Gravitational instability of discs with dissipative coronae around supermassive black holes

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ABSTRACT

We study the dynamical structure of a self-gravitating disc with coronae around a supermassive black hole. Assuming that the magnetorotational instability responsible for generating the turbulent stresses inside the disc is also the source for a magnetically dominated corona, a fraction of the power released when the disc matter accretes is transported to and dissipated in the corona. This has a major effect on the structure of the disc and its gravitational (in)stability according to our analytical and self-consistent solutions. We determine the radius where the disc crosses the inner radius of gravitational instability and forms the first stars. Not only the location of this radius which may extend to very large distances from the central black hole, but also the mass of the first stars highly depends on the input parameters, notably the viscosity coefficient, the mass of the central object and the accretion rate. For accretion discs around quasi-stellar objects (QSOs) and the Galactic Centre, we determine the self-gravitating radius and the mass of the first clumps. Comparing the cases with a corona and without a corona for typical discs around QSOs or the Galactic Centre, when the viscosity coefficient is around 0.3, we show that the self-gravitating radius decreases by a factor of approximately 2, but the mass of the fragments increases with more or less the same factor. The existence of a corona implies a more gravitationally unstable disc according to our results. The effect of a corona on the instability of the disc is more effective when the viscosity coefficient increases.

Key words: accretion, accretion discs – black hole physics – galaxies: active.

1 INTRODUCTION

Among various physical agents which play significant roles in accreting systems, self-gravity has a vital role in accretion discs around supermassive black holes. According to the standard theory of accretion discs (Shakura & Sunyaev 1973), the outer parts of steady, geometrically thin, optically thick discs are prone to self-gravity and they might be expected to fragment into stars (e.g. Shlosman & Begelman 1987; Goodman 2003). While many authors proposed possible solutions to this problem (e.g. Goodman 2003), it seems that such a gravitationally unstable disc is a good explanation for the existence of the first stars in galactic centres or the formation of supermassive stars in quasar accretion discs (Goodman & Tan 2004; Tan & Blackman 2005; Levin 2007). Quasar spectra show that near the black hole there is high metallicity (Hamann et al. 2002), presumably because of massive star formation. Interestingly, young and massive stars are observed within 0.5 pc of Sgr A* in our Galactic Centre (e.g. Bender et al. 2005; Lu et al. 2005; Paumard et al. 2006). Based on the observation of the Galactic Centre, Levin & Beloborodov (2003) proposed that a recent burst of star formation has occurred in a dense gaseous disc around Sgr A*. Also, the data of Paumard et al. (2006) strongly favour in situ star formation from a dense disc in the central regions of the Galactic Centre. Star formation activities are seen in the nuclei of Seyfert galaxies (Terlevich 1996).

These observations have motivated many authors to study star formation under extreme conditions in discs around massive black holes at the centre of galaxies. The suggestion of star formation in the self-gravitating parts of an accretion disc was first made by Kolykhalov & Sunyaev (1980). Shlosman & Begelman (1989) discussed in detail the conditions for star formation in the discs around black holes. Recently, star formation near the Galactic Centre has also been studied by Levin (2007) and Nayakshin (2006). Levin (2007) considers a disc that fragments into many stars of up to a few hundred solar masses and Nayakshin (2006) shows that the...
initial mass function for disc-born stars at the distances between 0.03 and 0.3 pc from the supermassive black hole should be top-heavy. Collin & Zahn (1999a,b) have also studied the formation of stars in active galactic nuclei (AGN) discs. Goodman & Tan (2004) proposed that supermassive stars may form in quasar discs. According to their model, while the disc may fragment and each fragment may start with only a handful of solar masses, it seems likely that they grow to a significant fraction of the disc mass, perhaps around $10^5 M_\odot$. Just recently Nayakshin, Cuadra & Springel (2007) studied star formation in the central parsec of our Galaxy using numerical simulations. However, they found that the masses of the first clumps are much smaller than $10^5 M_\odot$ which has been predicted by simple theoretical analysis (e.g. Goodman & Tan 2004).

