The birth properties of Galactic millisecond radio pulsars

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Accepted 2006 December 1. Received 2006 November 22; in original form 2006 November 6

ABSTRACT

We model the population characteristics of the sample of millisecond pulsars (MSPs) within a distance of 1.5 kpc. We find that for a braking index $n = 3$, the birth magnetic field distribution of the neutron stars as they switch on as radio-emitting MSPs can be represented by a Gaussian in the logarithm with mean $\log B = 8.1$ and $\sigma_{\log B} = 0.4$ and their birth spin period by a Gaussian with mean $P_0 = 4$ ms and $\sigma_{P_0} = 1.3$ ms. We assume no field decay during the lifetime of MSPs. Our study, which takes into consideration acceleration effects on the observed spin-down rate, shows that most MSPs are born with periods that are close to the currently observed values and with average characteristic ages that are typically larger by a factor of $\sim 1.5$ compared to the true age. The Galactic birth rate of the MSPs is deduced to be $\gtrsim 3.2 \times 10^{-7}$ yr$^{-1}$ near the upper end of previous estimates and larger than the semi-empirical birth rate $\sim 10^{-7}$ yr$^{-1}$ of the low-mass X-ray binaries (LMXBs), the currently favoured progenitors. The mean birth spin period deduced by us for the radio MSPs is a factor of $\sim 2$ higher than the mean spin period observed for the accretion and nuclear powered X-ray pulsars, although this discrepancy can be resolved if we use a braking index $n = 5$, the value appropriate to spin-down caused by angular momentum losses by gravitational radiation or magnetic multipolar radiation. We discuss the arguments for and against the hypothesis that accretion-induced collapse (AIC) may constitute the main route to the formation of the MSPs, pointing out that on the AIC scenario the low magnetic fields of the MSPs may simply reflect the field distribution in isolated magnetic white dwarfs which has recently been shown to be bi-modal with a dominant component that is likely to peak at fields below $10^3$ G which would scale to neutron star fields below $10^9$ G, under magnetic flux conservation.

Key words: stars: magnetic fields – stars: neutron – pulsars: general – X-rays: binaries.

1 INTRODUCTION

The properties of the millisecond pulsars (MSPs) and the ‘normal’ radio pulsars place them in two nearly disjoint regions in the spin period ($P$) period derivative ($\dot{P}$) diagram. In the normal pulsars, $P$ and $\dot{P}$ are distributed about mean values of $\sim 0.6$ s and $\sim 10^{-15}$ s s$^{-1}$, respectively, with implied magnetic field strengths in the range $\sim 10^{11}-10^{13}$ G. In contrast, in the MSPs, $P$ and $\dot{P}$ are distributed about $\sim 5$ ms and $\sim 10^{-20}$ s s$^{-1}$, respectively, with field strengths in the range $\sim 10^8-10^9$ G. This bi-modality in the field distributions of the radio pulsars has been an enigma which has still to be fully resolved.

There are also major differences in the population characteristics of these two groups of pulsars which provide important clues on their origin. Most ($\sim 85$ per cent) of the MSPs are in binary systems (MSPs) on nearly circular orbits in contrast to the normal pulsars which tend to be isolated and, when they are not, exhibit more eccentric orbits. Furthermore, proper motion studies have shown that while the average space velocity for normal pulsars is $\sim 400$ km s$^{-1}$ (Hobbs et al. 2005), the MSPs form a low-velocity population with typical transverse speeds of $\sim 85$ km s$^{-1}$ (Toscano et al. 1999; Hobbs et al. 2005). Differences in the incidence of binarity, eccentricity of orbits and space motions are usually attributed to differences in the kick velocity imparted to the neutron stars at birth (Shklovskii 1970). The magnitude of the kick, and its effect on the binary system, will depend on the nature of the system, and whether the neutron star originates from the core collapse (CC) of a massive star with a supernova explosion, or from the accretion-induced collapse (AIC) of a white dwarf.

In the standard model, the MSPs are considered to be the end product of the evolution of low-mass X-ray binaries (LMXBs) and intermediate-mass X-ray binaries (IMXBs) where it is assumed that the neutron star was formed by CC of a massive ($M > 8$ $M_\odot$) star, and is subsequently spun-up to millisecond periods during an accretion disc phase (Bisnovatyi-Kogan & Komberg 1974; Bhattacharya & van den Heuvel 1991). We will refer to this class of objects as the ‘core-collapsed LMXBs and IMXBs’ or, more briefly, the...
LMXBs(CC)/IMXBs(CC). Accretion-induced field decay is an integral part of this model which appears plausible from a theoretical viewpoint, particularly if the fields in neutron stars are of crustal origin [Konar & Bhattacharya 1997; but see Ruderman (2006) for an alternative model]. Regardless of the origin of the low fields in the MSPs, a long-standing problem with the LMXB/IMXB scenario has been the difficulty in reconciling their semi-empirical birth rates with those of the radio MSPs (e.g. Lorimer 1995; Cordes & Chernoff 1997). The problem with the birth rates has been confirmed by recent population synthesis calculations which have also highlighted the difficulties in explaining the observed orbital period distribution of MSPs on the LMXB(CC)/IMXB(CC) scenario (Pfahl, Rappaport & Podsiadlowski 2003).

