The metallicity dependence of the long-duration gamma-ray burst rate from host galaxy luminosities

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ABSTRACT

We investigate the difference between the host galaxy properties of core-collapse supernovae (CC SNe) and long-duration gamma-ray bursts (LGRBs), and quantify a possible metallicity dependence of the efficiency of producing LGRBs. We use a sample of 16 CC SNe and 16 LGRBs from Fruchter et al. which have similar redshift distributions to eliminate galaxy evolution biases. We make a forward prediction of their host galaxy luminosity distributions from the overall cosmic metallicity distribution of star formation. The latter is based on luminosity functions, star formation rates (SFRs) and luminosity–metallicity (L–Z) relations of galaxies. This approach is supported by the finding that LGRB hosts follow the L–Z relations of star-forming galaxies. We then compare predictions for metallicity-dependent event efficiencies with the observed host data. We find that ultraviolet-based SFR estimates predict the host distribution of CC SNe perfectly well in a metallicity-independent form. In contrast, LGRB hosts are on average fainter by one magnitude, almost as faint as the Large Magellanic Cloud. Assuming this to be a metallicity effect, the present data are insufficient to discriminate between a sharp cut-off and a soft decrease in efficiency towards higher metallicity. For a sharp cut-off, however, we find a best value for the cut-off metallicity, as reflected in the oxygen abundance, 12 + log (O/H)_{lim} ≃ 8.7 ± 0.3 at 95 per cent confidence including systematic uncertainties, in the calibration of Asplund, Grevesse & Sauval. This value is somewhat lower than the traditionally quoted value for the Sun, but is comparable to the revised solar oxygen abundance. LGRB models that require sharp metallicity cut-offs well below approximately one-half the revised solar metallicity appear to be effectively ruled out, as they would require fainter LGRB hosts than those that are observed. We also discuss the likely implications of the still ongoing metallicity ‘calibration debate’.

Key words: galaxies: high-redshift – gamma-rays: bursts.

1 INTRODUCTION

Since the first optical afterglow to a gamma-ray burst (GRB) was identified (van Paradijs et al. 1997, GRB 970228), the rest-frame properties of about 50 host galaxies of long-duration gamma-ray bursts (LGRBs) have been identified. Already with a small sample it became clear that the hosts are preferentially faint blue irregular galaxies (Fruchter et al. 1999; Le Floc’h et al. 2003; Christensen, Hjorth & Gorosabel 2004). This trend was recently confirmed by a comparison between hosts of LGRBs and hosts of core-collapse supernovae (CC SNe). CC SNe were found in late-type galaxies of all morphologies and luminosities, in contrast to LGRB hosts, which were still found to be almost exclusively irregular and on average fainter (Fruchter et al. 2006, hereafter F06).

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particular ‘collapsar’ models; MacFadyen & Woosley 1999), wind mass loss plays an important role in slowing down the rotation of massive stellar cores. Since lower metallicity leads to weaker stellar winds and hence less angular momentum loss, the stars are more likely to retain rapidly rotating cores at the time of the explosion, as required in the collapsar model.

LGRBs discovered by the *Swift* mission have a median redshift of 2.75 and apparently have a distribution expected for an unbiased tracer of star formation (Jakobsson et al. 2006). If both the cosmic star formation history and the cosmic evolution of metallicity in cold star-forming gas were known with high accuracy, this redshift distribution could provide clues about the metallicity dependence of the LGRB rate. However, the LGRBs discovered by *BeppoSAX* have a median redshift near \( z \approx 1 \), and predictions for the completeness function are very difficult to make, owing to the complexity of GRB trigger algorithms and detector intricacies. Even with a much larger sample, the redshift distribution of GRBs might be affected by systematic uncertainties such that no strong constraints on metallicity can be found.

Instead, first spectroscopic measurements of the metallicity of LGRB hosts at redshifts 0.4 < \( z < 1 \) from the GHOST Studies (Savaglio, Glazebrook & Le Borgne 2006) indicate that LGRB hosts follow the luminosity–metallicity relation of star-forming galaxies. A preference for low metallicity would then require LGRBs to favour less-luminous galaxies more than general star formation does. When working with gas-phase metallicities of galaxies, we need to be aware of the *calibration debate* between two basic methods which result in numbers that differ by a factor of \( \sim 2 \): spectra of low signal-to-noise ratio only facilitate an indirect strong-line relation of star-forming galaxies. When working with gas-phase metallicities of galaxies, we need to be aware of the *calibration debate* between two basic methods which result in numbers that differ by a factor of \( \sim 2 \): spectra of low signal-to-noise ratio only facilitate an indirect strong-line method (Pugl’s \( R_b \) indicator, 1979) to measure metallicity, which is calibrated against photoionization models. In contrast, high-S/N spectra allow for a direct measurement of metallicity taking into account the electron temperature measured from weak auroral lines (see e.g. Kennicutt et al. 2003). The \( T_e \)-based methods find typically 0.2–0.5 dex lower metallicities, but have been criticized to be biased by not taking inhomogeneities in the temperature into account.

In this paper, we wish to derive constraints on the metallicity dependence of LGRB progenitors by comparing forward predictions of host luminosity distributions with the observed host sample. We apply a minor modification to the F06 samples by restricting them to identical redshift ranges \( z = [0.2, 1] \) (see Section 2). Our quantitative predictions of the host luminosity distribution are based on luminosity functions (LFs) and star formation rates (SFRs) of galaxies and their luminosity–metallicity relations (see Section 3). Here, we need to make sure that all ingredients are on the exactly same metallic calibrator and opt for the strong-line method on the KK04 calibrator for consistency. We then fold in different prescriptions for the metallicity dependence of the LGRB event rate and constrain these by comparing our predictions to the observed sample in Section 4. In Section 5, we discuss the results and the implications of the calibration debate. Throughout this paper, we use Vega magnitudes and the cosmological parameters (\( \Omega_m, \Omega_\Lambda \)) = (0.3, 0.7) and \( H_0 = 70 \text{ km (s}^{-1}\text{ Mpc)} \). We use the revised solar oxygen abundance of 12 + log (O/H) = 8.66 (Asplund et al. 2005, see also Allende Prieto, Lambert & Asplund 2001).