The observation of the ultraviolet bump of AGN (e.g. Malkan & Sargent 1982) argues in favour of geometrically thin and optically thick discs, possibly embedded in a hot X-ray emitting corona. In fact, coexisting soft and hard X-ray components observed in ordinary AGN cannot be well described by simple one-zone hot-flow models and require a composite disc–corona structure. Considering some common features of the corona of a disc with a solar corona, interesting theoretical investigations have also been presented in the frame of a magnetized disc corona (Haardt & Maraschi 1991; Field & Rogers 1993; Nakamura & Osaki 1993; Merloni & Fabian 2002; Misra & Taam 2002; Liu, Mineshige & Shibata 2002; Merloni 2003). It has been suggested that the corona consists of localized active regions. It is likely that these are produced by magnetic fields in the disc amplified through differential rotation. When the disc’s magnetic field builds up significantly, buoyancy forces the field out and above the disc, giving rise to active regions of high magnetic field. Misra & Taam (2002) found that a significant part of the accretion flow (or dissipation rate) can take place in the corona if the scaleheight of the magnetic field is larger than that of the disc. According to their study, optically thick discs with dissipative coronae can provide an attractive explanation for the origin of the soft spectral component observed in black hole X-ray binary systems. Różańska et al. (1999) studied the vertical structure of a radiation pressure dominated disc with a hot corona. They showed that the presence of the corona modifies considerably the density and the opacity of the disc surface layers and concluded that the corona is not only essential from the point of view of the X-ray spectra formation but also helps us to remove the problems with the disc models for AGN. The disc–corona model for AGN has also been discussed by Field & Rogers (1993).

Merloni & Fabian (2002) showed that, if angular momentum transport in the disc is due to magnetic turbulent stresses, the magnetic energy density and effective viscous stresses inside the disc are proportional to the geometric mean of the gas pressure and the total pressure (gas plus radiation). They discussed why energetically dominant coronae are ideal sites for launching powerful jets/outflows, both magneto-hydrodynamically and thermally driven. Merloni (2003) presented thin disc solutions accompanied by powerful, magnetically dominated coronae and outflows, as models for black holes accreting at super-Eddington rates. Based on the assumption that the magnetorotational instability (MRI) responsible for generating the turbulent stresses inside the discs (Balbus 1991) is also the source for a magnetically dominated corona, Merloni & Merloni (2006) studied the limit-cycle instability in magnetized accretion discs. In these models, a fraction of the gravitational energy release is assumed to dissipate in a hot diffuse corona, above the main body of the disc, away from the majority of the accreted mass.

Thus, one may conclude that corona is an important part of the accretion processes around central black holes of AGN. Possible effects of a corona on its underlying disc have been studied using different models. For example, a dissipative corona may be able to thermally stabilize the underlying disc (Misra & Taam 2002). However, the possible effects of the corona on the gravitational stability of the disc have not been studied to our knowledge. In this study, we would like to address this question by a very simple model: what is the effect of the corona on the gravitational stability of a self-gravitating disc? We know that the corona may thermally stabilize the disc, but can we expect gravitational stabilization as a result of the presence of the corona? Considering the growing interest in star formation near our Galactic Centre and discs of AGN, any possible effects of the corona will have interesting implications in these studies. In our approach, the disc structure equations are reduced to simple algebraic relations in terms of average quantities of the disc and corona.

2 GENERAL FORMULATION

We assume that all angular momentum transport takes place in the disc and the mass-accretion rate $M$ is constant with radius and time. However, the microphysics mechanism of the angular momentum transfer remains unknown. Shakura & Sunyaev (1973) replaced all the missing physics by a parameter $\alpha$. This approach has been widely used for studying the dynamics and structure of the accretion flows. A promising mechanism for driving the turbulence responsible for angular momentum and energy transport is the action of the MRI that is expected to take place in such discs (Balbus 1991). However, for a disc–corona system it is typically thought that the viscous stress, assumed to be magnetic in nature, transports angular momentum and initially randomizes the gravitational binding energy near the mid-plane. The magnetized fluid elements, which are buoyant with respect to their surroundings, dissipate above the disc. Merloni & Fabian (2002) and Merloni (2003) presented very detailed discussions about dissipation in the corona and its relation with angular momentum transport in the disc itself. Our disc–corona model is developed along the line proposed by Merloni & Fabian (2002) and Merloni (2003).

We consider a more general prescription for the viscous stresses $\tau_{i\phi}$ (Taam & Lin 1984; Watarai & Mineshige 2003; Merloni & Merloni 2006):

$$\tau_{i\phi} = \alpha_0 \frac{\mu^{1/2}}{f_g} p_{\text{gas}}, \quad (1)$$

where $\alpha_0$ and $0 \leq \mu \leq 2$ are constants. Also, $p$ is the sum of the gas and radiation pressures. Phenomenological models generally assume that at each radius, a fraction $f$ of accretion energy is released in the reconnecting magnetic corona. Assuming that in MRI-turbulence discs such a fraction of the binding energy is transported from large to small depths by Poynting flux, Merloni & Merloni (2006) estimated the fraction $f$ as

$$f = \sqrt{2\alpha_0 \beta^{1/2}}, \quad (2)$$

where $\beta$ is the ratio of gas pressure to the total pressure. Thus, in this model, this fraction $f$ is not a free parameter.