Another often-discussed channel for the production of MSPs involves the AIC of an ONeMG white dwarf (Michel 1987). Here, during the course of mass transfer, a white dwarf reaches the Chandrasekhar limit and collapses to form a neutron star (Bhattacharya & van den Heuvel 1991). In the AIC scenario, we may expect the magnetic field distribution of the MSPs to reflect in some way the magnetic field distribution of their progenitor white dwarfs obviating the need for field decay. A LMXB or IMXB phase may follow the collapse of the white dwarf, and we will refer to this class of objects as the ‘AIC LMXBs and IMXBs’ or, more briefly, as the LMXBs(AIC)/IMXBs(AIC). Population synthesis calculations indicate that the expected birth rates from the AIC channel may be significantly higher than those from the LMXB(CC)/IMXB(CC) route (Hurley, Tout & Pols 2002; Tout et al., in preparation; Hurley, private communication).

In this paper, we present an analysis of the 1.5 kpc sample of MSPs which is considered to be sufficiently sampled out (Kramer et al. 1998; Lyne et al. 1998) with the aim of establishing the MSP birth properties and constraining the different models that have been proposed for their origin. Our estimate of the Galactic birth rate of the MSPs is at the upper end of previous estimates (e.g. Cordes & Chernoff 1997) and again brings into question the LMXB(CC)/IMXB(CC) scenario as being the dominant route for the origin of the MSPs. The paper is arranged as follows. In Section 2, we describe the data set and our model. Our results are presented and discussed in Section 3 where we also present the case for and against LMXB(CC)/IMXB(CC) progenitors and AIC progenitors for the MSPs. Our conclusions are presented in Section 4.

2 THE MODELLING OF THE RADIO PROPERTIES OF THE MSPs

In 1998, Kramer et al. conducted a very detailed study aimed at comparing the radio emission properties of MSPs to those of normal pulsars. In their work, they restricted their comparative studies to objects within 1.5 kpc on the grounds that the population of all radio pulsars is sufficiently sampled out up to this distance, as first pointed out by Lyne et al. (1998). Recently, this assumption has gained further strength through the high-latitude survey of Burgay et al. (2006) in the region of the sky limited by $220^\circ < l < 260^\circ$ and $|b| < 60^\circ$ conducted with the 20-cm multibeam receiver on the Parkes radio telescope. If we restrict the pulsars in this survey to a distance of up to 1.5 Kpc, we find that all but one previously known radio pulsars have been re-detected and four new objects, out of a total of 16, discovered. We therefore apply a correction factor of 1.25 to obtain an estimate of the total number of MSPs in the 1.5 kpc sample that we analyse, stressing the fact that this may only be a lower limit for the real number of objects up to this distance.

We have restricted our analysis to binaries and isolated MSPs with spin periods shorter than 30 ms. In the restricted sample to 1.5 kpc, the Australia Telescope National Facility (ATNF) catalogue (Manchester et al. 2005) gives 24 MSPs in binaries and 11 isolated MSPs. We do not distinguish between these two groups since the isolated MSPs are likely to be an end product of binary systems in which the companion has been tidally disrupted or ablated (Radhakrishnan & Shukre 1986) with an otherwise similar evolutionary history to the binary MSPs (see, however, Michel 1987; Bailyn & Grindlay 1990 for alternative points of view). They exhibit very similar observational properties, except, perhaps, for the radio luminosity which seems to be slightly lower in the isolated MSPs (Bailes et al. 1997). Of the 24 binary MSPs, 14 have periods above 10 d and most have lower limits to the companion masses in the range $\sim 0.2$–0.4 $M_\odot$ (Manchester et al. 2005) indicative of the end state of evolution of binary systems that evolve to longer periods (beyond the bifurcation period; see Podsiadlowski, Rappaport & Pfahl 2002) due to mass transfer from a low-mass giant (see Sections 3.1 and 3.2) leading to He white dwarfs. The remaining shorter period systems appear to have either He or CO white dwarfs or very low-mass companions.

The age of the MSP is calculated according to

$$ t = \frac{P}{(n-1)P} \left[ 1 - \left( \frac{P_0}{P} \right)^{n-1} \right], $$

where $n$ is the braking index, which is equal to 3 for the dipolar spin-down model, and $P$ and $P_0$ are the observed and the initial period of the MSP, respectively. If $P_0 \ll P$, we obtain the characteristic age of the MSP:

$$ \tau_c = \frac{P}{2P}. $$

In the following sections, we will show that $P_0$ is often too close to $P$ to be able to rely on $\tau_c$ for an estimate of the true age of a MSP.

