Lensing of 21-cm absorption haloes of \( z \sim 2–30 \) first galaxies

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ABSTRACT

Extended 21-cm absorption regions (dubbed ‘21-cm absorption haloes’) around first galaxies at \( z \sim 30 \) are likely the first distinctive structures accessible to radio observations. Though the radio array capable of detecting and resolving them must have \( \sim 200 \text{ km}^2 \) total collecting area, given the great impact of such detections to the understanding of the reionization process and cosmology, such radio survey would be extremely profitable. As an example, we point out a potentially useful byproduct of such survey. The resolved 21-cm absorption ‘haloes’, likely close to spherical, can serve as (almost) ideal sources for measuring the cosmic shear and mapping the matter distribution to \( z \sim 30 \). We investigate the expected lensing signal and consider a variety of noise contributions on the shear measurement. We find that signal-to-noise ratio \( (\text{S/N}) \sim 1 \) can be achieved for individual ‘haloes’. Given millions of 21-cm absorption ‘haloes’ across the sky, the total S/N will be comparable to traditional shear measurement of \( \sim 10^9 \) galaxies at \( z \sim 1 \).

Key words: gravitational lensing – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

Gravitational lensing (see, e.g. reviews of Bartelmann & Schneider 2001; Refregier 2003) directly probes the matter distribution of the universe and is becoming one of the most important probes of cosmology. Gravitational lensing distorts galaxy shape (cosmic shear), changes galaxy number density (cosmic magnification) and induces mode-coupling in cosmic backgrounds. These observable lensing effects enable several powerful methods to extract cosmological information. Cosmic microwave background (CMB) lensing (Seljak & Zaldarriaga 1999; Zaldarriaga & Seljak 1999; Hu 2001; Hu & Okamoto 2002), 21-cm background lensing (Cooray 2004; Pen 2004; Mandel & Zaldarriaga 2006; Zahn & Zaldarriaga 2006) and cosmic magnification measurements of 21-cm emitting galaxies (Zhang & Pen 2005, 2006) are expected to achieve a high signal-to-noise ratio \( (\text{S/N}) \) in the near future. On the other hand, cosmic shear measurements have achieved a high S/N (e.g., Van Waerbeke, Mellier & Hoekstra 2005; Hoekstra et al. 2006; Jarvis et al. 2006) and will be improved significantly by several ongoing or upcoming large lensing surveys.

Traditional cosmic shear measurements are fundamentally limited by intrinsic ellipticities of galaxies. Intrinsic ellipticities have a dispersion \( \sim 30 \) per cent, much larger than the typical \( 1 \) per cent lensing signal for source galaxies at \( z = 1 \). Even in the best case that ellipticities of galaxies do not correlate, one still needs to average over several hundred galaxies to achieve an S/N of 1. Furthermore, source galaxies at \( z \gtrsim 3 \) are difficult to detect optically, so it is hard to map the matter distribution at redshifts beyond 2 in optical lensing surveys. These two intrinsic problems can be overcome by the 21-cm absorption regions around first galaxies (Cen 2006). These regions are of arcminute size, or \( \sim \text{Mpc} \) in comoving scale. Since their sizes are much larger than the non-linear scale at corresponding redshifts, they are believed to be very close to spherical (Cen 2006). In addition, since they lie at redshifts \( z \sim 25 \), the lensing signal is strong. These two intrinsic advantages make these regions nearly ideal targets for shear measurement. Since these regions have distinctive structures, they are dubbed ‘21-cm absorption haloes’ or ‘21-cm haloes’. We caution the readers that these 21-cm ‘haloes’ are not virialized dark matter haloes. In fact, the density fluctuations in these ‘haloes’ are well in the linear regime. In this paper, we study the requirement and application of the shear measurement of these 21-cm ‘haloes’.

Observations of 21-cm absorption ‘haloes’ are challenging and are way beyond the capability of the planned square kilometer array (SKA1). However, a mission capable of resolving these ‘haloes’ will


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be extremely profitable. The 21-cm brightness temperature can be simultaneously measured to high accuracy over a large fraction of the sky and a wide redshift range, without extra cost. This will allow the measurement of related statistics, such as the brightness temperature power spectrum, bispectrum, etc., to unprecedented accuracy and significantly improve our understanding on reionization process and cosmology at high redshifts. The detection of 21-cm absorption ‘haloes’ will have potentially great impact on cosmology too (Cen 2006). Furthermore, the same mission can detect even smaller 21-cm absorption regions around first stars, the so called Lyman α spheres (because it is mainly the Lyman α scattering that couples the 21-cm spin temperature to the kinetic temperature, leading to the 21-cm absorption against the CMB, Chen & Miralda-Escudé (2006), through strong lensing of galaxy clusters (Li, Zhang & Chen 2007). Given these potentially powerful applications, such mission deserve detailed investigation and observation efforts. In this paper, we discuss a surprisingly valuable byproduct of such mission: mapping the matter distribution to $z \sim 25$ through weak lensing of 21-cm absorption ‘haloes’.