The rotation curve is dominated by a Newtonian point mass $M$, as relativistic effects are only important at small radii. Although self-gravity may be important for local stability at the outer radii of the disc, the rotation angular velocity is not much affected because the disc is thin and its mass is smaller than the central point mass (e.g. Goodman 2003). Thus, the rotational angular velocity of the disc is Keplerian, that is, $\Omega_k = \sqrt{GM/R^3}$. Having equation (1) as a prescription for the viscous stresses and equation (2) as an expression for the fraction of power dissipated in the corona, we can...
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The basic equations of the disc. Vertical hydrostatic equilibrium of the disc implies
\[
\frac{p}{\Sigma} = \frac{\Omega_k^2 H}{2},
\]
and the azimuthal component of the equation of motion gives
\[
8\pi a_0 H (\rho_{gas})^{1/2} \cdot \beta \cdot (\rho_{gas}^{1/2}) = 3\Omega_k M J(R),
\]
where \(J(R) = 1 - \sqrt{\frac{R_{in}}{R}}\). Here, \(R_{in}\) denotes the inner boundary of the disc. Since we are interested in the regions of the disc with radii much larger than \(R_{in}\), we can assume \(J \approx 1\). However, we will keep this factor in order to present solutions of the disc structure as general as possible.

The energy equation is given by
\[
\sigma T_{eff}^4 = \frac{3}{8\pi} \Omega_k^2 M J(R)(1 - f),
\]
where as we discussed \(f = \sqrt{2\rho_{gas}\beta/\rho}\). Also, we can assume that the vertical transport of heat is by radiative diffusion which implies that the mid-plane and the surface temperatures are related by
\[
T = \left(\frac{3}{8\pi} \kappa \Sigma \right)^{1/4} \cdot T_{eff},
\]
where \(\kappa\) is the opacity coefficient.

Now, equations (3), (4) and (5) are the main equations which enable us to find \(p\) and \(T\) as functions of \(R\) and \(\beta\) and the other input parameters. Thus,
\[
T = \left(\frac{4\pi \Omega_k}{3\kappa a_0}\right)^{-1/2} \left(\frac{16\pi^2 \alpha_0^3 c k_B}{3\sigma \mu_m M^2 \Omega_k^2 J^2}\right)^{-1/3} \times \left(1 - \sqrt{\frac{2\rho_{gas}\beta/\rho}{\beta(\rho_{gas})^{1/2}}}(1 - \beta)^{2/3}\right),
\]
\[
p = \left(\frac{4\pi \Omega_k}{3\kappa a_0}\right)^{-1/2} \left(\frac{16\pi^2 \alpha_0^3 c k_B}{3\sigma \mu_m M^2 \Omega_k^2 J^2}\right)^{-2/3} \times \left(1 - \sqrt{\frac{2\rho_{gas}\beta/\rho}{\beta(\rho_{gas})^{1/2}}}(1 - \beta)^{2/3}\right),
\]
there is an algebraic equation for \(\beta\) as follows:
\[
\frac{k_B}{\mu_m m_H} \left(\frac{4\pi \Omega_k}{3\kappa a_0}\right)^{-3/2} \left(\frac{8\pi a_0}{3\Omega_k^2 M J}\right)^2 \left(\frac{16\pi^2 \alpha_0^3 c k_B}{3\sigma \mu_m M^2 \Omega_k^2 J^2}\right)^{-5/3} \times \left(1 - \sqrt{\frac{2\rho_{gas}\beta/\rho}{\beta(\rho_{gas})^{1/2}}}(1 - \beta)^{2/3}\right)^{(4/3)} = 1,
\]
where \(\mu_m\) is the mean particle mass in units of the hydrogen atom mass, \(m_H\). The other constants have their usual meanings.

