The dynamics of subhaloes in warm dark matter models

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ABSTRACT
We present a comparison of the properties of substructure haloes (subhaloes) orbiting within host haloes that form in cold dark matter (CDM) and warm dark matter (WDM) cosmologies. Our study focuses on selected properties of these subhaloes, namely their anisotropic spatial distribution within the hosts; the existence of a ‘backsplash’ population; the age–distance relation; the degree to which they suffer mass loss; and the distribution of relative (infall) velocities with respect to the hosts. We find that the number density of subhaloes in our WDM model is suppressed relative to that in the CDM model, as we would expect. Interestingly, our analysis reveals that backsplash subhaloes exist in both the WDM and CDM models. Indeed, there are no statistically significant differences between the spatial distributions of subhaloes in the CDM and WDM models. There is evidence that subhaloes in the WDM model suffer enhanced mass loss relative to their counterparts in the CDM model, reflecting their lower central densities. We note also a tendency for the (infall) velocities of subhaloes in the WDM model to be higher than in the CDM model. Nevertheless, we conclude that observational tests based on either the spatial distribution or the kinematics of the subhalo population are unlikely to help us to differentiate between the CDM model and our adopted WDM model.

Key words: methods: N-body simulations – galaxies: evolution – galaxies: haloes – cosmology: theory – dark matter.

1 INTRODUCTION
The presently favoured Λ cold dark matter (CDM) model of cosmological structure formation has proven to be extremely successful at describing the clustering of matter on intermediate to large scales (e.g. Springel et al. 2005; Spergel et al. 2007). In contrast, it has been argued that the predictions of the ΛCDM model are at odds with observations on the scales of galaxies, on the basis of cosmological N-body simulations. CDM haloes are predicted to have ‘cuspy’ density profiles with inner logarithmic slopes of approximately −1.2 (e.g. Navarro et al. 2004; Tasitsiomi et al. 2004; Diemand, Moore & Stadel 2005; Reed et al. 2005b), whereas high-resolution observations of low surface brightness galaxies appear to require haloes with constant density cores (e.g. Gentile, Tonini & Salucci 2007; McGaugh et al. 2007). Furthermore, CDM haloes are predicted to contain a wealth of substructure, which we might expect to observe as satellite galaxies within galactic haloes, in sharp contrast to the observed abundance of satellites around our Galaxy and others (Klypin et al. 1999; Moore et al. 1999).

Suggested solutions to these problems have included allowing the dark matter to be collisional (i.e. self-interacting) rather than collisionless (Bento et al. 2000; Spergel & Steinhardt 2000), allowing it to be warm rather than cold (Avila-Reese et al. 2001; Bode, Ostriker & Turok 2001; Knebe et al. 2002), and introducing non-standard modifications to an otherwise unperturbed CDM power spectrum (e.g. Bullock 2001; Little, Knebe & Islam 2003). Arguably, the most promising (and least intrusive) modification to the dark matter paradigm is to allow the dark matter particle to be warm. In such a case, warm dark matter (WDM) particles will have a relatively high thermal velocity dispersion at decoupling and therefore a non-negligible free-streaming scale \( \lambda_{fs} \). This modification results in a change in the primordial matter power spectrum, corresponding to a damping of density perturbations on scales below a filtering scale \( R_f \) (which is related to the free-streaming scale \( \lambda_{fs} \)), which in turn is related to a filtering mass \( M_f \) (Bardeen et al. 1986; Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002).

Previous studies have revealed that WDM can resolve some of the tension between theoretical prediction and observation. In particular, the abundance of substructure haloes (subhaloes) is greatly reduced in WDM models (Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002) compared to the CDM model. However, the
simulations used in these studies did not have sufficient resolution to follow the orbits of subhaloes in detail, and so these results are based on a snapshot of the subhalo population, static in time rather than a dynamic entity.