As we show in this paper, the ellipticity induced by various contaminations is very likely to be controlled to below 10 per cent for 21-cm absorption regions considered here and we expect that shear measured from each such region would achieve an S/N $\sim 1$. At $z \sim 25$ with $\Delta z = 2$, for a lower mass cut $\sim 7 \times 10^7 M_\odot$, the total number of these 21-cm absorption ‘haloes’ is $\sim 10^6$ (Fig. 1). The total S/N will be equal to that with $\sim 10^5$ $z \sim 1$ source galaxies and thus comparable to what would be provided by planned ambitious cosmic shear surveys such as the Large Synoptic Survey Telescope. Moreover, the lensing applications of 21-cm absorption ‘haloes’ will fill the gap of source redshifts between traditional cosmic shear measurements (source redshifts $z_s \lesssim 3$), cosmic magnification of 21-cm emitting galaxies ($z_s \sim 1–6$) and CMB lensing ($z_s \simeq 1100$). They provide independent checks for 21-cm background lensing ($z_s \sim 10–30$).

Furthermore, lensing measurement to such high redshifts will provide a direct way of CMB delensing and improve the measurement of primordial inflationary gravitational waves (Sigurdson & Cooray 2005). Lensing reconstruction from 21-cm absorption ‘haloes’ will significantly improve our understanding of matter distribution to $z \sim 25$ and the nature of dark matter, dark energy, gravity and inflation.

This paper is structured as follows. After a brief introduction of the 21-cm absorption ‘haloes’ of first galaxies in Section 2, we discuss their detection prospect in Section 3. Then, in Section 4 we define the mean cosmic shear to be measured for these ‘haloes’. It would be ideal for cosmic shear measurement if the 21-cm ‘haloes’ are intrinsically spherical. Therefore, in Section 5, by investigating several sources that may induce ellipticity of these ‘haloes’, we estimate the expected ellipticity and show that these regions are quite close to spherical. Given the expected noise, in Section 6 we show the range of lensing power spectrum that can be precisely measured. Finally, we summarize and discuss the main results. Throughout this paper, we assume a spatially flat $\Lambda$ cold dark matter ($\Lambda$CDM) cosmology with matter density parameter $\Omega_m = 0.26$, baryon density parameter $\Omega_b = 0.044$, primordial power index $n_s = 0.95$, Hubble constant $H_0 = 100 h = 72$ km s$^{-1}$ Mpc$^{-1}$, and a fluctuation amplitude $\sigma_8 = 0.77$ on 8 h$^{-1}$ Mpc scales, which is consistent with the Wilkinson Microwave Anisotropy Probe (WMAP) three-year results (Spergel et al. 2006). For some plots, we also show cases with $\sigma_8 = 0.9$.

### 2 21-CM ABSORPTION ‘HALOES’ OF FIRST GALAXIES

Since the standard CDM cosmological model is by now accurately determined (Spergel et al. 2006), one could make testable predictions for the first generation of galaxies, expected to form within haloes of mass $M = 10^5–10^9 M_\odot$ at $z = 20–40$ (Fig. 1). The first galaxies are expected to be small and faint in terms of stellar optical light. The very first galaxies may have masses of $\sim 10^5 M_\odot$, limited by Jeans mass and molecular hydrogen cooling. However, star formation in minihaloes with molecular cooling might be quenched, when hydrogen molecules are destroyed by Lyman–Werner photons. In any case, because of the expected low star formation efficiency in the minihaloes, their 21-cm signals will be too weak to be relevant even if they form stars. However, as shown in Cen (2006), with the coupling of the 21-cm spin temperature to Lyman α scattering, some of the large first galaxies with total mass $M \gtrsim 10^{12} M_\odot$ may each display an extended hydrogen 21-cm absorption ‘halo’ against the CMB with a brightness temperature decrement of $\delta T = (100–150)$ mK at a radius of $0.3–3.0$ Mpc.2

