A thousand and one nova outbursts

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ABSTRACT

A full nova cycle includes mass accretion, thermonuclear runaway resulting in outburst and mass-loss, and finally, decline. Resumed accretion starts a new cycle, leading to another outburst. Multicycle nova evolution models have been calculated over the past twenty years, the number being limited by numerical constraints. Here we present a long-term evolution code that enables a continuous calculation through an unlimited number of nova cycles for an unlimited evolution time, even up to \(1.5 \times 10^{10}\) yr. Starting with two sets of the three independent nova parameters – the white dwarf (WD) mass, the temperature of its isothermal core, and the rate of mass transfer on to it – we have followed the evolution of two models, with initial masses of \(1\ M_\odot\) and \(0.65\ M_\odot\) through over 1000 and over 3000 cycles, respectively. The accretion rate was assumed constant throughout each calculation: \(10^{-11}\ M_\odot\ yr^{-1}\) for the \(1\ M_\odot\) WD, and \(10^{-9}\ M_\odot\ yr^{-1}\) for the \(0.65\ M_\odot\) one. The initial temperatures were taken to be relatively high: \(30 \times 10^6\) and \(50 \times 10^6\ K\), respectively, as they are likely to be at the onset of the outburst phase. The results show that although on the short-term consecutive outbursts are almost identical, on the long-term scale the characteristics change. This is mainly due to the changing core temperature, which decreases very similarly to that of a cooling WD for a time, but at a slower rate thereafter. As the WD’s mass continually decreases, since both models lose more mass than they accrete, the central pressure decreases accordingly. The outbursts on the massive WD change gradually from fast to moderately fast, and the other characteristics (velocity, abundance ratios, isotopic ratios) change, too. Very slowly, a steady state is reached, where all characteristics, both in quiescence and in outburst, remain almost constant. For the less massive WD accreting at a high rate, outbursts are similar throughout the evolution.

Key words: accretion, accretion discs – binaries: close – novae, cataclysmic variables – white dwarfs.

1 INTRODUCTION

It is now twenty years since the publication of the first numerical simulation of a full nova cycle (Prialnik 1986). During this time modelling of nova outbursts has advanced significantly and calculations have extended to continuous evolution thorough a series of several cycles (e.g. Shara, Prialnik & Kovetz 1993). Extensive parameter studies of the classical nova phenomenon have been carried out by Prialnik & Kovetz (1995) and Kovetz & Prialnik (1997), and more recently by Yaron et al. (2005). In parallel, computation capability has increased tremendously and thus the computing time required to simulate a full nova cycle has dropped from months to minutes.

Models have shown – and observations have confirmed – that the characteristics of a nova outburst are determined by three independent parameters (Prialnik & Kovetz 1994; Schwartzman, Kovetz & Prialnik 1994; Prialnik 1995). On the theoretical side, these are the mass and core temperature (or luminosity) of the white dwarf (WD), which accretes mass from a binary companion and eventually ignites it explosively and ejects it back into space, and the rate of accretion on to the WD. Changing these parameters independently over the entire ranges allowed by stellar structure theory reproduces the wide range of observed characteristics of novae (Yaron et al. 2005). So far, therefore, the theory of nova outbursts has proved extremely successful. However, the three free parameters are not truly independent. They are linked by the long-term evolution of the binary system, which changes all of them simultaneously. Thus the question remains whether the entire three-dimensional parameter space is accessible to nova binary systems. Is it possible for the WD to cool to low temperatures despite repeated outbursts and loss or gain of mass? Does the WD mass remain the exclusive structural parameter, or is the structure affected on the long-term scale by the events occurring at the surface? Such questions can only be answered by following the evolution of an accreting WD continuously, not through one or a few nova cycles, but through a thousand such cycles and more. This is the purpose of the present paper.
Long-term unabridged evolutionary calculations through repeated nova outburst cycles require some changes in the evolution code used in previous calculations, which will be described in Section 2. The results of two computation runs are described and analysed in Section 3, both from the point of view of the outbursts and of the WD on top of which they occur. A brief discussion, summary and conclusions are given in Section 4.

