Evolution of protoplanetary discs driven by the MRI, self-gravity and hydrodynamical turbulence

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ABSTRACT

We study the viscous evolution of protoplanetary discs driven by the combined action of magnetohydrodynamic turbulence, resulting from the magneto-rotational instability (MRI), self-gravity torques, parametrized in terms of an effective viscosity and an additional viscous agent of unspecified origin. The distribution of torques driving the evolution of the disc is calculated by analysing where in the disc the MRI develops and, to incorporate the effect of self-gravity, calculating the Toomre parameter. We find that, generally, discs rapidly evolve towards a configuration where the intermediate regions, from a fraction of an au to a few au, are stable against the MRI due to their low-ionization degree. As an additional source of viscosity is assumed to operate in those regions, subsequent evolution of the disc is eruptive. Brief episodes of high mass accretion ensue as the criterion for the development of the MRI is met in the low-ionization region. The radial distribution of mass and temperature in the disc differs considerably from disc models with constant $\alpha$ parameter or layered accretion models, with potentially important consequences on the process of planet formation.

Key words: accretion, accretion discs – magnetic fields – MHD – planetary systems: formation.

1 INTRODUCTION

For the past 30 yr theoretical models for protoplanetary discs have been constructed within the framework of viscous, geometrically thin accretion discs. In most models, the torques responsible for the transport of angular momentum across the disc are expressed in terms of an anomalous viscosity, parametrized in terms of the large-scale flow properties according to the Shakura & Sunyaev (1973) $\alpha$ prescription. Although several mechanisms have been proposed as responsible for the torques driving disc evolution, there is still great uncertainty in the detailed properties of the process. Hence, it seems only natural that in most protoplanetary accretion disc models the so-called $\alpha$ parameter is considered to be uniform and constant.

Several mechanisms have been proposed as the origin of angular momentum transfer in protoplanetary discs. Among these, the magneto-rotational instability (MRI) is currently considered the most important agent driving disc evolution during the so-called viscous stage (see e.g. the review by Balbus & Hawley 1998). The MRI has been shown to lead to self-sustained turbulence and gives rise to effective torques that, averaged over several dynamical periods, can be formulated in terms of the $\alpha$ prescription of Shakura & Sunyaev (1973). However, when, where and how the instability develops in conditions of low ionization, as those expected in extended regions of protoplanetary discs, is still the subject of debate.

Self-gravity instabilities are known to be the dominant agent driving disc evolution in the commonly called formation stage, when significant infall from the parent molecular cloud is still taking place and the mass in the disc is comparable to that of the central object. Although angular momentum transport due to self-gravity is a non-local process, near the end of the formation stage, physical properties in the protoplanetary disc are well represented in terms of constant $\alpha$ disc models (Bodenheimer & Laughlin 1995). A similar conclusion is reached by Lodato & Rice (2004) who study the conditions under which self-gravity may be treated as a local angular momentum transport process. The precise role of self-gravity instabilities in the transport of angular momentum at other times during the disc evolution is not clear at present. We address this issue briefly in this paper.

In addition to the mechanisms mentioned above, several authors have suggested that hydrodynamic processes, such as non-linear shear or baroclinic instabilities, may give rise to angular momentum through the disc (Dubrulle 1993; Klahr & Bodenheimer 2003; Richard 2003; Dubrulle et al. 2005), although probably with a reduced efficiency in comparison to that associated with magnetohydrodynamic (MHD) turbulence.

The evolving distribution of torques due to any of these mechanisms defines the physical conditions within protoplanetary discs and hence sets the stage for the process of planetary system formation.

The ability of any of these processes to transfer angular momentum depends on the density and the temperature profiles in the...
disc. The MRI additionally depends on the ionization degree within the disc and on its spatial variation, which may depend on external ionization sources. Several authors have studied the development of the MRI in low-ionization environment (Jin 1996; Blaes & Balbus 1994; Sano & Miyama 1999; Wardle 1999; Reyes-Ruiz 2001; Fromang, Terquem & Balbus 2002). Although specific details vary considerably, since different factors and models have been considered in these studies, most coincide in finding the existence of three regions of distinct ionization properties in protoplanetary discs. An inner region, extending up to a fraction of an au, where thermal ionization of alkali metals may lead to an ionization degree sufficient for the development of the MRI, we call this the inner active region (IAR). An outer region, starting after a few au, where external agents, namely energetic particles or photons from the central star, or Galactic cosmic rays, can penetrate the disc and lead to ionization degree sufficiently, we call this the outer active region (OAR). Finally, an intermediate region where, at most, the MRI can only be excited in thin layers (active layers) near the disc surfaces, where ionization due to external agents can lead to the emergence of the MRI. Between these active layers lies the so-called dead zone (Gammie 1996) where external ionizing agents cannot penetrate and hence the neutral gas is decoupled from the magnetic field.