![Figure 1](https://academic.oup.com/mnras/article-abstract/382/3/1087/1008629/1088-P.Zhang-Z.Zheng-and-R.Cen)
The detection of these radio ‘haloes’ around first galaxies will be extremely profitable, although admittedly difficult. If and when a 21-cm radio survey of first galaxies is carried out, some fundamental applications for cosmology and galaxy formation may be launched, as discussed in Cen (2006). Here, we recapitulate them. First, it may yield direct information on star formation physics in first galaxies. Secondly, it could provide a unique and sensitive probe of small-scale power in the standard cosmological model, hence physics of dark matter and inflation, by being able to, for example, constrain the primordial power index $n_s$ to an accuracy of $\Delta n_s = 0.01$ at a high confidence level. Constraints on the nature of dark matter particles, that is, mass or temperature, or running of index could be still tighter. Thirdly, clustering of galaxies that may be computed with such a survey will provide an independent set of characterizations of potentially interesting features on large scales in the power spectrum including the baryonic oscillations, which may be compared to local measurements (Eisenstein et al. 2005) to shed light on gravitational growth and other involved processes from $z = 30$ to 0. Finally, the 21-cm absorption haloes are expected to be highly spherical and trace the Hubble flow faithfully, and thus are ideal systems for an application of the Alcock–Paczynski test (Alcock & Paczyński 1979). Exceedingly accurate determinations of key cosmological parameters, in particular, the equation of state of the dark energy, may be finally realized. As an example, it does not seem excessively difficult to determine the dark energy equation of state $w$ to an accuracy of $\Delta w \sim 0.01$, if $\Omega_m$ has been determined to a high accuracy by different means. If achieved, it may have profound ramifications pertaining dark energy and fundamental particle physics (e.g. Upadhye, Ishak & Steinhardt 2005). This last property may also make them ideal sources for weak gravitational lensing measurements, as investigated here.

### 3 DETECTABILITY

The resolution and rms noise of a radio survey depend on the detailed array configuration. Without loss of generality, we focus on a configuration of $N \times N$ arrays homogeneously distributed over a square with side-length $L$. The resolution is $\theta_b = \sqrt{\pi \alpha^2/4} / L$, where $\alpha \simeq 1.2$ for this specific configuration and $\lambda = 0.21(1 + z)$ m is the redshifted wavelength of the 21-cm line. Foreground contaminations at the low frequencies of concern here are overwhelming. The dominant one is galactic synchrotron emission, which scales as $\nu^\beta$ with $\beta \sim -2.55$. For concreteness, we adopt the brightness temperature at $v \sim 54$ MHz (redshifted frequency of the 21-cm lines at $z = 25$) as $T_{\text{syn}} \simeq 3000$ K. This number is consistent with that of Keshet, Waxman & Loeb (2004) and Chen & Miralda-Escudé (2006). However, brightness temperature estimation at these low frequencies is quite uncertain (e.g. $T_{\text{syn}}^\text{bol}$ adopted by Bowman, Morales & Hewitt 2006 is a factor of 2 higher, if scaled to $z = 25$). A factor of 2 increase in $T_{\text{syn}}^\text{bol}$ would require a factor of 4 increase in integration time or a factor of 2 increase in the total collecting area, in order to achieve the same S/N.

Since the foreground is highly smooth in frequency space and the 21-cm signal has line features, showing as sharp fluctuations in the spectrum, the mean foreground contamination can be efficiently removed pixel by pixel (Wang et al. 2006). The residual noise per resolution pixel (with area $\delta^2$) caused by photon number fluctuations in foreground and instrumental noise can be worked out to be

\[
\sigma_T \simeq \frac{T_{\text{sys}}}{\sqrt{\Delta v t}} \frac{4\sqrt{2}}{\pi \alpha^2 f_{\text{cover}}} \approx 10 \text{ mK} \frac{T_{\text{sys}}}{3000 \text{ K}} \left( \frac{\Delta v}{300 \text{ kHz}} \right)^{-1/2} \left( \frac{f_{\text{cover}}}{14 \text{ per cent}} \right)^{-1}.
\]

Here, $f_{\text{cover}} = A_{\text{total}}/L^2 = N^2 A_{\text{dish}}/L^2$ and $A_{\text{dish}}$ is the collecting area of each dish. The bandwidth $\Delta v$ is related to the comoving separation $r$ by $\Delta v \simeq 46$ kHz$(26/(1 + z))^{1/2} (r/h^{-1} \text{ Mpc})$. The typical bandwidth $\Delta v = 300$ kHz corresponds to $\sim 6h^{-1}$ Mpc (comoving) at $z \sim 25$, which is about the diameter of the 21-cm absorption ‘haloes’ (see Fig. 2). The integration time $t$ per pixel is also the integration time per field of view (FOV). The system temperature $T_{\text{sys}} \simeq T_{\text{syn}}^\text{bol}$ in our case. We note that the noise estimate is much larger than that from the Poisson noise of photons. The reason is simple – in the Rayleigh–Jeans part of the emission spectrum, the mean occupation number $\bar{n}$ of photons per quantum state is much greater than unity and the variance, which is $\bar{n}(\bar{n} + 1)$, is super-Poisson.

