Supermassive black hole growth and merger rates from cosmological 
N-body simulations

Miroslav Micic,1* Kelly Holley-Bockelmann,2* Steinn Sigurdsson1* and Tom Abel3*

1Department of Astronomy & Astrophysics, Pennsylvania State University, University Park, PA 16802, USA
2IGPG, Pennsylvania State University, University Park, PA 16802, USA
3SLAC, Stanford University, Menlo Park, CA, USA

Accepted 2007 June 28. Received 2007 June 27; in original form 2007 March 20

ABSTRACT

Understanding how seed black holes (BHs) grow into intermediate-mass and supermassive
black holes (IMBHs and SMBHs, respectively) has important implications for the duty cycle of
active galactic nuclei (AGN), galaxy evolution and gravitational wave astronomy. Most studies
of the cosmological growth and merger history of BHs have used semianalytic models and have
concentrated on SMBH growth in luminous galaxies. We have developed a ‘hybrid method’
that combines high-resolution cosmological N-body simulations for the haloes’ merger history,
with semi-analytical recipes for BH pair dynamics and BH gas accretion. We track the assembly
of BHs over a large range of final masses – from seed BHs to SMBHs – over widely varying
dynamical histories. We used the dynamics of dark matter haloes to track the evolution of
seed BHs in three different gas accretion scenarios. We have found that growth of a Sagittarius
A* – size of SMBH reaches its maximum mass $M_{\text{SMBH}} = \sim10^6 M_\odot$ at $z \sim 6$ through early
gaseous accretion episodes, after which it stays at near constant mass. At the same redshift,
the duty cycle of the host AGN ends, hence redshift $z = 6$ marks the transition from an AGN
to a starburst galaxy which eventually becomes the Milky Way. By tracking BH growth as
a function of time and mass, we estimate that the IMBH merger rate reaches a maximum of
$R_{\text{max}} = 55$ yr$^{-1}$ at $z = 11$. From IMBH merger rates we calculate $N_{\text{ULX}} = 7$ per Milky Way
type galaxy per redshift range $2 \lesssim z \lesssim 6$.

Key words: gravitational waves.

1 INTRODUCTION

Supermassive black holes (SMBH) are thought to dwell at the centres
of most galaxies (Kormendy & Richstone 1995), with masses
within $10^6 \lesssim M \lesssim 10^9 M_\odot$. In principle, the abundance of SMBHs
today can be explained if they grow through mergers and early
accretion (Schneider et al. 2002), from a gaseous disc. The most
likely candidates for SMBH seeds are black holes (BHs) that form
as remnants of Population III (Pop III) stars at redshifts $z \gtrsim 12 - 20$
(Heger et al. 2003; Islam, Taylor & Silk 2003; Volonteri, Haardt &
Madau 2003; Wise & Abel 2005). These relic seeds are predicted to
form in the centres of dark matter haloes (DMHs), and have masses
$\lesssim 10^3 M_\odot$ (Abel, Bryan & Norman 2000, 2002). DMHs form in the
early Universe and hierarchically merge into larger bound objects.
As DMHs merge into massive haloes, the seed BHs sink to the centre
due to dynamical friction and eventually coalesce.

Although the seed formation stops at $z \sim 12$ as Pop III supernova rates drop to zero (Wise & Abel 2005), SMBH growth con-
tinues as DMH mergers proceed to low redshifts. From a combi-
nation of gas accretion and binary BH coalescence, seeds can
grow to intermediate-mass black holes (IMBHs, with masses $10^2 \lesssim
m \lesssim 10^3 M_\odot$). With continued mergers and gas accretion, it is
thought that these IMBHs may form the SMBHs we observe today.
The detection of IMBHs is a matter of debate, but possible candi-
dates are ultraluminous X-ray sources in young star-forming regions
(Fabbiano 1989; Roberts & Warwick 2000; Ptak & Colbert 2004;
Fabbiano & White 2006) and nearby extragalactic star clusters.

Although this scenario works in general, real detailed understand-
ing of how seed BHs grow is a remaining challenge. For example,
there are still debates on the seed BH mass, on the mass of haloes
that can host seeds, on the seed-formation redshift, on the type
and efficiency of gas accretion and on the dynamics of the BH mergers,
all of which compound to yield a huge range in BH merger rate es-
timates. Recent BH merger rate calculations span over three orders
of magnitude: Haehnelt (1994) calculates the merger rate to be $R \sim
0.1 - 100$ events yr$^{-1}$; Menou, Haiman & Narayanan (2001), $R \sim
1 - 100$ yr$^{-1}$; Wyithe & Loeb (2003), $R \sim 15 - 350$ yr$^{-1}$; Sesana
et al. (2004), $R \sim 10$ yr$^{-1}$ and Rhook & Wyithe (2005), $R \sim 15$ yr$^{-1}$.