2 DATA

This work is based on the host galaxy data of CC SNe and LGRBs collected by F06. The CC SNe were all discovered by the *Hubble* Higher Redshift Supernova Search (Strolger et al. 2004) in the two *HST GOODS* fields (Giavalisco et al. 2004). This search was only sensitive to CC SNe at \( z \approx 1 \) and may already be incomplete at the upper end of this redshift range (Dahlen et al. 2004). The LGRBs were taken by F06 from public *HST* data of host galaxy observations, where the list of objects extends to redshifts above 4.

At redshifts \( z < 0.2 \), the *HST GOODS* survey has too little volume to detect CC SNe efficiently. The same restriction applies to samples of LGRBs with typical gamma-ray luminosities. However, the GRB list contains a few very nearby objects with extremely low gamma-ray luminosities, which may be viewed at larger angles or be intrinsically faint events, and possibly an altogether different kind of explosion (see the ensuing discussion on GRB 060218, e.g. Soderberg et al. 2006). In order to exclude these peculiar GRB events, which we could only see at redshifts \( z \lesssim 0.1 \), we restrict the existing CC SN and LGRB data to a subsample in the redshift range \( z = [0.2, 1.0] \). We note that this is similar to but more conservative than the comparison considered by F06, which includes LGRBs up to \( z = 1.2 \).

As a result, we keep the full sample of 16 CC SNe with a mean redshift of 0.63, while the LGRB sample is restricted to 16 events with a mean redshift of 0.69, so they are on average from an extremely similar cosmic epoch.

3 MODELLING THE HOST LUMINOSITY DISTRIBUTION

We wish to model rest-frame V-band luminosity distributions for the host galaxies of both types of explosive events in order to compare them with the observed distribution from the restricted F06 sample. For this purpose, we ultimately need a distribution of star formation density over V-band host luminosity and potentially a metallicity-dependent efficiency function.

3.1 Galaxy luminosity function and star formation density

Wolf et al. (2005, hereafter W05) measured the rest-frame ultraviolet (UV) (280 nm) LF of galaxies from a sample of almost 1500 galaxies at a mean redshift of \( z \approx 0.70 \). Based on morphological classifications from the GEMS survey (Rix et al. 2004; Caldwell et al. 2006), the LFs were also split by galaxy type. The mean redshift of these LFs coincides with that of the host galaxy samples from the CC SNe and LGRBs considered here. Hence, they can serve as an unbiased description of the general galaxy population at the redshift of the events. The LF \( \phi(L_{280}) \) is parametrized as a Schechter function:

\[
\phi(L_{280}) \, dL = \phi^* \left( \frac{L}{L^*} \right)^{-\alpha} \exp \left( -\frac{L}{L^*} \right) \, dL.
\]

We chose the rest-frame UV luminosity at 280 nm, \( L_{280} \), since it can be used as a proxy for the unobscured SFR using the relation of Kennicutt (1998):

\[
\text{SFR}(M_{\odot} \text{ yr}^{-1}) = 1.4 \times 10^{-28} \frac{L_{280}}{\text{erg s}^{-1} \text{ Hz}^{-1}}.
\]

Evidently, the total SFR of a galaxy depends also on the amount of obscured star formation, which could be measured from the thermal far-infrared dust emission or crudely estimated using UV spectral slopes.

However, both CC SNe and LGRB afterglows are subject to host galaxy extinction as well, just as expected for events occurring in very young stellar populations. In fact, star-bursting galaxies do not only have an above average proportion of obscured star formation, but also have an increased rate of obscured, optically invisible CC SNe, which can be found by near-infrared (NIR) monitoring (Mannucci...
et al. 2003). Such SNe are absent in the sample used here, which is based on a rest-frame UV–optical search. Present data for LGRB afterglows show no evidence for high levels of dust obscuration, while a fraction of optically dark LGRBs may still be due to high dust extinction in the host galaxy (Klose et al. 2003). However, the host galaxies of known LGRBs show no significant signs of dust-obscured star formation (Le Floc’h et al. 2006).

Hence, we conclude that a comparison of UV-based star formation measures with UV–optical detection rates of stellar explosions is fair. It would be next to impossible to attempt an accurate extinction correction, which would need to take into account changes in the average extinction with progenitor lifetime as well as detailed detection efficiencies for events of different extinction levels.

For our model, we first calculate the distribution of UV luminosity density $j_{280}$ cumulative over UV host luminosity $L_{280}$ using the integral

$$j_{280}(L_{280,\lim}) = \int_{L_{280,\lim}}^{\infty} L_{280} \phi(L_{280}) \, dL_{280}. \quad (3)$$

Given the Kennicutt law, this quantity is an estimate of the unobscured star formation density in hosts of $L_{280} > L_{280,\lim}$. Hence, it also describes the host distribution of any explosive event which is proportional to the SFR. As an example, CC SNe are broadly expected to be an unbiased tracer of star formation, and should have a host distribution described by equation (3).