The very low spin-down rates of the MSPs have so far precluded any direct measurements of the braking index. An index of $n = 3$ is indicated for old normal radio pulsars, but values $n \sim 1.5$–2.8 have been measured in younger pulsars (Lyne 1996; Hobbs et al. 2004). In this work, we have adopted $n = 3$, but it is conceivable that a different value of $n$ may be appropriate for the MSPs. For instance, if spin-down is by multipolar radiation, the braking index will be somewhat larger than 3, while if angular momentum is lost mainly by gravitational radiation, we expect $n = 5$ (Camilo, Thorsett & Kulkarni 1994). In our modelling, we adopt $n = 3$ but we also discuss the implications of using a larger value of $n$.

We synthesize the properties of the MSPs using essentially the method described in Ferrario & Wickramasinghe (2006, hereafter FW). However, in the present study there are two main differences. First, we directly assume an initial magnetic field distribution for the MSPs without attempting to relate it back to the magnetic properties of the (main-sequence) progenitors. We therefore have, as our basic input, the MSP birth magnetic field distribution, which we describe by a Gaussian in the logarithm, and the birth spin distribution also described by a Gaussian. We stress that here with ‘birth’ characteristics of MSPs we refer to those characteristics that the MSPs have as they switch on as radio emitters, regardless of their previous history. Hence, the results of our calculations do not depend in any ways on the specific route(s) leading to the formation of the MSPs.

Secondly, we take into consideration the three Doppler acceleration effects cited by Damour & Taylor (1991) which affect the observed spin-down rate of the MSPs, namely, (i) the Galactic differential rotation, (ii) the vertical acceleration $K_z$ in the Galactic...
potential and (iii) the intrinsic transverse velocity of the pulsar. Thus, the observed spin-down rate is given by (e.g. Toscano et al. 1999)

\[ \dot{P}_{\text{obs}} = \dot{P} + \Delta \dot{P} \]  

(3)

where \( P \) is the ‘intrinsic’ spin-down rate and \( \Delta P \) is the term due to the aforementioned acceleration effects. Hence, when we compare our models to observations, we introduce these acceleration terms to our synthetic population to mimic the behaviour of the observed MSPs.

We follow the motions of the stars we generate by integrating the equations of motion in the Galactic potential of Kuijken & Gilmore (1989) assuming that the neutron stars are born with a kick velocity given by a Gaussian distribution with velocity dispersion \( \sigma_k \).

To fit the observations, we also model the radio luminosity at 1400 MHz and compare it to the members of our list with a measured value at this frequency. The studies of Kramer et al. (1998) and Kuz'min (2002) indicate that despite the large differences in periods and magnetic fields, normal pulsars and MSPs exhibit the same flux density spectra, therefore pointing towards the same emission mechanism, although the MSPs tend to be weaker sources on average (Kramer et al. 1998).

Hence, similarly to many previous investigators (e.g. FW; Narayan & Ostriker 1990), we have assumed that the luminosity \( L_{400} \) at 400 MHz can be described by a mean luminosity of the form

\[ \log(L_{400}) = \frac{1}{3} \log \left( \frac{P}{P_0^3} \right) + \log L_0. \]  

(4)

Here, the luminosities are in units of mJy kpc\(^2\). We have modelled the spread around \( L_{400} \) using the dithering function of Narayan & Ostriker (1990) to take into account the various intrinsic physical variations within the sources and also variations caused by different viewing geometries. This function is given by

\[ \rho_1(\lambda) = 0.5\lambda^2 \exp(-\lambda) \quad (\lambda \geq 0) \]  

(5)

where

\[ \lambda = b \left( \frac{\log L_{400}}{L_{400}} + a \right) \]  

(6)

and \( a \) and \( b \) are constants to be determined (Hartman et al. 1997).

Kramer et al. (1998) find that by restricting their comparison analysis of normal radio pulsars to MSPs to sources up to 1.5 kpc, the mean spectral indices of normal radio pulsars and MSPs are essentially the same, i.e. \(-1.6 \pm 0.2\) (MSPs) and \(-1.7 \pm 0.1\) (normal pulsars). Hence, our deduced radio luminosity at 400 MHz is scaled to 1400 MHz using a spectral index of \(-1.7\) (as in FW).

Once all the intrinsic properties of our model MSPs are determined, we check for pulsars detectability at 1400 MHz by the Parkes multibeam receiver (e.g. Manchester et al. 2001; Vranesic et al. 2004).