There are two general approaches to the problem: direct cosmo-
ological N-body simulations and analytical techniques based on a
Press–Schechter (PS) formalism (Press & Schechter 1974). Both methods extract merger rates from the DMHs’ merger tree. N-body simulations have the advantage that the evolution of density fluctuations is followed in complete generality, without the need for any of the assumptions involved in creating PS merger trees. Extended PS theory (EPS) combines the PS halo mass function with halo merger rates derived by Lacey & Cole (1993), and it stands as the most widely used method for calculating merger rates. Unfortunately, the mass and spatial resolution involved in creating PS merger trees, there are, in principle, no constraints on the halo mass distribution obtained with it proved to be consistent with observations at low redshifts and mass functions obtained by PS theory. However, very little is known about DMH mass functions at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts (Reed et al. 2007). With N-body simulations, there are, in principle, no constraints on the halo mass, structure or kinematics. Unfortunately, the mass and spatial resolution needed to be extremely high, and even in the best resolved N-body simulations of a large-volume Lambda cold dark matter (ΛCDM) universe (Nagashima et al. 2005), the mass resolution sets the minimum DMH mass to $3 \times 10^8 M_\odot$ for haloes with as few as 10 particles. Hence using direct N-body simulations to track the dynamics of seed BHs as they grow within a large comoving volume is out of reach of our current technology.

In this paper, we have developed a hybrid method to follow the merger history of seed BHs as they grow. We performed a high-resolution cosmological N-body simulation in the unexplored parameter space of a small cosmological volume but with very high mass resolution to achieve well-resolved haloes with a minimum of 32 particles and mass as low as $M_{\text{halo}} = 2.8 \times 10^7 M_\odot$. The goal was to look at a representative ‘Local Group’ structure, comparable to the one which hosts the Milky Way, and to resolve as low as a mass as feasible. We improved the algorithms for identifying dark matter structures and developed set of physically motivated criteria for seeding DMHs. We then constructed the DMH merger tree, which provides a test bed to study the effects of different gas accretion scenarios. In addition to the merger rate, we extracted observables such as the BH mass function over cosmological time. This method is a major improvement in calculations of MBH merger rates and presents the first step towards a full treatment of the BH growth problem. Massive BHs mergers will be one of the prime signals for the future space-based gravitational wave observatories like Laser Interferometer Space Antenna (LISA) and Big Bang Observer (BBO). Possible detection depends on a number of parameters: the mass ratio of merging BHs, the total mass of the BH binary and the redshift. We describe our simulation and DMH seeding criteria in Section 2; obtaining DMH and BH merger trees in Sections 3 and 4 together with description of BH growth models; BH merger rates in Section 5; finally, we discuss our results in Section 6.

## 2 Simulation

### 2.1 Simulation set-up

In our numerical simulations, we use GADGET (Springel, Yoshida & White 2001) to evolve a comoving 10 Mpc$^3$ section of a ΛCDM universe ($\Omega_M = 0.3, \Omega_\Lambda = 0.7, \sigma_8 = 0.9$ and $h = 0.7$) with periodic boundary conditions from $z = 40$ to 0. We refine a sphere of 2 Mpc in the box to simulate at a higher resolution with $4.9 \times 10^6$ high-resolution particles (softening length 2 kpc comoving). The rest of the box has $2.0 \times 10^6$ low-resolution particles (softening length 4 kpc comoving). The mass of each high-resolution particle in this simulation is $8.85 \times 10^8 M_\odot$, and the mass of each low-resolution particle is $5.66 \times 10^7 M_\odot$. A detailed description of this simulation can be found in Micic, Abel & Sigurdsson (2006).