Over the last decade cosmological N-body simulations have advanced and reached a stage where it is possible to study the dynamics of well-resolved subhaloes and satellite galaxies and use them as a probe of cosmology. This field—dubbed ‘near-field cosmology’ (e.g. Bland-Hawthorn & Peebles 2006)—has prompted numerous studies (e.g. Zentner & Bullock 2003; De Lucia et al. 2004; Gao et al. 2004; Gill et al. 2004b; Knebe et al. 2004; Kravtsov, Gnedin & Klypin 2004; Libeskind et al. 2005; Reed et al. 2005a; Agustsson & Brainerd 2006; Knebe et al. 2006; Warnick & Knebe 2006; Faltenbacher et al. 2007; Sales et al. 2007; Libeskind et al. 2007; Kang et al. 2007; Colin, Valenzuela & Avila-Reese 2008; Warnick, Knebe & Power 2008), but as yet there have been no in-depth comparisons of subhaloes in CDM and WDM models. We address this in this study, using high-resolution resimulations of a set of ‘identical’ galaxy clusters (see the next section) forming in CDM and WDM models. We focus on selected properties of the subhalo population. Namely, we set out to validate the existence of the so-called ‘backsplash’ population reported in Gill, Knebe & Gibson (2005) (see also Mamon et al. 2004; Balogh, Navarro & Morris 2000; Moore, Diemand & Stadel 2004), the anisotropic spatial distribution of satellites (e.g. Knebe et al. 2004; Libeskind et al. 2005; Zentner et al. 2005; Agustsson & Brainerd 2006; Bailin et al. 2007), the putative age–distance relation (cf. Gao et al. 2004; Moore et al. 2004), and the degree to which subhaloes suffer mass loss. We also examine the relative velocities of subhaloes with respect to their hosts. This is of particular interest because it has been argued that the collision velocity of the ‘Bullet Cluster’ (Markevitch et al. 2002; Milosavljević et al. 2007) may pose another challenge to CDM (e.g. Hayashi & White 2006; Springel & Farrar 2007). While it might be expected that the large scale tidal field will be more important for collision velocities in mergers, it is nevertheless interesting to see whether the precise nature of the dark matter will be more important for collision velocities in mergers, it is nevertheless interesting to see whether the precise nature of the dark matter substructure in galactic haloes (e.g. Moore et al. 1999; Dalcanton & Hogan 2001; Goerdt et al. 2006). By focusing on the lowest warmon mass that could be considered consistent with observational data, we can explore subhalo dynamics in a plausible model, yet one in which the effects of the warmon should be more pronounced and therefore easier to identify when comparing and contrasting with the CDM model.

We note that the three CDM haloes CDM1, CDM2, and CDM3 have appeared previously in both Warnick & Knebe (2006) and Warnick et al. (2008), in which they corresponded to the ‘C3’, ‘C7’ and ‘C8’ systems.

2 THE NUMERICAL SIMULATIONS

2.1 The raw data

Our analysis is based on suite of high-resolution N-body simulations. They were carried out using the publicly available adaptive mesh refinement code MLAPM (Knebe, Green & Binney 2001) focusing on the formation and evolution of dark matter galaxy clusters containing of the order of one million particles, with mass resolution of \(1.6 \times 10^9 \ h^{-1} \ M_\odot\) and force resolution \(\sim 2 \ h^{-1} \ kpc\). We first created a set of four independent initial conditions at redshift \(z = 45\) in a standard ΛCDM cosmology (\(\Omega_m = 0.3, \Omega_\Lambda = 0.7, \Omega_b = 0.04, h = 0.7, \sigma_8 = 0.9\)). 512\(^3\) particles were placed in a box of sidereal length \(64 \ h^{-1} \ Mpc\) giving a mass resolution of \(m_p = 1.6 \times 10^6 \ h^{-1} \ M_\odot\). For each of these initial conditions, we iteratively collapsed eight adjacent particles to one particle reducing our particle number to 128\(^3\) particles. These lower mass resolution initial conditions were then evolved until \(z = 0\). As described by Tormen (1997), for each cluster the particles within five times the virial radius were then tracked back to their Lagrangian positions at the initial redshift (\(z = 45\)). Those particles were then regenerated to their original mass resolution and positions, with the next layer of surrounding large particles regenerated only to one level (i.e. eight times the original mass resolution), and the remaining particles were left 64 times more massive than the particles resident with the host cluster. The conservative criterion was selected in order to minimize contamination of the final high-resolution haloes with massive particles.

The three WDM haloes were simulated using the same techniques. In fact, the only difference between the CDM and WDM haloes is the functional form of the primordial power spectrum used as an input for the initial conditions generator. We follow Bardeen et al. (1986) and modify the CDM power spectrum by multiplying it with a damping function \(F_{\text{WDM}}^2(k)\), where

\[
F_{\text{WDM}}(k) = \exp \left[ -\frac{k R_d}{2} - \frac{(k R_d)^2}{2} \right] \tag{1}
\]

Following Bardeen et al. (1986), we parametrize the damping scale \(R_d\) in terms of the warmon density parameter \(\Omega_{\text{wdm}}\), its mass \(m_{\text{wdm}}\) and the dimensionless Hubble parameter \(h\):

\[
R_d \simeq 0.074 \left( \frac{M_{\text{wdm}}}{\text{keV}} \right)^{-4/3} \left( \frac{\Omega_{\text{wdm}}}{0.3} \right)^{1/3} \left( \frac{h}{0.7} \right)^{5/3} \text{ h}^{-1} \text{ Mpc}. \tag{2}
\]