With assumptions on the star formation properties in the central sources, the brightness temperature profile of the 21-cm absorption ‘haloes’ can be computed (see Cen 2006, for details). At $z = 25$, 21-cm absorbing regions generated by first galaxies in $10^8 M_\odot$ haloes have angular size of several arcminutes with signal $\lesssim 50$ mK (top panels, Fig. 2). To reliably measure the shape of these ‘haloes’, a resolution of $\theta_b \sim 0.3$ arcmin and a system temperature noise $\sigma_T \sim 10$ mK are required. For $\Delta v = 300$ kHz and

\[
\sigma_T \simeq \frac{T_{\text{sys}}}{\sqrt{\Delta v t}} \frac{4\sqrt{2}}{\pi \alpha^2 f_{\text{cover}}} \approx 10 \text{ mK} \frac{T_{\text{sys}}}{3000 \text{ K}} \left( \frac{\Delta v}{300 \text{ kHz}} \right)^{-1/2} \left( \frac{f_{\text{cover}}}{14 \text{ per cent}} \right)^{-1}.
\]

**Figure 2.** Top left-hand panel: the brightness temperature of a 21-cm absorption ‘halo’ as a function of radius (in comoving unit), around a first galaxy in a $10^8 M_\odot$ halo at $z = 30, 25$ and 20, respectively. The star formation efficiency $\epsilon_s = 0.2$ is adopted. Top right-hand panel: the brightness temperature averaged over $\pm 3h^{-1}$ Mpc (comoving) along the line of sight. At these redshifts, 1 arcmin corresponds to $2.4h^{-1}$ Mpc (comoving). Bottom left-hand panel: ellipticities caused by random system noise (with an rms $\sigma_e$ per resolution pixel). See the text for details. Bottom right-hand panel: the lensing signal, $\kappa$ or $\gamma$, smoothed over an area $\delta^2$. The arrow denotes the size of the 21-cm absorption regions used for shear measurement. The dashed line is roughly the expected noise level.
where $q_T(T_{21})$ is a weighting function. We choose $q_T(T_{21}) = T_{21}$ when $T_{21} < T_0$ and $q_T = 0$ otherwise, and $T_0$ is a threshold of brightness temperature that defines the boundary of each region used for shear measurement. The position $\theta$ is measured with respect to the centre with $i, j = 1, 2$, the two orthogonal directions. For a round object, $\epsilon$ is induced by the cosmic shear and we have

$$\epsilon = \frac{2\gamma}{1 - \kappa + \gamma^2(1 - \kappa)} \sim \frac{2\gamma}{1 - \kappa} \equiv g \simeq 2\gamma. \quad (4)$$

Here, $\gamma$ and $\kappa$ are the lensing shear and convergence, respectively. With the presence of the ellipticity $\epsilon_N$ induced by other sources (see Section 5), we have $\epsilon \simeq \epsilon_N + 2\gamma$. In equation (4), we have approximated the reduced shear $g \simeq 2\gamma$ since in general $\kappa \ll 1$. The second-order term $\kappa \gamma$ in $g$ has a 1–10 per cent contribution to the power spectrum (Dodelson et al. 2006), which slightly improves the detectability. However, given the uncertainties in $\epsilon_N$, we neglect this higher order correction.

An implicit assumption of the estimator (equation 4) is that cosmic shear does not vary across the integral area. This assumption holds closely for galaxies, which are of arcsecond size. The 21-cm absorption ‘haloes’ are of arcminute size and the cosmic shear does vary across these regions. In this case, the estimator (equation 4) measures the averaged cosmic shear. Higher order moments can be explored to extract the gradient of the cosmic shear (e.g. Goldberg & Bacon 2005). The relatively large size of 21-cm absorption ‘haloes’ could improve the S/N of such measurements. In this paper, we only discuss the mean shear signal of each 21-cm absorption ‘halo’.