In a second step, we transform the UV luminosity axis $L_{280}$ into $L_V$ to match the given luminosity data of the host sample. For this purpose, we need to know the $(M_{280} - M_V)$ colours of star-forming galaxies. These data have been obtained by the COMBO-17 survey and are publicly available for the Chandra Deep Field South (Wolf et al. 2004) covering the entire GEMS field. The galaxy sample used for the above LFs can be split into a red sequence of mostly passively evolving galaxies and a blue cloud of mostly star-forming galaxies. For star-forming galaxies at $z \simeq 0.7$, the colour–magnitude relation is (see fig. 5 of W05):

$$M_{280} - M_V = -0.11(M_{280} - 5 \log h_{70} + 20). \quad (4)$$

Hence, the lower integration limit in the calculation of the luminosity density can be substituted as

$$M_{V,\lim} = M_{280,\lim} + 0.11(M_{280,\lim} - 5 \log h_{70} + 20). \quad (5)$$

This is equivalent to stretching the scale of the luminosity axis by 11 per cent and holding it fixed at $M_V = M_{280} = -20$, where the average colour of star-forming galaxies is zero.

### 3.2 Predictions by galaxy type

It has often been reported, most recently by F06, that CC SNe appear in all types of star-forming galaxies, while the afterglows of LGRBs are almost exclusively found in morphologically irregular, faint blue galaxies. Although galaxy morphology may not be a suitable physical factor for the evolution of high-mass stars in a galaxy, it may be correlated with physical factors: irregulars are mostly of lower-luminosity and hence tend to be of lower metallicity, which may well be the physical cause of the increased LGRB rate.

We now briefly try to confirm the picture suggested by F06, and test a toy model in which CC SNe arise unbiased from (unobscured) star formation in all galaxies, while LGRBs arise unbiased from irregular galaxies. We model the host luminosity distributions using the parameters of the LFs obtained by W05 for irregular galaxies and all galaxies (see Table 1). Note that the model is only a normalized cumulative luminosity distribution. Hence, it does not constrain overall rates, and it also does not depend on the normalization $\phi^*$ of the LF, but only on the slope $\alpha$ of the faint end and the location $M^*$ of the break in the LF.

This toy model is compared with the sample data in Fig. 1. We find that CC SNe match the prediction extremely well and conclude that they can be considered an unbiased proxy for star formation in all galaxies within the limits of this small data set. Also, the prediction for irregular galaxies matches the host data for LGRBs reasonably well, although there is some deviation at low luminosities. Formally, both comparisons are entirely acceptable: a Kolmogorov–Smirnov (KS) test yields values of $D_{KS} = 0.12$ and 0.16, respectively, to rejection probabilities of only 4 and 22 per cent.

We note that the LFs are well determined at $M_V < -18.5$ but extrapolated with a Schechter function at fainter magnitudes. Any errors in this extrapolation will translate into a simple rescaling of the cumulative host prediction at $M_V$ brighter than $-18.5$, whereas the fainter section would change its shape. We choose to ignore the errors in the previous qualitative analysis and present a more quantitative study, using metallicity as the physical factor controlling the LGRB efficiency, in the following sections.

### 3.3 Luminosity–metallicity relations and dispersion in the stellar population

In this section, we refine our toy model by considering as progenitors only subpopulations selected by metallicity. Our use of the term metallicity denotes the abundance of oxygen in the ionized gas phase as a tracer of metallicity in young high-mass stars. We use the O/H ratio measured via the $R_{23}$-index on the KK04 calibration. This index is determined from flux ratios in the prominent ionized oxygen and Hβ emission lines of star-forming galaxies, which are relatively easy to observe even in faint galaxies at higher redshifts. Of course, it would also be attractive to look at abundances of other elements. However, these are more challenging to measure, and we believe...
that, with the present small sample of objects, we would not be able to improve our study significantly. As an abbreviation, we will use the notation

\[ Z_O = 12 + \log (O/H). \]  

(6)

Recent determinations of the luminosity–metallicity \((L–Z_O)\) or the stellar mass–metallicity \((M_*–Z_O)\) relation include the work by KK04 on the basis of the GOODS survey, as well as Savaglio et al. (2005) using the GDDS survey. We opt for the \(L–Z_O\) relation, which relates more directly to our data and model. For star-forming galaxies at \(z \approx 0.7\), the COMBO-17 data suggest on average \(M_Z \approx M_V\), and we use the KK04 relations without further modifications. Their relations are determined in redshift slices, and conveniently their slice \(z = [0.6, 0.8]\) corresponds precisely to the redshift range in which the LF was determined by W05. Altogether, three linear fits are shown in their fig. 11: a \(Z_O = f(M_Z)\) fit (A), an \(M_Z = f(Z_O)\) fit (C) and a bisector fit (B), which can all be expressed as

\[ Z_{bb} = 12 + \log (O/H) = a + b \times M_B. \]  

(7)

The bisector fit has a slope of \(b = 0.239\), between the two one-sided fits, and is presumably the most realistic one of the three. We also use the two one-sided fits to define a confidence interval for the relation (see Table 2 for fit parameters).

A recent determination of the \(L–Z_O\) relation in local dwarf galaxies suggests a slope of \(b \approx 0.30\) extending across the luminosity range from \(M_B = -11\) to \(-19\) (Lee et al. 2006). At fixed luminosity, the metallicity has a 1σ scatter of 0.16 dex, the same as that determined in a large sample of luminous galaxies from the Sloan Digital Sky Survey (Tremonti et al. 2004). The scatter in the mass–metallicity relation is even less (0.10), and in the absence of any higher-redshift data we assume that it is well behaved also for dwarfs at \(z \approx 0.7\). In any case, both the \(L–Z_O\) relation and the UV LF are just extrapolated below \(M_B = -18.5\) in our analysis.