2 LONG-TERM EVOLUTIONARY CALCULATIONS

The hydrodynamic Lagrangian stellar evolution code used for the calculations presented here was described in some detail by Prialnik & Kovetz (1995). It includes OPAL opacities, a nuclear reactions network among 40 heavy element isotopes up to $^{11}$P, convection according to the mixing-length theory, diffusion for all elements, accretional heating and a mass-loss algorithm that applies a steady, optically thick supersonic wind solution. We note, in particular, that the dynamical phases are calculated by solving the equation of motion along with the energy balance equation (rather than imposing hydrostatic equilibrium). Mass-loss is calculated continuously, according to the mass-loss rate $\dot{M}_{\text{acc}}$, derived from the optically thick wind solution. At each time-step $\delta t$, an amount $M_{\text{out}, t} = M_{\text{out}, t-1} - \dot{M}_{\text{acc}} \delta t$ is subtracted from the outermost mass shell $\Delta m$. Time-steps are constrained during this phase by the requirement $M_{\text{out}, t} < \Delta m$. Whenever $\Delta m$ becomes very small, it is merged with the underlying mass shell. In a similar manner, during the accretion phase, an amount $M_{\text{acc}} \delta t$ is added to the outermost mass shell, and when $\Delta m$ exceeds a prescribed value, the shell is divided into two mass shells.

Although thousands of cycles are computed in the present study, each nova cycle, including all evolutionary phases and involving a large network of nuclear reactions, is calculated accurately, with no short-cuts or simplifications. A detailed description of the changing physical parameters during the evolution of a full nova outburst was given by Prialnik (1986). Here we refrain from dealing with any single outburst cycle in detail, but focus on the trends of change of typical characteristics and on long-term evolutionary effects.

The spherical grid adopted for the numerical solution, which includes the entire WD, is fixed, except for the outermost grid shells, which may grow or shrink, and be divided or merged, during the accretion and mass-loss episodes of the nova cycle. The fixed grid is, however, by no means an equally spaced one. The masses of shells vary by many orders of magnitude, according to the gradients of various physical properties (such as temperature, density, element abundances) that are expected to develop during evolution. Thus, the spatial grid must be prepared in advance in anticipation of the evolutionary course of the stellar model. In particular, the outer layers of the WD that are expected to take part in the various outburst processes (diffusion, convection, nuclear burning, acceleration, expansion, ejection, contraction and so forth) must be finely zoned. Since, as a rule, each outburst erodes the WD of some mass (in addition to the accreted mass), the grid is set in advance for a prescribed number of nova cycles. The larger the number of cycles, the larger the grid, and consequently, the larger the computation time. This was the procedure adopted in former multicycle calculations. Thus the number of cycles was restricted in the original form of the code by the fixed spatial grid of the numerical scheme.

The long-term evolution code should allow an unlimited number of cycles. A new algorithm is therefore devised whose purpose is to rezone the grid at the beginning of each nova cycle, thereby preparing it for the upcoming evolutionary stages of that cycle. This numerical process, which involves interpolation on a previous grid, is designed in such a way as to conserve energy and mass of each nuclear species. Hydrostatic equilibrium (which prevails between the dynamical phases of the outburst itself) is preserved as well. One of the problems that have to be overcome is the determination of the optimal mass shell size for the outer part of the grid. Since consecutive nova outbursts are very similar (almost identical), this is done at each cycle based on the characteristics of the previous one, namely, the amount of accreted mass and ejected mass, and the depth of the burning shell and convective zone. Another problem is to determine the depth of the finely zoned region and the radius at which we gradually increase shell masses until they eventually match the coarsely zoned bulk of the star. This is achieved by a geometrically increasing series, with a typical factor of 1.25. Since matrix inversion is required at each iteration of every time-step of the numerical calculation, a compromise must be reached between computing time and resolution (or accuracy). Typically, the grid included between 200 and 400 mass shells for the entire configuration and each continuous – hands-off – run took several weeks of CPU time on a fast PC.

Finally, the code is capable of correcting itself in case of failure, by going back to an earlier stage and continuing from that stage with adjusted numerical parameters, such as length of time-step, maximal number of iterations allowed for achieving convergence and so forth. Occasionally, a couple of times per thousand cycles, it does happen that the code goes astray, but it recovers without intervention. Given the complexity of the input physics, in particular the need to interpolate among a large number of opacity tables, this should not be surprising. Such stray points have been removed from the results.