5 STATISTICAL ERRORS

Both the system temperature noise and inhomogeneities in neutral hydrogen density3 and spin temperature induce effective ellipticity $\epsilon_N$ to 21-cm absorption regions. This $\epsilon_N$ has expectation value zero and does not correlate over large distance. Analogous to galaxy intrinsic ellipticity, it only induces shot noise to the shear measurement. Since inhomogeneities and anisotropies in spin temperature, if exist, are mainly caused by the central source, they are uncorrelated with inhomogeneities in neutral hydrogen density. The rms fluctuations in $\epsilon_N$ then has three independent contributions:

$$\sigma_{\epsilon}^2 \equiv \langle \epsilon_N^2 \rangle = \langle \epsilon^2 \rangle_{\text{noise}} + \langle \epsilon^2 \rangle_{\delta H} + \langle \epsilon^2 \rangle_{T}, \quad (5)$$

where $\langle \epsilon^2 \rangle_{\text{noise}}$, $\langle \epsilon^2 \rangle_{\delta H}$ and $\langle \epsilon^2 \rangle_{T}$ are ellipticity fluctuations caused by the system temperature noise, the inhomogeneity in the neutral hydrogen distribution and the anisotropic distribution of the spin temperature around the first galaxies, respectively.

5.1 Ellipticity induced by the noise in system temperature

The distribution of the system temperature noise is Gaussian with an rms $\sigma_T$ given by equation (1). To estimate the fluctuation $\langle \epsilon^2 \rangle_{\text{noise}}$ in the ellipticity induced by this noise, we generate a series of realizations of observations by adding Gaussian noise with an rms fluctuation $\sigma_T$ to the otherwise spherically distributed signal and measure the ellipticity using equation (2). The result is shown in the bottom left-hand panel of Fig. 2. For $\sigma_T \sim 10$ mK and a resolution of 0.3 arcmin, the induced ellipticity is $\sim 10$ per cent for $z = 25$. To

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3 Peculiar velocities are highly coherent across the 21-cm absorption region, so it is unlikely to induce a non-negligible ellipticity.
have \( (\epsilon^2)_{\text{noise}}^{1/2} < 0.1 \), \( \sigma_T < 7 \) mK is required. This translates to a total collecting area \( \geq 300 \text{ km}^2 \) for \( L = 40 \) km. The induced ellipticities at other redshifts (e.g. \( z = 20 \) or 30) are similar, due to their similar signals (Fig. 2).

We notice that even for \( \sigma_T \rightarrow 0 \), \( (\epsilon^2)_{\text{noise}} \) noise does not vanish, rather it reaches a plateau instead. This is caused by the limited spatial resolution. With a finite resolution, one cannot exactly identify the physical centre of each region, and any shift from the physical centre would result in an effective ellipticity. As shown in the bottom left-hand panel of Fig. 2, with fixed \( \sigma_T \), one can indeed reduce \( (\epsilon^2)_{\text{noise}} \) noise significantly by improving the angular resolution \( \theta_p \). However, in reality, the requirement to improve resolution is often in conflict with the requirement to reduce the noise. To improve the angular resolution, longer baseline is required (\( \theta_p \propto L^{-1} \)). If the total collecting area is fixed, \( f_{\text{core}} \) would decrease (\( f_{\text{core}} \propto L^{-2} \)). This would cause the noise \( \sigma_T \) (per resolution pixel) to increase (\( \sigma_T \propto f_{\text{core}}^{-1} \propto L^{-1} \propto \theta_p^{-1} \)) and \( (\epsilon^2)_{\text{noise}} \) noise to increase as well (Fig. 2). Therefore, improving angular resolution does not necessarily reduce errors in shear measurement, unless the total collecting area is increased accordingly. Optimal surveys may balance between the angular resolution and \( \sigma_T \) to reduce \( (\epsilon^2)_{\text{noise}}^{1/2} \) to a level well below the intrinsic ellipticity induced by the spin temperature anisotropy (Section 5.3).

### 5.2 Ellipticity induced by density inhomogeneities

The fluctuation \( (\epsilon^2)_{\text{inhom}} \) in the ellipticity induced by inhomogeneities in neutral hydrogen density can be estimated as

\[
(\epsilon^2)_{\text{inhom}} \simeq \left( \int q_1 \theta_p^2 d^2 \theta \right)^{-2} \times \left[ \sigma_h^2 \right] \times \left[ \left( \theta_1^2 - \theta_2^2 \right) \left( \theta_4^2 - \theta_5^2 \right) + 4 \theta_1 \theta_2 \theta_4 \theta_5 \right] d^2 \theta d^2 \theta'.
\]

Here, \( w(\theta) \) is the angular correlation function of the neutral hydrogen overdensity. In this expression, we have neglected fluctuations in the denominator \( Q_{11} + Q_{22} \) (equation 2) since they only cause second-order effect, due to non-vanishing \( Q_{11} + Q_{22} \). For a flat \( w(\theta), (\epsilon^2)_{\text{inhom}} \) vanishes due to the geometry terms in equation (6). In reality, \( w(\theta) \) varies slowly. So \( w(\theta) \) does not contribute much to the integral in equation (6), although itself can reach \( -10^{-2} \) at subarcminute scale. For a 21-cm absorption ‘halo’ of a galaxy in a \( 10^5 \Omega_\odot \) halo at \( z = 25 \), \( (\epsilon^2)_{\text{inhom}} \) is \( 10^{-4} \) \( \ll 4(\gamma^2) \) for \( T_{\text{th}} \sim 50 \) mK. Therefore, this source of error seems to be negligible.