Below we first investigate the effects of a sharp metallicity cut-off for the production of LGRBs in our model. If we ignore both the scatter in the \(L–Z_O\) relation and the internal variations of \(Z_O\) among the young stellar population within any given galaxy, a sharp \(Z_O\) cut-off would cause a sharp \(M_V\) cut-off in the host galaxy sample. While we do not have perfect knowledge of the overall \(Z_O\) scatter, it certainly needs to be taken into account for the prediction of cumulative host luminosity distributions in the presence of \(Z_O\)-dependent event efficiencies.

Even for the same data, the scatter in the \(L–Z_O\) relation for average galaxy metallicities varies with the slope of the fit. Roughly, it is around \(±0.3\) dex end to end for a given luminosity at \(z \approx 0.7\), consistent with a 1σ scatter of 0.16 at low redshift. As far as the \(Z_O\)-spread of star-forming gas within a galaxy is concerned, we lack useful knowledge at \(z \approx 0.7\) owing to the limited spatial resolution of spectroscopic data, especially for smaller galaxies.

Instead, we look at the present-day Magellanic Clouds for clues on the \(Z_O\) variation within irregulars. The LMC appears to resemble a typical LGRB host at \(z \sim 0.7\), given its luminosity of \(M_V \approx -18.5\). The LMC and SMC have measured metallicities in young stars and the gas phase of 0.5 and 0.2 solar, respectively, and both have internal 1σ variations of \(≤0.2\) dex (Russell & Bessell 1989; Russell & Dopita 1990). Dispersions in Milky Way size disc galaxies are not expected to be larger either, given efficient gas mixing.

In principle, we need to convolve the distribution of the internal \(Z_O\) variation and the scatter in the \(L–Z_O\) relation to obtain a realistic cosmic \(p(Z_O | M_V)\) distribution. However, in the absence of any accurate values for the former, we consider two alternative scenarios for the combined metallicity variation by using the form

\[ d_Z(Z_O, M_V) = Z_O - Z_{bb}(M_V), \]  

(8)

\[ p(Z_O | M_V) = \begin{cases} 0 & \text{if } |d_Z| > 1, \\ \cos d_Z & \text{if } |d_Z| \leq 1, \end{cases} \]  

(9)

where \(Z_{bb}\) is taken from the \(L–Z_O\) relation (7) and \(\pm w\) is the end-to-end spread in the variation applied at any fixed \(M_V\). We opted to use a cosine function, because it is trivial to integrate analytically. Two other simple alternatives appear unrealistic: a box function because of its steep edges and a Gaussian because of its infinitely far-reaching wings.

Overall, we consider \(w = 0.4\) dex a realistic value for the combined spread at fixed host luminosity, which corresponds to about \(±0.25\) dex for the full width at half-maximum of the distribution \(p\). For illustration, we also explore an extreme value of \(w = 1\), which translates into an extreme and unrealistic metallicity spread from ten-fold below to ten-fold above the fit value at any given luminosity. This will allow even very luminous galaxies in our model to form a fraction of very low-\(Z_O\) stars.

### 3.4 Event efficiencies and their dependence on metallicity

Here, we specify our descriptions for the dependence of the LGRB efficiency on the metallicity of young stars. We consider two simple one-parameter models and one two-parameter model, which is a compromise between the first two. Any ad hoc model that is more complicated than these makes little sense, given the small number of 16 LGRBs in the sample. Our adopted prescriptions are as follows.

(i) A sharp high-\(Z_O\) cut-off:

\[ \eta(Z_O) = \begin{cases} 1 & \text{if } Z_O \leq Z_{\text{um}}, \\ 0 & \text{if } Z_O > Z_{\text{um}}. \end{cases} \]  

(10)

(ii) A power law, which unfortunately diverges at low \(Z_O\):

\[ \log \eta(Z_O) = -c Z_O. \]  

(11)

(iii) A broken power law, which is flat as in equation (10) below a break metallicity \(Z_s\), and declines as in equation (11) above the break:

\[ \log \eta(Z_O) = \begin{cases} 0 & \text{if } Z_O - Z_s \leq 0, \\ -c (Z_O - Z_s) & \text{if } Z_O - Z_s > 0. \end{cases} \]  

(12)

Note that for the single power law no normalization is required as we are not dealing with absolute LGRB rates but only host distributions. Also, model (12) contains the other models as special cases, because it converges into (10) for \(c \to -\infty\), and into (11) for \(Z_s \to -\infty\). The prescriptions for event efficiencies \(\eta(Z_O)\) and for the metallicity distributions of star-forming gas \(p(Z_O | M_V)\) in hosts of given

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The metallicity dependence of the LGRB rate

Figure 2. Top left-hand panel: cut-off models for a sequence of metallicity cut-offs at $Z_{\text{lim}} = \{9.0, 8.7, 8.4, 8.1\}$. The reference model for all galaxies has no metallicity dependence. Bottom left-hand panel: power-law efficiencies with $c = \{-0.2, -0.4, -0.6, -0.8\}$. All models predict too many bursts in faint galaxies. This is a consequence of the divergence to infinity of the LGRB efficiency at low metallicity. Varying $w$ has no effect on power-law models. Right-hand panel: broken power laws: the softer the break in the efficiency function, the lower the break metallicity in the best-fitting models. A hard cut-off ($c = -3$, top) is best fitted with $Z_{\text{lim}} \simeq 8.7$ (i.e. solar), while a very soft break ($c = -1$, bottom panel) is best matched at $Z_{\text{lim}} \simeq 8.2$. In the latter case, the LGRB efficiency at solar metallicity is still one-third of the plateau value.

4 COMPARISON WITH THE DATA

In this section, we compare our metallicity-dependent models with the host galaxy data. However, in Section 5.2 we will discuss empirical evidence that the LGRB hosts discussed here are on the $L-Z$ relation as far as we know, supporting the validity of our basic approach.