A run spanning thousands of nova outburst cycles produces a prohibitive amount of data, which must be carefully selected, divided among a large number of files, stored, and analysed, mainly by graphical means. A great deal of information is necessarily lost in the process and can only be retrieved by repeating the calculation.

3 EVOLUTION THROUGH REPEATED NOVA CYCLES

Two evolution runs were carried out, starting from two different initial parameter combinations taken from the extended grid of nova models studied by Yaron et al. (2005). We chose a relatively massive WD, of relatively high temperature, accreting at a low rate (hereafter Model A), and a hot, low-mass WD, accreting at a relatively high rate (hereafter Model B). According to Prialnik & Kovetz (1995) and Yaron et al. (2005), the outburst characteristics of these models differ considerably, although in both cases they were found to be well within the observed range of nova properties.

3.1 Nova outbursts – Model A

The initial parameters of Model A are: $M_{\text{WD}} = 1 M_\odot$, $M = 10^{-11} M_\odot$ yr$^{-1}$ and $T_{\text{WD}} = 30 \times 10^6$ K. Evolution was followed for $1.5 \times 10^{10}$ yr (roughly a Hubble time), during which time the WD underwent 1001 nova outbursts.

The long-term evolution of the main physical characteristics related to the outbursts that occur on top of this accreting WD is shown in Figs 1–3. The top left-hand panel of Fig. 1 shows the accreted mass $m_{\text{ac}}$, ejected mass $m_{\text{e}}$, and the mass of the convective envelope that develops after hydrogen ignition $m_{\text{conv}}$. All three increase with time for the first ~240 cycles, or ~$10^9$ yr, but stabilize thereafter to a very slow rate of growth. At first, the ejected...
mass is almost identical to the envelope mass, which means that the remnant, hydrogen-rich layer at the end of the outburst is very small, and thus the time of decline of the bolometric magnitude is short as well. We recall that the decline starts when the remnant hydrogen is exhausted. Gradually, however, the trend shifts towards the envelope mass being almost equal to the accreted mass, and the ejected mass being larger than both. This means that WD material is ejected directly during the late stages of mass-loss, without being previously mixed with the accreted material. Observationally, this should be reflected by a change in composition of the ejecta, which may be detectable if/when the material ejected earlier becomes transparent. The reason for this effect is a change in the evolution of the nova outburst. When most of the mass contained in the convective envelope has been ejected, the star contracts back to almost WD size. The high (bolometric) luminosity persists for a while longer, until the hydrogen in the envelope remnant is burnt out. When the remnant is massive, however, contraction followed by heating of the hydrogen-rich remnant layer may ignite it explosively again, which will lead to a second mass-loss episode, so that the entire original envelope and even some additional WD material will be ejected. Such behaviour has already been encountered and discussed in previous nova outburst studies (e.g. Prialnik & Livio 1995).

For each outburst, we calculate the average abundance of any element \( n \) in the ejecta \( X_{e,j} \) by

\[
X_{e,j} = \frac{\int M_{m-j} X_{e,j}(t) \, dt / m_{ej}}{M_{m-j}}
\]

where \( X_{e,j}(t) \) is the respective abundance in the outermost mass shell from which mass is removed (see Section 2) at a given time; it is not necessarily constant (see also Kovetz & Prialnik 1997), as it depends upon the mixing history of the envelope following the thermonuclear runaway. The integration is carried over the entire mass-loss phase of the cycle. The ejecta average metallicity \( Z_{ej} \) decreases with repeated cycles, as the accreted mass increases. At the same time the helium mass fraction \( Y_{ej} \) increases, but both reach a plateau after 240 cycles, when the accreted and ejected masses stabilize as well. At this stage, the breakdown of \( Z_{ej} \) into the most abundant elements is: N – 0.09, O – 0.08 and C – 0.04. The difference between \( Z_{ej} \) and \( Z_{env} \) is a direct consequence of the relationship between \( m_{ej} \) and \( m_{env} \), explained above: \( Z_{ej} > Z_{env} \), when during the secondary mass-loss episode, WD material (\( Z \approx 1 \)) that was not previously mixed with the accreted material, is ejected as well.

The maximal temperature attained at outburst reaches a maximum of about \( 2.3 \times 10^6 \) K around the 400th cycle and declines very slowly thereafter.