Furthermore, pulsar radio emission is anisotropic with pulsars at shorter periods exhibiting wider beams, hence we need to correct for this factor, since this will influence the birth rates of MSPs. For example, large beams would require smaller birth rates, since the MSPs would have a greater chance to be detected. However, there is as yet no agreement on the beaming fraction–period relationship, particularly for the MSPs. Rankin (1993), Gil, Kija & Seiradakis (1993) and Kramer et al. (1994) pointed out that observational evidence seems to suggest that the opening angles of normal radio pulsars (i.e. the last open dipolar field line) are proportional to \( 1/\sqrt{P} \). In the absence of a consensus on this issue, we use Kramer’s (1994) model at a frequency of 1.4 GHz for the opening half-angle \( \theta \) (in degrees) of the pulsar beam:

\[ \theta = \frac{5.3}{P^{0.15}}. \]  

(7)

These values of \( \theta \) yield duty cycles of less than unity for periods down to about 1 ms. However, we would like to remark that our results are quite insensitive to slight modifications to the above \( \theta - P \) relationship. Then, by assuming that the viewing angles of MSPs are randomly distributed, the fraction \( f \) of the sky swept by the radiation beam is given by (Emmering & Chevalier 1989)

\[ f = (1 - \cos \theta_i) + \left( \frac{\pi}{2} - \theta_i \right) \sin \theta_i \]  

(8)

where \( \theta_i \) is the half-opening angle now in radians. We will use this \( f \) to compare our MSP synthetic population to the data sample under consideration.

3 RESULTS AND DISCUSSION

As our observational constraints, we have used the one-dimensional projections of the data comprising the number distributions in period \( P \), magnetic field \( B \), period derivative \( \dot{P} \), radio luminosity \( L_{400} \), Z-distribution and characteristic age \( \tau_c \), as determined from equation (3).

Similarly to FW, the best-fitting model to the observations of the MSPs was determined ‘by eye’ after conducting hundreds of trials. Our results are shown in Fig. 1. The fit was obtained by setting \( \sigma_k = 1.3 \) ms about a mean \( P_0 = 4 \) ms. The parameters for the luminosity model are \( (L_{400}) = 5.4, a = 1.5 \) and \( b = 3.0 \). Furthermore, the MSPs are impounded with a one-dimensional natal kick dispersion velocity of 50 km s\(^{-1}\), which yields an average transverse velocity of 83 km s\(^{-1}\). Our model reproduces the observed total number of MSPs with a local formation rate of \( \geq 4.5 \times 10^{-4} \) yr\(^{-1}\) kpc\(^{-3}\) which translates into a Galactic birth rate of \( \geq 3 \times 10^{-6} \) yr\(^{-1}\). We emphasize that this should be seen as a lower limit for the MSP birth rate, since the 1.5 kpc sample is still likely to be somewhat incomplete even after the correction factor that we have applied.

Our calculations show that the observed MSP magnetic field distribution can be modelled with a field which is initially (i.e. at the onset of the MSP radio-emission phase) Gaussian in the logarithm. We find that similarly to our modelling of normal isolated radio pulsars (see FW) it is not necessary to assume any spontaneous field decay during the lifetime of the radio-emitting MSPs. Thus, the ‘high’ magnetic field tail of MSPs arises from the dependence of the spin-down rate on the magnetic field, and not from a complex birth field distribution assigned to the parent population (e.g. as a result of accretion-induced field decay during a previous phase of mass accretion). In this context, we note that if the MSPs were born with higher field strengths than postulated by us and then decayed during their radio-emission lifetime towards weaker magnetic fields values, then we would expect a continuous field distribution filling up the gap between the normal radio pulsars and the MSPs. This was first noted by Camilo et al. (1994), who also point out that there is no indication that old globular cluster MSPs have magnetic fields which are lower than those of MSPs in the Galactic disc.

Another observational peculiarity of MSPs is that some of these objects appear to be older than the Galactic disc (10 Gyr) if one uses equation (2) to assign them an age. Toscano et al. (1999) found that by correcting their spin-down rates for the transverse velocity (Shklovskii) effect they exacerbated this paradox, since the correction resulted in a decrease in the spin-down rate \( \dot{P} \) (and thus of the derived magnetic field strength) and an increase in characteristic
age. As a consequence, nearly half of their corrected sample exhibited characteristic ages comparable to or greater than the age of the Galactic disc. Our theoretical results agree with their findings and are presented in Fig. 2 where we have plotted the observed $\dot{\Omega}_{\text{obs}}$ and intrinsic $\dot{\Omega}_i$ spin-down rates of our synthetic population. This clearly supports the view that transverse velocity effects do play an important role in the observations of MSPs.

Camilo et al. (1994) proposed three possibilities to solve this paradox. The first is that the magnetic field structure of MSPs is multipolar and thus the braking index that appears in equation (1) may be greater than 3. Alternatively, due to gravitational radiation, $n = 5$. The second is that the magnetic field decays, so that high values of $\tau_c$ could be attained by a certain choice for the decay timescale, although they discarded this possibility as outlined earlier. Finally, the third possibility that Camilo et al. (1994) proposed is that at least some MSPs may have been born with $P_0 \sim P$.