4.1 Cut-off models

Here, we use a constant efficiency up to a cut-off metallicity defined in equation (10) and consider a sequence of cut-off values of $Z_{\text{lim}} = \{9.0, 8.7, 8.4, 8.1\}$, that is, roughly twice solar, solar, half solar and a quarter solar. The top left-hand panel in Fig. 2 shows the resulting models alongside the reference model with all galaxies that fits the CC SN hosts. The most likely cut-off metallicity can be readily obtained from the plot as $\sim 8.7$. Already a cut-off at 8.4 clearly underpredicts the host luminosities.

We also investigated the dependence of these results on uncertainties in the $L-Z_O$ relation by using the limiting fits A and C from Table 2 to define fiducial confidence intervals. We find that changes in the slope of the $L-Z_O$ relation translate directly into changes in the luminosity spread of the curves predicted for a fixed set of cut-off metallicities, which is expected. Fortuitously, the data lie in a range that appears to be not very sensitive to the choice of the $L-Z_O$ fit. This is a result of the pivotal point in the $L-Z_O$ fit being close to $M = -20$. There are not enough GRB hosts far away from the pivotal point of the $L-Z_O$ fit to let the slope make a large difference for the best-fitting cut-off metallicity. Instead, the slope of the relation affects the formal errors on the cut-off, which one obtains from a fit to the data (see Section 4.4).

4.2 Power-law efficiencies

As an alternative to a sharp cut-off model, we now test an efficiency function that smoothly changes with metallicity and choose a simple power law. The reference model of a constant efficiency for all
galaxies is always shown for comparison, and corresponds to $c = 0$ in equation (11). Fig. 2 further shows the curves for power-law slopes of $-0.2, -0.4, -0.6$ and $-0.8$, which deviate increasingly from the reference model as high-$Z_0$ GRBs are more and more suppressed relative to low-$Z_0$ events.

Any variation in $w$ has no effect on the curves, because the symmetric form of the $Z_0$ distribution is averaged out with a power-law efficiency. Hence, there is no need to reproduce the curves for any other $w$. Furthermore, the slopes $c$ of the efficiency curves are degenerate with the slope of the $L-Z_0$ relation. Thus, fits A or C will produce the same curves, if suitable $c$-values are chosen. All curves for power laws predict a substantial contribution from extremely low-luminosity galaxies, which are not observed among GRB hosts. This is due to the unrealistic divergence of the GRB efficiency towards zero metallicity.

4.3 Broken power laws

The broken power law with flat low-$Z_0$ efficiency combines the properties of the two previous models, a gradual decline towards higher metallicity and a constant, non-diverging efficiency at lower metallicity. Hence, this model avoids the unrealistic and problematic divergence of the single power law, while being less restrictive than the cut-off model, providing the ‘best of both worlds’.

The model is now described by two parameters, which provides more freedom to fit the data, but also introduces some degeneracy. While hard breaks are very similar to the cut-off models, the metallicity break locations for softer breaks become increasingly less constrained. Especially, with only 16 GRBs the data do not provide sufficient constraints for the model. Conversely, we conclude that these data do not yet rule out a variety of models.

Fig. 2 shows results for two broken power laws to illustrate the trends with the slope $c$. The sequence of models shows breaks at $Z_a = \{9.0, 8.7, 8.4, 8.1\}$; a constant efficiency matching the CC SN data is also shown. We find that the data cannot differentiate between a soft and a hard break. Instead, a hardening of the break only leads to higher break metallicities. While the assumption of a hard cut-off suggests $Z_a \approx 8.7$, softening the break to $c = -1$ moves $Z_a \sim 8.2$.

4.4 Kolmogorov–Smirnov tests and results at redshift $<1$

In the present analysis, systematic uncertainties appear to dominate, ranging from the $L-Z_0$ relation over the typical metallicity spread in the star-forming gas of galaxies at $z \approx 0.7$ to the form of the faint end of the UV galaxy LF. However, it is straightforward to obtain quantitative confidence intervals on $Z_{\text{lim}}$ (as shown in Fig. 3) or $Z_a$ (Fig. 4) for a particular model, using KS tests. We repeat that the KS test yielded a highly acceptable value of $D_{\text{KS}} = 0.12$ for the match between the CC SNe and the reference model of ‘all’ galaxies.

Fig. 3 shows the results for the cut-off models. The solid curves show the three different $L-Z_0$ relations with a realistic metallicity spread of $w = 0.4$. All three favour a cut-off near solar metallicity and differ mostly in the width of the confidence interval, which is a result of the different slopes of the relations. We obtain $Z_{\text{lim}} = 8.7 \pm 0.22$ at 95 per cent confidence level for the most likely $L-Z_0$ relation B or $\pm 0.3$ if allowing for any relation.

But how would these results be affected if the internal metallicity dispersion of star-forming galaxies was truly much larger than ever anticipated? Would this allow the low-metallicity tails in luminous galaxies to produce GRBs and make the host data consistent with much lower metallicity cut-offs? For illustration, we modelled an extremely wide metallicity spread of the star-forming populations at given host $M_\text{V}$, from one-tenth of the mean to 10 times the mean. While such a model is unrealistic, the location of the cut-off metallicity is only lowered by $\sim 0.25$ dex (see dashed line in Fig. 3).
Invoking such extreme metallicity inhomogeneities among young stellar populations still rules out cut-offs of $Z_{\text{lim}} < 8.1 \approx Z_\odot/4$ at 95 per cent confidence.

Fig. 4 illustrates the broken power-law models. Softening the break, from a hard cut-off ($c = -\infty$) over $c = -3$ to a slope of $c = -1$, moves the favoured break metallicity to lower values and widens the confidence interval. A more gradual change in GRB efficiency leads to a more gradual rise in the cumulative host luminosity distribution. A soft break is also a less-promounced feature in the efficiency function $\eta(Z_\odot)$. The best-fitting $\eta$ functions shown in the bottom panel illustrate that even softer breaks require the $\eta(Z_\odot) \approx 1/3$.