In order to study the behaviour of our solutions, it is more convenient to introduce dimensionless variables. For the central mass \(M\), we introduce \(M_8 = M/(10^8 M_{\odot})\) and for the radial distance \(R\), we have \(r_1 = R/(10^3 R_0)\), where \(R_0 = 2GM/c^2\) is the Schwarzschild radius. The mass-accretion rate can be written as
\[
M = \frac{\dot{L}_K}{4\pi GM} = \frac{\dot{L}_K}{\kappa_{s, e} c L_{Edd}},
\]
where \(\dot{L}_K = L/K\) is the dimensionless disc luminosity relative to the Eddington limit and \(c = L/(M c^2)\) is the radiative efficiency. Also, \(\kappa_{s, e} \approx 0.04\) m² kg⁻¹ is the electron opacity. In our analysis, we will use the non-dimensional factor \(\dot{L}_K/c\) as a free parameter so that by changing this parameter we can consider appropriate values of the accretion rate. However, some authors introduce different forms for the accretion rate. For example, Nayakshin & Cuadra (2005), who studied gravitational stability of the Galactic Centre, introduced \(M = (\dot{m}/c^2)(L_K/c)^2\) with \(\epsilon \approx 0.06\) and \(\dot{m} = 0.03-1\). These values, which are appropriate for the Galactic Centre, correspond to \(\dot{L}_K/c \approx 0.5-16.6\) in our notation. Also, for a central mass with mass \(M = 10^9 M_{\odot}\), Goodman & Tan (2004) proposed \(\dot{L}_K/c = 10\). Thus, in our analysis, the chosen values of \(\dot{L}_K/c = 1\) and 10 are acceptable.

Now, we can rewrite our solutions as (in SI)
\[
\rho = 2.76 \times 10^{-3} \alpha_0^{-1/3} \kappa^{-1/2} M_8^{1/2} \left(\frac{L}{c}\right)^2 \sqrt{J_{eff}^3},
\]
\[
p = 27.15 \alpha_0^{-5/6} \kappa^{-1/2} M_8^{5/6} \left(\frac{L}{c}\right)^{4/3} \sqrt{J_{eff}^9 / r_1},
\]
\[
\frac{H}{R} = 4.67 \times 10^{-3} \alpha_0^{-1/6} \kappa^{-1/2} M_8^{1/6} \left(\frac{L}{c}\right)^{-1/3} \sqrt{J_{eff}^3 / r_1},
\]
and the ratio \(\beta\) is obtained from the non-dimensional form of equation (9), that is,
\[
\frac{0.16 \alpha_0^{1/6} \kappa^{3/2} M_8^{1/6} \left(\frac{L}{c}\right)^{4/3} \sqrt{J_{eff}^9 / r_1}}{\beta(1 - \beta)^{-5/3} \left(1 - \sqrt{2\rho_{gas}\beta/\rho}\right)^{2/3}} = 1,
\]
where \(\tilde{\kappa} = \kappa/\kappa_{s, e}\) and we assumed \(\mu_m \approx 0.6\). We can also calculate the surface density as
\[
\Sigma = 1.27 \times 10^3 \alpha_0^{-2/3} \kappa M_8^{1/3} \left(\frac{L}{c}\right)^{3/5} \sqrt{J_{eff}^3 / r_1^{11/4}},
\]
\[
\times \beta^{(4-\mu)/3} (1 - \beta)^{-4/3} \left(1 - \sqrt{2\rho_{gas}\beta/\rho}\right)^{2/3}.
\]

Equations (11), (12) and (13) along with equation (14) describe the structure of a disc with a dissipative corona. However, the physical variables depend not only on the radial distance but also on the ratio of the gas pressure to the total pressure which can be calculated at each radius from algebraic equation (14). For \(\mu = 1\), our solutions reduce to what has been obtained by Merloni (2003). In the next section, we will analyse our solution, in particular, the gravitational stability of the disc.

### 3 Analysis

Since we are interested in the gravitational stability of the disc, we can approximate \(J(R) \approx 1\) for \(R \gg R_0\). Having the input parameters, we can solve equation (14) numerically at every radius to obtain the ratio of the gas pressure to the total pressure. Equation (2) then gives the fraction of gravitational power associated with the angular momentum transport that is transported vertically and dissipated in the corona. As the systems tend towards the gas pressure dominated regime, the dissipated energy in the corona increases according to this equation. However, our solutions generally describe an inner radiation-dominated region with an outer gas pressure dominated region according to equation (14). Moreover, the size of the gas pressure dominated regime increases, as the accretion rate decreases. These results are valid irrespective of the value of \(\mu\), the exponent of the magnetic viscosity. For \(\mu = 1\), similar typical behaviours
have already been obtained by Merloni & Fabian (2002). We want to extend this analysis by studying the gravitational instability of the disc. In doing so, we should calculate the Toomre parameter of the disc.