On the assumption that no source of angular momentum transport is present in the dead zone, Gammie (1996) proposed a layered accretion scenario in which the resulting disc structure has significant consequences for the process of planet formation. However, as has been recently shown by Fleming & Stone (2003) and Reyes-Ruiz, P´erez-Tijerina & S´anchez-Salcedo (2003), turbulence in the active layers gives rise to significant viscous torques in the presumably dead zone. Consequently, we believe that the structure and the evolution of protoplanetary discs are not as that predicted by evolutionary models of layered discs (Stepinski 1999; Armitage et al. 2001). In this paper, we compute the evolution of protoplanetary discs taking into account these recent results on the distribution of viscous torques across the so-called dead zone. In our opinion, these imply that radial, one-dimensional models, in which the intermediate region is modelled as a low-viscosity region (also called LVR hereafter), may be sufficient to capture the essential features of protoplanetary disc structure and evolution.

The paper is organized as follows. In Section 2, we present the basic equations used to model protoplanetary discs. In Section 3, we present results for different model parameters. In Section 4, we discuss some of the implications of our results, and in Section 5 we summarize our main conclusions.

2 MODEL DESCRIPTION

According to the ideas presented above our model for protoplanetary discs is a one-dimensional, vertically averaged model. The IAR and the OAR are considered to be fully turbulent and characterized by a single $\alpha$ parameter. The intermediate region is also modelled as viscous region but with a reduced viscosity, lower $\alpha$, as indicated by the studies of angular momentum transport across the dead zone (Fleming & Stone 2003; Reyes-Ruiz et al. 2003). This LVR can also be taken to represent the absence of magnetically induced torques and the presence of hydrodynamic agents with a reduced efficiency in transporting angular momentum.

Viscous regions evolve under the action of Maxwell and/or Reynolds stresses generated by turbulence resulting from the MRI and/or hydrodynamical instabilities. In addition, our model allows the possibility for the existence of angular momentum transfer by self-gravity instabilities, and we model this process in terms of an effective viscosity. In the following, we describe the criteria for selecting the sources of viscosity at a given position in the disc and how the evolution is calculated in each of these regimes.

2.1 Turbulent viscosity

Viscous discs evolve under the action of Maxwell and/or Reynolds stresses generated by turbulence resulting from the MRI and/or hydrodynamical instabilities. We assume that MHD turbulence can be sustained at a given radius if the conditions for the linear development of the MRI, across the whole vertical extent of the disc, are met. If this is not the case, we assume the presence of either hydrodynamical turbulence or a net, vertically averaged viscosity resulting from the combined action of MHD turbulence in the active layers and induced velocity fluctuations in the dead zone.

We adopt as a criterion for the development of the MRI in the linear regime that given by Reyes-Ruiz (2001). The criterion includes the effect of ohmic diffusion and its vertical variation argued to be significant in protoplanetary discs. The effects of finite Hall conductivity and other diffusion processes are not considered at this stage.

According to Reyes-Ruiz (2001) quasi-global analysis, the MRI will develop depending on the magnetic diffusivity, its vertical profile and the magnitude of the seed magnetic field for the instability. For a given seed magnetic field and stratification parameter, a critical Reynolds number exists above which the MRI will develop.

The criterion cannot be analytically derived and written in the form of a simple mathematical expression. Instead, we adopt polynomial fits for the logarithm of the critical magnetic Reynolds number at the disc surfaces in terms of the stratification parameter. Once we assume a value for the strength of the seed magnetic field for the MRI, introduced through the parameter $\beta = P_{mag}/P_{gas}$, we can determine whether the disc is unstable in terms of the stratification parameter at a given disc radius.