In summary, we have explored for the $z = [0.2, 1.0]$ sample the impact of the two most important uncertainties by comparing results for different sets of assumptions. Three alternative $L-Z_\odot$ relations and different degrees of internal metallicity scatter in the star-forming gas are put in contrast in Fig. 3. Varying the $L-Z_\odot$ relations around the pivotal points provided by the observations of $z \approx 0.7$ galaxies appears to have very little effect. Changing the internal metallicity scatter of typical galaxies can have a significant effect when assuming unconventionally extreme degrees.

We further calculated the fractions of the young stellar population that can produce LGRB progenitors for $z \approx 0.7$, the mean redshift of our LGRB sample, given the various efficiency prescriptions. In the metallicity-independent model where all galaxies contribute and match the CC SN distribution, this fraction is defined as equal to 1. In the cut-off model, the best-fitting value $Z_{\text{lim}} = 8.7 \pm 0.3$ corresponds to fractions of $0.55 \pm 0.25$. This means that because of the metallicity constraint the LGRB production efficiency is reduced by a factor of about 1/2 compared to the efficiency if there were no metallicity constraint. This number is also consistent with half of the star formation and half of the CC SNe originating from galaxies with lower metallicity than solar, or with lower luminosity than $M_\star = -20$. In the broken power-law models, we find very similar fractions independent of the ambiguity between break softness and break location. In the single power-law model, the fractions cannot be normalized because the efficiency diverges at low $Z_\odot$. However, these models did not fit the data for the same reason.

4.5 High-redshift host galaxies

The purpose of restricting the GRB sample to low redshift was to compare it directly against the CC SN sample and to quantitatively investigate the role of metallicity in a regime where we have ample knowledge about the star formation and metallicity of the galaxy population. After obtaining a possible value for a metallicity cut-off around $Z_{\text{lim}} \approx 8.7$, we can check qualitatively whether this is in agreement with the observations of higher-redshift hosts. The FO6 sample contains eight host galaxies in the range $z = [1.5, 3.5]$, half of which have luminosities brighter than $M_\star = -20$. This reflects the fact that at higher redshift larger galaxies were more actively forming stars than they do at lower redshift (e.g. Perez-Gonzalez et al. 2005).

A recent study of $z \approx 2$ galaxies (Erb et al. 2006) showed that there is a mass–metallicity relation in place but no well-defined $L-Z_\odot$ relation due to strong variations in the mass-to-light ratios of galaxies. In their study, the most-massive galaxies reach $Z_\odot = 8.6$, which is on a $T_\text{eff}$-based calibration and probably equivalent to $\sim 8.9$ on the KK04 scale. A limit at $Z_\odot \leq 8.7$ would correspond to a mass limit of $M_\star/M_\odot \lesssim 10^{10.6}$. In fact, the sample contains plenty of luminous galaxies below this mass with $M_\star$ up to $-22$. Hence, our findings from the $z < 1$ sample are not in conflict with the luminosities of higher-redshift hosts. On the other hand, the Erb et al. sample contains galaxies with metallicities above our estimated cut-off, suggesting that at $z \approx 2$ LGRBs are not entirely unbiased tracers of star formation, which they might be at yet higher redshift.

5 DISCUSSION

5.1 Caveats

One difficulty with the present analysis could be host confusion. If the GRBs took preferentially place in low-luminosity companions to the formally identified hosts, then the observed host luminosity distribution would be biased to higher luminosities, suggesting higher metallicity cut-offs. It is beyond the scope of this paper to investigate whether $HST$ observations could resolve such dwarfs from their larger, mostly LMC-type companions at $z < 1$. However, if most LGRBs occurred in such small galaxies, one would also expect a large population of sufficiently isolated small dwarfs, where no confusion would take place. But the small number of observed isolated hosts with $M_V > -17$ leaves little room for any adjustment of our results.

On the contrary, it is possible that more-massive LGRB hosts are missing from the sample: luminous dust-obscured star-bursting galaxies harbour a significant fraction of the overall cosmic star formation. If LGRBs occur in these, their optical afterglows may be invisible, which would depopulate the host sample specifically at the bright end. The CC SNe host sample would be less affected by this effect, as CC SNe have a lower average progenitor mass and hence longer average time delay from formation to explosion. This allows the progenitors to leave their immediate formation environment and travel into less-obscured regions. The effect of progenitor mass on the distance to the likely formation site has been shown to apply in the comparison of SNe II and SNe Ib/c (James & Anderson 2006). The sample analysed here lacks by definition all so-called ‘dark bursts’, which may be related to massive dusty galaxies: GRB 020127 only produced an X-ray afterglow, and the host was identified as a dusty $z \sim 1.9$ extremely red object (ERO) with $5\sigma$ luminosity (Berger et al. 2006). Levan et al. (2006) observed an extremely red $(R - K \sim 6)$ afterglow to GRB 030115 in an ERO host galaxy with $R - K \sim 5$, suggesting that this burst was highly obscured by dust. The afterglow was only identified with NIR observations and would have been missed by optical follow-up alone. Again, for the more recent GRB 050223 only an X-ray afterglow was seen by $Swift$, but no optical afterglow. The host was shown to be a dusty galaxy with $A_V > 2$ (Pellizzi et al. 2006). If there is a significant observational bias against LGRBs in massive dust-enshrouded galaxies, then the metallicity-dependence of LGRB rates may be rather weaker than that our findings presently suggest.