Our modelling has shown that if we start with an initial period distribution that is Gaussian with mean at 4 ms, we obtain a current day period distribution that is close to what is observed. In particular, we can reproduce the sharp rise in the observed period distribution near $\sim 3$ ms. In our model, most pulsars have initial periods that are quite close to the observed periods. This yields an average characteristic age which is larger than the average true age (as given in equation 1) by nearly 50 per cent. Further observational support in favour of birth periods being close to observed periods in MSPs also comes from studies of individual systems. For instance, for PSR J0437-4715, Johnston et al. (1993) and Bell et al. (1995) find $P = 5.757$ ms, $P_{\text{orb}} = 5.741$ d and $\tau_c = 4.4 - 4.91$. Sarna, Ergma & Gerškevičiūtė-Antipova (2000) derive the mass of the companion of PSR J0437-4715 to be $0.21 \pm 0.01 M_\odot$ with a cooling age of 1.26–2.25 Gyr. This implies that PSR J0437−4715 is much younger than inferred through its characteristic age and was born with a period close to the current period.

The lack of sub-MSPs is apparent in the distribution of the MSPs and has also been noted in the millisecond X-ray pulsars (Chakrabarty 2005). Our study indicates that this is consistent with our assumption of a Gaussian distribution of initial birth periods that peaks at 4 ms and rules out the possibility that most MSPs are born at submillisecond periods. Here, selection effects may be weak.

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**Figure 1.** Our model (solid line) overlapped to the MSPs data sample of up to 1.5 Kpc (shaded) taken from the ATNF catalogue (http://www.atnf.csiro.au/research/pulsar/psrcat; Manchester et al. 2005). The error bars indicate the Poisson standard errors.

**Figure 2.** Intrinsic (solid histogram) and observed (dashed histogram) spin-down rates of our synthetic population of MSPs.
playing a role; however, Camilo et al. (2000) estimated a loss of sensitivity of only 20 per cent below about 2 ms, which is far too low to explain the sudden drop in the number of MSPs below this period. This may suggest that neutron stars can never achieve break-up spin periods (~0.4–0.7 ms depending on equation of state; Cook, Shapiro & Teukolsky 1994). Hence, loss of angular momentum caused by gravitational radiation may limit the neutron star spin rate (Wagoner 1984). Current estimates to the lower limits for spin periods are about 1.4 ms (Levin & Ushomirsky 2001), which is close to the spin of the recently discovered MSP in the globular cluster Terzan 5 (Hessels et al. 2006).

3.1 The LMXB(CC)/IMXB(CC) scenario and its relation to birth properties

In the LMXB(CC) route, matter is transferred from low-mass main-sequence donors, and binary evolution occurs towards shorter periods driven by magnetic braking and gravitational radiation. Mass transfer continues past the period minimum over a Hubble time or until the companion is evaporated. Binary MSPs that result from this route are expected to have very low mass companions with ultra-short orbital periods. In contrast, in the IMXB(CC) route the donor stars are of intermediate mass ≥2M⊙, and binary evolution is driven by nuclear evolution past the bifurcation period towards longer periods. For the lower mass donor stars, mass transfer phase ends when the helium core of the donor star is exposed as a low-mass (~0.2–0.4M⊙) helium white dwarf. For the more massive donor stars, mass transfer can terminate when a CO or an ONeMg white dwarf core is exposed. Although the observed sample of binary MSPs consists of systems with all of the above companions, recent population synthesis calculations have not been successful in modelling the observed orbital period distributions (Pfahl et al. 2003).

Thus, Pfahl et al. (2003) find that the LMXB(CC) route leads to a significant population of low period binary MSPs peaking at \( P_{\text{orb}} \sim 0.03 \) d, but this population is not represented in the observed sample of MSPs. Indeed, the shortest observed binary period for the MSPs is \( P_{\text{orb}} = 0.1 \) d for PSR J2051−0827 (Stappers et al. 2001). On the other hand, the binary periods of the ultra-compact X-ray binaries are generally significantly shorter with a few of them exhibiting orbital periods of 0.03 d. Hence, this may be an indication that either (i) as their LMXB(CC) evolution continues, they will end up ablating their companion and thus appearing as isolated MSPs, or (ii) the ultra-compact LMXBs(CC) and the MSPs are not evolutionarily linked.

In contrast, the IMXB(CC) route predicts a population of binaries that peaks roughly at the observed periods \( P_{\text{orb}} \sim 6–30 \) d, but with a width that falls short by a factor of ~10–100 (depending on assumptions on the common envelope parameter) in comparison with the observations of binary MSPs. Indeed, the majority of the binary MSPs in the ATNF sample (Manchester et al. 2005) do not have orbital periods that fall in the most probable region (Pfahl et al. 2003) predicted for either LMXB(CC) or IMXB(CC) evolution.