We adopt a warmon mass of \(M_{\text{wdm}} = 0.5 \text{ keV}\), which gives a damping scale of \(R_d = 0.186 \text{ h}^{-1} \text{ Mpc}\). The filtering scale \(R_f\) can be obtained by determining the wavenumber of the mode at which the amplitude of the linear density fluctuation is suppressed by a factor of 2, and then computing half the comoving wavelength. It is straightforward to evaluate this from the WDM and CDM power spectra. For our choice of warmon mass and cosmological parameters, we find that this wavenumber corresponds to \(k = 2.553 \text{ h Mpc}^{-1}\) and therefore the filtering scale (mass) is \(R_f = 1.24 \text{ h}^{-1} \text{Mpc} (M_f = 6.653 \times 10^{13} \text{ h}^{-1} \text{ M}_\odot)\).

Note that our choice of warmon mass is lower than recent lower limits derived from combined analysis of observed properties of the matter power spectrum as inferred from the Sloan Digital Sky Survey Lyman α flux power spectrum, cosmic microwave background data and the galaxy power spectrum, which vary between, for example, \(m_{\text{wdm}} \gtrsim 3 \text{ keV}\) (Abazajian 2006) and \(m_{\text{wdm}} \gtrsim 10 \text{ keV}\) (Viel et al. 2006). However, it is consistent with published estimates of the \sim 0.5 keV warmon mass that would resolve the ‘overabundance’ of dark matter substructure in galactic haloes (e.g. Moore et al. 1999; Dalcanton & Hogan 2001; Goerdt et al. 2006).

2.2 Discreteness effects

It has been argued in the recent study of Wang & White (2007) that WDM haloes below a given fraction of the filtering mass \(M_f\) are spurious, arising from the unphysical fragmentation of filaments. They provided the following expression (based on simulations of the Hot Dark Matter model):

\[
M_{\text{lim}} \approx 10.17 d \ k_{\text{peak}}^{-2}. \tag{3}
\]
where \( \overline{\rho} \) is the mean density, \( d \) is the mean interparticle separation, and \( k_{\text{peak}} \) is the wavenumber at which \( \Delta(k) = k^3 P(k) \) reaches its maximum.

We have taken care to compute \( M_{\text{lim}} \), noting that our simulations use boxes of side 64 \( h^{-1} \) Mpc, an effective number of particles 512\(^3\), a density parameter of \( \Omega_0 = 0.3 \) and a peak wavenumber of \( k_{\text{peak}} = 1.78 h \) Mpc\(^{-1}\). These numbers give \( d = 0.125 h^{-1} \) Mpc, \( \overline{\rho} = 8.3265 h^2 \) Mpc\(^{-3}\) and so \( M_{\text{lim}} = 3.317828 \times 10^{10} h^{-1} \) M\(_{\odot}\). Our particle mass is \( m_p = 1.626269 \times 10^9 h^{-1} \) M\(_{\odot}\), and so we find that \( M_{\text{lim}} \) is equivalent to 204 particles. Therefore, this corresponds to the mass cut applied in the following analysis.

We have also applied a cut-off of 2\( M_{\text{lim}} \) to our data and checked our results to ensure that they are unaffected by spurious haloes. This reveals that our results remain unchanged and therefore are stable. We do not find any systematic biases in our data if we employ mass cut-off of \( M_{\text{lim}} \) or 2\( M_{\text{lim}} \). We have looked for trends in our data that we would expect to be present if they were affected by spurious haloes (such as the distribution of concentrations) but we find no obvious signatures. Therefore, we conclude that our results are robust and unaffected by particle discreteness.