Uncertainties also exist with respect to the calibration of oxygen gas-phase metallicities and on how the oxygen abundance is representative of other elements. For example, $\alpha$-elements are often found to be enhanced in systems where the star formation time-scale is much shorter than the time-scale for Type Ia supernovae. Depending on the progenitor model for LGRBs, different elements might be most responsible for affecting opacities in stellar atmospheres/winds and hence stellar evolution. However, $\alpha$-enhancements are typically observed in old ellipticals which are not at all common LGRB hosts or typical star-forming galaxies, and thus should not affect our analysis appreciably.

In summary, solar metallicity appears to mark a pivotal point of roughly constant efficiency apparently independent of the softness of the break. At $z \sim 0.7$, this metallicity seems typical for galaxies.
of $M_B - 5 \log h_{70} = -20$ in either $L-Z_O$ relation. The number ratio of CC SNe to either side of this host luminosity cut is 1:1, whereas it is 1:3 for LGRBs. If there was any doubt about the calibration of the oxygen abundance at $z \sim 0.7$, or the relevance of oxygen in comparison to other elements, then we could rephrase our main conclusion such that a most likely cut-off for LGRBs is around the mean metallicity (by any measurement) of $M_B = -20$ galaxies at $z \sim 0.7$.

5.2 Are GRB hosts on the luminosity–metallicity relation?

Several recent studies have aimed at constraining LGRB progenitors from direct measurements of host metallicities (Sollerman et al. 2005; Savaglio et al. 2006; Stanek et al. 2006) using very small samples: Sollerman et al. (2005) presented measured metallicities for three LGRB hosts with values of $\sim$8.2–8.7. Savaglio et al. (2006) considered seven hosts at $z = [0.4, 1.0]$ and found mean galaxy metallicities of 8.3–8.55 for five of them, while two hosts are clearly supersolar. They furthermore claim, that the hosts fall right on to the regular mass–metallicity relation of normal star-forming galaxies. This host sample with measured metallicities lacks the lowest-luminosity hosts and so contains preferentially higher metallicity galaxies. Hence, their median metallicity is not representative, but the statement on consistency with the mass–metallicity relation is highly relevant.

Savaglio (2006) gave a number of host metallicity determinations, which are collected from the literature and converted on to the same (KK04) calibrator, which is also the calibrator used in this paper for the $L-Z_O$ relation. Five of the above hosts are actually part of the LGRB host sample used here. Their spectroscopic metallicities are compared to the estimated values we have assigned to them using $L-Z_O$ fit B. We find them all to be within $\pm 1/3$ dex of the relation with a mean of $-0.034$ dex, which is consistent with no bias within the statistical power of five objects.

This result is not surprising. A physical model for GRB progenitors may incorporate an explicit metallicity dependence, but we cannot expect the progenitor to know explicitly about galaxy parameters, such as luminosity or morphology. At a fixed metallicity level, we would then expect bursts predominantly from those galaxy mass bins containing the highest SFR. Here, both the shape of the mass function and the SFR trends with mass play a role. Presumably, the declining mass function and the increasing SFR with mass cancel to some extent. KK04 searched for trends of galaxy properties within the scatter of their $L-Z_O$ relation. While they found no such trends, they particularly did NOT find strongly star-forming galaxies on the bright or metal-poor side of the relation. However, among very low-mass dwarf galaxies, it is conceivable that strong variations in SFR with time lead to such strong variations in $M/L$, that CC SNe and LGRBs are found predominantly on the bright side of the relation. Presumably, this would have no effect for our analysis, as the mean metallicity bias would be small and remain within the flat portion of the parametric efficiency functions considered in this paper.

5.3 Global versus local metallicity measurements

We would like to point out that even spectroscopic, but spatially unresolved, measurements of a mean host metallicity do not directly reflect the metallicity of the young stellar population near the LGRB progenitor. In fact, the error in the progenitor metallicity is dominated by the internal $Z_O$ dispersion of the galaxy, which is estimated to be as large as the dispersion of galaxy-averaged $Z_O$ values in the $L-Z_O$ relation ($\leq 0.2$ dex).

If we make no prior assumptions on LGRB hosts and accept the result that LGRB hosts follow the usual $L-Z_O$ relation, then we can, in turn, estimate even individual host metallicities from this relation. These individual estimates then have errors of the order of the dispersion in the $L-Z_O$ relation, that is, $\leq 0.2$ dex.

In other words, owing to the internal metallicity dispersion, our indirect estimate of the progenitor metallicity via the $L-Z_O$ relation of the host galaxy should be almost as useful as an unresolved spectroscopic observation, with an error that is roughly larger by a factor of $\sqrt{2}$. If the internal metallicity dispersions exceeded 0.2 dex, then the determination of integrated metallicities of individual LGRB hosts would have very little value, because it could as well be estimated from an $L-Z_O$ relation, while the error was mostly internal in origin.

Therefore, major progress can mostly be expected from spatially resolved metallicity measurements that focus on the immediate burst environment. However, such observations are currently not possible for GRBs at cosmological distances.

5.4 A metallicity–LGRB energy relation?

Stanek et al. (2006) suggested a tentative relation between host metallicity $Z_O$ and the isotropic energy associated with the LGRB $E_{iso}$, which they interpreted as a metallicity cut-off around 0.15 solar for regular cosmological bursts. This conjecture is based on one object out of five, where a high LGRB energy coincides with low host metallicity. While this object was assigned a much higher host metallicity in a previous analysis (Sollerman et al. 2005), the mass of the galaxy makes that interpretation very unlikely. While the claimed relation relies on the significance of this single object, it is unclear whether the other (underluminous) LGRBs are representative of the more energetic and distant cosmological bursts. GRB 060218, for example, could not even have been detected by Swift at redshifts $z > 0.05$.