### 2.2 Linking-length problem

Identifying and following a real DMH in a cosmological simulation is technically challenging. If the minimum number of particles in a halo is set too low, the DMH identified at one time-step might later disperse. In the widely used FRIENDS-OF-FRIENDS ALGORITHM (FOF), if the linking length ($b$) is too large, large dark matter structures might bridge through a small number of tidally stripped particles. When this occurs, the fof algorithm cannot properly identify DMHs. A linking length of 0.2 has traditionally been used (Davis et al. 1985) since the DMH mass function obtained with it proved to be consistent with observations at low redshifts and mass functions obtained by PS theory. However, very little is known about DMH mass functions at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS reproduces the DMH mass function at high redshifts since it provides two equally valid merger rates for the same pair of DMHs. It is also unclear whether EPS repro...
following snapshot ‘i+1’, we identify haloes from the previous ‘i’ snapshot. At ‘i+1’ we again seed new haloes that satisfy our criteria. By ‘new’ haloes we consider only those haloes that do not have any particles that were part of any already seeded halo. This is because once a halo is seeded it cannot form a new BH due to feedback from the UV background of Pop III stars (Machacek, Bryan & Abel 2001). Meanwhile, we also trace the parent DMH at ‘i’ for every particle in every DMH at ‘i+1’, thus constructing a complete merger history. The method is repeated through the redshift range 12 \leq z \leq 19. Seeding stops at redshift z = 12 but we continue with creating a merger history until redshift z = 0. In total we have 1447 seed BHs.

4 ASSEMBLING A BLACK HOLE

After constructing the DMH merger tree, we turn our attention to the evolution of the seeds. The N-body simulation tracks all the seed BHs and their host haloes from z = 19 to 0. In order to study BH growth as a function of time, environment and merger type, we constructed three BH merger models which we detail below.

We parametrize the degree of gas accretion involved in growing the BH and the efficiency of binary coalescence. In the fiducial case, we modelled BH mergers in the simplest possible way. We assume that the hardening time-scale (the time for two BHs to form hard binary) is rapid, that the loss cone is full (Sigurdsson 2003; Berczik et al. 2006; Holley-Bockelmann & Sigurdsson 2006), that no BHs are ejected, and that there is no gas accretion involved. This gives us a strawman model with which to compare more realistic BH growth scenarios. We assign seed BH masses of 200 M⊙ to each of our BHs. When two DMH merge, their seeds form a binary and merge at the centre of the remnant with the total combined mass of its progenitors. If more than two DMHs merge, as often happens early on, the merger is sorted by DMH mass, since dynamical friction is stronger on higher masses.

We assume that dynamical friction is very efficient in merging the BHs once the parent haloes merge. Again, this yields an upper limit on the merger rate derived for this halo; we will explore the effects that can suppress this rate in our next paper. However, in a gas-rich, triaxial and/or highly non-equilibrium galaxy, the BH merger timescale may be quite rapid; Kazantzidis et al. (2005) shows that dynamical friction is efficient even in unequal-mass mergers because gaseous dissipation compacts the companion galaxy, enabling it to survive complete tidal disruption to more quickly sink to the host SMBH to the centre of the remnant. This can lead to very high pairing efficiency of SMBHs. Escala et al. (2005) suggests that equal-mass BH binaries will merge within a few \(10^7\) yr. Since DMHs typically merge in \(10^8\) yr, the MBHs will coalesce soon after the DMHs merge in this model. As the haloes evolve to the present day, BHs can only grow in mass through mergers, and in our (small cosmological volume) simulation reached a maximum mass of \(M_{\text{BH}} = 3 \times 10^7\) M⊙. We find that this value is too small to match observations.

For a sustained Eddington accretion, the mass growth rate is

\[ M_{\text{BH}} = \frac{M_{\text{BH}}}{t_{\text{Sal}}} \]

where \(t_{\text{Sal}}\) is the Salpeter time-scale, \(t_{\text{Sal}} \approx 4 \times 10^7\) yr from the recent observations (Hu et al. 2006). In our models, we assume that accretion is triggered by galaxy mergers and lasts \(\sim t_{\text{Sal}}\). During this time, the BHs approximately double their mass. However, we do not assume that every merger supplies enough gas for the BH to double its mass. For example, during minor galaxy mergers (mass ratios much larger than 10:1), the more massive galaxy will shred the satellite of its gas which, we assume, cools and dissipates rapidly (e.g. White 1983; Holley-Bockelmann & Richstone 1999), and the BH will sink towards the central BH without a large reservoir of gas. On the other hand, the merger of comparable (mass ratios less than 10:1) galaxies will generate ample gas accretion on to the central BH, as the satellite triggers a central starburst (Mihos & Hernquist 1994). Here we distinguish two cases, depending on the mass ratio of merging DMHs. The first is a more conservative criterion that allows BHs to accrete gas if the mass ratio of host DMHs is less than 4:1 (4:1 accretion). The second case sets an upper constraint on the mass of the final BH by allowing seeds to accrete gas as long as the merging DMHs have a mass ratio less than 10:1 (10:1 accretion). In the following section, we compare all three growth scenarios: BH growth through mergers only (dry growth); growth through mergers combined with 4:1 gas accretion (4:1 growth); and finally, growth through mergers combined with 10:1 gas accretion (10:1 growth). These are simplifying assumptions made to explore parametrically the range of solutions in our scenario.