There is also the problem with the birth rates mentioned in Section 1. Attempts at reconciling the LMXB(CC)/IMXB(CC) rates with the birth rates of MSPs have not been successful (Pfahl et al. 2003) and the present results go in the direction of making this discrepancy larger. It has been suggested that the above discrepancies may disappear when more realistic models are constructed that allow for limit cycles that may arise from X-ray irradiation of the donor stars, and for the intricacies in common envelope evolution. This remains a possibility.

However, it should be noted that the semi-empirical birth rates are based almost entirely on the observed LMXBs, which, given the arguments above, cannot be the dominant progenitors of the binary MSPs. The same comment also applies to our discussion of the AIC that undergoes a phase of mass transfer after collapse (see Section 3.2).

In the evolution that leads up to LMXBs(CC) and IMXBs(CC), one of the stellar components (usually the initially more massive) evolves into a neutron star through CC with a field distribution peaking near log \( B(\text{G}) = 12.5 \) (as observed in the isolated radio pulsars). The observed field distribution of the MSPs, on the other hand, peaks at log \( B(\text{G}) = 8.4 \). This discrepancy is often explained by accretion-induced field decay or evolution. The presence of MSPs in old systems suggests that if the low value of the magnetic field is due to field decay, it must do so mainly during the accretion phase prior to the neutron star becoming a radio MSP.

The manner in which the field is expected to decay in accreting neutron stars depends on the origin of the magnetic fields, and here there is no consensus. There is no clear evidence for field decay in ordinary pulsars on a time-scale of \( 10^7–10^8 \) yr. However, accretion can enhance field decay, particularly if the fields are of crustal origin. Two competing effects have been considered. Accretion raises the temperature and reduces the conductivity in regions of the crust that carry the current, thereby enhancing field decay. Accretion also pushes the current-forming region inward towards regions of higher density and conductivity where the field can be frozen. Konar & Bhattacharya (1997) have shown that these two effects, when taken together, could lead to an asymptotic ‘frozen’ field strength that is a factor of \( 10^{-1} \) to \( 10^{-4} \) below the initial field strength. The asymptotic value depends on the accretion rate and the total mass accreted. On the other hand, Wijers (1997) presented strong evidence against accretion-induced field decay which is proportional to the accreted mass on to the neutron star. Thus, the standard model does not explain the observed characteristics of the MSP birth field distribution as they play on as radio emitters. For instance, if we consider the route that contributes to the majority of binary MSPs, namely those having orbital periods \( P_{\text{orb}} \geq 10–1000 \) d with low-mass He white dwarf companions arising from the evolution of intermediate mass donors, we may expect the accretion history, and therefore the birth field distribution, to depend on the orbital period. It is therefore not immediately apparent why this field should have a nearly Gaussian distribution with such a narrow width. The problem becomes even more severe when more than one channel is considered (see discussion in Tout et al., in preparation).

The detection of coherent X-ray pulsations with millisecond periods in a handful of LMXBs (Lamb & Yu 2005) is often used in support of the idea of accretion-induced field decay (Wijnands & van der Klis 1998). However, whether this is evidence simply for field submersion and spin-up during an accretion disc phase, or for field decay and spin-up, remains to be established. Cumming, Zweibel & Bildsten (2001) have argued that the majority of the LMXBs do not show coherent pulsations because they may have fields significantly less than \( 10^8 \) G due to field submergence which, at face value, is inconsistent with the fields seen in the radio MSPs, but their calculations also indicate that the field will re-emerge on a time-scale of ~1000 yr although it is unclear to what value. Indeed, for the LMXB(CC)/IMXB(CC) standard scenario to be viable, the field would be required to re-emerge to values that are similar to those observed in the radio MSPs.

If we adopt the contentious viewpoint that magnetic fields do not decay due to accretion, but are simply temporarily submerged, and re-emerge to their original values of a few \( \times 10^{12} \) G at the end of the
LMXB(CC)/IMXB(CC) phase, then we may expect a population of high field MSPs. The objects in such a population would have a birth rate that is $10^{-4}$ times the birth rate of normal radio pulsars and would therefore be unlikely to be represented in the current sample of radio pulsars. Furthermore, since they would spin-down very rapidly to much longer periods (with characteristic time-scales of only a few hundred years), they would have an even smaller chance to be detected as high field radio MSPs.

Finally, we note that on the LMXB(CC)/IMXB(CC) hypothesis, we expect the birth spin period distribution of the MSPs, as they become radio emitters, to be similar to the observed spin period distribution of the LMXBs(CC). However, observations of accretion and nuclear powered LMXBs show that their spin periods peak at near 2 ms (Lamb & Yu 2005). This could indicate either a different origin for the radio MSPs (see Section 3.2) or that the braking index is significantly larger than adopted by us. We have carried out calculations for different braking indices and find that a braking index of $n = 5$ (appropriate to angular momentum loss by gravitational radiation or magnetic multipolar radiation) will bring the two distributions into closer agreement.