Perhaps the most interesting result of this long-term calculation is the relatively sharp transition from fast nova outbursts – for roughly the first 200 cycles – to moderately fast nova outbursts from the 250th outburst onwards, as shown by the duration of the mass-loss episode plotted in the lower panel of Fig. 2. Even more intriguing is the behaviour of the average expansion velocity, shown in the upper panel: as the fast nova outbursts progress, the velocity goes through a maximum and settles to an almost constant value of about 500 km s\(^{-1}\). It thus appears that fast novae may exhibit a range of expansion velocities extending between 500 and 2500 km s\(^{-1}\), while slower ones should have typically lower velocities. We should bear in mind, however, that a low average
velocity may still allow much higher peak velocities for short periods of time. Of course, these conclusions relate to outbursts obtained for a 1 M⊙ WD and may change with the WD mass.

The isotopic ratios shown in Fig. 3 undergo a similar evolution: a changing, non-monotonous pattern along the first 250 fast nova cycles, settling down to constant ratios for most of the evolution. We note significant overabundances of $^{13}$C and $^{17}$O, compared to solar abundances (taken from Lodders 2003), a somewhat lower overabundance of $^{15}$N, while $^{18}$O is underabundant. We also note that for the first few tens of outbursts the trends of change are different and variable. Thus, in this respect, results based on one or even a few nova cycles may be misleading. In other respects, however, a general trend is already apparent and maintained from the beginning.

3.2 Nova outbursts – Model B

The initial parameters of Model B are: $M_{WD} = 0.65 M_{\odot}$, $\dot{M} = 10^{-9} M_{\odot}$ yr$^{-1}$ and $T_{WD} = 50 \times 10^{8}$ K. Evolution was followed for a much shorter time, $\sim 5 \times 10^8$ yr, but during this time the WD underwent over 3000 nova outbursts. The total mass transferred to it by accretion amounted to $\sim 0.5 M_{\odot}$, which is of the order of a typical red dwarf binary companion mass. Thus, in spite of the relatively short evolution time, the companion mass may have been exhausted by its end.

The long-term evolution of the main physical characteristics related to nova outbursts that occur on top of this accreting WD is shown in Figs 4 and 5. Here, due to the high accretion rate, the evolutionary time-scale is much shorter; thus, for example, 1000 cycles which spanned $\sim 1.5 \times 10^9$ yr for Model A, take only $\sim 10^8$ yr for Model B. The times are not precisely inversely proportional to the corresponding accretion rates because the accreted masses required for outbursts to be triggered depend on the WD mass.

The striking difference between Models A and B, which is immediately obvious from the comparison of Figs 1 and 4 as well as Figs 3 and 5, is the constancy (or monotony) in the evolution of the outburst characteristics for the low-mass WD, in contrast to the sharp changes that take place in the course of evolution of the more massive one. But, in fact, indication for this trend of behaviour was already provided by the grid of nova models (Yaron et al. 2005).

We note, in particular, that for Model B the envelope mass is always larger than both the accreted and the ejected masses. The mass fractions of the most abundant elements of the ejecta are 0.065 for N, 0.055 for O and 0.004 for C. Regarding isotopic ratios, we note that $^{15}$N is underabundant in the present case, whereas for Model A it was overabundant at various levels throughout most of the evolution. The average expansion velocity oscillates between 150 and 220 km s$^{-1}$, with no particular trend. The outburst duration, as measured by the extent of the mass-loss phase, increases gradually from $\sim 80$ to $\sim 160$ d along the first 1000 cycles and oscillates between these limits thereafter.
luminosity, given by the accretion WD and a cooling WD of the same mass and initial core temperature: left-hand panels – Model A; right-hand panel – Model B. The accretion luminosity, equation (2), is also shown in the lower panels. An additional curve for the core temperature of Model B shows the long-term evolution through almost 3000 nova cycles, adopting the same mass and accretion rate, but a 10 times lower initial temperature, $T_{\text{WD}} = 5 \times 10^5$ K (see discussion in Section 4.1).