Even in the absence of spectroscopic metallicity measurements for the individual hosts, we can tentatively investigate such a relation for the full sample with known $E_{iso}$ and host luminosities using the $L-Z_O$ relation. Fig. 5 shows the result for 13 GRBs from our sample (black data points) which have $E_{iso}$ values in the literature (Amati et al. 2006). We also add the five local objects discussed by Stanek et al. (2006) based on their actual host $Z_O$-data as grey data points. At least for the cosmological bursts (black) no relation is apparent. However, due to the expected significant variations in jet geometry and viewing angle among the bursts, we would not expect the fiducial isotropic energy estimate to show very clear trends with other parameters. Ghirlanda, Ghisellini & Lazzati (2004) presented LGRB energies, $E_{corr}$, that are corrected for jet beaming effects. Correspondingly, their Ghirlanda relation, $E_{corr}$ versus $E_{peak}$, is more clearly defined than the original Amati relation $E_{iso}$ versus $E_{peak}$ (Amati et al. 2002), where $E_{peak}$ is the rest-frame photon energy at the peak in the $\nu F_{\nu}$ spectrum. In the bottom panel of Fig. 5, we show $E_{corr}$ versus the host metallicity for the bursts with available data. We find no trend in these data.

5.5 Theoretical implications

From a theoretical point of view, it is not surprising that metallicity plays an important role for LGRB progenitors. In one of the most promising progenitor scenarios, the collapsar model (Woosley 1993; MacFadyen & Woosley 1999), the progenitor is the rapidly
The metallicity dependence of the LGRB rate

Figure 5. Top panel: comparison of the fiducial isotropic LGRB energy $E_{iso}$ with the host metallicity, as estimated from the $L-Z_O$ relation for $z > 0.2$ bursts (black points) and as measured from spectroscopy for five $z < 0.2$ bursts (grey points, green online). Bottom panel: comparison of the LGRB energy $E_{corr}$ corrected for beaming by Ghirlanda et al. (2004) with the host metallicity.

rotating core of a massive star. On the other hand, the core of a massive star is believed to efficiently lose angular momentum during its evolution: both within the star by hydrodynamical (e.g. Heger, Langer & Woosley 2000) and magnetohydrodynamical processes (e.g. Spruit 2002) and subsequently from the surface of the star in the form of a stellar wind. As a consequence, it seems unlikely that most massive stars will still have cores at the time of core collapse that rotate as rapidly as is required in the collapsar model.

Indeed, this is also not necessary, since it is clear that LGRBs are rare events, associated with less than 1 in 1000 CC SNe (Podsiadlowski et al. 2004a). One of the key unresolved questions is what are the special circumstances that produce LGRB progenitors: are these special conditions in single stars (e.g. Yoon & Langer 2005) or do they require particular binary channels (e.g. Fryer & Woosley 1998; Izzard, Ramirez-Ruiz & Tout 2004; Petrovic et al. 2004; Fryer & Heger 2005; Detmers, Langer & Podsiadlowski, in preparation; Fryer et al. 2006; Podsiadlowski et al., in preparation)? One of the major problems in many of these models is that mass loss, both in the red-supergiant phase and, in particular, in the Wolf–Rayet phase, leads to very efficient angular momentum loss. Since these wind mass-loss rates are dependent on the metallicity (typically $M \propto Z^{0.5-0.7}$; e.g. Vink & de Koter 2005), massive stars with lower metallicity are more likely to have rapidly rotating cores at the time of core collapse. This is a generic advantage for many of the proposed models, both single and binary.

Specifically, Yoon & Langer (2005) and Woosley & Heger (2006) recently proposed that a low-metallicity, very rapidly rotating star may evolve essentially homogeneously during its early evolutionary phases and may avoid a red-supergiant phase altogether, in which most of the angular momentum loss from the core tends to take place. As a consequence, these stars would still preserve rapidly rotating cores at the time of collapse, fulfilling one of the key conditions in the collapsar model. Similarly, in some of the most promising binary scenarios, in which two massive stars (almost) merge or interact tidally, it is advantageous or even necessary that this occurs late in the evolution of one of the stars, that is, after helium core burning (so-called case C mass transfer). The range of masses for which case C mass transfer occurs and allows the formation of a black hole is a strong, non-linear function of metallicity (Juskhâm, Podsiadlowski & Rappaport, in preparation), again favouring low-metallicity progenitors.

As this discussion illustrates, the dependence of the LGRB rate on host metallicity can provide an important constraint on the various proposed progenitor models. Indeed, Hirschi, Meynet & Maeder (2005) and Yoon et al. (2006) have estimated that, when the effects of magnetic fields are included, the single, low-metallicity, high-rotation model (Yoon & Langer 2005; Woosley & Heger 2006) requires a metallicity less than one-fifth solar. This already appears inconsistent with the constraints derived in this paper, which seem to rule out any models that require a metallicity significantly less than one-half solar.

The ongoing metallicity calibration debate might render our limit to lie $\sim 0.3$ dex lower, if $T_e$-based methods were correct. There are now explorations of a third method for measuring metallicities, the O$_{bg}$ method (Peimbert et al. 2006), which is aimed at overcoming the shortcomings of both previous methods. Early results indicate that this method gives values in between the two debated versions, but closer to the $R_{32}$ method. This would leave our results largely unchanged.

ACKNOWLEDGMENTS

CW was supported by a PPARC Advanced Fellowship and appreciates the hospitality of the IAA in Granada, where this paper was written up. We acknowledge discussions with Andy Fruchter, Javier Gorosabel, Stephen Justham, Norbert Langer and Sung-Chul Yoon, as well as regular inspiration from the Stellar Coffee Group at Oxford. We thank the anonymous referee for his support in improving the clarity of this manuscript.

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