1999; Gusakov et al. 2004). Similarly, in CSSs, the PBF process also dominates the cooling when $T$ falls below $T_c$ (Jaikumar & Prakash 2001). However, differing from superfluid NSs, the dominating PBF process occurs at the earliest ages due to a much larger pairing gap in quark matter than normal nuclear matter. The PBF process may cancel out the early temperature rise of CSSs. The purpose of this paper is to investigate this possibility.

The star is cooler and the PBF process is suppressed if a CSS is bare. Inversely, a wrapped CSS with nuclear crust has a hot core. The temperature could be in the vicinity of the pairing temperature. Our analysis shows the PBF process cannot change the cooling behaviour of bare CSSs, but probably improves the early cooling curves of CSSs and significantly suppresses the stellar temperature for given parameters. The rest of our paper is arranged as follows. We recall neutrino emissivity and specific heat in colour superconductivity and the DH mechanism in Sections 2 and 3, respectively. The cooling curves and the corresponding explanations are presented in Section 4. Section 5 contains our conclusion and discussions.

2 NEUTRINO EMISSIVITIES AND SPECIFIC HEAT IN THE COLOUR SUPERCONDUCTIVITY PHASE

The most efficient cooling process in unpaired quark matter is the quark direct Urca (QDU) process $d \rightarrow u e \bar{v}$ and $u \rightarrow d \bar{v}$, given by Iwamoto (1982),

$$\epsilon^{(d)} \simeq 8.8 \times 10^{26} a_c \left( \frac{\rho_b}{\rho_0} \right) Y_e^{1/3} T_c^6 \text{erg cm}^{-3} \text{s}^{-1},$$

(1)

where $a_c$ is the strong coupling constant, $\rho_b$ is the baryon density and $\rho_0 = 0.17 \text{fm}^{-3}$ is the nuclear saturation density, $Y_e = (\rho_e/\rho_0)$ is the electron fraction, and $T_c$ is the temperature in units of $10^9$ K. When the QDU process is switched off due to a small electron fraction, the dominating contribution to the emissivities is the quark modified Urca (QMU) $dq \rightarrow u q e \bar{v}$ and the quark bremsstrahlung (QB) processes, estimated as (Iwamoto 1982)

$$\epsilon^{(m)} \simeq 2.83 \times 10^{19} a_c^2 \left( \frac{\rho_b}{\rho_0} \right) T_c^8 \text{erg cm}^{-3} \text{s}^{-1},$$

(2)

$$\epsilon^{(q)} \simeq 2.98 \times 10^{19} \left( \frac{\rho_b}{\rho_0} \right) T_c^8 \text{erg cm}^{-3} \text{s}^{-1}.$$  

(3)

Because of the pairing in the CFL colour superconducting phase, the emissivity of the QDU process is suppressed by a factor of $\exp(-\Delta/T)$ and the emissivity of the QMU and QB processes are suppressed by a factor $\exp(-2\Delta/T)$ for $T < T_c$. So, in our calculation below, we use equation (1) with a factor of $\exp(-\Delta/T)$ and equations (2) and (3) with a factor of $\exp(-2\Delta/T)$.

As $T$ falls below $T_c$, the PBF process dominates the cooling in quark matter. The neutrino emissivity from the PBF process in quark matter can be expressed as (Jaikumar & Prakash 2001)

$$\epsilon^{(PBF)}_0 \simeq 1.4 \times 10^{20} N_r F a_0 \left( \frac{\rho_b}{\rho_0} \right)^{2/3} T_c^7 \text{erg cm}^{-3} \text{s}^{-1},$$

(4)

$$\epsilon^{(PBF)}_m = \epsilon^{(PBF)}_0 \left[ 1 - \frac{m^2}{4P^2} + \frac{1}{7} \left( \frac{c_A}{c_V} \right)^2 \left( 1 + \frac{7m^2}{12P^2} \right) \right],$$

(5)

where $N_r$ is the number of neutrino flavours, $a_0 = c_A/\left[ 1 + 1/7 \left( \frac{c_A}{c_V} \right)^2 \right]$, $c_A$ and $c_V$ are flavour-dependent vector and axial-vector coupling constants respectively and we use the date in table 1 of Jaikumar & Prakash (2001). The correction factor

$$F = f(y) = y^2 \int_y^\infty dx \frac{x^2}{\sqrt{x^2 - y^2} \left( e^x + 1 \right)^2}.$$  