5 RESULTS

From z = 19 to 12, we identified all of the DMHs in the manner described in the previous section. With this approach, we obtained the initial positions and formation redshifts for 1477 seed black holes, approximately 100 new MBHs per snapshot. We assigned an initial mass of 200 M⊙ to each MBH and traced their merger history from 0 \leq z \leq 19. Fig. 1 shows that the mass density of SMBH in our models is well below the values obtained from observations at low z, that is, \(\rho_{\text{BH}}(z = 0) \approx 3 - 5 \times 10^5\) M⊙ Mpc\(^{-3}\). Aller & Richstone (2002). Most of the mass is in the \(\sim 10^6\) M⊙ BHs. Our values are smaller since we are exploring low-mass end of SMBH that contributes little to total mass density.

5.1 Growth of Sagittarius A*

Throughout the simulation, one DMH emerges as the largest in mass and dominates the dynamics of its group. Its mass at z = 0 is \(M_{\text{primary}} = 4.5 \times 10^{12}\) M⊙ which roughly corresponds to Andromeda’s DMH, and is somewhat higher than estimates for the Milky Way’s halo (Dehnen, McLaughlin & Sachani 2006). This halo grows through the smooth accretion of dark matter particles as well as DMH mergers. Meanwhile, the central BH grows through gas accretion (if we included it) and mergers. We traced the growth of both the primary halo and its massive BH as a function of

![Figure 1. BH mass density as a function of redshift. The largest BH at redshift z = 0 is in the 10:1 growth model, and has mass 3 \times 10^7 M⊙. Since we are testing low-mass end of SMBH formation, we obtain BH mass densities well below the observed values.](https://academic.oup.com/mnras/article-abstract/380/4/1533/1061348)
redshift (Fig. 2). Fig. 2 compares the growth of both the BH (dry) and its host DMH. Overplotted is a descriptive redshift dependence because it exists only where the curve has a negative slope. Initially, the primary halo grows in mass only through smooth accretion and reaches $M_{\text{primary}} = 10^9 \, M_\odot$ by $z = 15$ (Fig. 2). During this period, the BH remains at its initial mass. At all other redshifts, the growth of primary halo and any other DMH can be described as a cycle of steady accretion of the surrounding dark matter punctuated by rapid growth through mergers with incoming DMHs.

We present the DMH merger rates in Fig. 3. In comparing Figs 2 and 3, note that changes in merger rate catalyse both the BH (dry) and its host DMH. Overplotted is a descriptive redshift dependence because it exists only where the curve has a negative slope. Initially, the primary halo grows in mass only through smooth accretion and reaches $M_{\text{primary}} = 10^9 \, M_\odot$ by $z = 15$ (Fig. 2). During this period, the BH remains at its initial mass. At all other redshifts, the growth of primary halo and any other DMH can be described as a cycle of steady accretion of the surrounding dark matter punctuated by rapid growth through mergers with incoming DMHs.

As described before, we used the Salpeter approximation to model the bound DMH mass decreases. Merger rates decrease at $z < 10$ but experience two more peaks with periods of violent mergers around $z = 6$ and 3 (Fig. 3). Note that these two peaks in merger rates need to be tested for cosmic variance. We intend to do this by running a new simulation in a larger cosmological volume.

At $z = 0$, there is one SMBH with mass $M_{\text{SMBH}} = 2.9 \times 10^9 \, M_\odot$ at the centre of the primary halo (Figs 2 and 4, thick line). This mass is too small to match present-day observations, so we know, as has been stated, that mergers alone cannot create an SMBH. As described before, we used the Salpeter approximation to model different gas accretion scenarios where the accretion efficiency depends on the mass ratio of merging DMHs. In all three scenarios, the BH reaches the IMBH range at $z \sim 12$ (Fig. 4). As the number of DMH mergers increases, the IMBH continues growing – faster, naturally, if gas accretion is more efficient. In a 4:1 growth scenario, the IMBH grows to a $2.3 \times 10^6 \, M_\odot$ SMBH by $z = 5.5$. These redshifts, low mass ratio mergers of DMHs are depleted and SMBH growth through gas accretion stops. Therefore, the duty cycle of the active galactic nucleus (AGN) hosting this SMBH dies at $z = 5.5$, too. In this model, the SMBH mass remains essentially constant for $z < 5.5$, because it only experiences very high mass ratio mergers with $200-1000 \, M_\odot$ BHs. Finally, at $z = 0$ the AGN has evolved into galaxy hosting a $2.3 \times 10^6 \, M_\odot$ SMBH at its centre, strikingly similar to the mass of the Milky Way SMBH (Schoedel, Ott & Genzel 2002; Gehrz, Duchene & Matthews 2003; Ghez, Salim & Hornstein 2005). A highly efficient 10:1 growth scenario yields an upper constraint of $3.4 \times 10^5 \, M_\odot$ – comparable to the SMBH observed in M31 (Bender, Kormendy & Bower 2005).