3.3 Evolution of the WD – Models A and B

One of the most puzzling questions related to nova outbursts is their effect, if any, on the characteristics of the underlying WD, keeping in mind that the outbursts are confined to the outermost layers of the WD, less than a thousandth of its mass. Previous studies, even if they considered several cycles, did not span a sufficient fraction of the WD evolutionary time-scale for answering this question. An indication that the WD core may continue to cool despite the heat generated by nova outbursts was given by Prialnik (1987). In the present calculations the evolution of the WD was carried out for a period of $1.5 \times 10^{10}$ yr (about a Hubble time); we are thus in a position to answer this question.

Using the same code, we followed the evolution of the same initial WD configuration, omitting accretion, to obtain the cooling curve of the WD. The central temperatures of the undisturbed WD and of the accreting one are compared in Fig. 6. For the 1 $M_\odot$ WD (Model A), the accreting WD cools at the same rate as the single one for a period of $\sim 10^9$ yr (equivalent to about 250 cycles), but then the cooling trend slows down and the WD core tends to a constant temperature of $\sim 5 \times 10^5$ K. The evolutionary time-scale for Model B is short compared to the cooling time-scale of a WD, hence the cooling curves are still close even after several thousand cycles, which amount to only a few $10^9$ yr. The luminosities of the cooling WD and of the accreting WD in quiescence (during the accretion episodes separating outbursts) are compared in Fig. 6. The luminosity of the accreting WD has two sources: the heat emanated by the cooling WD core and the energy imparted by accretion, known as accretion luminosity, given by

$$L_{\text{acc}} = \frac{\alpha_{\text{acc}} G M_{\text{WD}} \dot{M}}{R_{\text{WD}}},$$

where $\alpha_{\text{acc}}$ is taken to be 0.15, following Regev & Shara (1989). The outer layers of the WD adjust so as to re-emit this influx of energy, in a similar manner to an irradiated star (Kovetz, Prialnik & Shara 1988). We note that only at the beginning, while the WD core is still relatively hot, does the core luminosity exceed the accretion luminosity.

In order to understand the thermal evolution of the WD, we plot in Fig. 7 temperature profiles at various times during evolution (marked by the cycle number). All profiles correspond to the end of a nova cycle, just before the onset of a new accretion episode. Thus the configuration includes the WD and the eventual remnant of the envelope at the end of the mass-loss phase. At first, for several hundred cycles, the temperature gradient is negative everywhere. Gradually, however, the outer layers of the WD, well beneath the outermost one that is directly affected by the outburst (mainly through diffusion, which changes its composition), are heated and a temperature inversion develops. Nevertheless, for many additional cycles, this temperature “wall” does not prevent the core from cooling, since the temperature still decreases between the centre and the inversion zone. But the slope of the core temperature profile slowly decreases and finally flattens. At this stage the WD core ceases to cool. It may subsequently start to heat up, but for this to happen, more than a Hubble time seems to be required for the massive WD, and several.
equation of state for degenerate matter to first order in temperature of Fig. 8), which drives it down. Applying to the WD centre the already mentioned cooling of the core, which drives the den-
er after. The reason for this behaviour lies in the competition between the cooling curves of an accreting and an undisturbed WD, shown in Fig. 6.

The evolution of the central density of the accreting WD is shown in the lower panel of Fig. 8, together with that of the single WD. While the latter increases approaching an asymptotic value, the former – for Model A – increases at first but decreases steadily thereafter. The reason for this behaviour lies in the competition between the already mentioned cooling of the core, which drives the density up, and decrease of the WD mass (shown in the upper panels of Fig. 8), which drives it down. Applying to the WD centre the equation of state for degenerate matter to first order in temperature (e.g. Landau & Lifshitz 1969), we have

\[ P_c = \beta \rho_c^{5/3} \left( 1 + \alpha \frac{T_c^2}{\rho_c^{5/3}} \right) \]

(3)

Figure 9. Left-hand panel: Evolution of the LHS terms of equation (8) for Model A (see text for details): F1 represents the first term, which includes the derivative of temperature, and F2 – the second term, which includes the derivative of the WD mass. Right-hand panel: Zoom in into the evolution of the central density (shown in Fig. 8), to show the rise and decline of \( \rho_c \) during early evolution.

where, in standard notation,

\[ \beta = \frac{h^2}{5m_e (m_1 \mu_e)^{5/3}} \quad \alpha = \frac{5m_e^2 k^2 (m_1 \mu_e)^{5/3}}{\hbar^2 (3\pi^2)^{1/3}} \]

(4)

and \( \mu_0 \approx 2 \).