### 3.2 The AIC scenario and its relation to birth properties

According to current models, AIC leads to the formation of a rapidly spinning (a few ms) neutron star when an ONeMg white dwarf accretes matter in a binary system and reaches the Chandrasekhar limit. Recent calculations have re-affirmed that an AIC is the expected outcome of thermal time-scale mass transfer in such systems with orbital periods of the order of a few days (Ivanova & Taam 2004). The magnetic fluxes in these cores may thus reflect the magnetic fluxes seen in the isolated white dwarfs.

Until recently, it was believed that the magnetic fields of the isolated white dwarfs could be described by a single distribution. However, it is now evident that the distribution is bi-modal, comprising a high and a low field component. The high field component (10–15 per cent of all white dwarfs) has a distribution that peaks at log $B(G) \sim 7.5$ with a half-width $\sigma \log B = 7.3$ (e.g. Wickramasinghe & Ferrario 2005). This distribution declines towards lower fields with very few stars detected in the field range $10^5$–$10^6$ G (magnetic field gap). The incidence of magnetism rises again towards lower magnetic fields with some 15–25 per cent of white dwarfs being magnetic at the kilo-Gauss level (Jordan et al. 2006). Since the new detections of Jordan et al. (2006) are at the limit of the sensitivity of current spectropolarimetric surveys, it appears likely that all white dwarfs will be found to be magnetic, with the majority (≈ 85 per cent) belonging to the low field group ($B < 1000$ G).

A field of a kilo-Gauss scales under magnetic flux conservation to a neutron star field of $10^9$ G. Although the peak of the low field distribution has still to be established observationally, it is conceivable that the fields will be distributed in a Gaussian manner about a peak that will map on to the observed field distribution of the radio MSPs. Given the high mass transfer rates required for AIC, the Ohmic diffusion time-scale will be much larger than the accretion time-scale (Cumming 2002), so we expect the white dwarf field to be submerged during the build up of the white dwarf mass prior to collapse. For the above scenario to be viable, we need to postulate that the submerged field will re-emerge without decay to its flux conserved value at the birth of the neutron star. In this context, we note that there is no evidence of accretion-induced field decay in the AM Herculis-type Cataclysmic Variables, where a highly magnetic white dwarf has been accreting mass over billion years from a companion. In fact, their well-studied field configurations are very similar to those observed and modelled in the isolated high field magnetic white dwarfs (e.g. Wickramasinghe & Ferrario 2000 and references therein).

We expect the white dwarf to be spun-up to near break-up velocity prior to collapse (e.g. like the white dwarfs observed in dwarf novae). However, angular momentum (and mass) must necessarily be lost during the subsequent collapse to a neutron star (Bailyn & Grindlay 1990) so that detailed models are required to establish the expected birth spin and mass distributions of the resulting neutron star. Dessart et al. (2006) have conducted 2.5-dimensional radiation-hydrodynamics simulations of the AIC of white dwarfs to neutron stars. Their calculations show that these lead to the formation of neutron stars with rotational periods of a few (2.2–6.3) ms. Hence, even if the binary system were to be disrupted following a kick during the AIC, these ‘runaway’ newly born neutron stars would appear as isolated radio MSPs of the type currently observed.

A proportion of the white dwarfs that could be subjected to AIC will inevitably belong to the high field group ($\sim 10^9$–$10^{10}$ G), and will result in rapidly rotating (millisecond) pulsars with fields in the range $10^{10}$–$10^{11}$ G on collapse, if we assume magnetic flux conservation (see fig. 1 in FW). This proportion could be as high as 50 per cent because highly magnetic white dwarfs tend to be more massive than their non-magnetic (or weakly magnetic) counterparts (mean mass of 0.92 M⊙; Wickramasinghe & Ferrario 2005). Assuming that the kicks are not field dependent and thus preferentially disrupt these systems, we may also expect a group of MSPs with high fields. However, given the much higher spin-down rates of these objects, and their low birth rates as compared to normal radio pulsars, we expect them to make a small contribution which would be dominated by the lowest field objects in the distribution which would have the longest lifetimes as radio pulsars. A possible candidate could be the binary radio pulsar PSR B0655+64 which has a relatively high magnetic field ($B = 1.17 \times 10^{10}$ G), short orbital period ($P_{\text{orb}} = 1.03$ d) and is on a nearly circular orbit (Damashek, Taylor & Hulse 1978; Edwards & Bailes 2001).

The population synthesis calculations of Hurley et al. (2002) yielded an AIC rate that is two orders of magnitude higher than the LMXB(CC) rate that results from the evolution of binary systems with primaries that are less massive than 2 M⊙. A more detailed investigation of the AIC and CC rates, and orbital period distributions expected from such calculations, has been presented by Tout et al. (in preparation). Here, it is shown that as with the CC route, the AIC route also generates binary MSPs of all of the observed types. We note in particular that a class of long period ($P_{\text{orb}} \geq 10$ d) binary MSPs with He white dwarf companions is predicted, and this closely follows the observed $P_{\text{orb}} - M_{\text{WD}}$ relationship (Van Kerkwijk et al. 2005).