### 2.3 The haloes

Both the haloes and their subhaloes are identified using AMIGA’s-Halo-Finder (AHF)\(^2\), a modification of the mHDF\(^3\) algorithm presented in Gill, Knebe & Gibson (2004a), which has been parallelized using the MPI (Message Passing Interface) libraries. AHF utilizes the adaptive grid hierarchy of MLAPM to locate (sub)haloes as peaks in the density profile shows this upturn to define the truncation radius for the virial overdensity \( \Delta_{\text{vir}} \), that is, \( M(<R_{\text{vir}})/(4\pi\rho_0 R_{\text{vir}}^3)/3 = \Delta_{\text{vir}} \rho_0 \). Here, \( \rho_0 \) is the mean density of the background (Universe). Following convention, we assume the cosmology- and redshift-dependent definition of \( \Delta_{\text{vir}} \): for a distinct (i.e. host) halo in a CDM cosmology with the cosmological parameters that we have adopted, \( \Delta_{\text{vir}} = 340 \) at \( z = 0 \). This prescription is not appropriate for subhaloes in the dense environs of their host halo, where the local density exceeds \( \Delta_{\text{vir}} \rho_0 \), and so the density profile will show a characteristic upturn at a radius \( R \lesssim R_{\text{vir}} \). In this case, we use the radius at which the density profile shows this upturn to define the truncation radius for the subhalo.

Further details of this approach (and especially the ‘halo tracking’ used to obtain the temporal evolution of subhaloes) can be found in Gill et al. (2004a).

In Table 1, we summarize some of the properties of the host haloes along with particulars of respectively subhalo populations.

### Table 1. Summary of the host haloes properties and their subhalo populations

<table>
<thead>
<tr>
<th>Model</th>
<th>( z_{\text{form}} )</th>
<th>Age</th>
<th>( R_{\text{vir}} )</th>
<th>( M_{\text{vir}} )</th>
<th>( \sigma_v )</th>
<th>( c_{1/5} )</th>
<th>( T )</th>
<th>( b_{\text{la}} )</th>
<th>( c_{\text{la}} )</th>
<th>( \max { M_{\text{sat}} } )</th>
<th>( N_{\text{sat}}^{&lt;2.5 R_{\text{vir}}} )</th>
<th>( N_{\text{lim}}^{\text{int}} )</th>
<th>( N_{\text{inf}}^{\text{infall}} )</th>
<th>( N_{\text{back}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDM1</td>
<td>0.805</td>
<td>6.9</td>
<td>973</td>
<td>1.1</td>
<td>833</td>
<td>6.57</td>
<td>0.952</td>
<td>0.749</td>
<td>0.212</td>
<td>0.12</td>
<td>54</td>
<td>26</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>CDM2</td>
<td>0.443</td>
<td>4.6</td>
<td>1341</td>
<td>2.9</td>
<td>1185</td>
<td>5.91</td>
<td>0.836</td>
<td>0.597</td>
<td>0.467</td>
<td>0.18</td>
<td>182</td>
<td>91</td>
<td>53</td>
<td>38</td>
</tr>
<tr>
<td>CDM3</td>
<td>0.237</td>
<td>2.8</td>
<td>1379</td>
<td>3.1</td>
<td>1092</td>
<td>5.84</td>
<td>0.867</td>
<td>0.818</td>
<td>0.749</td>
<td>0.49</td>
<td>159</td>
<td>126</td>
<td>31</td>
<td>2</td>
</tr>
<tr>
<td>WDM1</td>
<td>0.871</td>
<td>7.1</td>
<td>967</td>
<td>1.1</td>
<td>783</td>
<td>5.98</td>
<td>0.958</td>
<td>0.773</td>
<td>0.206</td>
<td>0.09</td>
<td>17</td>
<td>6</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>WDM2</td>
<td>0.643</td>
<td>5.9</td>
<td>1340</td>
<td>2.8</td>
<td>1093</td>
<td>6.09</td>
<td>0.887</td>
<td>0.705</td>
<td>0.423</td>
<td>0.45</td>
<td>77</td>
<td>52</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>WDM3</td>
<td>0.284</td>
<td>3.2</td>
<td>1352</td>
<td>3.0</td>
<td>1119</td>
<td>3.69</td>
<td>0.837</td>
<td>0.693</td>
<td>0.576</td>
<td>0.36</td>
<td>68</td>
<td>50</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

\(^1\)The peak value \( k_{\text{peak}} = 1.78 h \) Mpc\(^{-1}\) has been determined numerically from the (tabulated) WDM power spectrum used in this study based on a CDM power spectrum calculated with the publicly available CMBFAST code (Seljak & Zaldarriaga 1996) and modified according to equation (1).

\(^2\)AHF can be downloaded from http://www.aip.de/People/aknebe/AMIGA. AMIGA is the successor to MLAPM.

\(^3\)MLAPM’s-halo-Finder.