We use two different fits for $R_{m, cry}$ for different values of the $\beta$ parameter:

$$R_{m, cry} = \begin{cases} R_{m, 0} & \text{if } l \leq l_0, \\ \frac{R_{m, 0}}{(C_1 + C_2 \log[l])^2} & \text{otherwise}, \end{cases}$$

where the constants $R_{m, 0}, C_0, C_1$ and $C_2$ depend on the value of $\beta$ chosen, and $l_0$ is directly related to the cosmic ray shielding length, $l_0 = 2\Sigma_0$, where $\Sigma_0$ is typically taken to be 100 gm cm$^{-2}$ (see e.g. Stepinski 1992). For a discussion of the calculation of the vertical ionization profile in protoplanetary discs and its influence on the linear development of the MRI, the reader is referred to Reyes-Ruiz (2001).

If the criterion for the development of the instability is met at a given radius, we consider that MHD turbulence can develop there and assign a value for the viscosity parameter $\alpha = \alpha_o$. If a given region is found to be stable to the MRI, according to the criterion presented, we assume that a much smaller viscosity, characterized by a parameter $\alpha = \alpha_d$, operates at that radius.

2.2 Self-gravity

In addition to turbulent viscosity, we include the possible effect of self-gravity induced angular momentum transport following Lin & Pringle (1987). To do this, we monitor the evolution of the Toomre parameter, $Q_T = C_s \Omega_1/\pi G \Sigma$, where $C_s$ is the sound speed and $\Omega_1$ is the Keplerian angular velocity at a given radius. In those regions where $Q_T$ drops below a critical value, $Q_{crit}$, we turn on an effective...
viscosity parametrized according to Shakura & Sunyaev with \( \alpha \) given by

\[
\alpha_{\text{grav}} = 0.01 \left( \frac{Q_{\text{irr}}}{Q_{\text{1}}} - 1 \right). 
\]  

(2)

In this study, \( Q_{\text{irr}} \) is taken to be 1.5. While we do not expect this simple recipe to capture all the complicated dynamics ensuing from self-gravity perturbations in the disc, we believe the idea has some merit when studying long-term disc evolution, over many dynamical periods, as is the case in the present analysis. We leave for future work a more detailed study of the effect of self-gravity as a source of angular momentum transport in the disc.

With the prescription of Lin & Pringle (1987) the evolution of the disc can now be followed using equation (4) with viscosity given by

\[
v = (\alpha + \alpha_{\text{grav}}) C_s H, 
\]  

where \( H \) is the disc scaleheight, \( C_s \) is the sound speed and \( \alpha \) is either \( \alpha_{\text{a}} \) or \( \alpha_{\text{d}} \) as described above.

### 2.3 Viscous evolution

The evolution of the disc is considered to proceed in a viscous manner. The surface density, \( \Sigma \), evolves according to the well-known non-linear diffusion equation (Frank, King & Raine 1995):

\[
\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( R^{3/2} \frac{\partial}{\partial R} \left( v \Sigma R^{1/2} \right) \right), 
\]

(4)

where the turbulent viscosity, \( v \), is given by the Shakura and Sunyaev parametrization in terms of the \( \alpha \) parameter. The value of the \( \alpha \) parameter depends on whether we are in the IAR, OAR or the weakly ionized LVR. An important parameter in our calculations is the ratio of the \( \alpha \) in the IAR and the OAR to the value assumed for the LVR. We call this parameter \( \delta = \alpha_{\text{a}}/\alpha_{\text{d}} \).

The temperature in viscous regions is determined from the assumed equilibrium between radiative energy losses through the disc faces, in the diffusion approximation, and viscous energy dissipation.

The Rosseland mean opacity is calculated from the power-law formulae in terms of temperature and density, used in previous studies (Ruden & Lin 1986).