First galaxies are more likely to reside in high-density regions and one may think that this strong formation bias invalidates the above conclusion, which is based on the ensemble average of the correlation function. As the shear measurement only depends on the variation in the local density field, to account for the environmental effect, one should replace \( w(\theta) \) (in equation) with the conditional correlation function \( w(\theta)(\delta H) \), where \( \delta H \) is the overdensity averaged over the 21-cm absorption regions for shear measurement. Being several Mpc \( h^{-1} \) in size, these regions are much larger than the non-linear scale or filament size at \( z \sim 25 \), which implies that \( \langle \delta H \rangle \ll 1 \). Hence, we do not expect that \( w(\theta)(\delta H) \sigma_{\text{inhom}} \) could be 10 times larger than \( w(\theta) \), the induced \( (\epsilon^2)_{\text{inhom}} \) would be still less than 3 per cent. Based on these arguments, we conclude that the ellipticity induced by inhomogeneities in neutral hydrogen density, if not negligible, is at most subdominant to other error sources.

### 5.3 Ellipticity induced by spin temperature inhomogeneities

The fluctuation \( (\epsilon^2)_{T_\alpha} \) in the ellipticity induced by anisotropies in the spin temperature distribution is most uncertain. The spin temperature is a weighted average of the CMB temperature and the gas kinetic temperature. For the temperature range in our study, the CMB temperature is perfectly homogeneous. The coupling coefficients of the spin temperature to the gas temperature are determined by the spin-changing collisional rate and the Lyman \( \alpha \) scattering rate (Wouthuysen 1952; Field 1959). On scales we consider, the collisional rate should be isotropic because of isotropic distributions of the gas temperature and density. However, the spatial distribution of the Lyman \( \alpha \) scattering rate highly depends on the origin and propagation of Lyman \( \alpha \) photons, which could introduce considerable anisotropies in the spin temperature distribution.

Cen (2006) considers Lyman \( \alpha \) photons originated from the galaxy at the centre of each halo. Photons blueshifts Lyman \( \alpha \) from the continuum of Population III stars in the galaxy redshift to the Lyman \( \alpha \) line-centre frequency because of the Hubble expansion. At the radius where a blue photon redshifts to Lyman \( \alpha \) frequency, the photon is continuously scattered by neutral hydrogen atoms in the IGM and has little spatial diffusion. The photon would not fly freely until it encounters an atom that has the right velocity to make a large jump in its frequency. The escape of photons slightly blueshifts the Lyman \( \alpha \) line-centre frequency from the central galaxy is affected by the neutral hydrogen distribution near the centre. If the neutral hydrogen column density in the innermost 1 kpc of the halo is above \( 10^{17} \text{ cm}^{-2} \) (which is highly possible) and the column density distribution is not isotropic (e.g. caused by the geometry of the star forming region or the galactic wind), it would lead to anisotropic distributions of the Lyman \( \alpha \) scattering rate and the spin temperature at large radii.

However, before a photon reaches the radius where it redshifts to Lyman \( \alpha \) and becomes strongly scattered, there is a probability for it to be scattered and thus change its propagation direction. The place where the photon encounters a tremendous number of scatterings with little spatial diffusion is then no longer along the initial direction that the photon escapes from the central galaxy. Therefore, a small number of scatterings encountered by photons before they reach the region where strong scatterings happen tend to make the distribution of the overall scattering rate more isotropic than the initial angular distribution of the photons from the central galaxy.