On the other hand, hydrostatic equilibrium for a polytropic equation of state requires

\[ P_c = (4\pi)^{1/3} B_n G \left( \frac{M_{WD}}{m_1} \right)^{2/3} \frac{\rho_c}{\rho_0}^{4/3} \]

(5)

where \( B_n \) is a constant determined by the polytropic index \( n \) (e.g. Chandrasekhar 1967), thus \( B_{1.5} = 0.206 \) for \( P \propto \rho^{4/3} \). Combining equations (3) and (5), we obtain a relation between \( \rho_c, T_c \) and \( M_{WD} \), of the form

\[ \beta \rho_c^{1/3} + \alpha \beta \frac{T_c^2}{\rho_c} = \gamma \frac{M_{WD}}{3} \]

(6)

where

\[ \gamma = (4\pi)^{1/3} 0.206 \]

(7)

Taking the time derivative of equation (6), we obtain

\[ \frac{\beta}{2 \rho_c} \left( \frac{\rho_c^{1/3}}{3} - \frac{\alpha T_c^2}{\rho_c} \right) \frac{d\rho_c}{dt} = -\alpha \beta \frac{T_c}{\rho_c} \frac{dT_c}{dt} + \frac{\gamma}{3} \frac{M_{WD}^{1/3} dM_{WD}}{dt} \]

(8)

Since \( \alpha \) is a very small number, deriving from the small correction to the degenerate equation of state due to temperature, the coefficient of the density derivative on the RHS is negligible. The density maximum corresponds to the intersection.

However, since the rate of change of the WD mass is roughly proportional to the rate of accretion,

\[ \frac{dM_{WD}}{dt} \approx -\dot{M} \frac{m_1 - m_{acc}}{m_{acc}} \]

(9)
for a very high accretion rate the second term on the RHS of equation (8) will always dominate and the central density will decrease monotonically. This is the situation for Model B, where the accretion rate is two orders of magnitude higher than for Model A. Equally, if \( \dot{m}T_{/}/dr > 0 \), the central density will decrease monotonically.

### 3.4 Energy budget

It is instructive to examine the energy budget of nova outbursts and its eventual change with time. The nuclear energy generated during the thermonuclear runaway and during the equilibrium burning of the remnant hydrogen (if any), \( E_{\text{nuc}} \), is spent in radiation at close to (or exceeding) Eddington luminosity during the outburst, \( E_{\text{rad}} \), kinetic energy of the ejecta, \( E_{\text{kin}} \), and gravitational energy required to lift the ejected material out of the potential well of the WD, \( E_{\text{grav}} \). These are plotted in Fig. 10 for Model A.

The last term is not computed during calculations, but estimated by \( E_{\text{grav}} \approx M_{\text{WD}} \dot{m}_y R(M_{\text{WD}}) \), which is an upper limit to the actual value, since \( R(M_{\text{WD}}) \) is the radius at the lower boundary of the ejected mass. We note that the bulk of nuclear energy is spent in work against the gravitational field of the WD. We also note that the radiated energy exceeds the kinetic energy and, for most of the evolution, quite significantly. Finally, the sum of sinks is lower than the source, that is,

\[
E_{\text{nuc}} \gtrsim E_{\text{grav}} + E_{\text{rad}} + E_{\text{kin}}
\]

which means that a small fraction of the heat generated at outburst is conducted into the WD. We may estimate the total thermal energy that the WD gains for the entire evolution time, or more precisely, a lower limit for it, by \( \sum_{\text{cycles}} E_{\text{nuc}} - (E_{\text{grav}} + E_{\text{rad}} + E_{\text{kin}}) \gtrsim 2 \times 10^{46} \text{ erg} \). This energy represents about 1 per cent of the total energy of the WD and explains the very slow change in slope of the core temperature profile shown in Fig. 7. During evolution, the WD expands due to mass-loss (see Fig. 8), and therefore its gravitational potential (binding) energy – as computed by the evolution code – increases (i.e. becomes less negative), while the internal energy (mostly the energy of the degenerate electron gas) decreases by roughly the same amount (~15 per cent), as expected (cf. Menzel & Raftery 1967).