### 2.4 Heating from stellar irradiation

An additional effect introduced into our models is the heating of the disc due to stellar irradiation. This effect has been proven to be important, especially in the disc outer regions, in the works of D’Alessio et al. (1999) and D’Alessio, Calvet & Hartmann (2001) among others. In our model, the effect of stellar heating is introduced in a simple manner by comparing the mid-plane temperature of a viscously heated disc, \( T_{\text{visc}} \), to that resulting solely from stellar irradiation, \( T_{\text{irr}} \). Inwards of the radius \( R_{\text{visc}} \), where \( T_{\text{visc}} = T_{\text{irr}} \), the mid-plane temperature is that determined from viscous heating alone; outwards of the \( R_{\text{visc}} \) the temperature is set to that of a centrally heated disc. The latter is taken as a simplified version of the models of D’Alessio et al. (1999) for which the temperature profile goes as

\[
T_{\text{irr}} \approx 122 \mathrm{K} \left( \frac{T_*}{4000 \mathrm{K}} \right)^{8/7} \left( \frac{R_*}{2 R_\odot} \right)^{1/7} \left( \frac{R}{2 R_\odot} \right)^{-3/7}, 
\]

where \( T_* \) and \( R_* \) reflect the properties of the central young star, \( M_* \) is taken to be 1 M_\odot, \( R \) is given in au and the \(-3/7\) radial dependence is derived assuming that the disc is flared, as indicated by the detailed calculations of D’Alessio et al. (1998) for the disc outer regions. If one does not consider disc flaring the corresponding power-law dependence is \(-1/2\), so that no dramatic differences are expected using either form.

### 2.5 Initial conditions and numerical details

In uniform \( \alpha \) models, the viscous evolution of a disc, being a diffusive process, quickly ‘forgets’ the details of the initial configuration of mass and temperature and settles to a self-similar evolution. However, in our models, as a consequence of the manner in which we calculate the viscous torque to be applied at each radii, the prescription of initial conditions is an important issue.

We construct our initial condition by letting a constant \( \alpha \) disc, starting from an arbitrary, uniform \( \Sigma \) distribution, relax to a self-similar configuration. This is achieved in less than 10^4 yr. As has been noted by Bodenheimer & Laughlin (1995), the structure of an \( \alpha \) disc near such time can resemble the structure of a self-gravitating disc resulting from two-dimensional simulations. The precise value of \( \alpha \) necessary to mimic the behaviour of a self-gravitating disc depends on the details of simulations; a value around 0.01 apparently gives a good result.

Equation (4) is solved on a logarithmically spaced grid using an explicit finite difference scheme second-order accurate in space and first order in time. At the inner boundary, we apply a zero-torque condition by setting the surface density to zero. This assumption, that no coupling between the star and the disc exists, is made solely for simplicity and may be relaxed in future analysis. Our choice of grid coordinates allows us to place the outer boundary far enough so that the disc does not reach it and the boundary condition there is irrelevant. The computations for all results shown were made on a grid having 140 points extending from 0.1 to 10 000 au. Starting from the initial condition at each time-step the distribution of torques and the mid-plane temperature is determined from the criteria presented above and the disc is evolved accordingly using the minimum time-step that will ensure stability at all grid points.

### 3 RESULTS

Even for our relatively hot, initial disc configuration, the weakly ionized region, where the MRI cannot develop across the whole vertical extent, covers from about 1 au to approximately 4 au. We have carried out several calculations of the disc evolution using different values of the parameter \( \delta \). We also explore two different conditions for the critical Reynolds number, as given by equation (1), corresponding to the case of a weak ambient magnetic field and a strong ambient magnetic field. The ambient magnetic field serves as a seed for the development of the MRI. In the presence of ohmic diffusivity, the smaller the seed magnetic field the harder it is for the MRI to develop.

#### 3.1 Effect of \( \delta \) parameter

We have computed the evolution of disc models with several different values of the ratio of \( \alpha \) parameters in the LVR and the active regions. Here, we present results for three cases: \( \delta = 0.3, 0.1 \) and 0.03. In these calculations, we assume that the ambient magnetic field, which serves as seed for the MRI instability, corresponds to \( \beta = 10^{-3} \). Disc evolution is followed up to 10^6.5 yr and we show radial profiles of the disc surface density and the temperature at certain times in the following figures.
Figure 1. Temporal evolution of disc global properties for a model with $\alpha_0 = 0.01$ and $\alpha_0 = 0.003$. The parameter $\beta$ is $10^{-3}$ in this case as well as in that shown in Fig. 3. Top to bottom panels show total disc mass, disc radius and mass-accretion rate on to the star as a function of time, respectively. For comparison, the evolution of a uniform $\alpha = 0.01$ disc is shown by the dotted line in each panel.