Toomre (1964) showed that a rotating disc is subject to gravitational instabilities when the Q-parameter becomes smaller than a critical value, which is close to unity,

\[ Q = \frac{c_s \Omega}{\pi G \Sigma}, \]

where \( c_s \) is the sound speed inside the accretion disc and \( \Omega = \Omega_K \) is the angular velocity. Thus, the Toomre parameter of our model becomes

\[ Q = \frac{4 \alpha_0^{1/2} \kappa^{-3/2} M_8^{-3/2} \left( \frac{l_k}{\epsilon} \right)^{-2}}{J^{-2} \Sigma S^{-3/2}}. \]

This equation with algebraic equation (14) gives the Toomre parameter as a function of the radial distance. Generally, this parameter is much higher than unity in the inner parts of the disc which implies that these regions are gravitationally stable and do not fragment. However, the Toomre parameter decreases with increasing radial distance so that \( Q \) reaches the critical value of unity at a self-gravitating radius which we denote by \( R_{\text{sg}} \). Thus, all regions with \( R > R_{\text{sg}} \) are gravitationally unstable and may fragment to clumps and cores.

Different authors estimate the mass of fragments differently. Since the disc is marginally unstable, the initial sizes and masses of gravitationally bound fragments can be determined by Toomre’s dynamical instability (Toomre 1964). The most unstable wavelength for the \( Q \approx 1 \) disc is of the order of the disc vertical scaleheight \( H \) (Toomre 1964). Thus, the most unstable mode has radial wave number \( k_{\text{msu}} = (QH)^{-1/3} \) and so the mass of a fragment at \( R = R_{\text{sg}} \) becomes

\[ M_{\text{frag}} \approx \frac{2\pi}{k_{\text{msu}}} = 4\pi^2 \Sigma H^2. \]

### 3.1 Parameter study

Now, we can have a parameter study of the solutions. Fig. 1 shows the self-gravitating radius \( R_{\text{sg}} \) (top panel) and the mass of the fragments (bottom panel) versus the exponent \( \mu \) of the viscosity prescription for different input parameters. We fix the central mass (\( M_8 = 1 \)) and the opacity (\( \kappa = 1 \)), but vary the other input parameters. Each curve is labelled by the pair of the viscosity coefficient \( \alpha_0 \) and the accretion rate \( l_k/\epsilon \), as \( (\alpha_0, l_k/\epsilon) \). The top panel of Fig. 1 shows that the disc of our system becomes self-gravitating at distances close to the central black hole. While for high accretion rate the self-gravitating radius \( R_{\text{sg}} \) depends on the exponent \( \mu \), for lower accretion rate this radius is more or less independent of the exponent of the viscosity. For a fixed accretion rate, the self-gravitating radius \( R_{\text{sg}} \) increases with \( \alpha_0 \).

In fact, our viscosity prescription is different from the standard alpha model (Shakura & Sunyaev 1973). In the alpha model, Rice, Lodato & Armitage (2005) argue that the maximum viscosity in marginally stable self-gravitating discs is around 0.06. Unfortunately, there is not enough information about appropriate values of the coefficient \( \alpha_0 \) for studying the gravitational structure of discs. In analogy to previous studies of accretion discs based on the viscosity prescription of our model (e.g. Watarai & Mineshige 2003), we are using a range 0.03–0.3 for \( \alpha_0 \).

Using equation (18), we can calculate the mass of the first fragments for the above set of input parameters. For low accretion rate, the mass of the fragments is also more or less independent of the exponent of \( \mu \). However, low values of \( \alpha_0 \) correspond to fragments with higher masses so that for the pair of (0.03, 10) the masses will be between \( 10^4 \) and \( 10^5 \) \( M_\odot \) depending on the viscosity exponent \( \mu \). However, as we mentioned, we think low values of \( \alpha_0 \) are not acceptable in self-gravitating discs. For a fixed accretion rate, the mass of the first clumps decreases with increasing \( \alpha_0 \). However, as the accretion rate decreases, the mass of clumps decreases as well so that while for (0.3, 10) we have \( M_{\text{frag}} \approx 600 M_\odot \) for (0.3, 1) the mass of the fragments decreases approximately to \( M_{\text{frag}} \approx 30 M_\odot \).

It means that for the above set of input parameters, as the accretion rate decreases, fragments with lower masses are forming at larger distances from the central black hole. According to our results not
only the self-gravitating radius but also the mass of the first clumps shows wide ranges of variations depending on the input parameters.

In order to make easier comparison with other input parameters, Table 1 summarizes the self-gravitating radius and the mass of the first clumps for $M = 10^8 M_\odot$, $\mu = 1$ and $\hat{k} = 1$. We will discuss the fraction of dissipated energy in the corona; however, before that we can study the solutions for another set of input parameters.