Fig. 5 shows the SMBH mass as a function of central velocity dispersion and redshift for our three growth scenarios. We obtained the velocity dispersion for dark matter at 10 kpc comoving from the halo centre. Recall that 10 kpc comoving corresponds to 700 pc at $z = 19$ and 14 kpc at $z = 0$. Similar to Fig. 4, every decrease in dark matter velocity dispersion is a consequence of high merger rates at a specific redshift. The maximum value for the velocity dispersion is obtained at $z = 0$, $\sigma = 120 \, \text{km} \, \text{s}^{-1}$. When com-

---

**Figure 2.** Mass of the SMBH at the centre of primary halo as a function of primary halo’s mass and redshift in the dry growth scenario. The growth of primary halo and any other DMH can be described as a cycle of steady accretion of the surrounding dark matter followed by rapid growth through mergers with incoming DMHs. This process is best observed at $z = 11$.

**Figure 3.** Merger rates per unit time per unit redshift observed at $z = 0$ as a function of redshift. Merger rates reach maximum at $z = 11$, decrease at redshifts $z \lesssim 10$ but experience two more peaks with periods of violent mergers around redshifts $z = 6$ and 3.

**Figure 4.** Growth of SMBH in three different accretion scenarios presented as mass of the SMBH as a function of redshift. BH mergers are dominant way of growing IMBH at $z \gtrsim 12$. Gas accretion is important at $6 \lesssim z \lesssim 12$. At redshift $z = 6$, SMBH reaches the maximum mass marking the transition of its host AGN into the starburst galaxy.
SMBH growth and merger rates

5.2 Black hole merger rates and ULXs

Although there are no studies of AGN duty cycle for $z > 6$ and $M_{\text{SMBH}} \lesssim 10^6 M_\odot$ (Wang, Chen & Zhang 2006), it is probable that the AGN-fuelling mechanism – gas accretion – is similar to that of higher mass galaxies. The SMBH in 4:1 and 10:1 growth scenarios grows to its final mass of $M_{\text{SMBH}} \sim 10^7 M_\odot$ to few $\times 10^7 M_\odot$ in the redshift range $5.5 \lesssim z \lesssim 6$. As major mergers dictate the growth at $z > 6$, the SMBH evolves as a low-luminosity AGN with the duty cycle governed by on and off switching of accretion on to the BH. The AGN duty cycle drops to zero at $z \sim 6$ as the host galaxy becomes too large for major mergers to occur and both the SMBH and its host galaxy enter a quiet phase of their growth through mergers with much smaller galaxies.

Interestingly, all incoming small galaxies at $z \sim 6$ in our model carry an IMBH $\sim 1000 M_\odot$. This sets the stage for a phase where each new galaxy merger can be characterized as a dwarf starburst galaxy with a central IMBH accreting gas while sinking towards the SMBH at the centre. In other words, in these later phases the lower mass incoming BH may accrete gas, but we assume no significant gas accretion on to the central SMBH because of the high mass ratio of the BHs. With the exception of the largest BH in our simulation, all other BHs fall into the IMBH range: $200 \lesssim M_{\text{IMBH}} \lesssim 10^7 M_\odot$. These BHs are of the special interest as candidates for ULX sources (Mii & Totani 2005). ULXs are interpreted as massive BHs accreting gas in starburst galaxies, although this is still a matter of debate (King et al. 2001).