For the case of $\delta = 0.3$, Fig. 1 shows the evolution of global disc properties; the disc mass, $M_d$ (in solar masses), the disc radius, $R_d$ (in au) and the mass-accretion rate on to the central star, $\dot{M}$ (in solar masses per year). For the purpose of this presentation, the disc drops below 0.1 gm cm$^{-2}$. By $5 \times 10^5$ yr the mass-accretion rate on to the central star decreases to less than 6 $\times 10^{-9}$ M$_\odot$ yr$^{-1}$. In all cases shown, we start out with a mass distribution consistent with a uniform $\alpha$ having a total disc mass of 0.2 M$_\odot$ extending out to 60 au. For about $5 \times 10^4$ yr, the evolution of the disc is very similar to that of a uniform $\alpha = 0.01$ disc, its mass drops to less than 0.1 M$_\odot$ and it extends to nearly 350 au. By $5 \times 10^5$ yr the mass-accretion rate on to the central star decreases to less than 6 $\times 10^{-9}$ M$_\odot$ yr$^{-1}$. In this model, which corresponds to the case with a weak viscosity contrast between the active regions and the region of low viscosity, about 0.6 Myr into the disc evolution, a qualitatively different evolutionary regime is signalled by the behaviour of the mass-accretion rate.

This is better understood analysing Fig. 2 which shows radial profiles of surface density and mid-plane temperature at $10^4$, $10^5$ and $10^6$ yr for the same case, $\delta = 0.3$. In these profiles, the LVR is identified as a hump in the surface density and the temperature profiles, corresponding to the required increase in surface density consistent with an almost uniform accretion rate through the disc. The IAR and the OAR are the regions inwards and outwards of this surface density hump, respectively. The location of the LVR, extending from the outer radius of the IAR, $R_{\text{IAR}}$, to the inner radius of the OAR, $R_{\text{OAR}}$, varies with time as the disc cools down and becomes more tenuous.

Approximately $6 \times 10^5$ yr into the viscous evolution of the disc, a qualitatively different evolutionary phase begins as indicated by the eruptive behaviour of the mass-accretion rate. As the temperature throughout most of the disc, even the innermost parts, drops below $10^5$ K, there is a rapid decrease of $R_{\text{IAR}}$ reaching the disc inner radii. At such point the IAR disappears. The origin of the subsequent eruptive disc evolution is the following. In the innermost portions of the LVR the accumulation of mass leads to greater viscous heating. The temperature gradually increases up to $10^3$ K at which point some part of the region becomes MRI unstable. As the mass accumulated is rapidly accreted on to the central star, the temperature and the ionization degree drop accordingly rendering the region stable again. This cycle repeats for as long as there is sufficient mass in the disc to ‘fill’ the LVR to the point where the gas reaches temperatures in excess of about $10^3$ K.

The eruptive behaviour found in this example is present in all of our models, albeit at different times, duration and amplitude, depending mainly on the parameter $\delta$. Fig. 3 shows the evolution of a disc with a smaller ratio of viscosity parameters between the dead and the active regions. In this case, $\delta = 0.1$ and all other parameters remain the same as in the previous case. In contrast to the case with $\delta = 0.3$, in this model the eruptive behaviour is present from the beginning of the evolutionary sequence. The lower average mass-accretion rate resulting from the eruptive behaviour is manifest in the slower reduction of the total disc mass and disc spreading.

The smaller viscosity coefficient in the LVR for the case shown in Fig. 3 leads to a greater amplitude for the eruptions recorded in the mass-accretion rate. This follows from the greater accumulation of mass in the LVR until the gas temperature is sufficient for the ionization degree to allow the MRI to develop. This can be seen in Fig. 4 which shows how the surface density evolves for the case in which $\delta = 0.1$.

The evolution of a disc with an even smaller ratio of viscosity parameters between the dead and the active regions is shown in Fig. 5. In this case, $\delta = 0.03$ and all other parameters remain the same.
as in the previous two cases. The trend observed for the previous two cases, from \( \delta = 0.3 \) to 0.1, is confirmed. The average mass-accretion rate is still lower up to a fraction of \( 10^6 \) yr resulting in a greater disc mass at any given time. The amplitude for the eruptions also increases as, again, the smaller viscosity coefficient in the LVR leads to a greater accumulation of mass in the LVR, shown in Fig. 6, until the gas temperature exceeds the critical temperature.