3 ANALYSIS OF THE SUBHALO POPULATION

#### 3.1 The ‘backsplash’ population

It has been noted that a significant population of haloes on the outskirts of present-day galaxy- and cluster-mass host haloes once resided within the virial radii of these hosts at earlier times (Balogh et al. 2000; Mamom et al. 2004; Moore et al. 2004; Gill et al. 2005; Warnick et al. 2008). These results are based on simulations of the CDM model, but it is interesting to ask whether such a ‘backsplash’ population exists in the WDM model. We expect there to be fewer satellites in WDM models and these satellites will tend to have lower concentrations (Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002), which, when combined, should affect the numbers of ‘backsplash’ haloes. The lower the concentration of a (sub)halo, the greater the likelihood that it will be tidally disrupted within the host. Therefore, we might expect the numbers of backsplash haloes to be suppressed in WDM models.

In Fig. 1, we plot the minimum halocentric distance reached by a (sub)halo against its present-day halocentric radius. Note that we combine data for all three haloes in the CDM and WDM models, respectively. As expected, backsplash haloes are more present in both the CDM and WDM models, although the numbers are reduced in the WDM model. Table 1 reveals that the youngest system (i.e. host #3) has the smallest fraction of backsplash haloes; this reflects the fact that this system has experienced a recent triple merger (cf. Warnick & Knebe 2006; Warnick et al. 2008). Interestingly, we find again (cf. Gill et al. 2005) that the number of infalling haloes is of the...
same order as the number of backsplash haloes in both the CDM and WDM models.

3.2 Mass spectra

The mass spectrum of satellite galaxies in WDM and CDM has been studied previously (Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002), but we consider it here briefly for completeness. In Fig. 2, we show the cumulative mass functions of subhaloes normalized to the total number of subhaloes, where we normalize the subhalo mass by the host halo mass. It is readily apparent that the abundance of low-mass haloes is suppressed in WDM cosmologies. We fit a power law to this mass function:

\[ \frac{N_M}{N_{total,interior}} \propto \left( \frac{M_{sat}}{M_{vir,host}} \right)^\alpha, \]

and obtain logarithmic slopes of \( \alpha = -0.8 \) for CDM and \( \alpha = -0.6 \) for WDM. This is consistent with the findings of previous studies for CDM subhaloes (e.g. De Lucia et al. 2004; Gao et al. 2004; Gill et al. 2004a; Reed et al. 2005a; Shaw et al. 2007).

We also calculate the mass functions of the infalling and backsplash halo populations, which we show in Fig. 3. We observe a general trend that backsplash haloes contain fewer high-mass objects in comparison to the infalling haloes. This reflects the importance of tidally induced mass loss for backsplash haloes, which we quantify in the next section (cf. also Warnick et al. 2008).

3.3 Mass loss

There is a general consensus that subhaloes in WDM models are less concentrated than their counterparts in CDM models (Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002) and therefore more susceptible to tidal destruction while orbiting within the dense environs of their host halo. We verify this in Figs 4 and 5 where we plot the total (fractional) mass loss as a function of distance to the host for both the interior and the backsplash population. The mass loss is measured over the time period from infall on to the host (i.e. the first time a satellite crosses the virial radius of the host on an inward trajectory) until the present day.

In both dark matter models, the average mass loss (presented as histograms in Fig. 4) is a monotonic decreasing function of minimum distance. However, in the WDM model this function is pointwise greater than the corresponding curve in CDM, which would be expected if mass loss is enhanced as a result of the lower concentrations of subhaloes in the WDM model.
Subhaloes in WDM models

3.4 Spatial anisotropy

There is good reason to believe that the spatial distribution of subhaloes (and the subset corresponding to satellite galaxies) in both cluster-sized systems and galactic haloes is anisotropic in the CDM model (e.g. Knebe et al. 2004; Libeskind et al. 2005, 2007; Zentner et al. 2005; Agustsson & Brainerd 2006, 2007; Faltenbacher et al. 2007; Kang et al. 2007), and so it is interesting to ask whether the same can be said of the WDM model.

For each of our host haloes, we compute the cumulative fraction of subhaloes which have cosine of the angle

$$\cos \Phi = \frac{R_{\text{sat},z=0}}{E_1};$$

this measures the position of a subhalo relative to the host $R_{\text{sat},z=0}$ and the host’s major-axis $E_1$. The host’s major-axis is identified using the eigenvalues and eigenvectors of its moment of inertia tensor, where the eigenvector corresponding to the smallest eigenvalue defines the major-axis.

The resulting distributions of $\cos \Phi$ are shown in Fig. 6 for subhaloes within the host’s virial radius, which confirms that the spatial anisotropy is present in the WDM model although not as pronounced as for the CDM case. Although we do not show the result here as it (probably) lies below the credibility level of the WDM simulation (Wang & White 2007), we note that very low mass systems ($M_{\text{sat}} < 10^{-4} M_{\text{host}}$) correlate more strongly with the major-axis than the remainder of the satellites – for both dark matter models. This is consistent with the expectation that (especially low mass) objects are primarily channelled along the filaments feeding the cluster (cf. also Knebe et al. 2004). The thin dashed line represents an isotropic distribution or the ‘uniform continuous distribution function’ (UCDF).