Fig. 6 shows the merger rates for different mass ratio ranges and total binary masses. When deconstructed into ranges of mass ratios and log total binary mass [defined as $p = \log (m_1 + m_2)$], these merger rates can be used to predict the number of ULX sources in starburst galaxies throughout the galaxy evolution, assuming that

![Figure 5. Growth of SMBH in three different accretion scenarios presented as mass of the SMBH as a function of dark matter central velocity dispersion at $r = 10$ kpc ($\sigma$), and as a function of redshift. Redshift dependence is descriptive hence filled circles correspond to data points in terms of redshift. $\sigma = 120 \text{ km s}^{-1}$ at $z = 0$ and it fits the $M - \sigma$ relation for 10:1 growth accretion.](https://academic.oup.com/mnras/article-abstract/380/4/1533/1061348)

![Figure 6. IMBH merger rates as observed at $z = 0$ as a function of redshift and presented for different binary mass ratios and total binary mass ranges, $p = \log (m_1 + m_2)$. $1 \lesssim m_1 / m_2 < 10$ in (a), (b), (c); $10 \lesssim m_1 / m_2 < 100$ in (d), (e), (f); $100 \lesssim m_1 / m_2 < 6000$ in (g), (h), (i); dry growth in (a), (d), (g); 4:1 growth in (b), (e), (h); 10:1 growth in (c), (f), (i). Most of the mergers are low mass ratio mergers at $z > 10$. However, high mass ratio mergers at $z < 10$ (g, h) are in the LISA range and have large merger rates $R \sim 15 \text{ yr}^{-1}$ for a wide range of redshifts $2 < z < 10$.](https://academic.oup.com/mnras/article-abstract/380/4/1533/1061348)
merging galaxies are reasonably gas-rich. For the mass ratio $1 \lesssim m_1/m_2 < 10$, we see only minor differences between the three growth scenarios for any range of binary mass (Figs 6a–c). For $z \gtrsim 10$ BHs grow mostly through mergers. For $z \sim 8$, mass ratio $\sim 10$ and binary mass $10^3 < m_{\text{BH}} < 10^4 M_\odot$ ($3 \lesssim p < 4$) the merger rate is $R \sim 10$ yr$^{-1}$ in all three growth scenario – and it reaches a maximum $R_{\text{max}} \sim 30$ yr$^{-1}$ for $p < 3$. At lower redshifts the merger rates decrease rapidly (Figs 6a–c), and gas accretion becomes important (Fig. 4). Nevertheless, there are no changes in Figs 6(a)–(c) for $z < 10$ since the mass ratio of merging BHs becomes very large as the higher mass BHs gain mass preferentially (Fig. 7). In our scenario, only the BHs at the centres of merging DMHs grow through gas accretion; a large number of DMHs remain isolated for most of the simulation and merge late in high mass ratio encounters (middle and bottom panels of Fig. 6). The difference between the top and bottom panels in Fig. 6 shows that most of the BH mergers at high redshifts are with $1 \lesssim m_1/m_2 < 10$ mass ratio.

Since BHs grow faster with more efficient gas accretion, the merger rates for mass ratio $10 \lesssim m_1/m_2 < 100$ and total binary mass $10^4 < m_{\text{BH}} < 10^5 M_\odot$, shift towards higher redshifts in our accretion models (Figs 6d–f). In this mass ratio range, $3 \lesssim p < 4$ mergers peak at $8 \gtrsim z \gtrsim 10$, with a merger rate of $\sim 10$ depending on the growth scenario. Very high mass ratio mergers $10^2 \lesssim m_1/m_2 < 10^3$ are of special interest (Fig. 6. bottom panels). At $z < 6$, high mass ratio DMH mergers correspond to a dwarf galaxy or globular cluster being consumed by massive galaxy. The IMBH carried by smaller counterpart will eventually coalesce with SMBH at the centre by first forming a binary with total mass $p > 5$ and high mass ratio $10^2 \lesssim m_1/m_2 \lesssim 10^4$. This is an ideal LISA source, as will be explored in more detail in our next paper.

As a result, the merger rates in Figs 6(d)–(f) and 8 for the above total binary mass and mass ratio ranges directly correspond to the number of IMBHs per galaxy per redshift. Since the accretion time-scale is approximately half of the merger time-scale, the ULX number density is $N_{\text{ULX}} = R/2$. In the case of dry growth (Fig. 6g), a BH binary with mass $10^4 < m_{\text{BH}} < 10^5 M_\odot$ has an approximately constant and high merger rate of $R \sim 15$ yr$^{-1}$ for $2 < z < 6$. This corresponds to $\sim 7$ ULX sources per starburst galaxy at $2 < z < 6$. Similarly the $10^3 < m_{\text{BH}} < 10^4 M_\odot$ binary in the 4:1 growth scenario has $R \lesssim 10$ yr$^{-1}$ (Fig. 6h) and $N_{\text{ULX}} = 5$. More importantly, Fig. 6(h) shows $N_{\text{ULX}} > 2$ at $z < 10$. For even higher mass ratios $10^2 \lesssim m_1/m_2 < 10^3$, the merger rate is $R \lesssim 12$ yr$^{-1}$ for $z \lesssim 3$.