### 3.2 Effect of \( \beta \) parameter

The \( \beta \) parameter measures the strength of the seed magnetic field present in the disc required to destabilize the flow and lead to MHD turbulence via the MRI. The linear stability analysis (see e.g. Reyes-Ruiz 2001) indicates that weaker magnetic fields require a greater ionization degree in order for the flow to become unstable. For a given seed magnetic field, the emergence of the MRI is determined by the magnetic Reynolds number at the disc mid-plane, which in turn depend on the magnetic diffusivity, electrical conductivity and ionization degree. As shown by Reyes-Ruiz (2001), the growth rate for the MRI can also depend on the stratification of the ionization degree, measured by the parameter \( l_o \), which is dictated by the disc model.

Due to the exponential temperature dependence of the thermal ionization, the stability of the IAR is mainly determined by \( R_m \), which goes from subcritical to supercritical quickly as soon as the temperature rises above roughly 1000 K, almost irrespective of the value \( \beta \). However, in the disc outer regions, the development of the instability depends strongly on the magnitude of the seed magnetic field. Hence, the radius where the OAR starts is strongly affected by this parameter. This in turn has some effect on the evolution of the disc.

We consider two extreme cases for the parameter \( \beta \) to illustrate its effects. The higher value \( \beta = 10^{-2} \) represents an upper limit...
for this parameter as it corresponds to a seed magnetic field energy density approximately equal to the kinetic energy density in the turbulent velocity fluctuations due to the MRI. In other words, the seed magnetic field already has a strength comparable to that at which the turbulence due to the MRI will saturate. The case with lower $\beta = 10^{-4}$ roughly corresponds, at 1 au, to a seed magnetic field of 100 $\mu$G, similar to that observed in the densest regions of molecular clouds. We consider this case a lower limit for the parameter $\beta$. In both cases, we have used a ratio of 10 for the $\alpha$ parameters between the active regions and the LVR, that is, $\delta = 0.1$.

Results for the high-$\beta$ case are shown in Figs 7 and 8. On average, the global evolution of our model is very similar to that of a uniform $\alpha$ model. The evolution of disc mass and disc radius is almost the same. However, the mass-accretion rate for our model is qualitatively different than in the uniform $\alpha$ model, with eruptive behaviour around an average value more or less equal to the uniform $\alpha$ model. As in other cases reported above, marked differences between these two models can also be found in the radial profiles of surface density shown in Fig. 8. Once again, the LVR corresponds to a surface density hump extending from a fraction of an au to a few au. As the disc cools down the IAR shrinks, also, as the surface density decreases with time, energetic particles can better penetrate the outer portions of the disc so that the inner boundary of the OAR decreases. This is seen in Fig. 8 as the LVR, traced by the surface density hump, moves inward.

Finally, in Figs 9 and 10 we present the case when the seed magnetic field for the MRI is at its weakest, $\beta = 10^{-4}$. Fig. 9 shows the evolution of the disc global properties, disc mass and mass-accretion rate on to the central star. Comparing Fig. 10 to Fig. 8, corresponding to the radial profiles of $\Sigma$ for the low- and the high-$\beta$ cases, respectively, we find that the main difference is the extent of the LVR in the latter case. This follows from the fact that a lower value of $\beta$ makes it more difficult for the disc to become unstable, an effect mainly felt at the disc outer regions, as explained above. The slight differences in the properties of the IAR between both cases are related to the changing properties at the disc outer regions. In the low-$\beta$ model, at all times the LVR extends from a fraction of an au to more than 10 au.
the only source of viscosity in the disc is MHD turbulence due to accretion takes place only through active layers near the disc sur-

Armitage, Livio & Pringle (2001). These models are qualitatively disc models as those developed by Stepinski (1999) and has also disappeared. Hence, the models of Reyes-Ruiz & Stepinski leading to eruptive behaviour in the models presented here, the OAR as a consequence, by the time the IAR disappears in their models, has been proven incorrect by the work of Reyes-Ruiz et al. (2003) the presence of magnetic fields is determined from the criterion for the growth of an $\alpha - \Omega$ dynamo. Reyes-Ruiz & Stepinski (1995) find that a surface density hump results from the absence of magnetic fields in the region of lowest ionization. However, the models of Reyes-Ruiz & Stepinski (1995) start out from a different initial condition (uniform $\Sigma$ disc) and do not include the effect of stellar irradiation heating. As a consequence, by the time the IAR disappears in their models, leading to eruptive behaviour in the models presented here, the OAR has also disappeared. Hence, the models of Reyes-Ruiz & Stepinski (1995) evolve as uniform $\alpha$ discs.