We conclude this section by noting that neutron stars that are formed via AIC may also go through a mass transfer phase prior to their switching on as radio MSP. We therefore expect that the known sample of LMXBs/IMXBs will have a contribution from both neutron stars that have resulted from the CC of massive stars and the AICs. According to current estimates of the AIC rates, the LMXBs(AIC)/IMXBs(AIC) may dominate over the LMXBs(CC)/IMXBs(CC). However, because of field submersion, it may be difficult at present to distinguish between these two possibilities.

### 4 CONCLUSIONS

We have presented an analysis of the properties of the MSPs and placed constraints on the magnetic field and the spin period...
distributions of the MSPs at the time they turn on as radio emitters. We find that if we assume a braking index $n = 3$, the field distribution can be represented by a Gaussian in the logarithm with mean log $B(G) = 8.1$ and $\sigma_{\log B} = 0.4$ and the birth spin period by a Gaussian with mean $P_0 = 4$ ms and $\sigma_P = 1.3$ ms. Our study, which allows for acceleration effects on the observed spin-down rate, shows that (i) most MSPs are born with periods that are close to the currently observed values, (ii) the characteristic ages of MSPs are typically much larger than their true age and (iii) sub-MSPs are rare or do not exist. We also find a Galactic birth rate for the MSPs of $\gtrsim 3.2 \times 10^{-6}$ yr$^{-1}$.

We have used our results to discuss the relative merits of the LMXB(CC)/IMXB(CC) and AIC scenarios that have been proposed for explaining the origin of the MSPs. Our main conclusions can be summarized as follows.

(i) The birth rate that we deduce for the MSPs is significantly larger (by a factor of $\sim 100$) than the semi-empirical birth rates quoted for the LMXBs/IMXBs and is more in accord with the expected birth rates from the AIC route from population synthesis calculations.

(ii) The AIC scenario relates the MSP neutron star field distribution to their white dwarf ancestry without invoking any kind of field decay, and finds some support from the recently discovered bi-modality of the magnetic field distribution of the white dwarfs. The low fields of the MSPs may simply arise from the low field component of the white dwarf magnetic field distribution that is expected to peak below $10^7$ G and to scale to fields below $10^6$ G under magnetic flux conservation. The nearly Gaussian distribution that we deduce for the neutron star birth magnetic fields in the MSPs may thus have a more ready explanation on the AIC scenario.

(iii) On the standard LMXB(CC)/IMXB(CC) picture, the majority of the MSPs which have intermediate or long orbital periods come from the IMXB(CC) route. However, the predictions of the expected orbital period distributions from population synthesis calculations are not in good agreement with the observations of the MSPs. It remains to be seen if better agreement with the observed period distribution can be obtained with predictions from the AIC route which is expected to also produce systems with the same range of orbital periods and companion masses at the end of mass transfer prior to the radio MSP phase.

(iv) Population synthesis calculations predict that the standard LMXB(CC) route results in a significant class of LMXBs with periods less than 1 h, and these are observed as ultra-compact LMXBs. However, there is not as yet a single MSP with such a low period. A likely possibility is that the mass of the companion has been reduced to negligible values by mass transfer, or that the companion has been fully ablated by the time the pulsar turns on. These systems could result in isolated MSPs. However, the AIC route also leads to similar systems and therefore end products.

(v) The peak in the spin period distribution of accretion and nuclear powered X-ray pulsars occurs at $\sim 2$ ms, shorter than the $\sim 4$ ms that we have deduced for the birth spin period of MSPs assuming a braking index of $n = 3$. We find that the two distributions can be brought into closer agreement if we assume a braking index $n = 5$ which may suggest that the spin-down in the MSPs is dominated by angular momentum losses by gravitational radiation or by magnetic multipolar radiation. Alternatively, this may be an indicator that the two groups are not associated and have neutron stars with intrinsically different properties (e.g. mean mass) which is reflected in their birth spin periods.

We defer a more detailed discussion of the expected outcomes from the AIC route to a subsequent paper (Tout et al., in preparation), where we present a detailed comparison of the birth rates and orbital period distributions of the different types of radio MSPs that result from the usual LMXB(CC)/IMXB(CC) route and the AIC route.

We conclude by noting that if the AICs provide the dominant route leading to the MSPs, one has to make the proposition that neutron star fields do not decay, even in accreting stars, which remains contentious at the present time. This issue is likely to be resolved by detailed observations, as has been done in the case of white dwarfs where the general consensus appears to be that there is no evidence for field decay either in single magnetic white dwarfs or in accreting magnetic white dwarfs in binaries.

ACKNOWLEDGMENTS

We thank Chris Tout and Jarrod Hurley for helpful discussions and the anonymous referee for a careful reading of our manuscript and for numerous useful comments.

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