![Figure 7](https://example.com/figure7.png) Mass ratio of merging BHs averaged over all mergers at a specific redshift for all three growth models. BHs grow mostly through mergers at $z > 10$. Differences between three growing models become apparent at $z < 10$ where the increase in mass ratio of merging BHs is due to gas accretion.

![Figure 8](https://example.com/figure8.png) Merger rates observed at $z = 0$ as a function of redshift, presented is the case of 10:1 growth scenario and merger rates for large binary mass and extreme binary mass ratios, $p = \log (m_1 + m_2)$. Similar to Figs 5(g) and (h), mergers of SMBH with IMBH at $z > 2$ will be observed by LISA, $R \sim 10$ yr$^{-1}$.

Fig. 8 shows merger rates for the 10:1 growth scenario and high mass ratio $10^4 \lesssim m_1/m_2 < 10^6$ for $10^6 < m_{\text{BH}} < 10^7 M_\odot$ mergers. ULX sources in Figs 8 and 6(g), (h) have constant number density of $\sim 10$ per starburst galaxy at $z > 2$. The ULX number density drops rapidly at $z = 2$. Note that these ULX sources are not the same as the ULXB sources seen in the local Universe with much shorter accretion lifetimes.

5.3 Comparison to results from PS theory

We compare our results with those obtained by various BH merger models from EPS theory. Fig. 9 shows the merger rates for four PS models described in Sesana, Volonteri & Haardt (2007). In the Volontieri et al. (2003, hereafter VHM) model, massive DMHs ($M_{\text{DMH}} = 10^{11} – 10^{13} M_\odot$) are seeded with $m_{\text{BH}} \sim 200 M_\odot$ BHs at $z = 20$; in the Koushiappas, Bullock & Dekel (2004, hereafter KBD) model, low-mass haloes ($M_{\text{DMH}} = 10^9 – 10^10 M_\odot$) are seeded with $m_{\text{BH}} \sim 5 \times 10^5 M_\odot$ at $15 \lesssim z \lesssim 20$; and the Begelman, Volonteri & Rees (2006, hereafter BVR) models explore different redshift ranges for seeding BHs in haloes: $m_{\text{BH}} = 10^8 – 10^9 M_\odot$ at $18 \lesssim z \lesssim 20$ in the BVRhf model and $15 \lesssim z \lesssim 20$ in the BVRif model. Fig. 9(a) superimposes their merger rates with our results. Considering that these two methods have different approaches in choosing seeding redshift range, mass of the seeds, mass of DMHs and different constraints on number of mergers per halo, the results match remarkably. At $z \lesssim 10$ our merger rates match the KBD model. However, we have an overall higher merger rate, with multiple peaks at $z = 6$ and 3 from episodes of violent DMH dynamics (if proven not to be the consequence of cosmic variance) that is not depicted by EPS theory (Fig. 9a). In numerical simulations, there is no limit on number of progenitors for each halo, while in a standard EPS treatment every halo is the end-product of a merger of exactly two haloes. Therefore, an N-body simulation may naturally obtain a larger merger rate, BH growth rate and larger mass ratio for typical mergers than an EPS study. On the other hand, our BHs do not experience dynamical friction properly, so this artificially increases our merger rate with respect to EPS.

In order to achieve $10^4 \lesssim m_{\text{BH}} \lesssim 10^6 M_\odot$ (Fig. 9c) with EPS models, one must start at higher redshifts and assume higher initial BH masses. The KBD model starts with high-mass seeds, and as a result the KBD merger rates peak for $10^4 \lesssim m_{\text{BH}} \lesssim 10^5 M_\odot$, and...
are larger than the $N$-body rates, both in this mass range and at $z > 5$. We ‘spend’ BH seeds naturally during mergers so in this mass range our peak is $n_{\text{BH}} \lesssim 10^3 \, M_\odot$ at $z > 5$.

6 DISCUSSION

We used a high-resolution cosmological $N$-body simulation to study the formation and growth of seed BHs into SMBHs and derived BH merger rates. We used physically motivated formalisms for seeding DMHs with BHs. Better understanding of the initial mass function for Pop III BHs will improve the accuracy of our results in the future. Of course, BH seeds do not necessarily have to be Pop III BHs. We can use our approach to test different seeding scenarios. For example, it can be used to set constraints for primordial BH formation (Mack, Ostriker & Ricotti 2006).