In order to compute the cooling curves of the stars, we need to give the specific heat of the electrons and quarks (Iwamoto 1982):

$$c_e \simeq 2.5 \times 10^{20} \left( \frac{\rho_b}{\rho_0} \right)^{2/3} T_c \text{erg cm}^{-3} \text{K}^{-1},$$

(6)

$$c_q \simeq 0.6 \times 10^{20} \left( \frac{Y_e \rho_b}{\rho_0} \right)^{2/3} T_c \text{erg cm}^{-3} \text{K}^{-1}.$$  

(7)

However, in the colour superconductivity phase, the quark specific heat is changed exponentially (Blaschke et al. 2000):

$$c_{qL} = 3.2c_q \left( \frac{T_c}{T} \right) \times \left[ 2.5 - 1.7 \left( \frac{T}{T_c} \right) + 3.6 \left( \frac{T}{T_c} \right)^2 \right] \exp \left( -\frac{\Delta}{k_B T} \right),$$

(8)

where $T_c$ is related to $\Delta$ as $\Delta = 1.76T_c$ and $k_B$ is the Boltzmann constant. The quarks contribution to the specific heat is suppressed by colour superconductron while the temperature decreases and the specific heat becomes dominated by the electrons. Compared with the total mass of the stars, the mass of the crust is very small ($M_c \lesssim 10^{-3} M_\odot$). So we can neglect the crustal contribution to neutrino emissivity and specific heat (Lattimer et al. 1994).

3 DECONFINEMENT HEATING IN STRANGE STARS WITH CRUST

DH heating is affected by the surplus number of neutrons in the crust falling into the quark core with a strongly exothermic process, which changes the mass of the crust. The total heat released per unit time as a function of $\gamma$ is

$$H_{dH}(\gamma) = -q_{\alpha} \frac{1}{m_\alpha} \frac{dM_c}{d\gamma} v^2,$$

(9)

where $q_{\alpha}$, the heat released per absorbed neutron, is expected to be in the range $q_{\alpha} \sim 10^-40$ MeV, its specific value depending on the assumed strange quark matter (SQM) model, and $m_\alpha$ is the mass of baryon. Assuming the spin-down is induced by magnetic dipole radiation, the evolution of the rotation frequency $\nu$ is given by Yu & Zheng (2006)

$$\nu = -\frac{8\pi^2}{3Me} \mu_3 \gamma^3 \sin^2 \theta,$$

(10)

where $\mu_3$ is the stellar moment of inertia, $\mu = \frac{1}{2} BR^3$ is the magnetic dipole moment and $\theta$ is the inclination angle between magnetic and rotational axes. The mass of the crust $M_c$ can be approximated by a quadratic function of rotation frequency $\nu$. As discussed in Zdunik, Haensel & Gourgoulhon (2001) and Yu & Zheng (2006), the mass of the crust reads

$$M_c = M_c^0 \left( 1 + 0.24v_3^2 + 0.16v_5^4 \right),$$

(11)

where $v_3 = \nu/10^5$ Hz and $M_c^0 < 10^{-5} M_\odot$ is the mass of the crust in the static case.
4 COOLING SIMULATIONS
AND DISCUSSIONS

Considering the energy equation of the star, the cooling equation can be written as

\[ C_v \frac{dT}{dt} = -L_v - L_\gamma + H, \]

(12)

where \( C_v \) is the total specific heat, the term \( H \) indicates the heating energy per unit time, in our work \( H = H_{\text{dec}} \), \( L_v \) is the total neutrino luminosity and \( L_\gamma \) is the surface photon luminosity given by

\[ L_\gamma = 4\pi R^2 \sigma T^4, \]

(13)

where \( \sigma \) is the Stefan–Boltzmann constant and \( T_\gamma \) is the surface temperature.

The thermal evolution of bare SSs is investigated by Blaschke et al. (2000). They showed that the bare SSs are very cool objects. To simulate the cooling behaviour of CSSs with a nuclear crust, we first would like to go into the PBF effect in particular. Its internal temperature is below 10^5 K (see their fig. 3), so the PBF process is completely suppressed like those QDU, QMU and QB processes. The existence of the crust of the CSSs brings two effects: heat preservation due to the crust as a cover and DH due to the spin-down of the stars. DH causing the CSSs to be very hot objects has been investigated (Yu & Zheng 2006). The heat preservation deposits a vast amount of latent heat inside the stars, thus the temperature probably increases the level in the vicinity of paring quarks at the early cooling stage of some CSSs. We can foresee that the combination effect will significantly change the cooling behaviour of CSSs.