We have computed the same distribution for both the backsplash and infalling satellites in Fig. 7. Interestingly, we find that the spatial anisotropy is even stronger for the backsplash population than for the interior objects even though it is skewed towards $\cos \Phi \approx 0.55$ for the WDM model and hence no perfect alignment with $E_1$ anymore (but nevertheless an anisotropic distribution). One possible explanation for this could be that backsplash haloes tend to be on radial orbits that are either plunging through the host or grazing the ‘virial surface’. If this is the case, we might expect to observe...
a tendency for the infalling population to align with the major-axis as verified in the lower panel of Fig. 7. In addition, we note that the infalling WDM satellites show a more pronounced tendency to be aligned with the major-axis of the respective host. This goes along with the expectation for (sub)haloes to be concentrated in the filaments and channelled along them into the cluster (e.g. Knebe et al. 2004).

### 3.5 Age–distance relation

Is there is a correlation between the infall time of a satellite and its present-day halocentric radius? Previous studies using cosmological simulations have argued that ‘older’ subhaloes tend to lie closer to the centre of the host (Gao et al. 2004; Willman et al. 2004), but there are also claims to the contrary (Moore et al. 2004). Here, the age of a subhalo corresponds to the period of time that has elapsed since it was first accreted by the host (accretion redshift), entering the virial radius on an inward trajectory.

We investigate whether such a correlation between a subhalo’s age or accretion redshift and halocentric radius exists in our data in Fig. 8. The crosses represent ‘interior’ subhaloes within the virial radius at the present day, while the triangles represent backsplash haloes. Because we output our snapshots at discrete intervals, the times at which subhaloes are accreted appear discrete. It is possible to correct for this discreteness by interpolating the growth of the virial radius and the positions of subhaloes between snapshots, but we output snapshots sufficiently frequently to make the uncertainty introduced by discreteness negligible.

Figure 8 reveals that the correlation between radius and age is not a straightforward one. The CDM1/WDM1 system is hard to interpret because there is no strong trend for subhaloes within the virial radius. However, there are interesting trends in the CDM2/WDM2 and CDM3/WDM3 that suggest that there may be distinct populations following distinct age–distance relations. The most recently accreted subhaloes, with accretion redshifts $z \lesssim 0.1$, show the expected trend for accretion redshift to increase with decreasing redshift. However, subhaloes accreted at $z \gtrsim 0.1$ appear to follow an inverse relation, tending to have higher accretion redshifts for larger halocentric radii, and this trend continues beyond the virial radius into the backsplash population. Finally, we note that the ‘oldest’ subhaloes do not appear to follow any trend, instead forming a hard upper edge in each panel. However, this edge is an artefact of our method for tracking subhaloes (cf. Gill et al. 2004a) and corresponds to the formation redshift, $z_{\text{form}}$, of the respective host halo. This explains why there is a systematic shift to higher redshifts from the lower plot to the upper one – our halo tracker starts following subhaloes at $z_{\text{form}}$ and so it cannot ‘see’ (sub)haloes prior to $z_{\text{form}}$. Therefore, all subhaloes that resided within the host at this initial time appear as infalling ones.

This figure also reveals that no backsplash haloes have been accreted more recently than $z \approx 0.15$, which corresponds to a period of approximately two billion years. This might be considered the minimum time a backsplash halo spends within the virial radius of the host halo. We can compare this to the time-scale $t_{\text{dyn}}$ for a...
subhalo to complete one circular orbit at the virial radius:
\[ t_{\text{dyn}} = \sqrt{\frac{3\pi}{G\rho}} \approx \sqrt{\frac{3\pi}{G\Delta_{\text{vir}}}} \rho_b^{-1/2}, \]
where we used \( \rho \approx \Delta_{\text{vir}}\rho_b \). This leads to \( t_{\text{dyn}} \approx 6 \times 10^9 \) yr. Therefore, the minimum time \( t_{\text{outside}} \) a subhalo spends inside its host is approximately one-third the time it would take to complete one orbit at the virial radius. This suggests that backsplash haloes are on preferentially radial orbits, and explains why subhaloes accreted at earlier times are preferably found outside \( R_{\text{vir}} \) today.