To quantify such an effect of isotropization, we simulate the Lyman \( \alpha \) scattering process around the galaxy using a Monte Carlo code (Zheng & Miralda-Escudé 2002). Photons near Lyman \( \alpha \) frequency are launched from the centre along a cone with a given open angle to represent the anisotropic initial distribution. Every scattering of a photon is followed until the photon escapes to infinity. The scattering rate at each position in the IGM around the galaxy is then obtained with its normalization set by the luminosity of the central galaxy around Lyman \( \alpha \) frequency (see Cen 2006). We calculate the resultant spin temperature distribution and find that with a threshold temperature \( T_{\text{th}} = -50 \) mK of the 21-cm absorption region, the ellipticity fluctuation \( (\epsilon^2)_{T_\alpha} \) introduced by the initial anisotropic Lyman \( \alpha \) emission is 0.21, 0.20, 0.16, 0.14, 0.08 and 0.03 for open angles of 5°, 15°, 30°, 45°, 60° and 75°, respectively. We may expect that galaxy formation at such high redshifts is quite irregular and thus the formation of a well-defined disc is unlikely. If this is the case, it may be unlikely that the effective open angles are much smaller than \( \sim 30° \). However, even in such extreme cases, a small number of initial scatterings are quite efficient in
Here, the first term is the cosmic variance and the second term is the scale-invariant contribution due to the Lyman α photons with frequency \( f_{\alpha} \) that redshift to the Lyman α frequency, there are other sources of Lyman α photons that can contribute to the pumping rate of the 21-cm line. Photons originally emitted between Lyman γ and Lyman limit can redshift into one of the higher Lyman series resonances and, after a few scatterings, cascade to locally produce Lyman α photons near the line centre (Chuzhoy, Alvarez & Shapiro 2006; Chuzhoy & Shapiro 2006). Owing to the lower cross-section of the photons between Lyman γ and Lyman limit, they are expected to escape from the central galaxy more isotropically than those slightly blueshifted Lyman α. The small number of scatterings before they cascade to produce Lyman α photons would further isotropize the distribution. Another source of Lyman α photons is from soft X-ray photons. Chen & Miralda-Escudé (2006) discuss 21-cm absorption haloes around first stars and argue that soft X-ray photons from Population III stars play a significant role in (collisionally) generating Lyman α photons. These Lyman α photons produced locally in the inner IGM region surrounding the galaxy are likely to be largely isotropic because of the largely isotropic escape of soft X-ray photons from the galaxy.

Based on the above investigation, we conclude that the ellipticity induced by the anisotropies in the spin temperature is likely at the level of \( \lesssim 0.1 \) but may be much smaller. Depending on the nature of these objects and observation configurations, either the noise-induced ellipticity or the spin temperature-induced ellipticity can dominate the error budget of shear measurement. The overall induced ellipticity is thus expected to be \( \lesssim 10 \) per cent but could be smaller.

### 6 Lensing Measurements

The lensing signal on arcminute scales at \( z = 25 \) is \( 2 \times 10^{-10} \) per cent (lower right-hand panel, Fig. 2). From the previous section, the expected noise is likely \( \lesssim 10 \) per cent. We then expect that the S/N of the shear measurement for each 21-cm absorption ‘halo’ is \( \sim 1 \). This S/N is impressive, comparing to S/N \( \sim 1/30 \) of the conventional shear measurement of optical galaxies at \( z = 1 \).

Combining cosmic shear measurements of all ‘haloes’, one can measure the lensing power spectrum \( C_\ell \), where \( \ell \) is the multipole.

The statistical error in \( C_\ell \), assuming Gaussianity, is

\[
\Delta C_\ell = \sqrt{\frac{2}{(2\ell + 1)\Delta f_{\text{sky}}}} \left( C_\ell + \frac{4\pi f_{\text{sky}} \sigma_{\epsilon}^2}{N_g} \right),
\]

Here, the first term is the cosmic variance and the second term is the shot noise, \( f_{\text{sky}} \) is the sky coverage and \( \Delta f_{\text{sky}} \) is the size of the \( \ell \) bin, and \( N_g \) is the number of 21-cm absorption ‘haloes’. Since we measure the shear averaged over the size of each 21-cm absorption ‘halo’, one should replace \( C_\ell \) with the smoothed \( \tilde{C}_\ell \). We are also assuming that the Fourier transform of the window function describing each 21-cm absorption region used for lensing measurement.

For a lower mass cut of \( 7 \times 10^7 \) \( M_\odot \), there are about \( 10^4 \) first galaxies and thus \( 10^6 \) 21-cm absorption ‘haloes’ across the sky at \( 24 < z < 26 \). The shot noise power spectrum is proportional to the square of \( \eta \equiv \sigma_\epsilon / \sigma_{\text{sky}} \). With \( \sigma_{\text{sky}} \sim 0.1 \) and \( N_g \sim 10^6 \), the typical value of \( \eta \) is \( 10^{-4} \), for which precision measurement can be done up to \( \ell \sim 5000 \) (several arcminute scale; Fig. 3).

The lensing reconstruction from 21-cm ‘haloes’ can be further improved. Fig. 3 shows that there is significant power at small scales (\( \ell \gtrsim 5000 \)). Since the quadrupole moment shear estimator only measures the averaged shear, the smoothing effect caused by the arcminute size of the smoothing area does not allow the extraction of small scale lensing information. By measuring higher multipoles, however, smaller scale lensing information can be recovered. In the literature, the octopole moments have been proposed to measure the gradients of shear, or equivalently, the local shear (e.g. Goldberg & Natarajan 2002). The relative large size of 21-cm absorption ‘haloes’ will make this measurement much easier than that of \( z \sim 1 \) galaxies. Eventually, by measuring all necessary multipoles, one can recover all lensing information up to the limit of shot noise. This will allow the measurement of the lensing signal to \( \ell \gtrsim 10^4 \) (Fig. 3).