Eruptive behaviour has been found in the context of layered accretion disc models as those developed by Stepinski (1999) and Armitage, Livio & Pringle (2001). These models are qualitatively different from ours. In their model, at the region of lowest ionization accretion takes place only through active layers near the disc surfaces. This assumption follows from the consideration that (i) the only source of viscosity in the disc is MHD turbulence due to the MRI and (ii) no angular momentum transport occurs through the ‘dead zone’. We consider that at least the second assumption has been proven incorrect by the work of Reyes-Ruiz et al. (2003) and Fleming & Stone (2003).

The precise behaviour of the mass-accretion rate and other disc properties through an outburst is very sensitive to the distribution of mass in the disc as the outburst commences. As can be seen in Fig. 11 showing the behaviour of $M$ across an outburst in the low- $\beta$ model presented in Figs 9 and 10, the accretion rate varies in a complicated manner. The accretion rate may increase or decrease slowly as matter is re-distributed throughout the disc. At certain times, for example, around $t = 0$ in Fig. 11, the accretion rate may increase rapidly in an eruptive manner increasing by a factor of a few in a very short time of the order of a few years in this particular case.

The source of the eruptive behaviour is very similar to the case presented by Armitage et al. (2001), viscous evolution takes disc regions previously stable to the MRI, into the unstable regime due to an increase in disc temperature. This is illustrated in Fig. 12 where the evolution of the mid-plane temperature radial profile just before and after the beginning of an outburst at $t = 0$ of Fig. 11 is shown in detail. Fig. 12 corresponds to the disc model shown in Figs 9 and 10, characterized by $\alpha_d = 0.01, \alpha_d = 0.001$ and parameter $\beta = 10^{-4}$. The start of the outburst coincides with the time at which the temperature reaches 1000 K at about 0.2 au.

4.1 The role of self-gravity

Albeit in a simplified manner, our model takes into account the effect of self-gravity instabilities as a source of angular momentum transport. At each time-step, we compute the Toomre parameter to see if any disc region is gravitationally unstable. In most of our disc models, we find that at very early times, less than $10^5$ yr, self-gravity acts in a region extending from about 8 to 30 au. The precise location of the region unstable to self-gravity (SGUR) varies in time as the disc evolves and depends on the particular disc model, that is, on the choice of $\delta$ and $\beta$ parameters. In all models presented, except the one with $\delta = 0.003$ (shown in Figs 5 and 6), the SGUR disappears entirely, or almost, after less than $10^7$ yr.

As expected, the model with the lowest $\alpha_d$, which has the highest $\Sigma$ and lowest $T$ in the LVR, is where the effect of self-gravity is more pervasive and lasts longer. This can be seen in Fig. 13 showing the evolution of $R_{\text{AR}}, R_{\text{OAR}},$ the inner radius of the SGUR, $R_{\text{SGUR}}$, and the outer radius of the SGUR, $R_{\text{SGUR}}$, for a disc model with $\delta = 0.003$ and $\beta = 10^{-4}$. In this model, the SGUR reaches a more or less constant size, extending approximately from 10 to 20 au, and it remains so for most of the disc lifetime.
the mass-accretion rate on to the central star ensues as the accretion of the disc depends mainly on the value of the α parameter. This effect can be seen in layered protoplanetary disc models where MRI-generated MHD turbulence, self-gravity instabilities and an additional, low-dimensional viscosity parameter operate. The latter operates in the region of lowest ionization in the disc, where the MRI cannot develop across its whole vertical extent. We argue that a lower viscosity leads to a greater surface density in the disc, thus the accretion rate is reduced compared to the accretion rate of protoplanetary discs. Littlefair et al. (2004) interpret this as an indication that these systems have evolved from a state of high mass-accretion rate to one with lower accretion rate on to the central star, over such period. The time-scale for variability suggested by Littlefair et al. (2004) is comparable to that found in our models at certain periods, for example, during the eruption shown in Fig. 12. In future work, we will present an exploration of the relevant parameter space to study the application of the mechanisms presented here as the source of disc variability as that presented by Littlefair et al. (2004) and possibly FU Orionis outbursts.