We found that the self-gravitating radius and the mass of the fragments are not very sensitive to the exponent $\mu$ for a lower mass of the central black hole. Thus, we summarize our results for $M = 3 \times 10^9 M_\odot$, $\mu = 1$ and $\hat{k} = 1$ in Table 2 which is appropriate for modelling the Galactic Centre. Clearly, when the mass of the central black hole decreases, the disc is more gravitationally stable as $R_{\text{eq}}$ is much larger than that in Fig. 1 and the mass of the fragments is much smaller. In this case, also the typical behaviours of the solutions are similar to the case with $M = 10^9 M_\odot$. We see that as the viscosity coefficient $\alpha_0$ increases, not only does the self-gravitating radius increase, but also the mass of the fragments decreases. Also, as the accretion rate increases, the self-gravitating radius decreases and the mass of the fragments increases as well.

Goodman (2003) also estimated the mass of the first clumps in quasar discs. Clearly, the disc of their model is without a corona. Assuming that the viscosity is proportional to gas pressure, they found $R_{\text{eq}} \approx 1700 R_S$ to $R_{\text{eq}} \approx 2700 R_S$ with a few hundred solar masses for the fragments. Another point is that their model is radiation pressure dominated. In another study, Nayakshin (2006) studied star formation near our Galactic Centre. The mass of the first stars of this model is a few solar masses which may increase because of the subsequent accretion. However, our model shows wider ranges for the self-gravitating radius and the mass of the fragments. Moreover, when we reach the self-gravitating radius, the disc of our model is not necessarily radiation pressure dominated. In fact, we found the transition to the self-gravitating regime happens at radii where generally the gas pressure, if it is not dominant, is at least comparable to the radiation pressure.

We can calculate the fraction $f$ of dissipated energy in the corona. Instead of integrating the fraction $f$ over all the disc, for our purposes it is sufficient to plot this fraction as a function of the radial distance. Fig. 2 shows the typical behaviour of $f$ for our sets of input parameters. Note that since we are not interested in the very inner regions of the disc, this plot is not accurate near these regions as we approximated $J \approx 1$. This figure shows interesting behaviour. First of all, as the viscosity coefficient increases, the fraction $f$ increases as well. We also showed that the self-gravitating radius increases and the mass of the fragments decreases with increasing $\alpha_0$. It means that the disc is more gravitationally stable as $\alpha_0$ increases or correspondingly, the amount of dissipated energy in the corona increases.

In order to confirm this result, we should check the behaviour of $f$ and the disc for the other input parameters.

### Table 2. The same as Table 1, but for $M = 3 \times 10^9 M_\odot$.

<table>
<thead>
<tr>
<th></th>
<th>$l_\text{fr}/\epsilon = 1$</th>
<th>$l_\text{fr}/\epsilon = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.03 0.3 0.03 0.3</td>
<td>0.03 0.3</td>
</tr>
<tr>
<td>$R_{\text{eq}}/(10^3 R_S)$</td>
<td>470 6884 102 1484</td>
<td></td>
</tr>
<tr>
<td>$M_{\text{frag}}/M_\odot$</td>
<td>0.0045 5 $\times$ 10^{-5} 0.12 0.0016</td>
<td></td>
</tr>
</tbody>
</table>

### Figure 2. The integral fraction $f$ of accretion power dissipated into the corona versus radial location in the disc with $\mu = 1.5$ for an $M = 10^8 M_\odot$ (solid lines) and $M = 10^9 M_\odot$ (dashed lines) central black hole with $\mu_0 = 0.6$ and $\hat{k} = 1$. The curves are labelled by the ratio $l_\text{fr}/\epsilon$. As this ratio increases, the fraction $f$ of dissipated energy in the corona decreases, irrespective of the other input parameters.

Consider Fig. 2 for a fixed coefficient $\alpha_0$. We see that as the accretion rate decreases, the fraction $f$ increases which means that more energy dissipates in the corona. Also, as the mass of the central black hole decreases, the fraction $f$ increases for a fixed accretion rate. For these cases, we showed that the disc becomes self-gravitating at larger radii with lower masses for the fragments, as the mass of the central black hole decreases (compare with Fig. 1 and Table 1).

### 3.2 The effect of the corona

How can we understand the above results based on the existence of the corona? To address this question, we can compare our results directly either with previous authors or with our calculations for the standard case (no corona). Although we briefly mentioned previous studies in this field, for example, Goodman & Tan (2004) and Nayakshin (2006), we cannot make direct comparisons with those studies because our basic assumptions for the corona and viscosity prescription are different. We think a straightforward way to understand the effect of the corona on the gravitational instability of the disc is direct comparison with our solutions, but without a corona.