Depending on the assumed growth scenario, we showed that gas accretion (90 per cent of SMBH mass growth) combined with hierarchical mergers of massive BHs (10 per cent of SMBH mass growth) leads to the formation of a Sagittarius A* type BH in the centre of a Milky Way-sized DMH. In the case of very efficient gas accretion, an M31-sized SMBH can form. In both cases, SMBH growth is consistent with Soltan’s argument (Soltan 1982) that SMBH grow mostly through gas accretion. In our models SMBH reaches its final mass at $z \sim 6$. We showed that $z = 6$ may be a critical redshift for the transition from the AGN duty cycle dominated by high mass ratio DMH mergers to a starburst galaxy phase where low mass ratio DMH mergers supply a galaxy with a constant population of ULXs up to $z = 2$. We argue that the $z = 6$ turning point is consistent with the maximum of AGN luminosity function at $z = 2$ since the SMBHs in our simulation have masses and luminosities below the currently observable range. Also note that potential ULX sources in our simulation are at $z > 2$. They may correspond to nuclear clusters at the centres of dwarf galaxies, and therefore likely sites of IMBHs formation. This population is different from the low-redshift ULX population which may form from globular cluster dynamics and are likely undergoing short lived binary accretion. Our results are consistent with Lehmer et al. (2006) who predicts $\sim 5$–$10$ ULX sources per galaxy at $z \sim 3$. Future very large aperture X-ray satellites, such as Generation-X, may be capable of resolving these sources.

The final stages of the BH merger are followed by an emission of gravitational radiation, low in frequency but relatively high in amplitude. In one of our follow-up papers, for each binary BH inspiral and merger, the expected gravitational wave signal for the LISA will be determined, and the LISA event rate as a function of time calculated. In particular, we will study whether LISA observations will be able to distinguish between different assembly scenarios. One interesting source we will be able to constrain will be IMBH/SMBH mergers. We will calculate LISA detectability of IMBH mergers for different mass ranges and binary mass ratios. At this point, rough estimates can be made from Fig. 5, considering the known ranges for LISA sensitivity. In case of dry growth (Fig. 5g), a BH binary with mass $10^5 < m_{\text{BH}} < 10^6 \, M_\odot$ is in the LISA range with a very high merger rate of $\sim 15 \, \text{yr}^{-1}$. Interestingly, just as SMBHs dim electromagnetically at $z = 6$, they turn on in the gravitational wavebands.

There are a number of processes that might suppress massive BH merger rates. We assumed that every first star will produce BH as opposed to neutron star or pair detonation with no remnant. We also assume that BHs merge efficiently and that recoil ejection which is a
function of spin, orientation, eccentricity and mass ratio of merging BHs is negligible. All of these processes will be addressed in the follow-up paper.

Finally, since our $O(10^6) M_\odot$ BH was in place so early, it might be tempting to interpret our results as an indictment against downsizing, the recent observation that the number density of low-mass AGN peaks at $z < 1$ (Cowie et al. 2003; Heckman et al. 2004; Merloni 2004). However, we caution that in order to achieve such high mass resolution, we simulated a small volume, and are therefore plagued by small number statistics. In fact, if we were to scale our simulation up in mass, so that the most massive SMBHs were $O(10^{10}) M_\odot$, we find some indication that the more isolated DMHs will host low-mass SMBHs that gain most of their mass at late redshifts compared to the most massive SMBH. We are exploring the growth of BHs as a function of environment and smooth accretion efficiency in a follow-up paper.

This simulation has not been tested for cosmic variance which can change the merger rate’s profiles dramatically. In order to do this, we are planning a new simulation in a larger cosmological volume. In this manner, we will also test if our results are dependent on the size of simulated cosmological volume.

ACKNOWLEDGMENTS

We would like to thank NASA’s Columbia High End Computing program for a generous time allocation, and the Centre for Gravitational Wave Physics at Pennsylvania State University for sponsoring this research. This research was supported by a grant from the NSF, PHY 02-03046 and from NASA.

REFERENCES

Alder M. C., Richstone D., 2002, AJ, 124, 3035
Schodel R., Ott T., Genzel R., 2002, Nat, 419, 694
Sigurdsson S., 2003, Class. Quantum Gravity, 20S, 45
Springel V., 2000, MPA

This paper has been typeset from a TeX/LaTeX file prepared by the author.