For the 0.65 M\(_{\odot}\) WD, the budget is slightly different. All energy values are almost constant in time, as are most of the other nova characteristics. Typical values are: \( E_{\text{nuc}} \approx 7.6 \times 10^{46} \text{ erg} \), \( E_{\text{grav}} \approx 2.7 \times 10^{46} \text{ erg} \), \( E_{\text{rad}} \approx 4.2 \times 10^{46} \text{ erg} \), and finally, \( E_{\text{kin}} \approx 8 \times 10^{45} \text{ erg} \). We note that in this case the radiated energy is the major energy sink – due to the long duration of the outburst – while the total kinetic energy is only a very small fraction of the nuclear energy generated, considerably lower than the radiated energy.

### 4 DISCUSSION AND CONCLUSIONS

#### 4.1 Comparison with earlier studies

Model A starts at a core temperature of \( 30 \times 10^6 \text{ K} \), which decreases during the WD’s evolution. After 210 cycles (7.8 \times 10^8 \text{ yr} ), the temperature reaches \( 10^{10} \text{ K} \). This result enables a comparison with the grid of nova models calculated by Prialnik & Kovetz (1995) and Yaron et al. (2005), where the different values of WD core temperature were set as initial independent parameters. Similarly, the 0.65 M\(_{\odot}\) WD, which started at a core temperature of \( 50 \times 10^6 \text{ K} \), reaches \( 30 \times 10^6 \text{ K} \) after about 970 cycles (9.7 \times 10^7 \text{ yr} ). The results are compared in Table 1. We note a striking similarity between the results, allowing for the fact that the WD mass changes as well along with the core temperature. In the case of the 0.65 M\(_{\odot}\) WD, for example, it dropped to 0.637 M\(_{\odot}\) by the time the core temperature dropped to \( 30 \times 10^6 \text{ K} \).

Townsley & Bildsten (2004) studied the problem of accreting WDs analytically, assuming that the WD core reaches an equilibrium temperature and this is achieved by heating of the core due to release of gravitational energy during compression of the accreted material as well as quiet nuclear burning during the nova cycle. Their study, by its nature, does not take into consideration evolutionary effects, such as the decrease of the WD mass, and dynamical phases of the outburst, which determine to a large extent the energy budget. Our full-scale calculations show that, indeed, a steady state (or equilibrium) is eventually attained or approached asymptotically, but on a very long time-scale.

The equilibrium core temperatures that we find for the two cases considered here are considerably higher than those derived by Townsley & Bildsten (2004); the accreted masses, equivalent to

### Table 1. Comparison between present calculations and the grid of nova models (Yaron et al. 2005) for \( M_{\text{WD}} = 1 \text{ M}_{\odot} \) and \( M = 10^{-11} \text{ M}_{\odot} \text{ yr}^{-1} \) and for \( M_{\text{WD}} = 0.65 \text{ M}_{\odot} \) and \( M = 10^{-8} \text{ M}_{\odot} \text{ yr}^{-1} \) – characteristics of the nova outbursts.

<table>
<thead>
<tr>
<th></th>
<th>( m_{\text{nuc}} ) (M(_{\odot}))</th>
<th>( m_{\text{ej}} ) (M(_{\odot}))</th>
<th>( Y_{\text{env}} )</th>
<th>( Y_{\text{ej}} )</th>
<th>( Z_{\text{env}} )</th>
<th>( Z_{\text{ej}} )</th>
<th>( T_{\text{max}} ) (10(^6) K)</th>
<th>( v_{\text{avg}} ) (km s(^{-1}))</th>
<th>( t_{\text{on}} ) (d)</th>
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<td>Model A – grid</td>
<td>8.72E–05</td>
<td>1.00E–04</td>
<td>0.2475</td>
<td>0.2730</td>
<td>0.1529</td>
<td>0.1917</td>
<td>2.09</td>
<td>1063</td>
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<tr>
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<tr>
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<td>Model B – evolution</td>
<td>1.17E–04</td>
<td>1.33E–04</td>
<td>0.2831</td>
<td>0.2887</td>
<td>0.1248</td>
<td>0.1322</td>
<td>1.20</td>
<td>206</td>
<td>106</td>
</tr>
</tbody>
</table>
their ignition masses, for the same cases, are similar. In order to test whether the steady state temperature is, indeed, an equilibrium temperature, we have run a calculation similar to Model B, but for a very low initial WD temperature, lower than the asymptotic value. The evolution was again followed for close to 3000 full nova cycles and we found the WD temperature to rise slowly towards the same steady state value of about $1.5 \times 10^7$ K, as the cooling, initially hot, WD. We wish to stress, however, that in both cases the asymptotic value is approached very slowly compared with the evolutionary time-scale of the nova binary system, on which the mass ratio of the components changes very significantly, and the mass transfer rate is expected to change as well. Consequently, the equilibrium core temperature may not be assumed to occur a priori.