We conclude that an age–distance relation akin to the one reported by Gao et al. (2004) is valid only for ‘recently’ accreted subhaloes. According to Fig. 8, there is a clear correlation between a subhalo’s age and its distance apparent in CDM2/WDM2 and CDM3/WDM3, the two youngest sets of hosts in our sample, and this correlation is apparent for objects accreted after \( z = 0.2 \). However, as we note above, there is some evidence that there may be distinct subhalo populations, separated according to their accretion redshift, that follow distinct relations and inverse relations.

It is worth noting that Gao et al. (2004) observed an age–distance relation over a much greater time span ranging from \( z \sim 0.9 \) down to \( z \sim 0.3–0.4 \). These authors identify all subhaloes at \( z = 1 \) and track them forward in time, which is equivalent to our approach (we start tracking subhaloes at the formation redshift of the host, which varies between \( z \gtrsim 0.8 \) for CDM1/WDM1 to \( z \gtrsim 0.2 \) for CDM3/WDM3). We note that accretion in their data appears to be complete by \( z \sim 0.3 \) (see the upper panel of their fig. 15). However, it remains unclear why there is no further accretion apparent in that plot for smaller redshifts as the fraction of accreted subhaloes increases at least down to \( z \sim 0.1 \) (cf. upper left-hand panel in their fig. 12). Moore et al. (2004) deduced from their analysis that any age–distance relation present in their data had to be very weak with a large scatter. We note that these authors identify subhaloes at \( z = 0 \) and track their merger trees backwards in time.

It is clear from our analysis that any conclusions we draw must be tentative – if we are to gain greater insight into the age–distance relation then we must draw upon a larger (i.e. statistical) sample of host haloes. Nevertheless, we note that any age–distance relation is apparent in the subhalo populations in both the CDM and WDM models.

It is notable that there are no counterparts in the WDM1 system to the ‘old’ backsplash population we observe in the CDM1 model. To better understand why this might be, we checked the distribution of infall velocities for subhaloes that were accreted at early times in each of host (i.e. \( z > 0.75 \) for CDM1 and \( z > 0.80 \) for WDM1). We found that the typical subhalo velocity in the CDM1 run was approximately 30 per cent larger than in the WDM1 run. This explains why we do not find an ‘old’ backsplash population in the WDM1 run – the typical infall velocity of a subhalo is too low to allow it to escape the host and become a backsplash halo. It is interesting to note that this behaviour is peculiar to the CDM1/WDM1 set of hosts, but we consider it statistically insignificant.

### 3.6 Relative velocities

So far we have compared and contrasted the spatial distribution and subhaloes in the CDM and WDM models, and the degree to which these subhaloes suffer mass loss. It is also of interest to ask whether the kinematics of subhaloes in the respective models differs. We already learnt that the early infalling population in CDM1 is marginally faster than its counterpart in WDM1. Hence, if there are systematic differences between the models, what might the implication be for a system such as the ‘Bullet Cluster’? This is an extremely high velocity merger between two galaxy clusters (Markevitch et al. 2002; Milosavljevi´c et al. 2007) and it has prompted discussion as to whether such high relative velocities (of the order of \( \sim 4500 \text{ km s}^{-1} \)) can be accommodated within the CDM model (Gill et al. 2005; Hayashi & White 2006; Nusser 2007; Springel & Farrar 2007; Angus & McGaugh 2008). We might expect such high relative velocities to be sensitive to the large scale gravitational field. Furthermore, if there are differences in the relative velocity distributions in the WDM and CDM models, this could also allow limits to be placed on the nature of the dark matter.

In Fig. 9, we plot the cumulative distribution of relative velocities (\( V_{\text{rel}} = V_{\text{sat}} - V_{\text{host}} \)) for all interior subhaloes, where \( V_{\text{sat}} \) and \( V_{\text{host}} \) are the centre-of-mass velocities of all particles inside the virial radius of the subhalo and the host, respectively. Relative velocities have been normalized to the circular velocity \( V_{\text{vir}} \) of the host at the virial radius. If we compute this quantity for the ‘Bullet Cluster’ using the estimate of the mass deduced from weak lensing (Clowe, Gonzalez & Markevitch 2004), the normalized collision speed \( (V_{\text{rel}} - V_{\text{host}})/V_{\text{vir}} \) is approximately 1.9 (and shown as a thin vertical line). This figure reveals that \( \sim 6 \) per cent of subhaloes in the CDM model have normalized relative velocities in excess of 1.9, compared to \( \sim 10 \) per cent in the WDM model. In other words, the probability of a high-speed encounter is greater in the WDM model than in the CDM model.