Obviously, a certain amount of the generated energy in the disc is transported into the corona in our model. This leads to a cooler disc in comparison with a disc without corona. One may conclude that since the sound speed and consequently the Toomre parameter decreases, a disc with a corona will be more unstable gravitationally. This result is correct as long as the surface density does not change or increase. In our model, the surface density is also lower than in the case of a disc without a corona. Thus, this qualitative consideration does not help us to determine the typical behaviour of the Toomre parameter as a result of the existence of the corona.

Now, we can simplify equation (17) by calculating $r_1$ from equation (14):

$$Q = 9.14 \alpha_0^{9/14} \hat{k}^{-3/14} M_\odot^{-19/14} \left( \frac{l_\text{fr}}{\epsilon} \right)^{-6/7} \times \beta^{(9\mu-40)/28} (1-\beta)^{5/7} \left( 1 - \sqrt{2\alpha_0 \beta \mu / 3} \right)^{-3/14}.$$  (19)

Also, the Toomre parameter $Q_0$ of the disc without a corona becomes

$$Q_0 = 9.14 \alpha_0^{9/14} \hat{k}^{-3/14} M_\odot^{-19/14} \left( \frac{l_\text{fr}}{\epsilon} \right)^{-6/7} \times \beta^{(9\mu-40)/28} (1-\beta)^{5/7}.$$  (20)
the self-gravitating radius of a disc without a corona. In other words, the self-gravitating radius of a disc with a corona reaches unity at a smaller radius in comparison with a disc without a corona. When we plot the Toomre parameter versus the distance from the central black hole, we have always $Q_{bh} > Q$ at a fixed radius. Thus, the Toomre parameter $Q$ of the disc with a corona reaches unity at a smaller radius in comparison with a disc without a corona. In other words, the self-gravitating radius of a disc with a corona is smaller than that of a disc without a corona. We can confirm this general result by direct numerical solution of the equations. Fig. 3 shows typical behaviour of the Toomre parameter for the cases with and without a corona. Also, Tables 1 and 2 show the self-gravitating radius and the mass of the fragments of the discs with and without a corona. In the last section, we discussed general properties of discs with a corona. Now, we can compare the gravitational instability of the discs with and without a corona.

By comparing the tables, we can simply conclude that as the mass of the central black hole increases, the discs become more self-gravitating irrespective of the existence of a corona. In fact, while the self-gravitating radius $R_{q}$ decreases with the mass of the central object, the estimated mass of the clumps increases. These tables confirm our above analytical expectation that in discs with a corona the Toomre parameter is lower than that in the same disc but without a corona. For example, Table 1 compares the self-gravitating radius and the mass of the clumps in discs with and without a corona, but both with $M = 10^{8} M_{\odot}$. We see that not only does the self-gravitating disc decrease because of the corona, but also the mass of the clumps increases. However, the effect of a corona on the instability of the disc is more effective for $\alpha_0 = 0.3$ compared to $\alpha_0 = 0.03$. It means that as the viscosity coefficient $\alpha_0$ increases, the corona makes the disc more gravitationally unstable. In fact, in our model, the fraction of the energy dissipated into the corona is directly proportional to the viscosity coefficient. Thus, as this parameter increases, since more energy is dissipated into the corona, we see the effect of the corona more effectively. Comparing the cases with a corona and without a corona for $\alpha_0 = 0.3$, we see that $R_{q}$ decreases by a factor of approximately 2, but $M_{\text{frag}}$ increases with more or less the same factor because of the existence of the corona.

The above behaviours are also seen in Table 2, but the mass of the first clumps is much smaller in the case of $M = 3 \times 10^{6} M_{\odot}$ which corresponds to Sgr A*, compared to the other case. Also, in this case, the self-gravitating radius is much larger. However, evidently a corona increases the mass of the first clumps and reduces the self-gravitating radius. Massive stars were found at a distance of the order of 0.1–0.3 pc from Sgr A*. For this central black hole, we have 0.1 pc $\sim 3 \times 10^{7} R_{s}$. According to Table 2, the self-gravitating radius of our models is within the range 0.03–2.3 pc, depending on the existence of the corona and the input parameters. For example, consider a disc without a corona with $\ell_{E}/\epsilon = 10$ and $\alpha_0 = 0.3$: this table gives $R_{q} \sim 0.5$ pc. However, when we consider the same disc but with a corona, the self-gravitating radius decreases to $R_{q} \sim 0.25$ pc. This new location of the first clumps is consistent with the typical distance of the observed massive stars in the Galactic Centre. Nayakshin & Cuadra (2005) found that the self-gravitating radius for their standard disc is around 0.03 pc. We note that their standard disc model is not similar to our standard model, but since without a corona, we calculate a larger radial distance for the first clumps, if we modify the Nayakshin & Cuadra (2005) model to include a corona, then the self-gravitating radius decreases. If we consider higher values for the viscosity coefficient, the discrepancy between the cases with and without a corona would be more significant.