4.2 Summary

We have carried out long-term continuous evolutionary calculations of accreting WDs through thousands of nova cycles, where each cycle was followed in detail through its different phases: accretion, thermonuclear runaway, expansion, mass-loss and contraction.

This study shows, for the first time, the long-term effect nova outbursts have on the structure and evolution of the WD at the surface of which they occur. At the same time, it shows the effect of the changing WD characteristics, in turn, on the nova outbursts. We have chosen two very different combinations of WD mass and accretion rate in order to test these two types of effect. The main results and conclusions reached may be summarized as follows.

1. The WD physical parameters change considerably with time in both cases, in particular:
   (i) The WD cools as a single, undisturbed WD for a while, but after a time, cooling slows down and will eventually cease. The maximum temperature attained in the burning shell $T_{\text{max}}$ increases at first, but settles at an almost constant value as cooling of the WD slows down.
   (ii) The WD luminosity decreases with time, but in contrast to a single WD, only down to the accretion luminosity value, where it remains constant. This, of course, applies only to the quiescent phases of accretion between outbursts.
   (iii) The WD mass decreases steadily for the cases considered, but not by much. The mass of the donor star, by contrast, decreases considerably; by 0.16 M$_\odot$ for Model A, and by 0.51 M$_\odot$ for Model B.
   (iv) The WD becomes less dense as its mass decreases, as expected from the relation $\rho_\odot(M)$ characteristic of degenerate stars.

2. On the long-term scale, the outburst characteristics change considerably for the massive WD (Model A), but only slightly for the low-mass one (Model B). In particular:
   (i) The accreted and ejected masses increase with time, as the WD temperature decreases.
   (ii) The heavy element abundance (Z) decreases with time for Model A, while it oscillates around a constant value for Model B.
   (iii) The helium mass fraction increases with time for Model A, while again, it oscillates around a constant value for Model B.
   (iv) Isotopic ratios settle eventually into constant values for Model A and are practically constant throughout for Model B. In both cases $^{13}$C and $^{17}$O are overabundant as compared to solar values, while $^{18}$O is underabundant; $^{17}$N, however, is underabundant in one case (A) and underabundant in the other (B).

3. The changes in physical characteristics are not necessarily monotonic; some, such as the expansion velocity, go through an extremum. Consequently, one should be careful when using parametrized grids of models and interpolating between them. Nevertheless, the agreement between the long-term calculations and the parametrized models is quite good for many of the nova characteristics.

4. Both models settle, eventually, into almost steady state where both the WD properties and the outburst characteristics remain almost unchanged with many repeated cycles. However, this state is reached after hundreds of cycles, which means a long period of time, or a considerable change in the mass of the donor star. Although our calculations assume a constant accretion rate throughout, it is to be expected that spurious dynamical effects on the binary orbit that are bound to occur on a long time-scale, or the change in donor mass, or both, will affect the mass transfer rate significantly. A different accretion rate will tend to a different steady state. It is thus possible that, in reality, the system will never be able to achieve steady state.

5. For the first time in numerical modelling of nova outbursts, it is possible to estimate the accuracy (or reliability) of the results. The numerical ‘noise’ that appears in the various plots may be statistically interpreted as error bars.

Of the three independent nova parameters, only two are left – the WD mass and the accretion rate – while the third – the WD temperature – is determined by the evolutionary course of the accreting WD in the nova system. The present calculations were based on the arbitrary assumption that the accretion rate is constant (and prescribed). This assumption is not realistic; the evolutionary course of the nova system, taking into account the interaction between its components, determines the mass transfer rate and its evolution, and future studies should follow the evolution of both members of the binary system consistently. This leaves the WD mass as the single truly independent parameter of nova outburst evolution.

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