We now address the problem of cooling curves of CSSs with a crust. The surface temperature of the stars is related to internal temperature by a coefficient determined by the scattering processes occurring in the crust. We apply a formula that is demonstrated by Gudmundsson, Pethick & Epstein (1983). It reads

\[ T_\gamma = 3.08 \times 10^6 g_{14}^{1/4} R_{10^4 \text{yr}}^{0.5495}, \]

(14)

where \( g_{14} \) is the proper surface gravity of the star in units of 10^{14} cm s^{-2}. In principle, magnetic fields may change the expression of equation (14). However, Potekhin, Yakovlev & Prakash (2001) have proposed that the effect is negligible if the field strength is lower than 10^{13} G. So equation (14) is a good approximation for our case.

We consider a model of canonical SSs of 1.4 M⊙ at a constant density in our work, which is a very good approximation for SSs of mass \( M \leq 1.4 \) M⊙ (Alcock et al. 1986). We choose \( g_0 = 20 \text{ MeV}, \) the initial temperature \( T_0 = 10^5 \text{ K}, \) initial period \( P_0 = 0.78 \text{ ms}, \) and the magnetic tilt angle \( \theta = 45^\circ. \) We also considered the gravitational redshift, which gives the effective surface temperature detected by a distant observer \( T_\gamma = T_\gamma \sqrt{1 - R_\theta/R}, \) where \( R_\theta \) is the gravitational stellar radius.

We first plot the cooling curves in Fig. 1 showing the cooling behaviours of SSs with crust masses of 10^{-3} and 10^{-6} M⊙ and gaps from 0.1 to 100 MeV in various magnetic fields. The observation data are taken from Page et al. (2004) and are shown in order to give the reader a feeling of the position of the data in the logarithm \( T_\gamma^4 - t \) plane. As displayed, the X-ray data cannot rule out the existence of CSSs for a wide range of parameters. This is quite a different point from the suggestion in Blaschke et al. (2000).

Fig. 2 is depicted to illustrate the brightness constraint on models, suggested by Grigorian (2005). In our results, the curves for larger gaps (panels a and b) may show the slight contradiction with the fact that we do not see any very hot compact objects with a field of about 10^{11} \sim 10^{12} G because the core temperature is still much lower than the gaps. For the gap of 1 MeV (panel c), the PBF process significantly reduces the temperature rise at early periods. The cooling curves are improved at ages 10^{5} \sim 10^{7} \text{ yr} and conflict with the brightness constraint. For a smaller gap (panel d), the QDU and QB processes as well as the PBF process become somewhat important and hence the cooling curves lie below the brightness constraint. In accordance with our results, we can see that DH delays the cooling of CSSs but the PBF effect partially cancels out the heating at early ages. The cooling curves later are governed by the equilibrium between the photon and heating (\( L_\gamma = H_{\text{dec}} \)).

5 CONCLUSIONS

We have studied the cooling behaviours of rotating CSSs including the PBF process in the core of the stars and the DH effect due to the spin-down of the stars. DH greatly delays the cooling of colour superconducting stars. The X-ray data, thus, do not rule out...
the existence of colour superconducting stars for a wide range of parameters. The PBF effect suppresses the temperature rise of the stars at early ages for the smaller gaps so that very hot compact stars, which we have never seen, would be avoided.

Considering the poorly understood properties of the colour superconductivity of strange quark matter, we use a simple parametrized model for colour superconductivity to produce the cooling curves in a wide range of gap parameters (0.1 ∼ 100 MeV). Although we have been referring to the CFL superconduction phase in this paper, our approach and result is generally suitable for other possible superconduction phases, such as the spin-one colour conductivity etc. The smaller gap curves are in favour of the brightness constraint suggestion, and this fact will supply constraints on the existence of phases in strange stars.

Finally, we must point out that the stars may cool down too slowly at old ages over 10^6 yr for the existence of not only photon emission but also the heating effect. The cooling curves should be improved in the future if the decrease of the magnetic moment for the spin-down of stars, with both field decay and dipole alignment, is involved.

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