4 DISCUSSION

We have studied the effect of a corona on the gravitational stability of the disc by assuming that the hot corona and the disc coexist at the same radius from the central black hole. In our disc–corona model almost all the mass exists in the disc, but the dissipated energy in the corona depends on the input parameters, such as the mass-accretion rate, the mass of the central black hole and the exponent of the viscosity prescription. Our study shows that a corona has a destabilizing effect on the gravitational stability of the disc. However, our disc and corona system is still able to explain the formation of rings of massive stars observed in the Galactic Centre and in the nucleus of M31. However, star formation in a self-gravitating disc with a corona may have significant differences from discs without a corona. Considering the results of our study, one can try to determine the initial mass function for disc-born first clumps at the inner region of a self-gravitating disc with a corona around a supermassive black hole, though Nayakshin (2006) has already predicted a top-heavy initial mass function for the first stars at the distances between 0.03 and 0.3 pc from our Galactic Centre. We think that the estimated supermassive mass of the fragments according to Goodman & Tan (2004) will change because of the destabilizing effect of the corona.
which transports part of the generated energy inside the disc to the corona.

As we discussed, there is significant observational evidence in favour of star formation activities in the accretion discs of AGN. The mechanism of accretion is an important part of any theory for accretion discs. Most of the analytical and numerical simulations show that MRI is the most significant process in accretion discs. Almost all models of star formation in accretion discs use MRI as the main physical process of accretion. Does MRI have other consequences for the structure of the disc? As we discussed in the Introduction section, the idea of a hot corona lying above the AGN accretion disc has been mainly developed in order to explain substantial emission in the X-ray range. There are interesting analytical and numerical studies for understanding processes of corona formation (e.g. Heyvaerts & Priest 1989; Miller & Stone 2000). Interestingly, current models of disc and corona systems replace the fraction of accretion power transferred from the disc to the corona with a Poynting flux quantity estimated from a mean field buoyant velocity and an equi-partition, mean field magnetic energy density. In other words, MRI may lead to corona formation as well, even though all the details of the process itself are not yet understood. Heyvaerts & Priest (1989) have shown that loops and arcades can form also in AGN accretion discs; these structures, connecting remote points of the disc itself, can convert disc kinetic energy into magnetic energy and subsequently dissipate it by emitting the observed spectrum. Numerical simulations indicate that turbulent fluctuations in a vertically stratified disc are capable of driving the magnetogravitational modes of the Parker instability (e.g. Miller & Stone 2000). It seems that there is a coexistence between MRI and corona, at least in AGN for which the X-ray emission can be explained based on the existence of coronae. Since our model is also based on this idea that MRI inside the disc is also the source for a magnetically dominated corona, we think that this model is applicable to such AGN, if the accretion disc itself is massive enough.

One should note that we have not followed the evolution of the first clumps after formation. One of the main important factors is accretion on to these new fragments. In fact, although the typical mass of the first clumps corresponding to the Galactic Centre is low compared to the cases with higher central mass, we think that the accretion on to first clumps is able to increase the mass of first clumps significantly. This accretion process is believed to be similar to the growth of terrestrial planets in a planetesimal disc. In order to estimate the amount of the accreted mass on to the first clump, we should calculate the Hill radius which is directly proportional to the mass of the clump itself at the time of formation. Since a corona increases the mass of the first clumps, the corresponding Hill radius and subsequently the rate of the accretion on to it will increase. It means that the final mass of the first clumps is even higher because of the corona and subsequent enhanced accretion rate compared to a case without a corona.

Star formation feedback is indeed able to slow down disc fragmentation as suggested by several authors, but since our goal was only to study the possible effects of the corona on the gravitational instability of the disc, we did not include star formation feedback in our model. Also, we have not studied the evolution of the clumps after formation. In fact, we showed that a corona may have significant effects on the star formation in an AGN disc or even near our Galactic Centre which should be considered in any successful theoretical model. Since our model for the disc-corona system is a phenomenological model, a more careful description of the disc-corona transition is necessary. This transition zone may have an important effect on the emergent radiation flux. On the other hand, we did not investigate the structure of the corona itself, because in our simple model there is only energy exchange between the corona and the underlying optically thick disc. In future work, it would be interesting to study the gravitational stability of a system of disc and corona, in which not only can the corona itself transport the angular momentum, but also there is exchange of mass, energy and angular momentum between disc and corona.

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