A variability in the mass-accretion rate for a given disc, as that found in our models, could also be related to the spread of the observed accretion rate for protoplanetary discs of similar mass and age. As reported in Natta, Testi & Randich (2006), the measured accretion rates in a complete sample of T Tauri stars in ρ-Ophiuchi, show a large spread, of two orders of magnitude at least, for stars of similar mass and age. They report that data for pre-main-sequence stars in Taurus show similar behaviour. Such observations are expected in the context of variable disc models as the ones presented here.

4.2 Relation to observations

Some indication that protoplanetary discs present variability on a scale of tens of years can be found in the observations of Littlefair et al. (2004, and references therein). They report significant differences in observations made 20 yr apart of several T Tauri stars, showing strong variability in spectral features traditionally related to the accretion rate of protoplanetary discs. Littlefair et al. (2004) interpret this as an indication that these systems have evolved from a state of high mass-accretion rate to one with lower accretion rate on to the central star, over such period. The time-scale for variability suggested by Littlefair et al. (2004) is comparable to that found in our models at certain periods, for example, during the eruption shown in Fig. 12. In future work, we will present an exploration of the relevant parameter space to study the application of the mechanisms presented here as the source of disc variability as that presented by Littlefair et al. (2004) and possibly FU Orionis outbursts.

A variability in the mass-accretion rate for a given disc, as that found in our models, could also be related to the spread of the observed accretion rate for protoplanetary discs of similar mass and age. As reported in Natta, Testi & Randich (2006), the measured accretion rates in a complete sample of T Tauri stars in ρ-Ophiuchi, show a large spread, of two orders of magnitude at least, for stars of similar mass and age. They report that data for pre-main-sequence stars in Taurus show similar behaviour. Such observations are expected in the context of variable disc models as the ones presented here.

5 CONCLUSIONS

We have computed the evolution of one-dimensional, vertically averaged protoplanetary disc models that take into account the effect of MRI-generated MHD turbulence, self-gravity instabilities and an additional, low-α viscosity of unspecified origin. The latter operates in the region of lowest ionization in the disc, where the MRI cannot develop across its whole vertical extent. We argue that a lower viscosity may result from the action of hydrodynamical turbulence or from the vertically averaged Reynolds stress in an active layer/dead layer scenario.

Within the framework of our model, we find that the evolution of the disc depends mainly on the value of the α parameter in the active regions and in the LVRs. Eruptive behaviour of the mass-accretion rate on to the central star ensues as the α parameter in the LVRs decreases. The smaller the α parameter, the greater the amplitude of the variability in the mass-accretion rate. These dependences are explained considering that a smaller viscosity leads to a greater surface density in the LVR as the disc tends to maintain a uniform mass-accretion rate.

Our results may have important consequences for the process of planet formation in protoplanetary discs. On one side, the rapidly changing physical properties in the disc, on the scale of thousands of years, significantly modify current planet formation scenarios which assume more or less stationary disc configuration up to its dispersal. Additionally, due to the temporary accumulation of gas in the LVR, there is a greater amount of gas (and possibly dust particles also) available in that region, in comparison to that found in uniform α disc models. Finally, the existence of an extended disc region where self-gravity could be an important dynamical agent warrants a more detailed investigation of this process and its relevance for planet formation.

However, before continuing to speculate about the possible consequences of our results there are two issues, at least, that warrant a further investigation. First in importance is the issue of the ionization degree. It has been shown by several authors (Ilgner & Nelson 2006, and references therein) that the ionization degree in protoplanetary discs is strongly dependent on the characteristics of the dust grain population in the disc and on the gas phase and gas-grain reactions assumed to take place. We have taken a simplified approach to this issue. A more detailed treatment will be conducted as some of the uncertainties regarding the detailed evolution of dust and the effect of external sources of ionization are clarified. Another important and related issue concerns the development of the MRI into MHD turbulence, and the transfer of angular momentum, in disc regions where there is a strong stratification of the ionization degree. Further work must be conducted to determine under what conditions, if any, layered accretion can occur.

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Figure 13. Evolution of characteristic radii for a disc model with α = 0.01, αd = 0.0003 and parameter β = 10^{-3}.
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