We have computed the same distributions for infalling and backsplash (sub)haloes and the results are shown in Fig. 10. There we observe that the infalling population in the WDM model is marginally faster than its CDM counterpart. However, we also note that the fastest infalling satellite always be found in the CDM model, or in other words, in WDM there are more infalling subhaloes with relative velocities up to about \( 1.2V_{\text{vir}} \), but in CDM there exists the odd satellite with a velocity as high as \( \sim 1.5V_{\text{vir}} \). One potential explanation of this may be that subhaloes suffer ‘dynamical friction

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4 We caution the reader that the ‘Bullet’ cluster is a system where the host (and the ‘Bullet’ itself) is an order of magnitude more massive than the hosts (and satellites) presented in this study. While the existence of the ‘Bullet’ cluster may serve as a motivation for the study of relative velocities, any conclusions drawn from our results are to be extrapolated to the ‘Bullet’ system with care. This is especially so because the differences between CDM and WDM are less prominent on scales corresponding to the ‘Bullet’ cluster.
within the filaments'. As shown by Knebe et al. (2003), more mass in CDM filaments is found in gravitationally bound objects whereas the mass in a WDM filament is more uniformly distributed, which may lead to enhanced dynamical friction. Therefore, subhaloes falling along filaments may have their infall velocities reduced and hence (i) do not find exceedingly fast subhaloes and (ii) observe an increase in the number of objects in the range 0.75–1.2 \( V_{\text{vir}} \). The situation though is different for the backsplash population that appears to be slower than its CDM counterpart.

In Fig. 11, we plot the (unnormalized) relative velocities of subhaloes as a function of their present-day halocentric radii. Interior and backsplash populations are represented by the crosses, while infalling subhaloes are represented by the diamonds; we also highlight the 20 most massive subhaloes by the green squares.

There are a number of points to note in this figure. The first is that the subhaloes with the highest relative velocities are concentrated towards the centre of the host, nestled in its potential well. The second is that the most massive subhaloes (green squares) are not responsible for the high-velocity tail that we observe in Fig. 9. The third point is that the WDM backsplash population (i) does not extend spatially as far out as for its CDM counterpart and (ii) has lower velocities leading to the observed steeper decline of the velocities with increasing clustercentric distance. The fourth and final point is that infalling subhaloes are a distinct population kinematically, tending to have higher velocities than backsplash galaxies (see also Gill et al. 2005). We conclude that the kinematics of subhaloes is unlikely to allow us to differentiate between the WDM and CDM models.

### 4 DISCUSSION AND CONCLUSIONS

We have compared and contrasted the properties of subhaloes orbiting in a set of simulated galaxy cluster hosts in the CDM and WDM models. The mass and force resolution of our simulations were sufficient to our host haloes with \( \sim 10^6 \) particles within the virial radius at \( z = 0 \), and we could follow the orbital evolution of hundreds of subhaloes in detail using outputs finely spaced in time (\( \Delta t \approx 170 \text{ Myr} \)) from the formation time of the host to the present day.

Our study has revealed that many of the properties of subhaloes in the CDM model and the WDM model we have studied are similar. Subhaloes in both the CDM and WDM models are distributed anisotropically with respect to the major-axis of their host, and the phenomenon of ‘backsplash’ haloes is common to both models. Other studies have shown that low-mass haloes in WDM models tend to be less centrally concentrated than their counterparts in the CDM model (Colin et al. 2008), and this leads to enhanced mass loss via tidal stripping for subhaloes in WDM models. We find no evidence for a well-pronounced correlation between the age of a subhalo and its present-day halocentric radius (\( \Delta r \approx 170 \text{ Myr} \)) from the formation time of the host to the present day.

Our results nevertheless suggest that it is unlikely that the spatial distribution and kinematics of subhaloes can be used to differentiate between the CDM and WDM models at \( z = 0 \). It might be possible to detect differences at higher redshifts, when the effect of the filtering mass is more pronounced (e.g. Power, Bland-Hawthorn & Lewis 2008). Furthermore, it might be possible to detect differences in the stellar populations and star formation histories of the satellite galaxies that are hosted by subhaloes (e.g. Gao & Theuns 2007), which will be sensitive to the mass assembly histories of the (sub)haloes.
However, such measures depend explicitly on the veracity of galaxy formation modelling, and so it seems more likely that estimates of the small-scale power spectrum deduced from the Lyman \( \alpha \) forest (e.g. Viel et al. 2008) may provide stronger constraints. Nevertheless, it is important to consider the various strands of observational evidence when piecing together the dark matter puzzle.

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