In the above estimation, we have assumed that intrinsic ellipticities do not correlate over relevant scales and thus only contribute to shot noise. This is obviously true for that induced by local processes discussed in previous sections, such as local density inhomogeneities and spin temperature anisotropy caused by the central sources. Could a fluctuating large-scale field coupled to the brightness temperature, such as the fluctuating soft X-ray background (e.g. Pritchard & Furlanetto 2007), invalidate the above assumption? The answer is probably no. The modes capable of inducing ellipticities must be smaller than the \( \sim 10 \) Mpc size of 21-cm absorption ‘haloes’. On the other hand, the modes capable of inducing correlations in intrinsic ellipticities are of size \( \sim 2 \times 10^4/1h^{-1} \) Mpc. Thus, a fluctuating background can only induce correlations in intrinsic...
ellipticities for those modes of $l > 10^3$, which are of little relevance to this paper.

In the directions along the line of sight and perpendicular to the line of sight, 21-cm absorption haloes may appear different, due to the evolution of these haloes and the light cone effect, the redshift distortion, the beam and foreground (which are frequency- and thus redshift-dependent), etc. However, these physics do not induce asymmetry in the 2D plane perpendicular to the line of sight. Since we only use the shape distortion in the same 2D plane to measure cosmic shear, these physics do not induce systematics in the shear measurement.

Thus, with reasonable estimation of the 21-cm absorption ‘halo’ signal at high redshifts, rather conservative estimates for the intrinsic ellipticities caused by initial anisotropic Lyman $\alpha$ emission from galaxies and reasonable estimates on the level of foreground-induced ellipticities, the weak-lensing application of these ‘haloes’ are very promising. The combined lensing S/N of $10^6$ 21-cm ‘haloes’ at $z=24$–$26$ is comparable to those with traditional cosmic shear measurements, with cosmic magnification of 21-cm emitting galaxies, and with CMB lensing and 21-cm background lensing.

7 SUMMARY AND DISCUSSION

In this paper, we investigate the potential of using 21-cm absorption ‘haloes’ of first galaxies at $z \sim 25$ as background sources for gravitational weak-lensing reconstruction. We show that the accuracy obtained using this method can be comparable, and may be superior, to some of the conventional methods. It is potentially very rewarding scientifically to detect and survey the first galaxies using future 21-cm radio experiments for this and other applications.

There are several major uncertainties that may affect the results in this paper. The first one is the star formation efficiency $c_*$ in large haloes at $z \sim 25$. We have adopted $c_*=0.2$ in the calculation. A smaller $c_*$ would reduce the signal and size of the 21-cm ‘haloes’ and thus make the observations and shear measurement more difficult. We have checked that adopting $c_*=0.1$ doubles the rms of the ellipticity induced by the system noise for the same noise level $\sigma_T=7$ mK. Secondly, there are uncertainties in the hard X-ray heating, which depends on the energy extraction efficiency from black holes and the fraction of the released energy in the form of hard X-rays (see Cen 2006, for more details). If the real values are lower than what are adopted in our calculation, the X-ray heating would be less important and the 21-cm absorption ‘haloes’ may form at redshifts lower than $z=25$. As a consequence, we would have much more observable 21-cm absorption ‘haloes’ and the statistical errors of the lensing measurement would be significantly improved. If the X-ray heating is more efficient than we assume, the detection of 21-cm absorption ‘haloes’ would be more demanding and it becomes more difficult to use these ‘haloes’ for the lensing application. To reduce the above two types of uncertainties, we have to advance our understanding of the formation of the first objects. On the other hand, the observation (or even null observation) of the 21-cm absorption ‘haloes’ would lead to valuable constraints on star formation and X-ray heating at $z \sim 20$–30. Lastly, we have neglected any errors in the shear measurement induced by map making. Although, in principle, these errors can be corrected (Chang et al. 2004), it is not clear how well the correction is for unprecedented radio arrays required for 21-cm absorption ‘halo’ observations. The discussion of such residual errors is certainly beyond the scope of this paper.

While we focus here on the weak-lensing application of the 21-cm absorption ‘haloes’, because of their arccminute size and high redshifts, these ‘haloes’ also have interesting strong lensing signatures, which is discussed in Li et al. (2007).

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